ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
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3M Question Bank (2022 – 23)
CLASS – XII - MATHEMATICS

3M QUESTIONS

1) **Theorem 1.1**
Prove that, for every square matrix A of order n, $A(\text{adj } A) = (\text{adj } A)A = |A| I_n$.
Prove for n = 2.

2) **Theorem 1.3**
For any square matrix A of order n, prove that, A^{-1} exists if and only if A is non-singular.

3) **Example 1.3**
Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.

4) **Example 1.5**
Find a matrix A if $\text{adj}(A) = \begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$.

5) **EXERCISE 1.1**
1. Find the adjoint of the following:
(iii) $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

6) 3. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

7) 10. Find $\text{adj}(\text{adj}(A))$ if $\text{adj } A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

8) 6. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A| I_2$.

9) 7. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

10) 11. $A=\begin{bmatrix}1 & \tan x\\ -\tan x & 1\end{bmatrix}$, show that $A^T A^{-1}=\begin{bmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{bmatrix}$.

11) 12. Find the matrix A for which $A\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}14 & 7\\ 7 & 7\end{bmatrix}$.

12) 13. Given $A=\begin{bmatrix}1 & -1\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}3 & -2\\ 1 & 1\end{bmatrix}$ and $C=\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}$, find a matrix X such that $AXB=C$.

13) 14. If $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$, show that $A^{-1}=\frac{1}{2}\left(A^2-3I\right)$.

14) 15. Decrypt the received encoded message $\begin{bmatrix}2 & -3\end{bmatrix}\begin{bmatrix}20 & 4\end{bmatrix}$ with the encryption matrix $\begin{bmatrix}-1 & -1\\ 2 & 1\end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1-26 to the letters $A-Z$ respectively, and the number 0 to a blank space.

15) **Example 1.14**

Reduce the matrix $\begin{bmatrix}0 & 3 & 1 & 6\\ -1 & 0 & 2 & 5\\ 4 & 2 & 0 & 0\end{bmatrix}$ to a row-echelon form.

16) **Example 1.19**

Show that the matrix $\begin{bmatrix}3 & 1 & 4\\ 2 & 0 & -1\\ 5 & 2 & 1\end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations.

17) 3. Find the inverse of each of the following by Gauss-Jordan method:

$$\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ 6 & -2 & -3\end{bmatrix}$$

18) **EXERCISE 1.3**

1. Solve the following system of linear equations by matrix inversion method:

(iii) $2x+3y-z=9,\ x+y+z=9,\ 3x-y-z=-1$

19) 4. Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

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20) **Example 1.25**
Solve, by Cramer's rule, the system of equations
$x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17, x_2 + 2x_3 = 7.$

21) **EXERCISE 1.4**
2. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

22) 3. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution ? (Use Cramer's rule to solve the problem).

23) **Example 1.27**
Solve the following system of linear equations, by Gaussian elimination method :
$4x + 3y + 6z = 25,\ x + 5y + 7z = 13,\ 2x + 9y + z = 1.$

24) **Example 1.31**
Test for consistency of the following system of linear equations and if possible solve: $x - y + z = -9,\ 2x - 2y + 2z = -18,\ 3x - 3y + 3z + 27 = 0.$

25) **Example 1.32**
Test the consistency of the following system of linear equations
$x - y + z = -9,\ 2x - y + z = 4,\ 3x - y + z = 6, 4x - y + 2z = 7.$

26) **Example 1.35**
Solve the following system:
$x + 2y + 3z = 0,\ 3x + 4y + 4z = 0,\ 7x + 10y + 12z = 0.$

27) **EXERCISE 2.2**
3. Find the values of the real numbers x and y, if the complex numbers
$(3-i)x - (2-i)y + 2i + 5$ and $2x + (-1+2i)y + 3 + 2i$ are equal.

28) Write any five properties of Complex Conjugates

29) **Example 2.3**
Write $\frac{3+4i}{5-12i}$ in rectangular form, hence find its real and imaginary parts.

30) **Example 2.4**
Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$. into rectangular form

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31) **Example 2.8**

Show that $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

32) **2.5.1** Write any five properties of modulus of complex numbers

33) **Example 2.11**

Which one of the points $i, -2+i$, and 3 is farthest from the origin?

34) **Example 2.12**

If z_1, z_2, and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1+z_2+z_3| = 1$,

35) **Example 2.13**

If $|z| = 2$ show that $3 \le |z+3+4i| \le 7$

36) **Example 2.16**

Show that the equation $z^2 = \bar{z}$ has four solutions.

37) **EXERCISE 2.5**

1. Find the modulus of the following complex numbers

$$\frac{2-i}{1+i}+\frac{1-2i}{1-i}$$

38) 2. For any two complex numbers z_1 and z_2, such that $|z_1| = |z_2| = 1$ and $z_1z_2 \ne -1$, then show that $\frac{z_1+z_2}{1+z_1z_2}$ is a real number.

39) 8. If the area of the triangle formed by the vertices z, iz, and $z+iz$ is 50 square units, find the value of $|z|$.

40) **EXERCISE 2.6**

1. If $z = x+iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$ show that the locus of z is real axis.

41) 3. Obtain the Cartesian form of the locus of $z = x+iy$ in each of the following cases:

$$|z+i| = |z-1|$$

42) 5. Obtain the Cartesian equation for the locus of $z = x+iy$ in each of the following cases:

$$|z-4|^2 - |z-1|^2 = 16.$$

43) Prove that, $\arg(z_1z_2) = \theta_1+\theta_2 = \arg(z_1)+\arg(z_2)$.

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44) Prove that, $\arg\left(\frac{z_1}{z_2}\right) = \theta_1 - \theta_2 = \arg(z_1) - \arg(z_2)$.

45) **Example 2.26**

Find the quotient $\dfrac{2\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right) + i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form.

46) **Example 2.27**

If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$.

47) **EXERCISE 2.7**

3. If $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\cdots(x_n+iy_n) = a+ib$, show that

(i) $(x_1^2+y_1^2)(x_2^2+y_2^2)(x_3^2+y_3^2)\cdots(x_n^2+y_n^2) = a^2+b^2$

(ii) $\sum_{r=1}^{n} \tan^{-1}\left(\frac{y_r}{x_r}\right) = \tan^{-1}\left(\frac{b}{a}\right) + 2k\pi,\ k \in \mathbb{Z}$.

48) 4. If $\frac{1+z}{1-z} = \cos 2\theta + i\sin 2\theta$, show that $z = i\tan\theta$.

49) 5. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ and

50) 6. If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

51) **Example 2.28**

If $z = (\cos\theta + i\sin\theta)$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$.

52) **Example 2.29**

Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$.

53) **EXERCISE 2.8**

1. If $\omega \neq 1$ is a cube root of unity, show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$.

54) **EXERCISE 2.8**

4. If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that

(iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$ (iv) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$.

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55) **Example 3.5**
Find the condition that the roots of cubic equation $x^3+ax^2+bx+c=0$ are in the ratio $p:q:r$.

56) **Example 3.6**
Form the equation whose roots are the squares of the roots of the cubic equation $x^3+ax^2+bx+c=0$.

57) EXERCISE 3.1
6. Solve the equation $x^3-9x^2+14x+24=0$ if it is given that two of its roots are in the ratio $3:2$.

58) 7. If α, β, and γ are the roots of the polynomial equation $ax^3+bx^2+cx+d=0$, find the value of $\sum \frac{\alpha}{\beta\gamma}$ in terms of the coefficients.

59) 8. If α, β, γ, and δ are the roots of the polynomial equation $2x^4+5x^3-7x^2+8=0$, find a quadratic equation with integer coefficients whose roots are $\alpha+\beta+\gamma+\delta$ and $\alpha\beta\gamma\delta$.

60) 9. If p and q are the roots of the equation $lx^2+nx+n=0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}}=0$.

61) 10. If the equations $x^2+px+q=0$ and $x^2+p'x+q'=0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$.

62) **Example 3.10**
Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

63) EXERCISE 3.2
1. If k is real, discuss the nature of the roots of the polynomial equation $2x^2+kx+k=0$, in terms of k.

64) 4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

65) **Example 3.15**
If $2+i$ and $3-\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$, find all roots.

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66) **Example 3.20**

Find the condition that the roots of $ax^3+bx^2+cx+d=0$ are in geometric progression. Assume $a,b,c,d\neq 0$

67) **Example 3.21**

If the roots of $x^3+px^2+qx+r=0$ are in H.P., prove that $9pqr=27r^2+2q^3$. Assume $p, q, r\neq 0$

68) **EXERCISE 3.4**

1. Solve : (i) $(x-5)(x-7)(x+6)(x+4)=504$

69) **Example 3.26**

Find the roots of $2x^3+3x^2+2x+3=0$.

70) **Example 3.27**

Solve the equation $7x^3-43x^2=43x-7$.

71) **Example 3.29:** Find solution, if any, of the equation

$$2\cos^2 x-9\cos x+4 = 0.$$

72) **EXERCISE 3.5**

1. Solve the following equations

(i) $\sin^2 x-5\sin x+4=0$

73) 3. Solve : $8x^{\frac{3}{2n}}-8x^{\frac{-3}{2n}}=63$

74) 4. Solve : $2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}$.

75) 6. Find all real numbers satisfying $4^x-3(2^{x+2})+2^5=0$.

76) **EXERCISE 3.6**

1. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9-4x^8+4x^7-3x^6+2x^5+x^3+7x^2+7x+2=0$.

77) 3. Show that the equation $x^9-5x^5+4x^4+2x^2+1=0$ has atleast 6 imaginary solutions.

78) 4. Determine the number of positive and negative roots of the equation $x^9-5x^8-14x^7=0$.

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79) Sketch the graph of $y = \sin^{-1} x$ in the interval [-1,1] and also write its domain and range.

80) Sketch the graph of $y = \cos^{-1} x$ in the interval [-1,1] and also write its domain and range.

81) Sketch $y = \tan^{-1} x$ in its principal its domain.

82) **Example 4.4**
Find the domain of $\sin^{-1}\left(2-3x^2\right)$

83) **Example 4.3**
Find the principal value of
(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (ii) $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$ (iii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$.

84) **Example 4.6**
Find (i) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ (ii) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$ (iii) $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

85) **EXERCISE 4.2**
5. Find the value of
(iii) $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17}-\sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$.

86) **EXERCISE 4.1**
6. Find the domain of the following
(i) $f(x)=\sin^{-1}\left(\frac{x^2+1}{2x}\right)$ (ii) $g(x)=2\sin^{-1}(2x-1)-\frac{\pi}{4}$.

87) 7. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.

88) **EXERCISE 4.2**
6. Find the domain of (i) $f(x)=\sin^{-1}\left(\frac{|x|-2}{3}\right)+\cos^{-1}\left(\frac{1-|x|}{4}\right)$ (ii) $g(x)=\sin^{-1}x+\cos^{-1}x$

89) **Example 4.9**
Find (i) $\tan^{-1}\left(-\sqrt{3}\right)$ (ii) $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$ (iii) $\tan\left(\tan^{-1}(2019)\right)$

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90) **Example 4.10**
Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$.

91) **Example 4.11**
Prove that $\tan\left(\sin^{-1}x\right)=\frac{x}{\sqrt{1-x^2}}$, $-1<x<1$.

92) **Example 4.14**
If $\cot^{-1}\left(\frac{1}{7}\right)=\theta$, find the value of $\cos\theta$.

93) **Example 4.15**
Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)=\sec^{-1}x$, $|x|>1$.

94) **EXERCISE 4.4**
2. Find the value of
(i) $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)$ (ii) $\sin^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\cot^{-1}(2)$
(iii) $\cot^{-1}(1)+\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-\sec^{-1}\left(-\sqrt{2}\right)$

95) **Example 4.16**
Prove that $\frac{\pi}{2}\leq\sin^{-1}x+2\cos^{-1}x\leq\frac{3\pi}{2}$.

96) **Example 4.19**
Prove that $\tan(\sin^{-1}x)=\frac{x}{\sqrt{1-x^2}}$ for $|x|<1$.

97) **Example 4.20**
Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sec^{-1}\left(\frac{5}{4}\right)\right]$

98) **Example 4.21**
Prove that (i) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\frac{\pi}{4}$ (ii) $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

99) **Example 4.22**
If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x,y,z<1$, show that
$x^2+y^2+z^2+2xyz=1$

100) **Example 4.24** Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$ for $x>0$.

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101) **Example 4.25** Solve $\sin^{-1} x > \cos^{-1} x$

102) **Example 4.26**
Show that $\cot(\sin^{-1} x) = \frac{\sqrt{1-x^2}}{x}, -1 \le x \le 1$ and $x \ne 0$

103) **Example 4.27:** Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, if $6x^2 < 1$.

104) **Example 4.28**
Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

105) **Example 4.29**
Solve $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$.

106) **EXERCISE 4.5**
2. Find the value of the expression in terms of x, with the help of a reference triangle.
(i) $\sin\left(\cos^{-1}(1-x)\right)$ (iii) $\cos\left(\tan^{-1}(3x-1)\right)$ (iii) $\tan\left(\sin^{-1}\left(x+\frac{1}{2}\right)\right)$.

107) 3. Find the value of
(i) $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$ (ii) $\cot\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right)$ (iii) $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$.

108) 5. Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.

109) 6. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$.

110) 7. Prove that $\tan^{-1} x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{3x-x^3}{1-3x^2}, |x| < \frac{1}{\sqrt{3}}$.

111) 8. Simplify: $\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$.

112) 9. Solve:
(i) $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$ (ii) $2\tan^{-1} x = \cos^{-1}\frac{1-a^2}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2}$, $a > 0$, $b > 0$.
(iii) $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$ (iv) $\cot^{-1} x - \cot^{-1}(x+2) = \frac{\pi}{12}$, $x > 0$.

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113) 10. Find the number of solutions of the equation $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}(3x)$.

114) **Example 5.2**
Find the equation of the circle described on the chord $3x+y+5=0$ of the circle $x^2+y^2=16$ as diameter.

115) **Example 5.7**
A line $3x+4y+10=0$ cuts a chord of length 6 units on a circle with centre of the circle $(2,1)$. Find the equation of the circle in general form.

116) **Example 5.9**
Find the centre and radius of the circle $3x^2+(a+1)y^2+6x-9y+a+4=0$.

117) EXERCISE 5.1
4. Find the equation of the circle with centre $(2,3)$ and passing through the intersection of the lines $3x-2y-1=0$ and $4x+y-27=0$.

118) EXERCISE 5.1
7. A circle of area 9π square units has two of its diameters along the lines $x+y=5$ and $x-y=1$. Find the equation of the circle.

119) 11. Find centre and radius of the following circles.
(i) $x^2+(y+2)^2=0$ (ii) $x^2+y^2+6x-4y+4=0$
(iii) $x^2+y^2-x+2y-3=0$ (iv) $2x^2+2y^2-6x+4y+2=0$

120) 12. If the equation $3x^2+(3-p)xy+qy^2-2px=8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.

121) **Example 5.15**
Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

122) **Example 5.19**
Find the vertex, focus, directrix, and length of the latus rectum of the parabola $x^2-4x-5y-1=0$.

123) **Example 5.21**
Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, one of the foci is $(2,3)$ and a directrix is

124) **Example 5.22**
Find the foci, vertices and length of major and minor axis of the conic
$4x^2+36y^2+40x-288y+532=0$.

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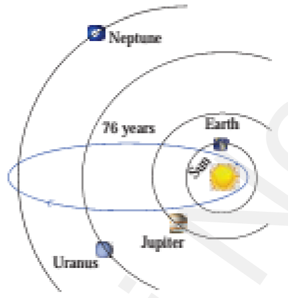
125) **Example 5.25**
Find the vertices, foci for the hyperbola $9x^2 - 16y^2 = 144$.

126) **Example 5.27**
The orbit of Halley's Comet (Fig.) is an ellipse which is 36.18 astronomical units long and by 9 astronomical units wide. Find its eccentricity.



The elliptical orbit of Halley's Comet

127) **EXERCISE 5.2**
4. Find the vertex, focus, equation of directrix and length of the latus rectum of the following:
(iv) $x^2 - 2x + 8y + 17 = 0$ (v) $y^2 - 4y - 8x + 12 = 0$

128) 6. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

129) 7. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis.

130) **Example 5.29**
Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at $(1, -3)$.

131) **EXERCISE 5.4**
2. Find the equations of tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$ which are parallel to $10x - 3y + 9 = 0$.

132) 3. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

133) 4. Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to $2x + 2y + 3 = 0$.

134) 5. Find the equation of the tangent at $t = 2$ to the parabola $y^2 = 8x$. (Hint: use parametric form)

135) 7. Prove that the point of intersection of the tangents at 't_1' and 't_2' on the parabola $y^2 = 4ax$ is $\left[at_1t_2,\ a(t_1 + t_2)\right]$.

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136) 8. If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2', then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.

137) **Example 5.32**
The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

138) **Example 5.33**
A concrete bridge is designed as a parabolic arch. The road over bridge is $40\ m$ long and the maximum height of the arch is $15m$. Write the equation of the parabolic arch.

139) **Example 5.34**
The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex.

140) **EXERCISE 5.5**
1. A bridge has a parabolic arch that is $10m$ high in the centre and $30m$ wide at the bottom. Find the height of the arch $6m$ from the centre, on either sides.

141) 3. At a water fountain, water attains a maximum height of $4m$ at horizontal distance of $0.5m$ from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of $0.75m$ from the point of origin.

142) 8. Assume that water issuing from the end of a horizontal pipe, $7.5m$ above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position $2.5\ m$ below the line of the pipe, the flow of water has curved outward $3m$ beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

143) **Example 6.1 (Cosine formulae)**
With usual notations, in any triangle ABC, prove the following by vector method.

(i) $a^2 = b^2 + c^2 - 2bc\cos A$ (ii) $b^2 = c^2 + a^2 - 2ca\cos B$

(iii) $c^2 = a^2 + b^2 - 2ab\cos C$

144) **Example 6.2**
With usual notations, in any triangle ABC, prove the following by vector method.

(i) $a = b\cos C + c\cos B$ (ii) $b = c\cos A + a\cos C$

(iii) $c = a\cos B + b\cos A$

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145) **Example 6.4**
With usual notations, in any triangle *ABC*, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.

146) **EXERCISE 6.1**
5. Using vector method, prove that if the diagonals of a parallelogram are equal, then it is a rectangle.

147) 6. Prove by vector method that the area of the quadrilateral *ABCD* having diagonals *AC* and *BD* is $\frac{1}{2}|\overline{AC}\times\overline{BD}|$.

148) 7. Prove by vector method that the parallelograms on the same base and between the same parallels are equal in area.

149) **Example 6.16**
Show that the four points $(6,-7,0)$, $(16,-19,-4)$, $(0,3,-6)$, $(2,-5,10)$ lie on a same plane.

150) **Example 6.17**
If the vectors $\vec{a},\vec{b},\vec{c}$ are coplanar, then prove that the vectors $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$ are also coplanar.

151) **EXERCISE 6.2**
4. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors represented by concurrent edges of a parallelepiped of volume 4 cubic units, find the value of $(\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{c})+(\vec{b}+\vec{c})\cdot(\vec{c}\times\vec{a})+(\vec{c}+\vec{a})\cdot(\vec{a}\times\vec{b})$.

152) 5. Find the altitude of a parallelepiped determined by the vectors $\vec{a}=-2\hat{i}+5\hat{j}+3\hat{k}$, $\hat{b}=\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{c}=-3\vec{i}+\vec{j}+4\vec{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$.

153) 9. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar, prove that c is the geometric mean of a and b.

154) 10. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a},\vec{b},\vec{c}]^2=\frac{1}{4}|\vec{a}|^2\,|\vec{b}|^2$.

155) **Example 6.19**
Prove that $[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$.

156) **Example 6.21**
For any four vectors $\vec{a},\ \vec{b},\ \vec{c},\ \vec{d}$, prove that,

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}.$$

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157) **EXERCISE 6.3**

3. Prove that $[\vec{a}-\vec{b},\ \vec{b}-\vec{c},\ \vec{c}-\vec{a}]=0$.

158) 6. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are coplanar vectors, show that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$.

159) 7. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k},\ \vec{b}=2\hat{i}-\hat{j}+\hat{k},\ \vec{c}=3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{a}\times(\vec{b}\times\vec{c})=l\vec{a}+m\vec{b}+n\vec{c}$, find the values of l,m,n.

160) 8. If $\hat{a},\hat{b},\hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a}\times(\hat{b}\times\hat{c})=\frac{1}{2}\hat{b}$, find the angle between $\hat{a}$ and $\hat{c}$.

161) **Example 6.24**

A straight line passes through the point $(1,2,-3)$ and parallel to $4\hat{i}+5\hat{j}-7\hat{k}$. Find (i) vector equation in parametric form (ii) vector equation in non-parametric form (iii) Cartesian equations of the straight line.

162) **Example 6.26**

Find the vector equation in parametric form and Cartesian equations of the line passing through $(-4,2,-3)$ and is parallel to the line $\frac{-x-2}{4}=\frac{y+3}{-2}=\frac{2z-6}{3}$.

163) **Example 6.8**

In triangle ABC, the points D,E,F are the midpoints of the sides BC,CA, and AB respectively. Using vector method, show that the area of ΔDEF is equal to $\frac{1}{4}$ (area of ΔABC).

164) **Example 6.31**

Show that the straight line passing through the points $A(6,7,5)$ and $B(8,10,6)$ is perpendicular to the straight line passing through the points $C(10,2,-5)$ and $D(8,3,-4)$.

165) **EXERCISE 6.4**

1. Find the non-parametric form of vector equation and Cartesian equations of the straight line passing through the point with position vector $4\hat{i}+3\hat{j}-7\hat{k}$ and parallel to the vector

166) 3. Find the points where the straight line passes through $(6,7,4)$ and $(8,4,9)$ cuts the xz and yz planes.

167) 4. Find the direction cosines of the straight line passing through the points $(5,6,7)$ and $(7,9,13)$. Also, find the parametric form of vector equation and Cartesian equations of the straight line passing through two given points.

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168) 5. Find the acute angle between the following lines.
(iii) $2x = 3y = -z$ and $6x = -y = -4z$.

169) 7. If the straight line joining the points $(2,1,4)$ and $(a-1,4,-1)$ is parallel to the line joining the points $(0,2,b-1)$ and $(5,3,-2)$, find the values of a and b.

170) 8. If the straight lines $\frac{x-5}{5m+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $x = \frac{2y+1}{4m} = \frac{1-z}{-3}$ are perpendicular to each other, find the value of m.

171) **Example 6.33**
Find the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

172) **Example 6.35**
Determine whether the pair of straight lines $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + t(2\hat{i} + 3\hat{j} + 4\hat{k})$, $\vec{r} = (2\hat{j} - 3\hat{k}) + s(\hat{i} + 2\hat{j} + 3\hat{k})$ are parallel. Find the shortest distance between them.

173) **Example 6.36**
Find the shortest distance between the two given straight lines $\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + t(-2\hat{i} + \hat{j} - 2\hat{k})$ and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+2}{2}$.

174) **EXERCISE 6.5**
5. Show that the straight lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ are skew and hence find the shortest distance between them.

175) 6. Find the parametric form of vector equation of the straight line passing through $(-1,2,1)$ and parallel to the straight line $\vec{r} = (2\hat{i} + 3\hat{j} - \hat{k}) + t(\hat{i} - 2\hat{j} + \hat{k})$ and hence find the shortest distance between the lines.

176) **Example 6.38**
Find the vector and Cartesian form of the equations of a plane which is at a distance of 12 units from the origin and perpendicular to $6\hat{i} + 2\hat{j} - 3\hat{k}$

177) **Example 6.40**
Find the direction cosines of the normal to the plane and length of the perpendicular from the origin to the plane $\vec{r} \cdot (3\hat{i} - 4\hat{j} + 12\hat{k}) = 5$.

178) **Example 6.42**
A variable plane moves in such a way that the sum of the reciprocals of its intercepts on the coordinate axes is a constant. Show that the plane passes through a fixed point

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179) EXERCISE 6.6

4. A plane passes through the point $(-1,1,2)$ and the normal to the plane of magnitude $3\sqrt{3}$ makes equal acute angles with the coordinate axes. Find the equation of the plane.

180) EXERCISE 6.8

1. Show that the straight lines $\vec{r}=(5\hat{i}+7\hat{j}-3\hat{k})+s(4\hat{i}+4\hat{j}-5\hat{k})$ and $\vec{r}=\left(8\hat{i}+4\hat{j}+5\hat{k}\right)+t\left(7\hat{i}+\hat{j}+3\hat{k}\right)$ are coplanar. Find the vector equation of the plane in which

181) 3. If the straight lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{m^2}$ and $\frac{x-3}{1}=\frac{y-2}{m^2}=\frac{z-1}{2}$ are coplanar, find the distinct real values of m.

182) Example 6.53

Find the equation of the plane passing through the intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)+1=0$ and $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+5\hat{k}\right)=2$ and the point $(-1,2,1)$.

183) Example 6.54

Find the equation of the plane passing through the intersection of the planes $2x+3y-z+7=0$ and $x+y-2z+5=0$ and is perpendicular to the plane $x+y-3z-5=0$.

184) EXERCISE 6.9

1. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot\left(2\hat{i}-7\hat{j}+4\hat{k}\right)=3$ and $3x-5y+4z+11=0$, and the point $(-2,1,3)$.

185) 2. Find the equation of the plane passing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z=3$, and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$.

186) EXERCISE 6.9

1. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot\left(2\hat{i}-7\hat{j}+4\hat{k}\right)=3$ and $3x-5y+4z+11=0$, and the point $(-2,1,3)$.

187) 2. Find the equation of the plane passing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z=3$, and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$.

188) 5. Find the equation of the plane which passes through the point $(3,4,-1)$ and is parallel to the plane $2x-3y+5z+7=0$. Also, find the distance between the two planes.

189) 7. Find the point of intersection of the line $x-1=\frac{y}{2}=z+1$ with the plane $2x-y+2z=2$. Also, find the angle between the line and the plane.

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