CENTUM ACHIEVERS' ACADEMY

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XII STD(MATHS)	FULL PORTION – 4	TIME : 2 ½ Hrs MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : (20× 1 = 20)

1. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are

   (1) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (2) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

   (3) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (4)) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

2. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

   (1) 2 (2) 4 (3) 3 (4) 1

3. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

   (1) cis $\frac{2\pi}{3}$ (2) cis $\frac{4\pi}{3}$ (3) $-$cis $\frac{2\pi}{3}$ (4) $-$cis $\frac{4\pi}{3}$

4. The principal argument of $\frac{3}{-1+i}$ is

   (1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

5. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

   (1) $a \geq 0$ (2) $a > 0$ (3) $a < 0$ (4) $a \leq 0$

6. If $x = \frac{1}{5}$, the value of $\cos(\cos^{-1} x + 2\sin^{-1} x)$ is

   (1) $-\sqrt{\frac{24}{25}}$ (2) $\sqrt{\frac{24}{25}}$ (3) $\frac{1}{5}$ (4) $-\frac{1}{5}$

7. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1} x$ is

   (1) $-\frac{\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $-\frac{\pi}{5}$

8. The circle passing through $(1,-2)$ and touching the axis of x at $(3,0)$ passing through the point

   (1) $(-5,2)$ (2) $(2,-5)$ (3) $(5,-2)$ (4) $(-2,5)$

9. The length of the diameter of the circle which touches the x-axis at the point $(1,0)$ and passes through the point $(2,3)$.

   (1) $\frac{6}{5}$ (2) $\frac{5}{3}$ (3) $\frac{10}{3}$ (4) $\frac{3}{5}$

10. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then

(1) $c = \pm 3$ (2) $c = \pm\sqrt{3}$ (3) $c > 0$ (4) $0 < c < 1$

11. If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$, where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{b} \cdot \vec{c} \neq 0$ and $\vec{a} \cdot \vec{b} \neq 0$, then $\vec{a}$ and $\vec{c}$ are

(1) perpendicular (2) parallel (3) inclined at angle $\frac{\pi}{3}$ (4) inclined at angle $\frac{\pi}{6}$

12. The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is -0.25 ?

(1) -8 (2) -4 (3) -2 (4) 0

13. The maximum value of the product of two positive numbers, when their sum of the squares is 200 , is

(1) 100 (2) $25\sqrt{7}$ (3) 28 (4) $24\sqrt{14}$

14. The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{2x^7 - 3x^5 + 7x^3 - x + 1}{\cos^2 x}\right) dx$ is

(1) 4 (2) 3 (3) 2 (4) 0

15. If $u(x, y) = x^2 + 3xy + y - 2019$, then $\left.\frac{\partial u}{\partial x}\right|_{(4,-5)}$ is equal to

(1) -4 (2) -3 (3) -7 (4) 13

16. For any value of $n \in \mathbb{Z}$, $\int_0^{\pi} e^{\cos^2 x} \cos^3 [(2n+1)x] dx$ is

(1) $\frac{\pi}{2}$ (2) π (3) 0 (4) 2

17. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is

(1) $x\phi\left(\frac{y}{x}\right) = k$ (2) $\phi\left(\frac{y}{x}\right) = kx$ (3) $y\phi\left(\frac{y}{x}\right) = k$ (4) $\phi\left(\frac{y}{x}\right) = ky$

18. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of a is

(1) 2 (2) -2 (3) 1 (4) -1

19. On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

(1) $\frac{11}{243}$ (2) $\frac{3}{8}$ (3) $\frac{1}{243}$ (4) $\frac{5}{243}$

20. If a compound statement involves 3 simple statements, then the number of rows in the truth table is

(1) 9 (2) 8 (3) 6 (4) 3

PART-II

(i) Answer any SEVEN questions. **$(7\times 2 = 14)$**

(ii) Qn.No.30 is compulsory

21. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

22. Find $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$

23. Prove that a straight line and parabola cannot intersect at more than two points.

24. Evaluate the limit $\lim_{x\to 0}\left(\frac{\sin x}{x^2}\right)$

25. Let $V(x,y,z) = xy + yz + zx,\ x,y,z \in \mathbb{R}$. Find the differential dV.

26. Evaluate $\int_0^{2\pi} \sin^7\frac{x}{4}\,dx$

27. Show that $y = a\cos b\,x$ is a solution of the differential equation $\frac{d^2y}{dx^2} + b^2y = 0$.

28. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function $f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$ Find the expected life of this electronic equipment.

29. Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation $+$ on $\mathbb{Z}$.

30. Find the centre and radius of the circle $3x^2 + (a+1)y^2 + 6x - 9y + a + 4 = 0$.

PART-III

(i) Answer any SEVEN questions. **$(7\times 3 = 21)$**

(ii) Qn.No.40 is compulsory

31. Solve $x + 2y + 3z = 0,\ 3x + 4y + 4z = 0,\ 7x + 10y + 12z = 0$.

32. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x, |x| > 1$.

33. Prove that the point of intersection of the tangents at ' t_1 ' and ' t_2 ' on the parabola $y^2 = 4ax$ is $[at_1t_2, a(t_1+t_2)]$.

34. Find the image of the point whose position vector is $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ in the plane $\vec{r}\cdot(\hat{\imath} + 2\hat{\jmath} + 4\hat{k}) = 38.$,

35. A Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally.

36. Let $u(x,y) = e^{-2y}\cos(2x)$ for all $(x,y) \in \mathbb{R}^2$. Prove that u is a harmonic function in $\mathbb{R}^2$.

37. Solve the Linear differential equations: $x\frac{dy}{dx} + y = x\log x$

38. Find the probability mass function and cumulative distribution function of number of girl child in families with 4 children, assuming equal probabilities for boys and girls.

39. Verify (i) closure property (ii) commutative property, and (iii) associative property of the following operation on the given set. $(a * b) = a^b; \forall a, b \in \mathbb{N}$ (exponentiation property)

40. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.

PART-IV

Answer the following questions. **$(7\times 5 = 35)$**

41. a) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8), (-2,-12)$, and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.) **(OR)**

 b) Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots.

42. a) Identify the type of conic and find centre, foci, vertices, and directrices
 $18x^2 + 12y^2 - 144x + 48y + 120 = 0$ **(OR)**

 b) Determine the values of λ for which the following system of equations
 $(3\lambda - 8)x + 3y + 3z = 0, 3x + (3\lambda - 8)y + 3z = 0, 3x + 3y + (3\lambda - 8)z = 0$

43. a) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$. **(OR)**

 b) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that
 $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \cdots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n-a_1}{1+a_1a_n}$.

44. a) Solve the differential equations: $\frac{dy}{dx} = \tan^2(x+y)$ **(OR)**

 b) Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r} = (\hat{\imath} - \hat{\jmath} + 3\hat{k}) + t(2\hat{\imath} - \hat{\jmath} + 4\hat{k})$ and perpendicular to plane $\vec{r}\cdot(\hat{\imath} + 2\hat{\jmath} + \hat{k}) = 8$.

45. a) Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$. **(OR)**

 b) A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) the probability of getting at least one correct answer.

46. a) The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. find the following in calculating the area of the circular plate:
 (i) Absolute error (ii) Relative error (iii) Percentage error **(OR)**

 b) Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

47. a) Let A be $\mathbb{Q}\setminus\{1\}$.Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on ? If so, examine the commutative associative and the existence of identity, existence of inverse properties for the operation $*$ on A**(OR)**

 b) For the function $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right)$ show that $f_{xy} = f_{yx}$.

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