COMMON SECOND REVISION TEST – 2023

Standard XII

Reg.No. : | | | | | | |

MATHEMATICS

Time: 3.00 hrs. Part - I Marks: 90

I. Choose the correct answer: 20 x 1 = 20

1. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj(AB)|$ =

a) –40 b) –80 c) –60 d) –20

2. If A^TA^{-1} is symmetric, then A^2 =

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

3. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

a) 0 b) 1 c) –1 d) i

4. The conjugate of a complex number is $\frac{1}{i-2}$

i) $\frac{1}{i+2}$ b) $\frac{-1}{i+2}$ c) $\frac{-1}{i-2}$ d) $\frac{1}{i-2}$

5. A zero of $x^3 + 64$ is

a) 0 b) 4 c) 4i d) –4

6. The value of $\sin^{-1}(\cos x)$, $0 \le x \le \pi$ is

a) $\pi - x$ b) $x - \pi/2$ c) $\pi/2 - x$ d) $x - \pi$

7. $\tan^{-1}(1/4) + \tan^{-1}(2/9)$ is equal to

a) $\frac{1}{2}\cos^{-1}(3/5)$ b) $\frac{1}{2}\sin^{-1}(3/5)$ c) $\frac{1}{2}\tan^{-1}(3/5)$ d) $\tan^{-1}(1/2)$

8. The sum of the focal distances of any point on an ellipse is equal to length of the

a) major axis b) minor axis

c) latus rectum d) line joining the foci

9. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k

a) 3 b) –1 c) 3 d) 9

10. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vector such that $\vec{a}$ is perpendicular to $\vec{b}$ and is parallel to $\vec{c}$, then $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to

a) $\vec{a}$ b) $\vec{b}$ c) $\vec{c}$ d) $\vec{0}$

11. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

a) $\tan^{-1}(3/4)$ b) $\tan^{-1}(4/3)$ c) $\pi/2$ d) $\pi/4$

12. The point of inflection of the curve $y = (x - 1)^3$ is

a) (0,0) b) (0,1) c) (1,0) d) (1,1)

13. The solution of $\frac{\partial y}{\partial x} + y \sin x = 0$ is

a) $y = c\,e^{-\cos x}$ b) $y = c\,e^{\cos x}$ c) $y = c\,e^{\sin x}$ d) $y = c\,e^{-\sin x}$

14. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

a) $\frac{1}{31}$ b) $\frac{1}{5}$ c) 5 d) 31

15. The value of $\int_{-1}^{2} |x|\,dx$ is

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$

16. The value of $\int_{0}^{a} \sqrt{(a^2 - x^2)^3}\,dx$ is

a) $\frac{\pi a^3}{16}$ b) $\frac{3\pi a^4}{16}$ c) $\frac{3\pi a^2}{8}$ d) $\frac{3\pi a^4}{8}$

17. If a die is thrown once, the expectation of the number on it is

a) 3 b) 6 c) $\frac{1}{6}$ d) 3.5

18. The general solution of the differential equation $\frac{\partial y}{\partial x} = \frac{y}{x}$ is

a) $xy = k$ b) $y = k \log x$ c) $y = kx$ d) $\log y = kx$

19. If $P(X = 0) = 1 - P(X = 1)$, if $E(X) = 3\,Var(X)$, then $P(X = 0)$ is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

20. Subtraction is not a binary operation in

a) R b) Z c) N d) Q

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) 7 x 2 = 14

21. Solve by Cramer's rule, the system of equation $5x - 2y + 16 = 0 ; x + 3y - 7 = 0$

22. Write in rectangular form: $\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

23. Construct a cubic equation with roots –1, 1 and 2

24. Find the value of $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

25. Find the centre and radius of the circle $x^2 + y^2 - x + 2y - 3 = 0$

26. Prove that $\left[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}\right] = 0$

27. Calculate $\lim_{x \to a}\left(\frac{x^n - a^n}{x - a}\right)$

28. Evaluate : $\int_{0}^{1} x^3\left(1 - x^4\right)dx$

29. $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

30. Find differential dy for the function $y = (3 + \sin 2x)^2 3$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) 7 x 3 = 21

31. If $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$

32. Obtain the cartesian equation for the locus of $z = x + iy$ in the following case $|z-4|^2 - |z-1|^2 = 16$

33. Find the real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$

34. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1}(x)$, $|x| > 1$

35. A concrete bridge is designed as a parabola arch. The road over bridge is 40 m long and the maximum height of the arch is 15 m. Write the equation of the parabolic arch.

36. Find the equation of tangent and normal to the curve $y = x^2 + 3x - 2$ at the point (1,2)

37. Solve: $\frac{dy}{dx} + 2y = e^{-x}$

38. Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent.

39. The probability density function of X is given by $f(x) = \begin{cases} ke^{-x}2 & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$

Find (i) The value of k (ii) The distribution function

40. If the straight lines $\frac{x-5}{5m+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $x = \frac{2y+1}{4m} = \frac{1-z}{-3}$ are perpendicular to each other. Find the value of m.

Part - IV

IV. Answer all the questions. 7 x 5 = 35

41. a) Investigate the values of λ and μ the system of linear equation $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$, have
(i) no solution (ii) a unique solution (iii) an infinite number of solutions

(OR)

b) Prove that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = \sqrt{-3}$

42. a) If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation
$x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$

(OR)

b) Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$

43. a) Find the vertex, focus, directrix and length of the latus rectum of the parabola $x^2 - 4x - 5y - 1 = 0$

(OR)

b) If D is the mid-point of the side BC of a triangle ABC, show by vector method that $|\overline{AB}|^2 + |\overline{AC}|^2 = 2\left(|\overline{AD}|^2 + |\overline{BD}|^2\right)$

44. a) A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point p on the rod, which is 0.3 m from the end in contact with x-axis is an ellipse. Find the eccentricity.

(OR)

b) Find the non parametric form of vector equation and cartesian equation of the plane passing through the point (1,–2,4) and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$

45. a) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each orthogonally, then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

(OR)

b) If $V(x, y) = \log\left(\frac{x^2 + y^2}{\sqrt{x^2 + y^2}}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$

46. a) The region enclosed by the circle $x^2 + y^2 = a^2$ is divided into two segments by the line $x = h$. Find the area of the smaller segment.

(OR)

b) Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 and 4,00,000.

47. a) A random variable X has the following probability mass function.

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find (i) $P(2 < x < 6)$ (ii) $P(2 \le x < 5)$ (iii) $P(x \le 4)$ (iv) $P(3 < x)$

(OR)

b) Define an operation * on Q as follow $a * b = \frac{a+b}{2}$; $a, b \in Q$. Examine the closure, commutative, associative, identity and the existence of inverse for the operation * on Q.

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