TIRUVALLUR DISTRICT
COMMON SECOND REVISION TEST - 2023

06|02|2023

Standard XII

Reg.No. ☐☐☐☐☐☐

MATHEMATICS

Time: 3.00 hours

Marks: 90

Part - I

I. Choose the correct answer:

20 x 1 = 20

1. If $A^T A^{-1}$ is symmetric, then $A^2 =$
   a) A^{-1} 　　b) $\left(A^T\right)^2$ 　　c) A^T 　　d) $\left(A^{-1}\right)^2$

2. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$
   a) 0 　　b) $\sin\theta$ 　　c) $\cos\theta$ 　　d) 1

3. If $(1+i)(1+2i)(1+3i)\dots(1+ni) = x + iy$ then $2.5.10\dots(1+n^2)$ is
   a) 1 　　b) i 　　c) $x^2 + y^2$ 　　d) $1 + n^2$

4. A zero of $x^3 - 64 = 0$ is
   a) 0 　　b) 4 　　c) 4i 　　d) −4

5. If the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belongs to
   a) $[-1,1]$ 　　b) $\left[\sqrt{2}, 2\right]$
   c) $\left[-2, -\sqrt{2}\right] \cup \left[\sqrt{2}, 2\right]$ 　　d) $\left[-2, -\sqrt{2}\right]$

6. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$; then $\cos^{-1}x + \cos^{-1}y =$
   a) $\frac{2\pi}{3}$ 　　b) $\frac{\pi}{3}$ 　　c) $\frac{\pi}{6}$ 　　d) π

7. The area of the quadrilateral formed with the foci of the hyperbolas
   $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is
   a) $4(a^2 + b^2)$ 　　b) $2(a^2 + b^2)$ 　　c) $a^2 + b^2$ 　　d) $\frac{1}{2}(a^2 + b^2)$

8. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is
   a) 3 　　b) −1 　　c) 1 　　d) 9

9. If a vector $\bar{\alpha}$ lies in the plane of $\bar{\beta}$ and $\bar{\gamma}$, then
   a) $\left[\bar{\alpha},\bar{\beta},\bar{\gamma}\right] = 1$ 　　b) $\left[\bar{\alpha},\bar{\beta},\bar{\gamma}\right] = -1$ 　　c) $\left[\bar{\alpha},\bar{\beta},\bar{\gamma}\right] = 0$ 　　d) $\left[\bar{\alpha},\bar{\beta},\bar{\gamma}\right] = 2$

10. In any triangle ABC, the cosine formula is
    a) $a^2 = b^2 + c^2 + 2bc \cos A$ 　　b) $a^2 = b^2 + c^2 - 2bc \cos A$
    c) $a^2 = b^2 + c^2 + bc \cos A$ 　　d) $a^2 = b + c - ab \cos A$

11. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when
    a) $y = 0$ 　　b) $y = \pm\sqrt{3}$ 　　c) $y = \frac{1}{2}$ 　　d) $y = \pm 3$

12. If $\lim\limits_{\theta \to 0} \left(\frac{1 - \cos m\theta}{1 - \cos n\theta}\right) = 1$ then
    a) $m = n$ 　　b) $m = -n$ 　　c) $m = \pm n$ 　　d) $m = \sqrt{n}$

13. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

   a) $\frac{1}{31}$ b) $\frac{1}{5}$ c) 5   d) 31

14. If $w(x,y) = x^y$, $x > 0$, then $\frac{\partial w}{\partial x} =$

   a) $x^y \log x$ b) $y \log x$ c) yx^{y-1} d) $x \log y$

15. The area between $y^2 = 4x$ and its latus rectum is

   a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) $\frac{5}{3}$

16. The value of $\int_0^a \left(\sqrt{a^2 - x^2}\right)^3 dx$ is

   a) $\frac{\pi a^3}{16}$ b) $\frac{3\pi a^4}{16}$ c) $\frac{3\pi a^2}{8}$ d) $\frac{3\pi a^4}{8}$

17. The general solution of the differential equation $\frac{dy}{dx} = -\frac{y}{x}$ is

   a) $xy = k$ b) $y = k \log x$ c) $y = kx$ d) $\log y = kx$

18. Integrating factor of the differential equation $\frac{dy}{dx} = \frac{x + y + 1}{x + 1}$ is

   a) $\frac{1}{x+1}$ b) $x + 1$ c) $\frac{1}{\sqrt{x+1}}$ d) $\sqrt{x+1}$

19. If in 6 trials, X is a binomial variable which follows the relation $9P(X = 4) = P(X = 2)$ then the probability of success is

   a) 0.125   b) 0.25   c) 0.375   d) 0.75

20. The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on

   a) Q^+ b) Z c) R d) C

Part - II

II. **Answer any 7 questions. (Q.No.30 is compulsory)** $7 \times 2 = 14$

21. Find the rank of the matrix by row reduction method : $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$

22. Evaluate $(z + w)^2$ if $z = 5 - 2i$ and $w = -1 + 3i$

23. If α, β, γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, form a cubic equation whose roots are $2\alpha, 2\beta, 2\gamma$.

24. Find the centre and radius of the circle $2x^2 + 2y^2 - 6x + 4y + 2 = 0$

25. The volume of the parallelepiped whose coterminous edges are $7\hat{i} + \lambda\hat{j} - 3\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$, $-3\hat{i} + 7\hat{j} + 5\hat{k}$ is 90 cubic units. Find the value of λ.

(3)   XII Mathematics

26. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the function $f(x) = x^2 - x$, $x \in [0, 1]$

27. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$

28. Find the value of $\int\limits_0^\infty x^5 e^{-3x}\, dx$

29. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occured.

30. Determine whether * is a binary operation on R, defined by $a * b = a\sqrt{b}$

Part - III

$7 \times 3 = 21$

III. **Answer any 7 questions. (Q.No.40 is compulsory)**

31. Solve the system of linear equations $2x + 5y = -2$; $x + 2y = -3$ by matrix inversion method.

32. Obtain the Cartesian of the locus of z in $|z| = |z - i|$

33. Solve the equation : $x^4 - 14x^2 + 45 = 0$

34. Find the vertex, focus and length of the latus rectum of $y^2 = -8x$

35. A particle moves so that the distance moved is according to the law $s(t) = \dfrac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero.

36. Evaluate : $\int\limits_{-\pi/4}^{\pi/4} \sin^2 x\, dx$

37. Determine the other and degree of the differential equation $3\left(\dfrac{d^2y}{dx^2}\right) = \left[4 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}$

38. Compute $P(X = k)$ for the Binomial Distribution, $B(n, p)$ where $n = 6$, $p = \frac{1}{3}$, $k = 3$

39. Let $g(x,y) = 2y + x^2$, $x = 2r - s$, $y = r^2 + 2s$, $s \in R$. Find $\dfrac{\partial g}{\partial r}$

40. Find the value of $\cos^{-1}\left(\cos \frac{\pi}{7} \cos \frac{\pi}{17} - \sin \frac{\pi}{7} \sin \frac{\pi}{17}\right)$

Part - IV

$7 \times 5 = 35$

IV. **Answer all the questions.**

41. a)   Solve the system of linear equations by Cramer's rule.

$\dfrac{3}{x} - \dfrac{4}{y} - \dfrac{2}{z} - 1 = 0$, $\dfrac{1}{x} + \dfrac{2}{y} + \dfrac{1}{z} - 2 = 0$, $\dfrac{2}{x} - \dfrac{5}{y} - \dfrac{4}{z} + 1 = 0$ **(OR)**

   b)   Find the value of $\tan\left(\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)\right)$

(4)   XII Mathematics

42. a) Solve the equation $z^3 + 8i = 0$, where $z \in C$

(OR)

b) Solve the linear differential equation $\cos x . \dfrac{dy}{dx} + y \sin x = 1$

43. a) The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

(OR)

b) Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum.

44. a) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

(OR)

b) For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$, find the intervals of monotonocity, local extrema, the intervals of concavity and points of inflection.

45. a) Find the non-parametric form of vector equation and cartesian equation of the plane $\vec{r} = \left(6\hat{i} - \hat{j} + \hat{k}\right) + s\left(-\hat{i} + 2\hat{j} + \hat{k}\right) + t\left(-5\hat{i} - 4\hat{j} - 5\hat{k}\right)$

(OR)

b) Find the probability mass function $f(x)$ of the discrete random variable X whose cumulative distribution function $F(x)$ is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \leq x < -1 \\ 0.60 & -1 \leq x < 0 \\ 0.90 & 0 \leq x < 1 \\ 1 & 1 \leq x < \infty \end{cases}$$

Also find (i) $P(X < 0)$ and (ii) $P(X \geq -1)$

46. a) If $v(x,y) = e^x (x \cos y - y \sin y)$ then prove that $\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} = 0$

(OR)

b) Construct that truth table for $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

47. a) The rate of increase in the number of bacteria in a certain culture is proportional to the number prsent. Given the number triples in 5 hours, find how many bacteria will be present after 10 hours?

(OR)

b) If $\vec{a} = -2\hat{i} + 3\hat{j} - 2\hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + 3\hat{k}$, $\vec{c} = 2\hat{i} - 5\hat{j} + \hat{k}$, find $\left(\vec{a} \times \vec{b}\right) \times \vec{c}$ and $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$. State whether they are equal.

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