# CENTUM ACHIEVERS' ACADEMY

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| XII STD(MATHS) | FULL PORTION –6 | TIME : 2 ½ Hrs |
|---|---|---|
| | | MARKS : 90 |

## PART-I

**Choose the correct answer from the given four alternatives :** **(20× 1 = 20)**

1. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

   (1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1

2. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If $B$ is the inverse of $A$, then the value of $x$ is

   (1) 2 (2) 4 (3) 3 (4) 1

3. If $z = x + iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of $z$ is

   (1) real axis (2) imaginary axis (3) ellipse (4) circle

4. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

   (1) cis $\frac{2\pi}{3}$ (2) cis $\frac{4\pi}{3}$ (3) $-$cis $\frac{2\pi}{3}$ (4) $-$cis $\frac{4\pi}{3}$

5. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

   (1) $a \geq 0$ (2) $a > 0$ (3) $a < 0$ (4) $a \leq 0$

6. If $\cot^{-1} 2$ and $\cot^{-1} 3$ are two angles of a triangle, then the third angle is

   (1) $\frac{\pi}{4}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

7. If $\sin^{-1}\frac{x}{5} + \text{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then the value of $x$ is

   (1) 4 (2) 5 (3) 2 (4) 3

8. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of $r^2$ is

   (1) 2 (2) 3 (3) 1 (4) 4

9. The locus of a point whose distance from $(-2,0)$ is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is

   (1) a parabola (2) a hyperbola (3) an ellipse (4) a circle

10. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath}$, $\vec{c} = \hat{\imath}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value of $\lambda + \mu$ is

    (1) 0 (2) 1 (3) 6 (4) 3

11. If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1, \lambda > 0$ is $\frac{1}{5}$, then the value of $\lambda$ is

(1) $2\sqrt{3}$ (2) $3\sqrt{2}$ (3) 0 (4) 1

12. The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is $-0.25$ ?

(1) $-8$ (2) $-4$ (3) $-2$ (4) 0

13. The maximum value of the product of two positive numbers, when their sum of the squares is 200 , is

(1) 100 (2) $25\sqrt{7}$ (3) 28 (4) $24\sqrt{14}$

14. If $w(x, y, z) = x^2(y - z) + y^2(z - x) + z^2(x - y)$, then $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$ is

(1) $xy + yz + zx$ (2) $x(y + z)$ (3) $y(z + x)$ (4) 0

15. The volume of solid of revolution of the region bounded by $y^2 = x(a - x)$ about x-axis is

(1) $\pi a^3$ (2) $\frac{\pi a^3}{4}$ (3) $\frac{\pi a^3}{5}$ (4) $\frac{\pi a^3}{6}$

16. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the value of $f(1)$ is

(1) $\frac{1}{2}$ (2) 2 (3) 1 (4) $\frac{3}{4}$

17. If sin $x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then $P$ is

(1) log sin $x$ (2) cos $x$ (3) tan $x$ (4) cot $x$

18. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of $a$ is

(1) 2 (2) $-2$ (3) 1 (4) $-1$

19. If $P(X = 0) = 1 - P(X = 1)$. If $E(X) = 3\text{Var}(X)$, then $P(X = 0)$ is

(1) $\frac{2}{3}$ (2) $\frac{2}{5}$ (3) $\frac{1}{5}$ (4) $\frac{1}{3}$

20. In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value ' $F$ ' are

(1) 1 (2) 2 (3) 3 (4) 4

## PART-II

**(i) Answer any SEVEN questions.** **$(7 \times 2 = 14)$**

**(ii) Qn.No.30 is compulsory**

21. Find the rank of the matrix by minor method $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

22. Obtain the Cartesian form of the locus of : $|2z - 3 - i| = 3$

23. Solve the equation: $x^4 - 14x^2 + 45 = 0$.

24. Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

25. Find the equation of the tangent at $t = 2$ to the parabola $y^2 = 8x$. (Hint: use parametric form.

26. Prove that the function $f(x) = x^2 + 2$ is strictly increasing in the interval (2,7) and strictly decreasing in the interval $(-2,0)$.

27. Evaluate $\int_0^1 x^3(1-x)^4 dx$

28. The mean and variance of a binomial variate $X$ are respectively 2 and 1.5. Find $P(X=0)$

29. Show that $p \to q$ and $q \to p$ are not equivalent

30. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$.

## PART-III

**(i) Answer any SEVEN questions.** **(7× 3 = 21)**

**(ii) Qn.No.40 is compulsory**

31. If $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\cdots(x_n+iy_n) = a+ib$, show that

(i) $(x_1^2+y_1^2)(x_2^2+y_2{}^2)(x_3^2+y_3^2)\cdots(x_n{}^2+y_n^2) = a^2+b^2$

(ii) $\sum_{r=1}^{n}\tan^{-1}\left(\frac{y_r}{x_r}\right) = \tan^{-1}\left(\frac{b}{a}\right) + 2k\pi, k \in \mathbb{Z}$.

32. Solve : $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

33. Find the value of $\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$

34. The maximum and minimum distances of the Earth from the Sun respectively are $152 \times 10^6$ km and $94.5 \times 10^6$km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

35. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot(2\hat{\imath}-7\hat{\jmath}+4\hat{k}) = 3$ and $3x-5y+4z+11=0$, and the point $(-2,1,3)$.

36. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

37. Show that $y = ax + \frac{b}{x}, x \neq 0$ is a solution of the differential equation $x^2y'' + xy' - y = 0$.

38. If $\mu$ and $\sigma^2$ are the mean and variance of the discrete random variable $X$, and $E(X+3) = 10$ and $E(X+3)^2 = 116$, find $\mu$ and $\sigma^2$.

39. Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

40. Find the volume of a right-circular cone of base radius $r$ and height

## PART-IV

**Answer the following questions.** **(7× 5 = 35)**

41. a) Find the condition on $a, b$ and $c$ so that the following system of linear equations has one parameter family of solutions: $x+y+z=a,\ x+2y+3z=b,\ 3x+5y+7z=c$. **(OR)**

b) Find the value of $\cos\left(\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right)$.

42. a) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution **(OR)**

b) If $z_1, z_2$, and $z_3$ are three complex numbers such that $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|z_1+z_2+z_3| = 1$, show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$.

43. a) Find centre, foci, vertices, and directrices of the conic $9x^2 - y^2 - 36x - 6y + 18 = 0$ **(OR)**

b) Find the number of solutions of the equation $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}(3x)$.

44. a) Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where $c, r$ are constants, cut orthogonally. **(OR)**

b) Find the non-parametric form and cartesian equation of the plane passing through the point $(1, -2, 4)$ and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$.

45. a) If $w(x, y) = 6x^3 - 3xy + 2y^2, x = e^s, y = \cos s, s \in \mathbb{R}$, find $\frac{dw}{ds}$, and evaluate at $s = 0$, **(OR)**

b) Father of a family wishes to divide his square field bounded by $x = 0, x = 4, y = 4$ and $y = 0$ along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them.

46. a) For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.**(OR)**

b) Solve the differential equation $(y^2 - 2xy)dx = (x^2 - 2xy)dy$

47. a) Verify (i) closure property (ii) commutative property, and (iii) associative property of the following operation on the given set. $(a * b) = a^b; \forall a, b \in \mathbb{N}$ **(OR)**

b) Suppose that $f(x)$ given below represents a probability mass function,

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$ | $c^2$ | $2c^2$ | $3c^2$ | $4c^2$ | $c$ | $2c$ |

Find (i) the value of c (ii) Mean and variance.

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