ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

5 Mark Questions
CLASS – XII - MATHEMATICS

Time Allowed : 3 Hrs | Maximum Marks : 90

5M QUESTION BANK (Volume 1)

I. Answer ALL questions. **20x1 = 20**

1) **Example 1.10**

If $A=\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$, find x and y such that $A^2+xA+yI_2=O_2$. Hence, find A^{-1}.

2) **Example 1.12**

If $A=\frac{1}{7}\begin{bmatrix}6 & -3 & a\\ b & -2 & 6\\ 2 & c & 3\end{bmatrix}$ is orthogonal, find a,b and c, and hence A^{-1}.

3) **EXERCISE 1.1**

3. If $F(\alpha)=\begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$.

4) 4. If $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$, show that $A^2-3A-7I_2=O_2$. Hence find A^{-1}.

5) 5. If $A=\frac{1}{9}\begin{bmatrix}-8 & 1 & 4\\ 4 & 4 & 7\\ 1 & -8 & 4\end{bmatrix}$, prove that $A^{-1}=A^T$.

6) **Example 1.26**

In a T20 match, a team needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y=ax^2+bx+c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points $(10,8),(20,16),(40,22)$, can you conclude that the team won the match?

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7) EXERCISE 1.4

1. Solve the following systems of linear equations by Cramer's rule:

(iii) $3x+3y-z=11,\ 2x-y+2z=9,\ 4x+3y+2z=25$

(iv) $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0, \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0, \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$

8) 5. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹ 150. The cost of the two dosai, two idlies and four vadais is ₹ 200. The cost of five dosai, four idlies and two vadais is ₹ 250. The family has ₹ 350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

9) Example 1.23

Solve the following system of equations, using matrix inversion method:

$2x_1+3x_2+3x_3=5,\quad x_1-2x_2+x_3=-4,\quad 3x_1-x_2-2x_3=3.$

10) Example 1.24

If $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the

11) EXERCISE 1.3

2. If $A=\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the

12) 1. Solve the following system of linear equations by matrix inversion method:

(iii) $2x+3y-z=9,\ x+y+z=9,\ 3x-y-z=-1$

(iv) $x+y+z-2=0,\ 6x-4y+5z-31=0,\ 5x+2y+2z=13$

13) 3. A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹ 19,800 per month at the end of the first month after 3 years of service and ₹ 23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem.)

14) 5. The prices of three commodities A, B and C are ₹ x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P, Q and R earn ₹ 15,000, ₹ 1,000 and ₹ 4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem.)

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15) **Example 1.28**

The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c,\ 0 \le t \le 100$ where $a, b,$ and c are constants. It has been found that the speed at times $t = 3, t = 6$, and $t = 9$ seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian elimination method.)

16) **EXERCISE 1.5**

1. Solve the following systems of linear equations by Gaussian elimination method:

(ii) $2x + 4y + 6z = 22,\quad 3x + 8y + 5z = 27,\quad -x + y + 2z = 2$

17) 2. If $ax^2 + bx + c$ is divided by $x + 3, x - 5,$ and $x - 1,$ the remainders are $21, 61$ and 9 respectively. Find a, b and c. (Use Gaussian elimination method.)

18) 3. An amount of ₹ 65,000 is invested in three bonds at the rates of 6%, 8% and 9% per annum respectively. The total annual income is ₹ 4,800. The income from the third bond is ₹ 600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)

19) 4. A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8), (-2, -12)$, and $(3, 8)$. He wants to meet his friend at $P(7, 60)$. Will he meet his friend? (Use Gaussian elimination method.)

20) **Example 1.29**

Test for consistency of the following system of linear equations and if possible solve: $x + 2y - z = 3,\ 3x - y + 2z = 1,\ x - 2y + 3z = 3,\ x - y + z + 1 = 0$.

21) **Example 1.30**

Test for consistency of the following system of linear equations and if possible solve:
$4x - 2y + 6z = 8,\ x + y - 3z = -1,\ 15x - 3y + 9z = 21$.

22) **Example 1.32**

Test the consistency of the following system of linear equations
$x - y + z = -9,\ 2x - y + z = 4,\ 3x - y + z = 6, 4x - y + 2z = 7$.

23) **Example 1.33**

Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: $x + y + z = a,\ x + 2y + 3z = b,\ 3x + 5y + 7z = c$.

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24) **Example 1.34**
Investigate for what values of λ and μ the system of linear equations
$x+2y+z=7,\ x+y+\lambda z=\mu, x+3y-5z=5$
has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

25) **EXERCISE 1.6**
1. Test for consistency and if possible, solve the following systems of equations by rank method.
(i) $x-y+2z=2,\quad 2x+y+4z=7,\quad 4x-y+z=4$ (ii) $3x+y+z=2,\quad x-3y+2z=1,\quad 7x-y+4z=5$
(iii) $2x+2y+z=5,\quad x-y+z=1,\quad 3x+y+2z=4$ (iv) $2x-y+z=2,\quad 6x-3y+3z=6,\quad 4x-2y+2z=4$

26) 2. Find the value of k for which the equations $kx-2y+z=1,\quad x-2ky+z=-2,\quad x-2y+kz=1$ have
(i) no solution (ii) unique solution (iii) infinitely many solution

27) 3. Investigate the values of λ and μ the system of linear equations $2x+3y+5z=9$,
$7x+3y-5z=8,\ 2x+3y+\lambda z=\mu$, have
(i) no solution (ii) a unique solution (iii) an infinite number of solutions.

28) **Example 1.36**
Solve the system: $x+3y-2z=0,\ 2x-y+4z=0,\ x-11y+14z=0.$

29) **Example 1.38**
Determine the values of λ for which the following system of equations
$(3\lambda-8)x+3y+3z=0,\ 3x+(3\lambda-8)y+3z=0,\ 3x+3y+(3\lambda-8)z=0$
has a non-trivial solution.

30) **Example 1.39**
By using Gaussian elimination method, balance the chemical reaction equation:
$C_5H_8+O_2 \rightarrow CO_2+H_2O.$

31) **Example 1.40**
If the system of equations $px+by+cz=0,\ ax+qy+cz=0,\ ax+by+rz=0$ has a non-trivial solution and $p\neq a, q\neq b, r\neq c,$ prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2.$

32) **EXERCISE 1.7**
2. Determine the values of λ for which the following system of equations
$x+y+3z=0,\ 4x+3y+\lambda z=0,\ 2x+y+2z=0$ has
(i) a unique solution (ii) a non-trivial solution.

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33) 3. By using Gaussian elimination method, balance the chemical reaction equation:
$C_2H_6 + O_2 \rightarrow H_2O + CO_2$

34) **Example 2.8**

Show that (i) $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real and (ii) $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

35) **EXERCISE 2.4**

7. Show that (i) $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary

(ii) $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is real.

36) **Example 2.14**

Show that the points 1, $\frac{-1}{2}+i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

37) **Example 2.15**

Let z_1, z_2, and z_3 be complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3 \neq 0$.

Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$.

38) **EXERCISE 2.5**

7. If z_1, z_2, and z_3 are three complex numbers such that $|z_1|=1, |z_2|=2, |z_3|=3$ and $|z_1+z_2+z_3|=1$, show that $|9z_1z_2+4z_1z_3+z_2z_3|=6$.

39) **EXERCISE 2.6**

2. If $z=x+iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right)=0$, show that the locus of z is $2x^2+2y^2+x-2y=0$.

40) 3. Obtain the Cartesian form of the locus of $z=x+iy$ in each of the following cases:

(i) $\left[\text{Re}(iz)\right]^2=3$ (ii) $\text{Im}[(1-i)z+1]=0$ (iii) $|z+i|=|z-1|$ (iv) $\bar{z}=z^{-1}$.

41) **Example 2.27**

If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$, show that $x^2+y^2=1$.

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42) **EXERCISE 2.7**

5. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ and

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$.

43) 6. If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

44) **Example 2.31**

Simplify (i) $(1+i)^{18}$ (ii) $(-\sqrt{3}+3i)^{31}$.

45) **Example 2.32**

Find the cube roots of unity.

46) **Example 2.33**

Find the fourth roots of unity.

47) **Example 2.34**

Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$.

48) **Example 2.35**

Find all cube roots of $\sqrt{3} + i$.

49) **Example 2.36**

Suppose z_1, z_2, and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

50) **EXERCISE 2.8**

3. Find the value of $\left(\dfrac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right)^{10}$.

51) 4. If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that

(i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$ (ii) $xy - \frac{1}{xy} = 2i\sin(\alpha+\beta)$

(iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$ (iv) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$.

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52) 5. Solve the equation $z^3+27=0$.

53) 6. If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3+8=0$ are $-1, 1-2\omega, 1-2\omega^2$.

54) 7. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9}+i\sin\frac{2k\pi}{9}\right)$.

55) **Example 3.5**
Find the condition that the roots of cubic equation $x^3+ax^2+bx+c=0$ are in the ratio $p:q:r$.

56) **Example 3.6**
Form the equation whose roots are the squares of the roots of the cubic equation $x^3+ax^2+bx+c=0$.

57) **Example 3.7**
If p is real, discuss the nature of the roots of the equation $4x^2+4px+p+2=0$, in terms of p.

58) **EXERCISE 3.1**
6. Solve the equation $x^3-9x^2+14x+24=0$ if it is given that two of its roots are in the ratio $3:2$.

59) 10. If the equations $x^2+px+q=0$ and $x^2+p'x+q'=0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$.

60) **EXERCISE 3.2**
4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

61) **Example 3.15**
If $2+i$ and $3-\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$, find all roots.

62) **Example 3.20**
Find the condition that the roots of $ax^3+bx^2+cx+d=0$ are in geometric progression. Assume $a,b,c,d \neq 0$

63) **Example 3.21**
If the roots of $x^3+px^2+qx+r=0$ are in H.P., prove that $9pqr=27r^2+2q^3$. Assume $p, q, r \neq 0$

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64) EXERCISE 3.3

4. Determine k and solve the equation $2x^3-6x^2+3x+k=0$ if one of its roots is twice the sum of the other two roots.

65) 5. Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$, if it is known that $1+2i$ and $\sqrt{3}$ are two of its zeros.

66) Example 3.24

Solve the equation $(2x-3)(6x-1)(3x-2)(x-2)-5=0$.

67) EXERCISE 3.4

2. Solve : $(2x-1)(x+3)(x-2)(2x+3)+20=0$

68) Example 3.27

Solve the equation $7x^3-43x^2=43x-7$.

69) EXERCISE 3.5

5. Solve the equations

(i) $6x^4-35x^3+62x^2-35x+6=0$ (ii) $x^4+3x^3-3x-1=0$

70) 7. Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution.

71) Example 4.4

Find the domain of $\sin^{-1}(2-3x^2)$

72) EXERCISE 4.1

6. Find the domain of the following

(i) $f(x)=\sin^{-1}\left(\frac{x^2+1}{2x}\right)$ (ii) $g(x)=2\sin^{-1}(2x-1)-\frac{\pi}{4}$.

73) Example 4.7

Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$.

74) EXERCISE

6. Find the domain of (i) $f(x)=\sin^{-1}\left(\frac{|x|-2}{3}\right)+\cos^{-1}\left(\frac{1-|x|}{4}\right)$ (ii) $g(x)=\sin^{-1}x+\cos^{-1}x$

75) EXERCISE 4.3

4. Find the value of (i) $\tan\left(\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ (ii) $\sin\left(\tan^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right)$.

(iii) $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right)$.

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76) **Example 4.17**

Simplify (i) $\cos^{-1}\left(\cos\left(\frac{13\pi}{3}\right)\right)$ (ii) $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

(iii) $\sec^{-1}\left(\sec\left(\frac{5\pi}{3}\right)\right)$ (iv) $\sin^{-1}[\sin 10]$

77) **Example 4.18**

Find the value of (i) $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ (ii) $\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$

(iii) $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$.

78) **Example 4.20**

Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sec^{-1}\left(\frac{5}{4}\right)\right]$

79) **Example 4.23**

If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n-a_1}{1+a_1a_n}$.

80) **Example 4.29**

Solve $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$.

81) **EXERCISE 4.5**

5. Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.

82) 9. Solve:

(i) $\sin^{-1}\frac{5}{x}+\sin^{-1}\frac{12}{x}=\frac{\pi}{2}$ (ii) $2\tan^{-1}x=\cos^{-1}\frac{1-a^2}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}$, $a>0,\ b>0$.

(iii) $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$ (iv) $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12}$, $x>0$.

83) **Example 5.10**

Find the equation of the circle passing through the points $(1,1)$, $(2,-1)$, and $(3,2)$.

84) **EXERCISE 5.1**

10. Determine whether the points $(-2,1), (0,0)$ and $(-4,-3)$ lie outside, on or inside the circle $x^2+y^2-5x+2y-5=0$.

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85) **Example 5.15**

Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

86) **Example 5.21**

Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, one of the foci is $(2,3)$ and a directrix is $x=7$. Also find the length of the major and minor axes of the ellipse.

87) **Example 5.22**

Find the foci, vertices and length of major and minor axis of the conic $4x^2+36y^2+40x-288y+532=0$.

88) **Example 5.23**

For the ellipse $4x^2+y^2+24x-2y+21=0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2.

89) **Example 5.26**

Find the centre, foci, and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$

90) **EXERCISE 5.2**

4. Find the vertex, focus, equation of directrix and length of the latus rectum of the following:

(iv) $x^2-2x+8y+17=0$ (v) $y^2-4y-8x+12=0$

91) 6. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$.

92) 8. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

(i) $\frac{(x-3)^2}{225}+\frac{(y-4)^2}{289}=1$ (ii) $\frac{(x+1)^2}{100}+\frac{(y-2)^2}{64}=1$ (iii) $\frac{(x+3)^2}{225}-\frac{(y-4)^2}{64}=1$

(iv) $\frac{(y-2)^2}{25}-\frac{(x+1)^2}{16}=1$ (v) $18x^2+12y^2-144x+48y+120=0$

(vi) $9x^2-y^2-36x-6y+18=0$

93) **Example 5.30**

Find the equations of tangent and normal to the ellipse $x^2+4y^2=32$ when $\theta=\frac{\pi}{4}$.

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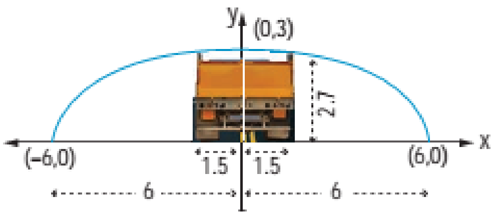
94) **EXERCISE 5.4**

6. Find the equations of the tangent and normal to hyperbola $12x^2 - 9y^2 = 108$ at $\theta = \frac{\pi}{3}$. (Hint: use parametric form)

95) **Example 5.31**

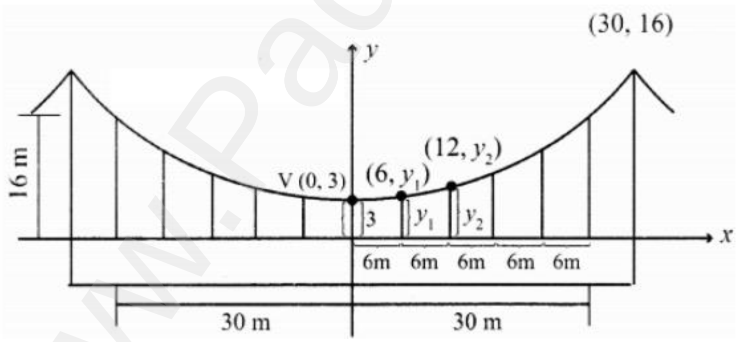
A semielliptical archway over a one-way road has a height of $3m$ and a width of $12m$. The truck has a width of $3m$ and a height of $2.7m$. Will the truck clear the opening of the archway?



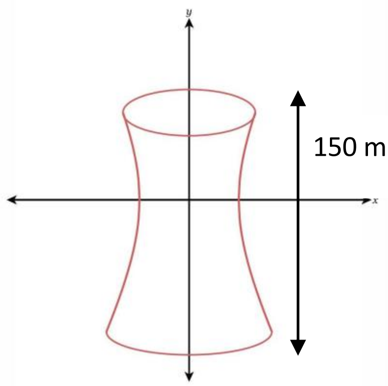
96) **EXERCISE 5.5**

2. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be $16m$, and the height at the edge of the road must be sufficient for a truck $4m$ high to clear if the highest point of the opening is to be $5m$ approximately . How wide must the opening be?

97) 5. Parabolic cable of a $60m$ portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every $6m$ along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



98) 6. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is $150m$ tall and the distance from the top of the tower to the centre



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99) 7. A rod of length $1.2m$ moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is $0.3m$ from the end in contact with x-axis is an ellipse. Find the eccentricity.

100) 8. Assume that water issuing from the end of a horizontal pipe, $7.5m$ above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position $2.5\,m$ below the line of the pipe, the flow of water has curved outward $3m$ beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

101) 9. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground $12m$ away from the starting point. Find the angle of projection.

102) 10. Points A and B are $10km$ apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 km closer to A than B. Show that the location of the explosion is restricted to a particular curve and find an equation of it.

103) **Example 6.3**
By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

104) **Example 6.4**
With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.

105) **Example 6.5**
Prove by vector method that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.

106) **Example 6.6 (Apollonius's theorem)**
If D is the midpoint of the side BC of a triangle ABC, show by vector method that $|\overline{AB}|^2+|\overline{AC}|^2=2(|\overline{AD}|^2+|\overline{BD}|^2)$.

107) **Example 6.7**
Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

108) **Example 6.8**
In triangle ABC, the points D,E,F are the midpoints of the sides BC, CA, and AB respectively. Using vector method, show that the area of ΔDEF is equal to $\frac{1}{4}$ (area of ΔABC).

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109) EXERCISE 6.1

8. If G is the centroid of a ΔABC, prove that (area of ΔGAB) = (area of ΔGBC) = (area of ΔGCA) $=\frac{1}{3}$ (area of ΔABC).

110) 9. Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

111) 10. Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$.

112) Example 6.16

Show that the four points $(6,-7,0)$, $(16,-19,-4)$, $(0,3,-6)$, $(2,-5,10)$ lie on a same plan

113) EXERCISE 6.2

10. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a},\vec{b},\vec{c}]^2=\frac{1}{4}|\vec{a}|^2\,|\vec{b}|^2$.

114) Example 6.21

For any four vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$, we have

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}$.

115) EXERCISE 6.3

8. If $\hat{a},\hat{b},\hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a}\times(\hat{b}\times\hat{c})=\frac{1}{2}\hat{b}$, find the angle between $\hat{a}$ and $\hat{c}$.

116) Example 6.24

A straight line passes through the point $(1,2,-3)$ and parallel to $4\hat{i}+5\hat{j}-7\hat{k}$. Find (i) vector equation in parametric form (ii) vector equation in non-parametric form (iii) Cartesian equations of the straight line.

117) Example 6.25

The vector equation in parametric form of a line is $\vec{r}=(3\hat{i}-2\hat{j}+6\hat{k})+t(2\hat{i}-\hat{j}+3\hat{k})$. Find (i) the direction cosines of the straight line (ii) vector equation in non-parametric form of the line (iii) Cartesian equations of the line.

118) Example 6.27

Find the vector equation in parametric form and Cartesian equations of a straight passing through the points $(-5,7,-4)$ and $(13,-5,2)$. Find the point where the straight line crosses the xy-plane.

119) 3. Find the points where the straight line passes through $(6,7,4)$ and $(8,4,9)$ cuts the xz and yz planes.

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120) 6. The vertices of ΔABC are $A(7,2,1), B(6,0,3)$, and $C(4,2,4)$. Find $\angle ABC$.

121) 7. If the straight line joining the points $(2,1,4)$ and $(a-1,4,-1)$ is parallel to the line joining the points $(0,2,b-1)$ and $(5,3,-2)$, find the values of a and b.

122) 8. If the straight lines $\frac{x-5}{5m+2}=\frac{2-y}{5}=\frac{1-z}{-1}$ and $x=\frac{2y+1}{4m}=\frac{1-z}{-3}$ are perpendicular to each other, find the value of m.

123) **Example 6.33**

Find the point of intersection of the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$.

124) **Example 6.34**

Find the parametric form of vector equation of a straight line passing through the point of intersection of the straight lines $\vec{r}=(\hat{i}+3\hat{j}-\hat{k})+t(2\hat{i}+3\hat{j}+2\hat{k})$ and $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z+3}{4}$, and perpendicular to both straight lines.

125) **Example 6.35**

Determine whether the pair of straight lines $\vec{r}=(2\hat{i}+6\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}+4\hat{k})$, $\vec{r}=(2\hat{j}-3\hat{k})+s(\hat{i}+2\hat{j}+3\hat{k})$ are parallel. Find the shortest distance between them.

126) **Example 6.37**

Find the coordinate of the foot of the perpendicular drawn from the point $(-1,2,3)$ to the straight line $\vec{r}=(\hat{i}-4\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}+\hat{k})$. Also, find the shortest distance from the given point to the straight line.

127) **EXERCISE 6.5**

1. Find the parametric form of vector equation and Cartesian equations of a straight line passing through $(5,2,8)$ and is perpendicular to the straight lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+s(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+t(\hat{i}+2\hat{j}+2\hat{k})$.

128) 2. Show that the lines $\vec{r}=(6\hat{i}+\hat{j}+2\hat{k})+s(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{r}=(3\hat{i}+2\hat{j}-2\hat{k})+t(2\hat{i}+4\hat{j}-5\hat{k})$ are skew lines and hence find the shortest distance between them.

129) 3. If the two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-m}{2}=z$ intersect at a point, find the value of m.

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130) 4. Show that the lines $\frac{x-3}{3}=\frac{y-3}{-1}, z-1=0$ and $\frac{x-6}{2}=\frac{z-1}{3}, y-2=0$ intersect. Also find the point of intersection.

131) 5. Show that the straight lines $x+1=2y=-12z$ and $x=y+2=6z-6$ are skew and hence find the shortest distance between them.

132) 7. Find the foot of the perpendicular drawn from the point $(5,4,2)$ to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$. Also, find the equation of the perpendicular.

133) **Example 6.40**

Find the direction cosines of the normal to the plane and length of the perpendicular from the origin to the plane $\vec{r}\cdot(3\hat{i}-4\hat{j}+12\hat{k})=5$.

134) **Example 6.43**

Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+s(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=(\hat{i}-3\hat{j}+5\hat{k})+t(\hat{i}+\hat{j}-\hat{k})$.

135) **Example 6.44**

Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0)$, $(2,2-1)$ and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.

136) **EXERCISE 6.7**

1. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(2,3,6)$ and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$

137) 2. Find the non-parametric form of vector equation, and Cartesian equations of the plane passing through the points $(2,2,1)$, $(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$.

138) 3. Find parametric form of vector equation and Cartesian equations of the plane passing through the points $(2,2,1),(1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$.

139) 4. Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$.

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140) 5. Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + t(2\hat{i} - \hat{j} + 4\hat{k})$ and perpendicular to plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = 8$.

141) 6. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points $(3, 6, -2), (-1, -2, 6)$, and $(6, 4, -2)$.

142) 7. Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r} = \left(6\hat{i} - \hat{j} + \hat{k}\right) + s\left(-\hat{i} + 2\hat{j} + \hat{k}\right) + t\left(-5\hat{i} - 4\hat{j} - 5\hat{k}\right)$.

143) **Example 6.46**

Show that the lines $\vec{r} = \left(-\hat{i} - 3\hat{j} - 5\hat{k}\right) + s\left(3\hat{i} + 5\hat{j} + 7\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} + 4\hat{j} + 6\hat{k}\right) + t\left(\hat{i} + 4\hat{j} + 7\hat{k}\right)$ are coplanar. Also,find the non-parametric form of vector equation of the plane containing these lines.

144) **EXERCISE 6.8**

1. Show that the straight lines $\vec{r} = (5\hat{i} + 7\hat{j} - 3\hat{k}) + s(4\hat{i} + 4\hat{j} - 5\hat{k})$ and $\vec{r} = \left(8\hat{i} + 4\hat{j} + 5\hat{k}\right) + t\left(7\hat{i} + \hat{j} + 3\hat{k}\right)$ are coplanar. Find the vector equation of the plane in which they lie.

145) 2. Show that the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{3}$ and $\frac{x-1}{-3} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar. Also, find the plane containing these lines.

146) 3. If the straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{m^2}$ and $\frac{x-3}{1} = \frac{y-2}{m^2} = \frac{z-1}{2}$ are coplanar, find the distinct real values of m.

147) 4. If the straight lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines.

148) **Example 6.53**

Find the equation of the plane passing through the intersection of the planes $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) + 1 = 0$ and $\vec{r} \cdot \left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) = 2$ and the point $(-1, 2, 1)$.

149) **Example 6.54**

Find the equation of the plane passing through the intersection of the planes $2x + 3y - z + 7 = 0$ and $x + y - 2z + 5 = 0$ and is perpendicular to the plane $x + y - 3z - 5 = 0$.

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150) **Example 6.56**

Find the coordinates of the point where the straight line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + t(3\hat{i} + 4\hat{j} + 2\hat{k})$ intersects the plane $x - y + z - 5 = 0$.

151) **EXERCISE 6.9**

1. Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (2\hat{i} - 7\hat{j} + 4\hat{k}) = 3$ and $3x - 5y + 4z + 11 = 0$, and the point $(-2,1,3)$.

152) 2. Find the equation of the plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$, and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$.

153) 5. Find the equation of the plane which passes through the point $(3,4,-1)$ and is parallel to the plane $2x - 3y + 5z + 7 = 0$. Also, find the distance between the two planes.

154) 7. Find the point of intersection of the line $x - 1 = \frac{y}{2} = z + 1$ with the plane $2x - y + 2z = 2$. Also, find the angle between the line and the plane.

155) 8. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane $x + 2y + 3z = 2$.