CENTUM ACHIEVERS' ACADEMY

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XII STD(MATHS)	FULL PORTION – 3	TIME : 2 ½ Hrs MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : **($20\times 1 = 20$)**

1. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^2 = A^{-1}$, then the value of x is

   (1) $\frac{-4}{5}$ (2) $\frac{-3}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

2. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

   (1) 2 (2) 4 (3) 3 (4) 1

3. If $(1+i)(1+2i)(1+3i)\cdots(1+ni) = x+iy$, then $2\cdot 5\cdot 10\cdots(1+n^2)$ is

   (1) 1 (2) i (3) x^2+y^2 (4) $1+n^2$

4. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f\circ g)(x)$, then the degree of h is

   (1) mn (2) $m+n$ (3) m^n (4) n^m

5. If $\sin^{-1} x = 2\sin^{-1} \alpha$ has a solution, then

   (1) $|\alpha| \le \frac{1}{\sqrt{2}}$ (2) $|\alpha| \ge \frac{1}{\sqrt{2}}$ (3) $|\alpha| < \frac{1}{\sqrt{2}}$ (4) $|\alpha| > \frac{1}{\sqrt{2}}$

6. $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1-2\sin^2 x) =$

   (1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

7. If $x+y=k$ is a normal to the parabola $y^2 = 12x$, then the value of k is

   (1) 3 (2) -1 (3) 1 (4) 9

8. The eccentricity of the ellipse $(x-3)^2+(y-4)^2 = \frac{y^2}{9}$ is

   (1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

9. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a},\vec{b},\vec{c}] = 3$, then $\{[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}]\}^2$ is equal to

   (1) 81 (2) 9 (3) 27 (4) 18

10. If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z = 1, \lambda > 0$ is $\frac{1}{5}$, then the value of λ is

    (1) $2\sqrt{3}$ (2) $3\sqrt{2}$ (3) 0 (4) 1

11. The volume of a sphere is increasing in volume at the rate of $3\pi \text{cm}^3/\text{sec}$. The rate of change of its radius when radius is $\frac{1}{2}$ cm

(1) 3 cm/s (2) 2 cm/s (3) 1 cm/s (4) $\frac{1}{2}$ cm/s

12. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(1) $\tan^{-1}\frac{3}{4}$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

13. If $u(x, y) = x^2 + 3xy + y - 2019$, then $\left.\frac{\partial u}{\partial x}\right|_{(4,-5)}$ is equal to

(1) -4 (2) -3 (3) -7 (4) 13

14. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x\,\partial y}$ is equal to

(1) xye^{xy} (2) $(1 + xy)e^{xy}$ (3) $(1 + y)e^{xy}$ (4) $(1 + x)e^{xy}$

15. If $f(x) = \int_0^x t\cos t\, dt$, then $\frac{df}{dx} =$

(1) $\cos x - x\sin x$ (2) $\sin x + x\cos x$ (3) $x\cos x$ (4) $x\sin x$

16. If $\int_0^a \frac{1}{4+x^2}dx = \frac{\pi}{8}$ then a is

(1) 4 (2) 1 (3) 3 (4) 2

17. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents

(1) straight lines (2) circles (3) parabola (4) ellipse

18. If the function $f(x) = \frac{1}{12}$ for $a < x < b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b ?

(1) 0 and 12 (2) 5 and 17 (3) 7 and 19 (4) 16 and 24

19. If in 6 trials, X is a binomial variable which follows the relation $9P(X = 4) = P(X = 2)$, then the probability of success is

(1) 0.125 (2) 0.25 (3) 0.375 (4) 0.75

20. A binary operation on a set S is a function from

(1) $S \to S$ (2) $(S \times S) \to S$ (3) $S \to (S \times S)$ (4) $(S \times S) \to (S \times S)$

PART-II

(i) Answer any SEVEN questions. (7× 2 = 14)

(ii) Qn.No.30 is compulsory

21. If A is a non-singular matrix of odd order, prove that $|\text{adj } A|$ is positive.

22. Prove : Re $(z) = \frac{z+\bar{z}}{2}$ and Im $(z) = \frac{z-\bar{z}}{2i}$

23. Find the equation of the hyperbola , foci $(\pm 2,0)$, eccentricity $= \frac{3}{2}$.

24. Verify whether the line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{12}$ lies in the plane $5x - y + z = 8$.

25. Let $g(x,y)=\frac{e^y \sin x}{x}$, for $x \neq 0$ and $g(0,0)=1$. Show that g is continuous at $(0,0)$.

26. Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^4 x\, dx$

27. Solve the differential equation $\sin \frac{dy}{dx}=a, y(0)=1$

28. The probability density function of X is given by $f(x)=\begin{cases} kxe^{-2x} & \text{for } x>0 \\ 0 & \text{for } x \leq 0 \end{cases}$. Find the value of k.

29. Let $*$ be defined on $\mathbb{R}$ by $(a*b)=a+b+ab-7$. Is $*$ binary on $\mathbb{R}$? If so, find $3*\left(\frac{-7}{15}\right)$.

30. Find the domain of $\sin^{-1}(2-3x^2)$

PART-III

(i) Answer any SEVEN questions. **$(7\times 3=21)$**

(ii) Qn.No.40 is compulsory

31. Find the rank of the matrix by minor method $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$

32. Show that the equation $z^2=\bar{z}$ has four solutions.

33. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

34. If $\cot^{-1}\left(\frac{1}{7}\right)=\theta$, find the value of $\cos\theta$.

35. A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t)=128t-16t^2$. (i) Compute the maximum height of the particle reached.
(ii) What is the velocity when the particle hits the ground?

36. Evaluate $\int_0^1 \frac{\sin(3\tan^{-1} x)\tan^{-1} x}{1+x^2} dx$

37. Solve: $\frac{dy}{dx}+2y\cot x=3x^2\operatorname{cosec}^2 x$.

38. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win 15 for each red ball selected and we lose ₹ 10 for each black ball selected. If X denotes the winning amount, find the values of X and number of points in its inverse images.

39. Check whether the statement $p \rightarrow (q \rightarrow p)$ is a tautology or a contradiction without using the truth table.

40. Show that the straight lines $\vec{r}=(5\hat{\imath}+7\hat{\jmath}-3\hat{k})+s(4\hat{\imath}+4\hat{\jmath}-5\hat{k})$ and $\vec{r}=(8\hat{\imath}+4\hat{\jmath}+5\hat{k})+t(7\hat{\imath}+\hat{\jmath}+3\hat{k})$ are coplanar. Find the vector equation of the plane in which they lie.

PART-IV

Answer the following questions. **$(7\times 5=35)$**

41. a) Investigate for what values of λ and μ the system of linear equations $x+2y+z=7,\ x+y+\lambda z=\mu,\ x+3y-5z=5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(OR)**

b) Find the cube roots of unity.

42. a) Form the equation whose roots are the squares of the roots of the cubic equation $x^3 + ax^2 + bx + c = 0$.
(OR)

b) Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

43. a) Write the Maclaurin series expansion $\tan^{-1}(x); -1 \le x \le 1$ **(OR)**

b) At a water fountain, water attains a maximum height of 4 m at horizontal distance of 0.5 m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 m from the point of origin.

44. a) Prove that $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)dx = \frac{\pi}{8}\log 2$. **(OR)**

b) Let $U(x, y, z) = xyz, x = e^{-t}, y = e^{-t}\cos t, z = \sin t, t \in \mathbb{R}$. Find $\frac{dU}{dt}$.

45. a) Solve $\left(1 + 2e^{x/y}\right)dx + 2e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$.**(OR)**

b) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall,
(i) how fast is the top of the ladder moving down the wall?
(ii) at what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

46. a) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0), (2,2-1)$ and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.**(OR)**

b) On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denotes the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective.

47. a) Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years?**(OR)**

b) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the commutative and associative properties satisfied by the existence of identity, existence of inverse properties for the operation $*$ on M.

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