CENTUM ACHIEVERS' ACADEMY

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XII STD(MATHS)	FULL PORTION – 2	TIME : 2 ½ Hrs MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : **(20× 1 = 20)**

1. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

   (1) 15 (2) 12 (3) 14 (4) 11

2. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{ adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

   (1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1

3. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

   (1) Z (2) $\bar{Z}$ (3) $\frac{1}{Z}$ (4) 1

4. If z is a complex number such that $z \in \mathbb{C} \setminus \mathbb{R}$ and $z + \frac{1}{z} \in \mathbb{R}$, then $|z|$ is

   (1) 0 (2) 1 (3) 2 (4) 3

5. If α, β, and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is

   (1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

6. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is

   (1) 0 (2) 1 (3) 2 (4) 3

7. If $\cot^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha}) = u$, then $\cos 2u$ is equal to

   (1) $\tan^2 \alpha$ (2) 0 (3) -1 (4) $\tan 2\alpha$

8. The radius of the circle passing through the point (6,2) two of whose diameter are $x + y = 6$ and $x + 2y = 4$ is

   (1) 10 (2) $2\sqrt{5}$ (3) 6 (4) 4

9. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]\}^2$ is equal to

   (1) 81 (2) 9 (3) 27 (4) 18

10. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$, then (α, β) is

    (1) $(-5,5)$ (2) $(-6,7)$ (3) $(5,-5)$ (4) $(6,-7)$

11. The maximum slope of the tangent to the curve $y = e^x \sin x, x \in [0,2\pi]$ is at

(1) $x = \frac{\pi}{4}$ (2) $x = \frac{\pi}{2}$ (3) $x = \pi$ (4) $x = \frac{3\pi}{2}$

12. The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0,3]$ is

(1) 1 (2) $\sqrt{2}$ (3) $\frac{3}{2}$ (4) 2

13. If $f(x) = \frac{x}{x+1}$, then its differential is given by

(1) $\frac{-1}{(x+1)^2}dx$ (2) $\frac{1}{(x+1)^2}dx$ (3) $\frac{1}{x+1}dx$ (4) $\frac{-1}{x+1}dx$

14. If $f(x,y,z) = xy + yz + zx$, then $f_x - f_z$ is equal to

(1) $z - x$ (2) $y - z$ (3) $x - z$ (4) $y - x$

15. If $f(x) = \int_0^x t \cos t \, dt$, then $\frac{df}{dx} =$

(1) $\cos x - x \sin x$ (2) $\sin x + x \cos x$ (3) $x \cos x$ (4) $x \sin x$

16. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

(1) 10 (2) 5 (3) 8 (4) 9

17. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(1) $xy = k$ (2) $y = k\log x$ (3) $y = kx$ (4) $\log y = kx$

18. If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is

(1) $\log \sin x$ (2) $\cos x$ (3) $\tan x$ (4) $\cot x$

19. If $P(X = 0) = 1 - P(X = 1)$. If $E(X) = 3\text{Var}(X)$, then $P(X = 0)$ is

(1) $\frac{2}{3}$ (2) $\frac{2}{5}$ (3) $\frac{1}{5}$ (4) $\frac{1}{3}$

20. Which one is the contrapositive of the statement $(p \vee q) \to r$?

(1) $\neg r \to (\neg p \wedge \neg q)$ (2) $\neg r \to (p \vee q)$

(3) $r \to (p \wedge q)$ (4) $p \to (q \vee r)$

PART-II

(i) Answer any SEVEN questions. **$(7\times 2 = 14)$**

(ii) Qn.No.30 is compulsory

21. If A is symmetric, prove that adj A is also symmetric.

22. Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$.

23. Find the equation of the ellipse with foci $(\pm 2,0)$, vertices $(\pm 3,0)$.

24. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a}, \vec{b}, \vec{c}]^2 = \frac{1}{4}|\vec{a}|^2|\vec{b}|^2$.

25. Find the values in the interval (1,2) of the mean value theorem satisfied by the function $f(x) = x - x^2$ for $1 \le x \le 2$.

26. An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm and radius to the outside of the shell is 5.3 mm, find the volume of the shell approximately.

27. Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|} dx$.

28. Show that $y = e^{-x} + mx + n$ is a solution of the differential equation $e^{x}\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.

29. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.

30. State and prove the uniqueness of identity.

PART-III

(i) Answer any SEVEN questions. **$(7\times 3 = 21)$**

(ii) Qn.No.40 is compulsory

31. Find the inverse of the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$, by Gauss-Jordan method

32. If $z = x + iy$, find in rectangular form Im $(3z + 4\bar{z} - 4i)$

33. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

34. For any vector $\vec{a}$, prove that $\hat{\imath} \times (\vec{a} \times \hat{\imath}) + \hat{\jmath} \times (\vec{a} \times \hat{\jmath}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$.

35. Find the equations of the tangents to the curve $y = 1 + x^3$ for which the tangent is orthogonal with the line $x + 12y = 12$.

36. If $w(x, y) = x^3 - 3xy + 2y^2,\ x, y \in \mathbb{R}$, find the linear approximation for w at $(1, -1)$.

37. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{5+4\sin^2 x}$

38. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$.

39. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the random variable.

40. Find the differential equation corresponding to the family of curves represented by the equation $y = \mathrm{A}e^{8x} + \mathrm{B}e^{-8x}$, where A and B are arbitrary constants.

PART-IV

Answer the following questions. **$(7\times 5 = 35)$**

41. a) If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}. **(OR)**

    b) Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3: 2.

42. a) Find the equations of tangent and normal to the ellipse $x^2 + 4y^2 = 32$ when $\theta = \frac{\pi}{4}$. **(OR)**

    b) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

43. a) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2 . **(OR)**

b) Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$.

44. a) If $w(x, y, z) = \log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$, find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$. **(OR)**

b) Solve: $(x^2 + y^2)dy = xy\,dx$. It is given that $y(1) = 1$ and $y(x_0) = e$. Find the value of x_0.

45. a) Evaluate $\int_1^2 (4x^2 - 1)dx$ **(OR)**

b) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$

46. a) Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000. **(OR)**

b) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

47. a) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1,3,4,5,9\}$ of the set of remainders {0,1,2,3,4,5,6,7,8,9,10}.**(OR)**

b) A retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 5%.The inspector of the retailer randomly picks 10 items from a shipment. What is the probability that there will be (i) at least one defective item (ii) exactly two defective items?

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