www.Padasalai.Net - Centum Special Question Paper 2023

Model Exam (2022 – 23) – Question
CLASS – XII - MATHEMATICS

Time Allowed : 3 Hrs | Maximum Marks : 90

PART – I

I. Answer ALL questions. **20x1 = 20**

1) If $f(x)=\frac{x}{x+1}$, then its differential is given by

(1) $\frac{-1}{(x+1)^2}dx$ (2) $\frac{1}{(x+1)^2}dx$ (3) $\frac{1}{x+1}dx$ (4) $\frac{-1}{x+1}dx$

2) If $f(x,y,z)=xy+yz+zx$, then f_x-f_z is equal to

(1) $z-x$ (2) $y-z$ (3) $x-z$ (4) $y-x$

3) If $A=\begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$ then $|\text{adj}(AB)|=$

(1) –40 (2) –80 (3) –60 (4) –20

4) The number of arbitrary constants in the general solutions of order n and $n+1$ are respectively

(1) $n-1,\ n$ (2) $n,\ n+1$ (3) $n+1,\ n+2$ (4) $n+1,\ n$

5) If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

(1) 3 (2) –1 (3) 1 (4) 9

6) P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount remaining, then

(1) $P=Ce^{kt}$ (2) $P=Ce^{-kt}$ (3) $P=Ckt$ (4) $Pt=C$

7) The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

(1) –2 (2) –1 (3) 1 (4) 2

8) The dual of $\neg(p\vee q)\vee[p\vee(p\wedge\neg r)]$ is

(1) $\neg(p\wedge q)\wedge[p\vee(p\wedge\neg r)]$ (2) $(p\wedge q)\wedge[p\wedge(p\vee\neg r)]$

(3) $\neg(p\wedge q)\wedge[p\wedge(p\wedge r)]$ (4) $\neg(p\wedge q)\wedge[p\wedge(p\vee\neg r)]$

9) Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X–3)$ is

(1) 0.24 b) 0.48 (3) 0.6 (4) 0.96

10) If z is a non zero complex number, such that $2iz^2=\bar{z}$ then $|z|$ is

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 3

Ph: 948 99 00 886 E-mail: berkmansja@gmail.com

ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

11) The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

12) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(1) 0.4 cu.cm (2) 0.45 cu.cm (3) 2 cu.cm (4) 4.8 cu.cm

13) A random variable X has binomial distribution with $n=25$ and $p=0.8$ then standard deviation of X is

(1) 6 (2) 4 (3) 3 (4) 2

14) $\sin^{-1}\left(\tan\frac{\pi}{4}\right)-\sin^{-1}\left(\sqrt{\frac{3}{x}}\right)=\frac{\pi}{6}$. Then x is a root of the equation

(1) $x^2-x-6=0$ (2) $x^2-x-12=0$ (3) $x^2+x-12=0$ (4) $x^2+x-6=0$

15) The polynomial x^3-kx^2+9x has three real zeros if and only if, k satisfies

(1) $|k|\le 6$ (2) $k=0$ (3) $|k|>6$ (4) $|k|\ge 6$

16) The value of $\int_0^1\left(\sin^{-1}x\right)^2\,dx$ is

(1) $\frac{\pi^2}{4}-1$ (2) $\frac{\pi^2}{4}+2$ (3) $\frac{\pi^2}{4}+1$ (4) $\frac{\pi^2}{4}-2$

17) The domain of the function defined by $f(x)=\sin^{-1}\sqrt{x-1}$ is

(1) $[1,\ 2]$ (2) $[-1,\ 1]$ (3) $[0,\ 1]$ (4) $[-1,\ 0]$

18) If $f(x,y,z)=xy+yz+zx$, then f_x-f_z is equal to

(1) $z-x$ (2) $y-z$ (3) $x-z$ (4) $y-x$

19) If a compound statement involves 3 simple statements, then the number of rows in the tru table is

(1) 9 (2) 8 (3) 6 (4) 3

20) A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?

(1) $\frac{57}{\ldots}$ (2) $\frac{57}{\ldots}$ (3) $\frac{19^3}{\ldots}$ (4) $\frac{57}{\ldots}$

ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

PART – II

II. Answer any SEVEN questions. Question number 30 is Compulsory **7x2 = 14**

21) The probability density function of X is given by $f(x)=\begin{cases} k\,x\,e^{-2x} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$.

Find the value of k.

22) Determine the order and degree (if exists) of the following differential equations:

$$3\left(\frac{d^2y}{dx^2}\right)=\left[4+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$$

23) If α, β and γ are the roots of the cubic equation $x^3+2x^2+3x+4=0$, form a cubic equation whose roots are $\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma}$

24) Evaluate the following: $\int_0^{\frac{\pi}{4}} \sin^6 2x\,dx$

25) Find the value in the interval $\left(\frac{1}{2},2\right)$ satisfied by the Rolle's theorem for th function $f(x)=x+\frac{1}{x}, x\in\left[\frac{1}{2},2\right]$.

26) If $2\hat{i}-\hat{j}+3\hat{k}, 3\hat{i}+2\hat{j}+\hat{k}, \hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m.

27) If $\text{adj}(A)=\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

28) Obtain the equation of the circle for which $(3,4)$ and $(2,-7)$ are the ends of a diameter.

29) Verify whether the following compound proposition is tautology or contradiction or contingency $((p\vee q)\wedge\neg p)\rightarrow q$

ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

30) Draw the graph of the function $y = \cos^{-1} x$ in the interval $[0, \pi]$ and also write its principal domain and range.

PART – III

II. Answer any SEVEN questions. Question 40 is compulsory **7x3 = 21**

31) For any two complex numbers z_1 and z_2, such that $|z_1| = |z_2| = 1$ and $z_1 z_2 \neq -1$, then show that $\frac{z_1 + z_2}{1 + z_1 z_2}$ is a real number.

32) The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.

33) Check whether the following system of equations can be solved by matrix inversion method or not. If not state the reason.
$4x - 2y + 6z = 8,\ x + y - 3z = -1,\ 15x - 3y + 9z = 21.$

34) Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at $(1, -3)$.

35) Write down the (i) conditional statement (ii) converse statement (iii) inverse statement, and (iv) contrapositive statement for the two statements p and q given below.
p : The number of primes is infinite. q: Ooty is in Kerala.

36) Evaluate the following limit, if necessary use l'Hôpital Rule : $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

37) If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights
(i) exactly 10 will have a useful life of at least 600 hours;
(ii) at least 2 will *not* have a useful life of at least 600 hours.

38) Prove that $[\vec{a} \times \vec{b},\ \vec{b} \times \vec{c},\ \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$.

ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

39) Solve $\frac{dy}{dx}+2y=e^{-x}$.

40) Evaluate the following definite integral :

$$\int_0^{\frac{\pi}{2}} e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$$

PART – IV

IV. Answer ALL the questions. **7x5 = 35**

41) a) Find the intervals of monotonicities and hence find the local extremum for the following function:

$$f(x)=\frac{e^x}{1-e^x}$$

OR

b) If the system of equations $px+by+cz=0,\ ax+qy+cz=0,\ ax+by+rz=0$ has a non-trivial solution and $p\neq a, q\neq b, r\neq c,$ prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2.$

42) a) If X is the random variable with probability density function $f(x)$ given by,

$$f(x)=\begin{cases} x-1, & 1\le x<2 \\ -x+3, & 2\le x<3 \\ 0 & \text{otherwise} \end{cases}$$

find (i) the distribution function $F(x)$
(ii) $P(1.5\le X\le 2.5)$

OR

b) Find the area of the region bounded between the curves $y=\sin x$ and $y=\cos x$ and the lines $x=0$ and $x=\pi$.

43) a) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

$$\frac{(x+3)^2}{225}-\frac{(y-4)^2}{64}=1$$

OR

ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

b) Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$.

44) a) Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$.

OR

b) Solve the equation $7x^3 - 43x^2 = 43x - 7$.

45) a) Verify De-Morgans Laws using Truth Table:

OR

b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground $12m$ away from the starting point. Find the angle of projection.

46) a) Let $U(x,y) = e^x \sin y$, where $x = st^2$, $y = s^2t$, $s,t \in \mathbb{R}$. Find $\frac{\partial U}{\partial s}, \frac{\partial U}{\partial t}$ and evaluate them at $s = t = 1$.

OR

b) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove:

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n-a_1}{1+a_1a_n}.$$

47) a) The equation of electromotive force for an electric circuit containing resistance and self-inductance is $E = Ri + L\frac{di}{dt}$, where E is the electromotive force is given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when $E = 0$.

OR

b) If D is the midpoint of the side BC of a triangle ABC, show by vector method that $|\overline{AB}|^2 + |\overline{AC}|^2 = 2(|\overline{AD}|^2 + |\overline{BD}|^2)$.

****