Tsi12M

Tenkasi District Common Examinations

Third Revision Test - February 2023

27.02.2023

Standard - 12

MATHEMATICS

Time Allowed: 3.00 Hours

Maximum Marks: 90

PART - I

Note: 1. **All questions are compulsory.** **20×1=20**

2. **Choose the correct or most suitable answer from the given four alternatives. Write the option code and the corresponding answer.**

1. If A^TA^{-1} is symmetric, then $A^2 =$

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

2. Which one of the following is not true in the case of discrete random variable X?

a) $\lim_{x\to\infty} F(x) = F(\infty) = 1$

b) $0 \le F(x) \le 1$ for all $x \in R$

c) F(x) is real valued decreasing function

d) $\lim_{x\to-\infty} F(x) = F(-\infty) = 0$

3. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^{\frac{1}{4}} = 0$ are:

a) 2,6 b) 2,3 c) 2,4 d) 3,3

4. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$

c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

5. If P(x,y) be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3, 0)$ and $F_2(-3, 0)$ then $PF_1 + PF_2$ is

a) 8 b) 6 c) 10 d) 12

6. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$ is

a) 0 b) $\frac{3}{2}$ c) $\frac{2}{3}$ d) $\frac{1}{2}$

7. If the planes $\vec{r}(2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r}(4\hat{i} + \hat{j} + \mu\hat{k}) = 5$ are parallel, then the value of λ and μ are

a) $\frac{1}{2}, -2$ b) $-\frac{1}{2}, 2$ c) $-\frac{1}{2}, -2$ d) $\frac{1}{2}, 2$

8. The length of the latus rectum of the parabola $y^2 - 4x + 4y + 8 = 0$ is:

a) 8 b) 6 c) 4 d) 2

9. The differential of y if $y = \sqrt{x^4 + x^2 + 1}$ is:

a) $\frac{1}{2}(4x^3 + 2x)^{-\frac{1}{2}}$ b) $\frac{1}{2}(4x^3 + 2x)^{-\frac{1}{2}} dx$

c) $\frac{1}{2}(x^4 + x^2 + 1)^{-\frac{1}{2}}(4x^3 + 2x)$ d) $\frac{1}{2}(x^4 + x^2 + 1)^{-\frac{1}{2}}(4x^3 + 2x)dx$

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10. The value of $\sum_{n-1}^{12} i^n$ is

a) 0 b) 1 c) −1 d) i

11. If the vectors $2\hat{i}-\hat{j}+3\hat{k}$, $3\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+m\hat{j}+4\hat{k}$ are coplanar, then the value of m is

a) 2 b) 3 c) −2 d) −3

12. $p \leftrightarrow q$ is equivalent to

a) $p \to q$ b) $q \to p$

c) $(p \to q) \vee (q \to p)$ d) $(p \to q) \wedge (q \to p)$

13. The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0, 3]$ is:

a) 1 b) $\sqrt{2}$ c) $\frac{3}{2}$ d) 2

14. A zero of $x^3 + 64$ is:

a) 4i b) 0 c) −4 d) 4

15. The point of inflection of the curve $y = (x - 1)^3$ is:

a) (1, 0) b) (0, 0) c) (1, 1) d) (0, 1)

16. Which of the following statement is correct regarding homogeneous system?

a) has only non−trivial solutions

b) always inconsistent

c) has only trivial solution only if rank of the coefficient matrix is equal to the number of unknowns

d) has only trivial solution

17. If cos x is an integrating factor of the differential equation $\frac{dy}{dx} + py = Q$, then P =

a) − cot x b) cot x c) tan x d) − tan x

18. Variance of the random variable X is 4. Its mean is 2. Then $E(X^2)$ is

a) 2 b) 4 c) 6 d) 8

19. The value of the complex number $(i^{25})^3$ is equal to :

a) 1 b) i c) −i d) −1

20. The operation * defined by $a*b = \frac{ab}{7}$ is not a binary operation on:

a) Q^+ b) Z c) R d) C

PART - II

Note: Answer any seven questions. **7×2=14**
Question Number 30 is compulsory.

21. Find the least positive integer n such that $\left(\frac{1+i}{1-i}\right)^n - 1$.

22. Which one of the points 10 − 8i, 11 + 6i is closest to 1 + i?

23. For what value of x does $\sin x = \sin^{-1}x$?

24. If $\text{adj}(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ find A^{-1}.

25. Show that the three vectors $2\hat{i}+3\hat{j}+\hat{k}$, $\hat{i}-2\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}+3\hat{k}$ are coplanar.

26. Find df for $f(x) = x^2 + 3x$ and evaluate it for x = 2 and dx = 0.1.

27. Evaluate : $\lim\limits_{x \to \infty} \dfrac{2x^2 - 3}{x^2 - 5x + 3}$.

28. Form the differential equation by eliminating the arbitrary constants A and B from $y = A\cos x + B\sin x$.

29. The probability density function of X is given by $f(x) = \begin{cases} kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$ Prove that the value of k is 4.

30. Prove that $\int_0^1 x e^x \, dx = 1$.

PART - III

Note: Answer any seven questions.
Question Number 40 is compulsory. **7×3=21**

31. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A| I$.

32. If $\omega \ne 1$ is a cube root of unity, show that $\dfrac{a + b\omega + c\omega^2}{b + c\omega + a\omega^2} + \dfrac{a + b\omega + c\omega^2}{c + a\omega + b\omega^2} = -1$.

33. Solve the equation $2x^3 - 9x^2 + 10x = 3$, if 1 is a root, find the other roots.

34. Find centre and radius of the circle $x^2 + y^2 - 5x + 2y - 5 = 0$.

35. Find the domain of $\sin^{-1}(2 - 3x^2)$.

36. With usual notations, in any triangle ABC, prove by vector method that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

37. Evaluate: $\int_0^{\frac{\pi}{2}} \begin{vmatrix} \cos^4 x & 7 \\ \sin^5 x & 3 \end{vmatrix} dx$.

38. Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

39. Two fair coins are tossed simultaneously. Find the probability mass function for number of heads occured.

40. If $f(x, y) = \dfrac{1}{\sqrt{x^2 + y^2}}$ then, prove that $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = -f$.

PART - IV

Note: Answer all the questions. **7×5=35**

41. a) For what values of μ the system of homogeneous equations $x + y + 3z = 0$, $4x + 3y + \mu z = 0$, $2x + y + 2z = 0$ have:
   i) only trivial solution
   ii) infinitely many solutions

(OR)

b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ If it is known that $\frac{1}{3}$ is a solution.

42. a) If $z = x + iy$ is a complex number such that $\left|\dfrac{z - 4i}{z + 4i}\right| = 1$, show that the locus of z is real axis or $y = 0$.

(OR)

b) The rate at which the population of a city increases at any time is proportional to the population at that time. If there were 1,30,000 people in the city in 1960 and 1,60,000 in 1990, what approximate population may be anticipated in 2020?

$\left[\log_e\left(\frac{16}{13}\right) = 0.2070,\ e^{0.42} = 1.52\right]$

43. a) Prove by vector method $\sin(A+B) = \sin A.\cos B + \cos A.\sin B$.

(OR)

b) Compute the area between the curve $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$.

44. a) Find the cartesian equation of the plane containing the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$ and passing through the point $(-1, 1, -1)$.

(OR)

b) If $w = x + 2y + z^2$ and $x = \cos t$, $y = \sin t$, $z = t$ find $\frac{dw}{dt}$ by using chain rule. Also find $\frac{dw}{dt}$ by substitution of x, y and z in w and hence verify the result.

45. a) Find the centre, foci and vertices of the hyperbola $16x^2 - 9y^2 - 32x - 18y + 151 = 0$ and draw the diagram.

(OR)

b) A Car A is travelling from west at 50 km/hr and Car B is travelling towards north at 60 km/hr. Both are headed for the intersection of the two roads. At what rate are the Cars approaching each other when Car A is 0.3 kilometers and Car B is 0.4 kilometers from the intersection?

46. a) Solve $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$.

(OR)

b) A random variable X has the following probability mass function:

X	1	2	3	4	5
f(x)	k^2	$2k^2$	$3k^2$	2k	3k

Find:

i) the value of k

ii) $P(2 \le X < 5)$

iii) $P(3 < X)$

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47. a) Prove that among all the rectangles of the given perimeter, the square has the maximum area.

(OR)

b) i) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let * be the matrix multiplication. Determine whether M is closed under *. If so, examine the commutative and associative properties satisfied by * on M.

ii) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let * be the matrix multiplication. Determine whether M is closed under *. If so, examine the existence of identity, existence of inverse properties for the operation * on M.