Class : 12

Register Number

THIRD REVISION EXAMINATION, FEBRUARY - 2023

Time Allowed : 3.00 Hours]

MATHEMATICS

[Max. Marks : 90

PART I

i) All questions are compulsory. 20 X 1 = 20

ii) Choose the most appropriate answer from the given **four** alternatives and write the answer along with the code.

1. If $A\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix} = \begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$, then $A =$

(1)$\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}$ (2)$\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$ (3)$\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$ (4)$\begin{bmatrix}4 & -1\\2 & 1\end{bmatrix}$

2. If Crammar's rule can be applied then
(1) $\Delta \neq 0$ (2) $\Delta = 0$ (3) $\Delta_x = 0$ (4) $\Delta_x \neq 0$

3. Find the value of $\sum_{n=1}^{13}(i^n + i^{n-1})$
(1)$1+i$ (2)i (3)1 (4)0

4. A zero of $x^3 + 64$ is
(1)0 (2)4 (3)$4i$ (4)-4

5. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$; Then $\cos^{-1} x + \cos^{-1} y$ is equal to
(1)$\frac{2\pi}{3}$ (2)$\frac{\pi}{3}$ (3)$\frac{\pi}{6}$ (4)π

6. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$. One of the points of contact of tangents on the hyperbola is
(1)$\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (2)$\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (3)$\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (4)$(3\sqrt{3}, -2\sqrt{2})$

7. *The angle between two lines* $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$; *and* $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$
(1)$\frac{\pi}{6}$ (2)$\frac{\pi}{4}$ (3)$\frac{\pi}{3}$ (4)$\frac{\pi}{2}$

8 The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0,3]$ is
[1]1 [2]$\sqrt{2}$ [3]$\frac{3}{2}$ [4]2

9. If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is
[1]0.4 cu. cm [2]0.45 cu.cm [3]2 cu. cm [4]4.8 cu. cm

10. *find the value* $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$
[1]$\frac{\pi}{6}$ [2]$\frac{\pi}{2}$ [3]$\frac{\pi}{4}$ [4]π

11. The integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is x, then $P(x)$
[1]x [2]$\frac{x^2}{2}$ [3]$\frac{1}{x}$ [4]$\frac{1}{x^2}$

12. The probability mass function of a random variable is defined as

x	-2	-1	0	1	2
$f(x)$	k	$2k$	$3k$	$4k$	$5k$

The $E(X)$ is equal to
[1]$\frac{1}{15}$ [2]$\frac{1}{10}$ [3]$\frac{1}{3}$ [4]$\frac{2}{3}$

13. Subtraction is not a binary operation in
(1)$\mathbb{R}$ (2) $\mathbb{Z}$ (3) $\mathbb{N}$ (4) $\mathbb{Q}$

14. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(1)$[1,2]$ (2)$[-1,1]$ (3)$[0,1]$ (4)$[-1,0]$

15. Find the general equation of a circle with centre $(-3,-4)$ and radius 3units.

(1)$x^2 + y^2 + 6x + 8y + 6 = 0$ (2)$x^2 + y^2 + 6x + 8y + 16 = 0$

(3)$x^2 + y^2 = 0$ (4)$x^2 + y^2 + 16 = 0$

16. If $2\hat{\imath} - \hat{\jmath} + 3\hat{k}, 3\hat{\imath} + 2\hat{\jmath} + \hat{k}, \hat{\imath} + m\hat{\jmath} + 4\hat{k}$ are coplanar, find the value of m

[1]3 [2]-3 [3]$\frac{3}{2}$ [4]0

17. Evaluate: $\lim_{x \to 0}\left(\frac{\sin mx}{x}\right)$.

[1]0 [2]$-m$ [3]m [4]∞

18. $g(x) = x^2 + \sin x$ the find dg.

[1]$dg = (2x + cosx)dx$ [2]$dg = (2x - cosx)dx$

[3]$dg = (x + 2cosx)dx$ [4]$dg = (2x + sinx)dx$

19. $find\ order\ and\ degree$ $\frac{d^3y}{dx^3} = (xy)^2 + \cos x\left(\frac{d^2y}{dx^2}\right)$

[1] 3,0 [2]3,1 [3]2,1 [4]2,2

20. $z = 5 - 2i$ and; $w = -1 + 3i$ then find $z + w$

[1] $4 + i$ [2]$-4 + i$ [3]]$-6 + 5i$ [4]]$-5 + 6i$

PART II

Answer any seven questions. question number 30 is Compulsory . 7 X 2 = 14

21. If adj $A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

22. Show that $Z = (2 + i\sqrt{3})^{10} + (2 - i\sqrt{3})^{10}$ is real .

23. Find the monic polynomial equation of minimum degree with real coefficients having $2 - \sqrt{3}i$ as a root..

24. Identify the type of conic sections for the equations $y^2 + 4x + 3y - 4 = 0$

25. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$

26. $y = Ae^{12x} + Be^{-12x}$ Show that the differential equation is $\frac{d^2y}{dx^2} - 144y = 0$ corresponding to the family of curves represented by the equation where A and B are arbitrary constants

27. Find the slope of the tangent to the curves at the respective given points.

$y = x^4 + 2x^2 - x$ at $x = 1$

28. Evaluate: $\int_0^3 (3x^2 - 4x + 5)\, dx$

29. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images..

30. Find the length of the perpendicular from the point $(1,-2,3)$ to the plane $x-y+z=5$

PART III .

Answer any seven questions. question number 40 is Compulsory . 7 X 3 = 21

31. Show that the rank matrix: $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$; is 3.

32. The complex numbers u, v, and w are related by $\frac{1}{u}=\frac{1}{v}+\frac{1}{w}$. If $v=3-4i$ and $w=4+3i$, find u in rectangular form.

33. Find the value of $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)+\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$

34. If p and q are the roots of the equation $lx^2+nx+n=0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}}=0$.

35. $U(x,y,z)=xyz, x=e^{-t}, y=e^{t}\cos t, z=\sin t, t\in\mathbb{R}$. ; *then find* $\frac{dU}{dt}$

36 Find the volume of a right-circular cone of base radius r and height h.

37. Construct the truth table $(p \vee q) \vee \neg q$.

38. If the equation $3x^2+(3-p)xy+qy^2-2px=8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.

39. If $\vec{a}=\hat{\imath}+2\hat{\jmath}+3\hat{k}, \vec{b}=2\hat{\imath}-\hat{\jmath}+\hat{k}, \vec{c}=3\hat{\imath}+2\hat{\jmath}+\hat{k}$ and $\vec{a}\times(\vec{b}\times\vec{c})=l\vec{a}+m\vec{b}+n\vec{c}$, find the values of l, m, n.

40. Solve $(1+x^2)\frac{dy}{dx}=1+y^2$.

PART IV

Answer all the questions . **7 X 5 = 35**

41.a) Investigate for what values of λ and μ the system of linear equations $x+2y+z=7, x+y+\lambda z=\mu, x+3y-5z=5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

b) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

42. a) If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$, show that $x^2+y^2=1$.

(OR)

b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

43. a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Show that the angle of projection is $tan^{-1}\left(\frac{4}{3}\right)$.

(OR)

b) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?

44.a) Prove by vector method that $sin(\alpha - \beta) = sin\,\alpha\, cos\,\beta - cos\,\alpha\, sin\,\beta$.

(OR)

b) A rectangular page is to contain 24cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum?

45. a) Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$..

(OR)

b) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5.

46. a). Father of a family wishes to divide his square field bounded by $x = 0, x = 4, y = 4$ and $y = 0$ along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them.

(OR)

b). A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find (i) the probability mass function (ii)the cumulative distribution function (iii) $P(4 \leq X \leq 10)$ (iv) $P(X \geq 6)$

47. **a)** if $u = sec^{-1}\left[\frac{x^3-y^3}{x+y}\right]$ then prove that; $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$.

(OR)

b) Solve the following differential equations: $(x^3 + y^3)dy - x^2y\,dx = 0$.

STD: XII

THIRD REVISION EXAM - FEB '23

MATHS

ANSWER KEY

PART - I

1. (3) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$
2. (1) $\Delta \neq 0$
3. (3) 1
4. (4) -4
5. (2) $\frac{\pi}{3}$
6. (3) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
7. (4) $\frac{\pi}{2}$
8. (4) 2
9. (4) 4.8 cu.cm
10. (1) $\frac{\pi}{6}$
11. (3) $\frac{1}{x}$
12. (4) $\frac{2}{3}$
13. (3) N
14. (1) (1, 2)
15. (2) $x^2+y^2+6x+8y+16=0$
16. (2) -3
17. (3) m
18. (1) $(2x+\cos x)dx$
19. (2) 3, 1
20. (1) $4+i$

PART - II

(21) Soln,

Given $\operatorname{adj} A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

To find A^{-1}

$$|\operatorname{adj} A| = \begin{vmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= -1[1-4] - 2[1-4] + 2[2-2]$$

$$= -1[-3] - 2[-3] + 2[0]$$

$$= 3 + 6 + 0 = 9$$

$$A^{-1} = \pm \frac{1}{\sqrt{|\operatorname{adj} A|}} \operatorname{adj} A$$

$$= \pm \frac{1}{\sqrt{9}} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\therefore \boxed{A^{-1} = \pm \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}}$$

(22) Soln,

$$z = (2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$$

$$\bar{z} = \overline{(2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}}$$

$$= \overline{(2+i\sqrt{3})^{10}} + \overline{(2-i\sqrt{3})^{10}} \quad \because \left(\overline{z_1+z_2} = \bar{z}_1 + \bar{z}_2\right)$$

$$= \left(\overline{2+i\sqrt{3}}\right)^{10} + \left(\overline{2-i\sqrt{3}}\right)^{10} \quad \left[\because \overline{(z^n)} = (\bar{z})^n\right]$$

$$= (2-i\sqrt{3})^{10} + (2+i\sqrt{3})^{10}$$

$$= z \quad \therefore \bar{z} = z \quad \therefore \boxed{z \text{ is real}}$$

(23) Soln:

Given: $2-i\sqrt{3}$ is a root

So, $2+i\sqrt{3}$ is also a root

Sum of the roots

$= 2-i\sqrt{3} + 2+i\sqrt{3}$

$= 4$

Product of roots.

$= (2-i\sqrt{3})(2+i\sqrt{3})$

$= 4+(\sqrt{3})^2$

$= 4+3 = 7$

∴ required monic polynomial equation is

$$\boxed{x^2-4x+7=0}$$

(24) Soln:

The given question in 9th. paper is wrong $y^2+4x+3y-4=0$ +

Given: $y^2+4x+3y+4=0$

Here,

$A=0$; $B=0$; $C=1$; $D=4$; $E=3$; $F=+4$

$\Rightarrow$ $B=0$ and $A=0$

[either A or C = 0]

∴ The given equation is Parabola.

(25) Soln:

Given: $f(x) = x^2+3x$.

$df(x) = (2x+3)\,dx$

~~df = 0.1~~

Given $x=2$; $dx=0.1$

$df = [2(2)+3](0.1)$

$= (4+3)(0.1)$

$= 7(0.1)$

$$\boxed{df = 0.7}$$

(26) Soln:

Given:

$y = Ae^{12x} + Be^{-12x}$

Diff 'y' w.r. to 'x'

$\frac{dy}{dx} = 12Ae^{12x} + (-12)Be^{-12x}$

$= 12Ae^{12x} - 12Be^{-12x}$

$\frac{dy}{dx} = 12[Ae^{12x} - Be^{-12x}]$

Again diff. $\frac{dy}{dx}$ w.r.t x

$\frac{d^2y}{dx^2} = 12[12Ae^{12x} - (-12)Be^{-12x}]$

$= 12[12Ae^{12x} + 12Be^{-12x}]$

$= 12(12)[Ae^{12x} + Be^{-12x}]$

$\frac{d^2y}{dx^2} = 144y$

$$\boxed{\frac{d^2y}{dx^2} - 144y = 0}$$

(27) Soln,

Given : $y = x^4 + 2x^2 - x$

Diff y w.r.t 'x'

$$\frac{dy}{dx} = 4x^3 + 2(2x) - 1$$

$$\frac{dy}{dx} = 4x^3 + 4x - 1$$

$$\boxed{\text{Slope}(m) = \frac{dy}{dx}}$$

$$m = 4x^3 + 4x - 1$$

At $x = 1$

$$m = 4(1)^3 + 4(1) - 1$$
$$= 4 + 4 - 1$$
$$\boxed{m = 7}$$

(28) Soln,

Given: $\int_0^3 (3x^2 - 4x + 5)\, dx$

$$= \left\{ \cancel{3}\left[\frac{x^3}{\cancel{3}}\right] - \overset{2}{\cancel{4}}\left[\frac{x^2}{\cancel{2}}\right] + 5(x) \right\}_0^3$$

$$= \left[x^3 - 2x^2 + 5x\right]_0^3$$

$$= \left[(3)^3 - 2(3)^2 + 5(3)\right] - [0]$$

$$= 27 - 2(9) + 15$$
$$= 27 - 18 + 15$$
$$= 27 - 3$$
$$= 24$$

$$\boxed{\int_0^3 (3x^2 - 4x + 5)\, dx = 24}$$

(29) Soln,

let X be the random Variable of a number of tails when three coins tossed.

$X = \{0, 1, 2, 3\}$

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$n(S) = 8$

$X^{-1}(0) = \{TTT\}$

$X^{-1}(1) = \{HHT, HTH, THH\}$

$X^{-1}(2) = \{HTT, THT, TTH\}$

$X^{-1}(3) = \{HHH\}$

Value of X	0	1	2	3	Total
No. of elements in Inverse image	1	3	3	1	8

(30) Soln,

Given:

Point $(1, -2, 3)$ (x_1, y_1, z_1)

Plane: $x - y + z = 5$

Perpendicular length:

$$= \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

Here, $a = 1$; $b = -1$; $c = 1$; $d = -5$

$$= \left| \frac{1(1) + (-1)(-2) + (1)(3) + (-5)}{\sqrt{(1)^2 + (-1)^2 + (1)^2}} \right|$$

$$= \left| \frac{1 + 2 + 3 - 5}{\sqrt{1 + 1 + 1}} \right|$$

$$= \left| \frac{1}{\sqrt{3}} \right|$$

$$\therefore \boxed{\text{Length of Perpendicular} = \frac{1}{\sqrt{3}} \text{ units}}$$

PART - III

31) Soln.

Given.

$$A = \begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & -8 & 5 & 2 \\ 0 & 1 & -7 & 8 \\ 0 & -2 & +14 & -4 \end{bmatrix} \begin{matrix} \\ R_2 \to 3R_2 - 2R_1 \\ R_3 \to 3R_3 + R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 3 & -8 & 5 & 2 \\ 0 & 1 & -7 & 8 \\ 0 & 0 & 0 & 12 \end{pmatrix} R_3 \to R_3 + 2R_2$$

The last equivalent matrix is in row-echleon form.

No. of non-zero row is 3

$\therefore \boxed{\rho(A) = 3}$

32) Soln.

Given: $v = 3-4i$; $w = 4+3i$

$$\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$$

$$= \frac{1}{3-4i} + \frac{1}{4+3i}$$

$$= \frac{4+3i+3-4i}{12+9i-16i+12}$$

$$\frac{1}{u} = \frac{7-i}{24-7i} \times \frac{7+i}{7+i}$$

$$\frac{1}{u} = \frac{49+1}{168+24i-49i+7}$$

$$\frac{1}{u} = \frac{50}{175-25i}$$

$$\frac{1}{u} = \frac{50^2}{25(7-i)}$$

$$\frac{1}{u} = \frac{2}{7-i}$$

$$u = \frac{7-i}{2}$$

$$\therefore \boxed{u = \frac{7}{2} - \frac{i}{2}}$$

33) Soln.

Given

$$\cos^{-1}\left[\cos\left(\frac{4\pi}{3}\right)\right] + \cos^{-1}\left[\cos\left(\frac{5\pi}{4}\right)\right]$$

$$\Rightarrow \cos\left(\frac{4\pi}{3}\right) = \cos\left(\pi + \frac{\pi}{3}\right)$$

$$= -\cos\frac{\pi}{3} \quad \boxed{\cos(180+\theta) = -\cos\theta}$$

$$\cos\left(\frac{5\pi}{4}\right) = \cos\left(\pi + \frac{\pi}{4}\right)$$

$$= -\cos\frac{\pi}{4}$$

$$\Rightarrow \cos^{-1}\left(-\cos\frac{\pi}{3}\right) + \cos^{-1}\left(-\cos\frac{\pi}{4}\right)$$

$$= -\frac{\pi}{3} - \frac{\pi}{4} = \boxed{\frac{-7\pi}{12}}$$

34) Soln.

Given $lx^2 + nx + n = 0$

If p and q are roots.

$p+q = \frac{-n}{l}$; $pq = \frac{n}{l}$

Now, $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}}$

$$\frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} + \sqrt{\frac{n}{l}}$$

$$\frac{p+q}{\sqrt{pq}} + \sqrt{\frac{n}{l}}$$

$$\frac{-n/l}{\sqrt{\frac{n}{l}}} + \sqrt{\frac{n}{l}} \qquad \boxed{\because \frac{a}{\sqrt{a}} = \sqrt{a}}$$

$$-\sqrt{\frac{n}{l}} + \sqrt{\frac{n}{l}}$$

$$= 0$$

(35) Soln.

$\boxed{\begin{array}{l} y = e^{t}\cos t \ (\times) \\ y = e^{-t}\cos t \ (\checkmark) \end{array}}$

$u(x,y,z) = xyz$

$\frac{\partial u}{\partial x} = yz;\ \frac{\partial u}{\partial y} = xz;\ \frac{\partial u}{\partial z} = xy$

$x = e^{-t}$ | $y = e^{-t}\cos t$

$\frac{\partial x}{\partial t} = -e^{-t}$ | $\frac{\partial y}{\partial t} = e^{-t}(-\sin t) + \cos t\, e^{-t}$

$= -e^{-t}(\cos t - \sin t)$

$z = \sin t \Rightarrow \frac{\partial z}{\partial t} = \cos t$

To find $\frac{du}{dt}$

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

$$= yz(-e^{-t}) + xz[-e^{-t}(\cos t - \sin t)] + xy[\cos t]$$

$$= e^{-t}[-yz - xz(\cos t - \sin t)] + xy[\cos t]$$

~~$= e^{-t}[-(e^{-t}\cos t)(\sin t) - e^{-t}($~~

$$= e^{-t}[-yz - xz\cos t - xz\sin t + xy\cos t\, e^{t}]$$

$$= e^{-t}[-e^{-t}\cos t\sin t - e^{-t}\sin t\cos t - e^{-t}\sin^2 t + e^{-t}e^{-t}\cos^2 t\, e^{t}]$$

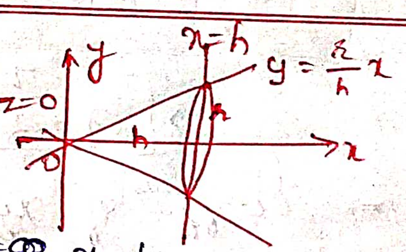
$$= -e^{-t}.e^{-t}[\cos t\sin t + \sin t\cos t + \sin^2 t - \cos^2 t]$$

$$= -e^{-2t}[2\cos t\sin t - (\cos^2 t - \sin^2 t)]$$

$$= -e^{-2t}[\sin 2t - \cos 2t]$$

$$\boxed{\therefore \frac{du}{dt} = -e^{-2t}[\sin 2t - \cos 2t]}$$

(36) Soln.



$y = \frac{r}{h}x,$

$x = 0 \Rightarrow$ ~~beco~~ $x = h$

Vol. of cone given by.

$$V = \pi\int_0^h y^2\,dx = \pi\int_0^h \left(\frac{r}{h}x\right)^2 dx$$

$$= \frac{\pi r^2}{h^2}\int_0^h x^2\,dx$$

$$= \frac{\pi r^2}{h^2}\left[\frac{x^3}{3}\right]_0^h$$

$$= \frac{\pi r^2}{\cancel{h^2}}\left[\frac{h^{\cancel{3}}}{3}\right]$$

$$= \frac{\pi r^2 h}{3}$$

$$\boxed{\therefore \text{Vol. of cone} = \frac{1}{3}\pi r^2 h}$$

37. Soln,

p	q	¬q	p∨q	(p∨q)∨¬q
T	T	F	T	T
T	F	T	T	T
F	T	F	T	T
F	F	T	F	T

Centre $(-g, -f)$

$(-(-1), -0)$

$= (1, 0)$ $\boxed{\text{Centre } (1,0)}$

radius $= \sqrt{g^2+f^2-c}$

$= \sqrt{(-1)^2 + (0)^2 - (-24)}$

$= \sqrt{1+0+24} = \sqrt{25}$

$\boxed{\text{Radius} = 5}$

38. Soln,

Given equation:

$3x^2 + (3-p)xy + qy^2 - 2px = 8pq$

For circle,

$\Rightarrow$ Coeff of x^2 = coeff of y^2

$3 = q$

$\Rightarrow \boxed{q=3}$

$\Rightarrow$ Coeff of $xy = 0$

$3-p=0$

$\boxed{p=3}$

$\therefore$ The equation becomes,

$3x^2 + 3y^2 - 6x - 72 = 0$

$\div$ by 3,

$x^2+y^2-2x-24=0$.

The general equation of circle

$\boxed{x^2+y^2+2gx+2fy+c=0}$

On comparing

$2g = -2$ | $2f=0$ | $\boxed{c=-24}$

$\boxed{g=-1}$ | $\boxed{f=0}$

39. Soln,

Given $\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$

$\vec{b} = 2\hat{i}-\hat{j}+\hat{k}$

$\vec{c} = 3\hat{i}+2\hat{j}+\hat{k}$

$\vec{a}\times(\vec{b}\times\vec{c}) = l\vec{a}+m\vec{b}+n\vec{c}$

Now,

LHS $\Rightarrow \vec{a}\times(\vec{b}\times\vec{c})$

$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{vmatrix}$

$= \hat{i}[-1-2] - \hat{j}[2-3] + \hat{k}[4+3]$

$= -3\hat{i}+\hat{j}+7\hat{k}$

$\vec{a}\times(\vec{b}\times\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & 1 & 7 \end{vmatrix}$

$= \hat{i}[14-3] - \hat{j}[7+9] + \hat{k}[1+6]$

$= 11\hat{i} - 16\hat{j} + 7\hat{k} \longrightarrow (1)$

~~$\vec{a}\times(\vec{b}\times\vec{c}) = l\vec{a}$~~

RHS $\Rightarrow l\vec{a}+m\vec{b}+n\vec{c}$

$= l(\hat{i}+2\hat{j}+3\hat{k}) + m(2\hat{i}-\hat{j}+\hat{k}) + n(3\hat{i}+2\hat{j}+\hat{k})$

$= (l+2m+3n)\hat{i} + (2l-m+2n)\hat{j} + (3l+m+n)\hat{k} \longrightarrow (2)$

On ~~comparing~~ Equating (1) & (2).

$l+2m+3n=11 \longrightarrow (i)$

$2l-m+2n=-16 \longrightarrow (ii)$

$3l+m+n=7 \longrightarrow (iii)$

Solving (ii) & (iii)

$2l-\cancel{m}+2n=-16$
$3l+\cancel{m}+n=7$

$5l+3n=-9 \longrightarrow (iv)$

Solving (i) & (ii)

$l+2m+3n=11$
(ii)×2⇒ $4l-2m+4n=-32$

$5l+7n=-21 \longrightarrow (v)$

Solving (iv) & (v)

$5l+3n=-9$
$5l+7n=-21$
$(-)\quad(-)\quad(+)$

$-4n=12$

$\boxed{n=-3}$

(iv)⇒ $5l+3(-3)=-9$

$5l-9=-9$

$5l=-9+9$

$5l=0$

$\boxed{l=0}$

(i)⇒ $l+2m+3n=11$

$0+2m+3(-3)=11$

$2m-9=11$

$2m=11+9$

$2m=20$

$\boxed{m=10}$

∴ $\boxed{l=0}$; $\boxed{m=10}$ $\boxed{n=-3}$

40) Soln:

Given $(1+x^2)\frac{dy}{dx}=1+y^2$

$(1+x^2)\,dy=(1+y^2)\,dx$

$\frac{dy}{1+y^2}=\frac{dx}{1+x^2}$

Now, Integrating,

$\int\frac{dy}{1+y^2}=\int\frac{dx}{1+x^2}$

$\tan^{-1}(y)=\tan^{-1}x+c$

$\tan^{-1}y-\tan^{-1}x=c$

$\tan^{-1}\left(\frac{y-x}{1+xy}\right)=c$

$\frac{y-x}{1+xy}=\tan c$

$\frac{y-x}{1+xy}=K$ [∵ $\tan c=K$ (say)]

$\boxed{y-x=K(1+xy)}$

PART-IV

41) (a) Soln.

Given: $x+2y+z=7$
$x+y+\lambda z=\mu$
$x+3y-5z=5$

$$\begin{bmatrix}1&2&1\\1&1&\lambda\\1&3&-5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}7\\\mu\\5\end{bmatrix}$$

A X = B

Augmented Matrix $[A|B]$

$$=\left[\begin{array}{ccc|c}1&2&1&7\\1&1&\lambda&\mu\\1&3&-5&5\end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 3 & -5 & 5 \\ 1 & 1 & \lambda & \mu \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & -6 & -2 \\ 0 & -1 & \lambda-1 & \mu-7 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & -6 & -2 \\ 0 & 0 & \lambda-7 & \mu-9 \end{array}\right] \quad R_3 \to R_3 + R_2$$

(i) If $\lambda = 7$ & $\mu \neq 9$

$\therefore \rho(A) = 2$

$\rho(A/B) = 3$

So $\rho(A,B) \neq \rho(A)$

The given system is inconsistent and it has no solution.

(ii) If $\lambda \neq 7$ & μ is any real num

$\rho(A) = 3$; $\rho(A,B) = 3$

$\rho(A) = \rho(A,B) =$ No. of unknowns.

The given system is consistent and it has unique solution.

(iii) If $\lambda = 7$ & $\mu = 9$

$\rho(A,B) = 2$; $\rho(A) = 2$

$\rho(A) = \rho(A,B) <$ No. of unknowns

The given system is consistent and it has infinite number of solutions.

(b) Soln,

Let h and r be the height and base radius

$\therefore h = 2r$

Let V be the volume of the salt

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{12}\pi h^3$$

Diff V w.r.t 't'

$$\frac{dV}{dt} = \frac{1}{\cancel{12}\,4}\pi(\cancel{3}h^2) \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$30 = \frac{1}{4}\pi h^2 \frac{dh}{dt} \qquad \left[\because \frac{dV}{dt} = 30\, m^3/min\right]$$

$$30 \times 4 = \pi (10)^2 \frac{dh}{dt}$$

$$\frac{3\cancel{0} \times \cancel{4}\,2}{1\cancel{0} \times \cancel{10}_{5} \times \pi} = \frac{dh}{dt} \qquad [\because h = 10m]$$

$$\frac{6}{5\pi} = \frac{dh}{dt}$$

$$\therefore \boxed{\frac{dh}{dt} = \frac{6}{5\pi}\ m/min}$$

42 (a) Soln:

Given: $z = x+iy$

$$\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$$

Now,

$$\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1}$$

$$= \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$= \frac{(x-1)(x+1)-iy(x-1)+iy(x+1)+y^2}{(x+1)^2+y^2}$$

$$= \frac{(x^2-1+y^2)+i(-\cancel{xy}+y+\cancel{xy}+y)}{(x+1)^2+y^2}$$

$$\frac{z-1}{z+1} = \frac{(x^2+y^2-1)+i(2y)}{(x+1)^2+y^2}$$

$$\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$$

$$\tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right) = \frac{\pi}{2}$$

$$\frac{2y}{x^2+y^2-1} = \tan\left(\frac{\pi}{2}\right)$$

$$\frac{2y}{x^2+y^2-1} = \infty$$

$$\frac{2y}{x^2+y^2-1} = \frac{1}{0} \quad \left[\because \frac{1}{0} = \infty\right]$$

$$x^2+y^2-1 = 0.$$

$$\Rightarrow \boxed{x^2+y^2=1}$$

(b) Soln:

Given:

$$6x^4-5x^3-38x^2-5x+6=0 \longrightarrow (1)$$

$x = \frac{1}{3}$ is a solution

$\Rightarrow (3x-1)$ is a factor.

eqn (1) is a reciprocal equation of even degree

So, $\div$ by x^2

$$6x^2-5x-38-\frac{5}{x}+\frac{6}{x^2}=0$$

$$6\left(x^2+\frac{1}{x^2}\right)-5\left(x+\frac{1}{x}\right)-38=0$$

let $y = x+\frac{1}{x}$

$$y^2-2 = x^2+\frac{1}{x^2}$$

$$6(y^2-2)-5y-38=0.$$

$$6y^2-12-5y-38=0$$

$$6y^2-5y-50=0.$$

$$\left(y+\frac{5}{2}\right)\left(y-\frac{10}{3}\right)=0$$

-300; -5; $+\frac{15}{6}_{2}^{5}$; $-\frac{20}{6}_{3}^{10}$

Case (i) $y+\frac{5}{2}=0$

$$x+\frac{1}{x}+\frac{5}{2}=0.$$

$$\frac{2x^2+2+5x}{2x}=0.$$

$$2x^2+5x+2=0.$$

$$(x+2)\left(x+\frac{1}{2}\right)=0.$$

4; $+5$; $\frac{4}{1}$; $\frac{1}{2}$

$\boxed{x=-2}$; $\boxed{x=\frac{-1}{2}}$

Case (ii) $y-\frac{10}{3}=0$

$$x+\frac{1}{x}-\frac{10}{3}=0.$$

$$3x^2+3-10x=0.$$

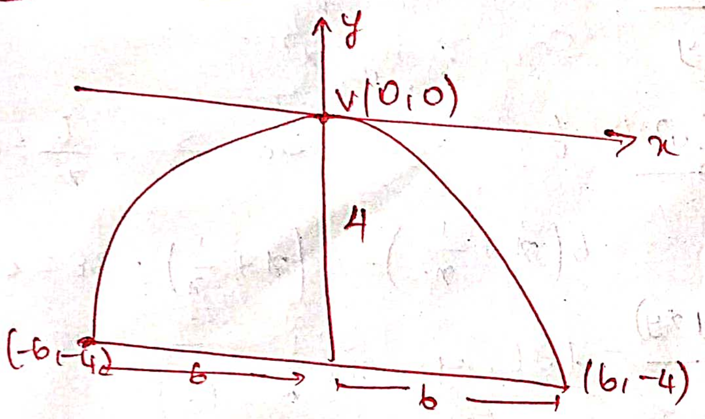
$$3x^2 - 10x + 3 = 0$$

$$(x - 3)\left(x - \frac{1}{3}\right) = 0$$

$\boxed{x = 3}$; $\boxed{x = \frac{1}{3}}$

∴ The roots are $3, \frac{1}{3}, 2, \frac{1}{2}$

43 (a) Soln.



The given curve is open downward

$$\rightarrow x^2 = -4ay \longrightarrow (1)$$

At $(6,-4)$

$$6^2 = -4a(-4)$$

$$\boxed{\frac{36}{16} = a}$$

$$(1) \Rightarrow x^2 = -4\left(\frac{36}{16}\right)y$$

$$x^2 = -9y \longrightarrow (2)$$

Diff w.r.t 'x'

$$2x = -9\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x}{-9}$$

$$m = \frac{2x}{-9} \quad \left[\because \frac{dy}{dx} = m\right]$$

$$\tan\theta = \frac{2x}{-9} \quad [\because m = \tan\theta]$$

At $(-6,-4)$

$$\tan\theta = \frac{2(-\cancel{6})^{2}}{\cancel{-9}\,3}$$

$$\tan\theta = \frac{4}{3}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{4}{3}\right)}$$

(b) Soln.

Let p be the number of bacteria present at time 't'.

Given: $\frac{dp}{dt} \propto p$

$$\frac{dp}{dt} = kp \quad [\because k \text{ is const.}]$$

$$\frac{dp}{p} = k\,dt$$

$$\int \frac{dp}{p} = k\int dt$$

$$\log p = kt + c$$

Taking exponential,

$$p = e^{kt+c}$$

$$p = e^{kt}.c$$

Now, when $t = 0$; $p = p_0$

$$p_0 = e^{k(0)}.c$$

$$\boxed{c = p_0}$$

$$\rightarrow p = e^{kt}.p_0$$

$t = 5 \; ; \; P = 3P_0$

$3\cancel{P_0} = e^{K(5)} \cdot \cancel{P_0}$

$3 = e^{5K}$

Again, $t = 10 \; ; \; P = ?$

$P = e^{K(10)} \cdot P_0$

$P = e^{10K} P_0$

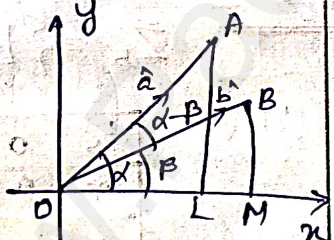
$P = (e^{5K})^2 P_0$

$P = (3)^2 P_0$

$P = 9P_0$

After 10 hours, the number of bacteria is 9 times the original amount.

44 (a) Soln.



let $\hat{a} = \vec{OA}$

$\hat{b} = \vec{OB}$

be the unit vectors making angles α and β respectively.

$\hat{a} = \cos\alpha \hat{i} + \sin\alpha \hat{j}$

$\hat{b} = \cos\beta \hat{i} + \sin\beta \hat{j}$

Hence,

$\hat{b} \times \hat{a} = |\hat{b}||\hat{a}| \sin(\alpha-\beta) \hat{k}$

$= (1)(1) \sin(\alpha-\beta) \hat{k}$

$\hat{b} \times \hat{a} = \sin(\alpha-\beta) \hat{k} \longrightarrow (1)$

Now,

$$\hat{b} \times \hat{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\beta & \sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix}$$

$= \hat{k}[\sin\alpha\cos\beta - \cos\alpha\sin\beta] \longrightarrow (2)$

equation (1) & (2).

$\sin(\alpha-\beta) \cancel{\hat{k}} = \cancel{\hat{k}}[\sin\alpha\cos\beta - \cos\alpha\sin\beta]$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

(b) Soln.

let the length of the printed page be x cm and breadth y cm.

Now, Area of rectangle $= 24 \text{ cm}^2$

$xy = 24$

$y = \frac{24}{x} \longrightarrow (1)$

length of paper $= y + 3$

Area $= (x+2)(y+3)$

$= xy + 3x + 2y + 6$

$= 24 + 3x + 2y + 6 \quad [\because xy = 24]$

$A = 3x + 2y + 30 \longrightarrow (2)$

Sub (1) in (2):

$A = 3x + 2\left(\frac{24}{x}\right) + 30$

Diff A w.r.t 'x'

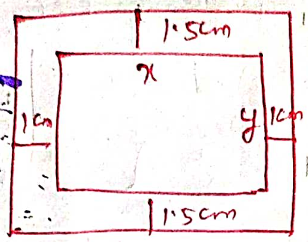
$A'(x) = 3 + 48\left(\frac{-1}{x^2}\right)$

$= 3 - \frac{48}{x^2}$

Again diff,

$A''(x) = -48\left(\frac{-2}{x^3}\right) = \frac{96}{x^3}$

$A'(x) = 0$



$3 - \frac{48}{x^2} = 0$

$3 = \frac{48}{x^2}$

$x^2 = \frac{48}{3}$

$x^2 = 16 \quad \boxed{x = 4}$

At $x=4$, $A''(x) = \frac{96}{x^3}$

$A''(x) = \frac{96}{4^3} = +ve$ is a min. point

So, $x = 4 \Rightarrow y = \frac{24}{x}$

$y = \frac{24}{4} \Rightarrow \boxed{y = 6}$

$\therefore$ Dimensions of the paper

$x + 2 = 4 + 2 = 6\text{cm}$

$y + 3 = 6 + 3 = 9\text{cm}.$

45 (a) Soln.

Given: $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = 9\hat{i} + 3\hat{j} + 6\hat{k}$

$\vec{c} = 2\hat{i} + 6\hat{j} + 6\hat{k}$

parametric form of vector equation.

$\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$

$\vec{r} = (2\hat{i} + 2\hat{j} + \hat{k}) + s(9\hat{i} + 3\hat{j} + 6\hat{k}) + t(2\hat{i} + 6\hat{j} + 6\hat{k})$

Cartesian form of equation.

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

Here,

$(x_1, y_1, z_1) = (2, 2, 1)$

$(b_1, b_2, b_3) = (9, 3, 6)$

$(c_1, c_2, c_3) = (2, 6, 6)$

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 9 & 3 & 6 \\ 2 & 6 & 6 \end{vmatrix} = 0.$$

$(x-2)[18-36] - (y-2)[54-12] + (z-1)[54-6] = 0$

$(x-2)(-18) - (y-2)[42] + (z-1)(48) = 0.$

$\div$ by 6,

$(x-2)(-3) - (y-2)(7) + (z-1)(8) = 0$

$-3x + 6 - 7y + 14 + 8z - 8 = 0.$

$-3x - 7y + 8z + 12 = 0$

$3x + 7y - 8z - 12 = 0.$

(b) Soln.

$Z_5 = \{[0], [1], [2], [3], [4]\}$

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	1	1	2
4	4	0	0	2	3

(i) each box filled by exactly one element, $+_5$ is a binary operation

(ii) Symmetrically placed to the main diagonal

$\therefore +_5$ is a commutative prop.

(iii) $(2 +_5 3) +_5 4 = 0 +_5 4 = 4 \pmod 5$

$2 +_5 (3 +_5 4) = 2 +_5 2 = 4 \pmod 5$

Hence, $(2 +_5 3) +_5 4 = 2 +_5 (3 +_5 4)$

$\therefore +_5$ is associative.

(iv) Row headed by 0 and column headed by 0.

$\therefore$ 0 is the identity element

(v) The inverses is

0 is 0,

1 is 4,

2 is 3,

3 is 2

4 is 1

$\therefore$ Henced proved.

46(a) Soln,

Given : $y^2 = 4x$ & $x^2 = 4y$

To find point of intersection

$y^2 = 4x \Rightarrow \frac{y^2}{4} = x.$

$\Rightarrow x^2 = 4y$

$\left(\frac{y^2}{4}\right)^2 = 4y$

$\frac{y^4}{16} = 4y$

$y^4 = 64y$

$y^3 = 64$

$\boxed{y = 4}$ $\rightarrow$ $x = \frac{(4)^2}{4}$

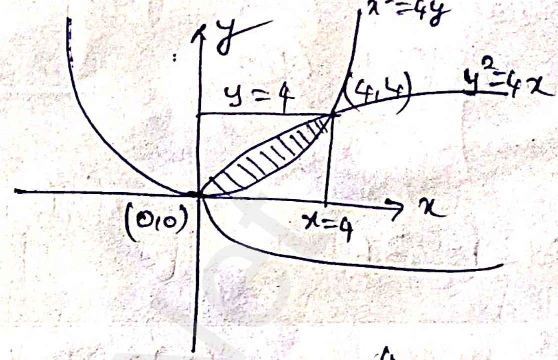
$\boxed{x = 4}$

$\therefore (0,0)$ & $(4,4)$

Area of square field $= 4 \times 4$

$= 16$ sq.units

$\therefore$ Each of three portions must be $= \frac{16}{3}$ sq.units



$\left.\begin{array}{l}\text{Area of} \\ \text{middle portion}\end{array}\right\} = \int_0^4 [f(x) - g(x)]dx$

$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx.$

$= \left[\frac{2x^{3/2}}{3/2} - \frac{x^3}{12}\right]_0^4$

$= \frac{4}{3}(4)^{3/2} - \frac{4^3}{12}$

$= \frac{4}{3}(2^2)^{3/2} - \frac{4 \times 4 \times 4}{12_3}$

$= \frac{4}{3}(8) - \frac{16}{3}$

$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$

$\therefore$ Yes, it is possible to divide the square field.

(b) Soln.

Given.

$S = \{1, 3, 3, 5, 5, 5\}$

$S = \{(1,1), (1,3), \ldots (5,5)\}$

$n(S) = 36$

Sum of faces $(X) = 2, 4, 6, 8, 10$.

$P(X=2) = \frac{1}{36}$; $P(X=4) = \frac{4}{36}$

$P(X=6) = \frac{10}{36}$; $P(X=8) = \frac{12}{36}$; $P(X=10) = \frac{9}{36}$

(i) Probability mass function.

x	2	4	6	8	10	Total
$f(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$	1

(ii) Cummulative distribution func.

$$F(x) = \begin{cases} 0 & ; x < 2 \\ 1/36 & ; x \le 2 \\ 5/36 & ; x \le 4 \\ 15/36 & ; x \le 6 \\ 27/36 & ; x \le 8 \\ 1 & ; x \le 10 \end{cases}$$

(iii) $P(4 \le X \le 10)$

$= P(X=4) + P(X=6) + P(X=8) + P(X=10)$

$= \frac{4}{36} + \frac{10}{36} + \frac{12}{36} + \frac{9}{36} = \frac{35}{36}$

(iv) $P(X \ge 6) = P(X=6) + P(X=8) + P(X=10)$

$= \frac{10}{36} + \frac{12}{36} + \frac{9}{36} = \frac{31}{36}$

47 (a) Soln.

Given $u = \sec^{-1}\left[\frac{x^3-y^3}{x+y}\right]$

$\sec u = \frac{x^3-y^3}{x+y}$

$f(x,y) = \sec u$

$f(x,y) = \frac{x^3-y^3}{x+y}$

$f(tx,ty) = \frac{t^3x^3 - t^3y^3}{tx+ty}$

$= \frac{t^3(x^3-y^3)}{\cancel{t}(x+y)}$

$= t^2 \frac{x^3-y^3}{x+y}$

f is a homogenous function of degree 2

By, Euler's theorem.

$x \cdot \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = nf$

$x \cdot \frac{\partial}{\partial x}(\sec u) + y \frac{\partial}{\partial y}(\sec u) = 2 \sec u$

$x \cdot \sec u \tan u \cdot \frac{\partial u}{\partial x} + y \sec u \tan u \cdot \frac{\partial u}{\partial y} = 2 \sec u.$

$\sec u \tan u \left[x \cdot \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}\right] = 2 \sec u$

$x \cdot \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{2}{\tan u}.$

$x \cdot \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \cot u$

Hence proved.

(b) Soln.

Given: $(x^3+y^3)dy - x^2y\,dx = 0$

$$(x^3+y^3)dy = x^2y\,dx$$

$$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3} \longrightarrow (1)$$

This is a homogenous diff. eqn.

put $y = vx$

$$\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$$

$(1) \Rightarrow$ $$v + x\frac{dv}{dx} = \frac{x^2y}{x^3+y^3}$$

$$v + x\frac{dv}{dx} = \frac{x^2(vx)}{x^3+(vx)^3}$$

$$v + x\frac{dv}{dx} = \frac{vx^3}{x^3+v^3x^3}$$

$$v + x\frac{dv}{dx} = \frac{v}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$x\frac{dv}{dx} = \frac{\cancel{v}-\cancel{v}-v^4}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$$

$$x\,dv = \frac{-v^4}{1+v^3}dx$$

$$\frac{1+v^3}{v^4}dv = -\frac{dx}{x}.$$

Prepared by,
I. Mohammed Iliyas
M.Sc., B.Ed., M.Phil
EDUCARE MATHS TUITION CENTRE
Ph: No: 7299205907.

$$\int \frac{1+v^3}{v^4}dv = \int -\frac{1}{x}dx.$$

$$\int \frac{1}{v^4} + \frac{v^3}{v^4}\,dv = \int -\frac{1}{x}dx.$$

$$\int v^{-4}dv + \int \frac{1}{v}dv = \int -\frac{1}{x}dx$$

$$\frac{v^{-3}}{-3} + \log v = -\log x + \log c$$

$$\frac{-1}{3v^3} = \log c - \log x - \log v$$

$$\frac{-1}{3v^3} = \log\left(\frac{c}{vx}\right)$$

$$\frac{-1}{3v^3} = \log\left(\frac{c}{\frac{y}{\cancel{x}}\cancel{x}}\right)$$

$$\frac{-1}{3\frac{y^3}{x^3}} = \log\left(\frac{c}{y}\right)$$

$$\frac{y}{c} = e^{\frac{x^3}{3y^3}} \Rightarrow \boxed{y = ce^{\frac{x^3}{3y^3}}}$$