CENTUM ACHIEVERS' ACADEMY

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XII STD(MATHS)	FULL PORTION – 5	TIME : 2 ½ Hrs MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : **(20× 1 = 20)**

1. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

   (1) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (2) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

2. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

   (1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1

3. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

   (1) (1,0) (2) (−1,1) (3) (0,1) (4) (1,1)

4. The principal argument of $(\sin 40° + i\cos 40°)^5$ is

   (1) −110° (2) −70° (3) 70° (4) 110°

5. A zero of $x^3 + 64$ is

   (1) 0 (2) 4 (3) $4i$ (4) -4

6. f the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belongs to

   (1) $[-1,1]$ (2) $[\sqrt{2}, 2]$ (3) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ (4) $[-2, -\sqrt{2}]$

7. The equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

   (1) no solution (2) unique solution (3) two solutions (4) infinite number of solutions

8. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is

   (1) $4(a^2 + b^2)$ (2) $2(a^2 + b^2)$ (3) $a^2 + b^2$ (4) $\frac{1}{2}(a^2 + b^2)$

9. An ellipse has OB as semi minor axes, F and F' its foct and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

   (1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{\sqrt{3}}$

10. The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a+b)x - 4 = 0$, then the value of $(a+b)$ is

    (1) 2 (2) 4 (3) 0 (4) −2

11. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath}$, $\vec{c} = \hat{\imath}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value of $\lambda + \mu$ is

(1) 0 (2) 1 (3) 6 (4) 3

12. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$, then (α, β) is

(1) $(-5,5)$ (2) $(-6,7)$ (3) $(5,-5)$ (4) $(6,-7)$

13. The value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

(1) 0 (2) 1 (3) 2 (4) ∞

14. The curve $y = ax^4 + bx^2$ with $ab > 0$

(1) has no horizontal tangent (2) is concave up

(3) is concave down (4) has no points of inflection

15. The change in the surface area $S = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$ is

(1) $12x_0 + dx$ (2) $12x_0 dx$ (3) $6x_0 dx$ (4) $6x_0 + dx$

16. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

(1) 10 (2) 5 (3) 8 (4) 9

17. The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through $(-1,1)$. Then the equation of the curve is

(1) $y = x^3 + 2$ (2) $y = 3x^2 + 4$ (3) $y = 3x^3 + 4$ (4) $y = x^3 + 5$

18. A rod of length $2l$ is broken into two pieces at random. The probability density function of the shorter of the two pieces is $f(x) = \begin{cases} \frac{1}{l} & 0 < x < l \\ 0 & l \le x < 2l \end{cases}$. The mean and variance of the shorter of the two pieces are

(1) $\frac{l}{2}, \frac{l^2}{3}$ (2) $\frac{l}{2}, \frac{l^2}{6}$ (3) $l, \frac{l^2}{12}$ (4) $\frac{l}{2}, \frac{l^2}{12}$

19. Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with Probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of $E(X)$ is

(1) 0.11 (2) 1.1 (3) 11 (4) 1

20. The dual of $\neg(p \vee q) \vee [p \vee (p \wedge \neg r)]$ is

(1) $\neg(p \wedge q) \wedge [p \vee (p \wedge \neg r)]$ (2) $(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

(3) $\neg(p \wedge q) \wedge [p \wedge (p \wedge r)]$ (4) $\neg(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

PART-II

(i) **Answer any SEVEN questions.** $(7\times 2 = 14)$

(ii) **Qn.No.30 is compulsory**

21. Solve by Cramer's rule: $\frac{3}{x} + 2y = 12,\ \frac{2}{x} + 3y = 13$

22. Which one of the points $10 - 8i, 11 + 6i$ is closest to $1 + i$.

23. If α, β, and γ are the roots of the polynomial equation $ax^3 + bx^2 + cx + d = 0$, find the value of $\sum \frac{\alpha}{\beta\gamma}$ in terms of the coefficients.

24. Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$.

25. Prove that $[\vec{a} - \vec{b},\ \vec{b} - \vec{c},\ \vec{c} - \vec{a}] = 0$.

26. Use the linear approximation to find approximate values of $\sqrt[4]{15}$

27. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x \, dx$.

28. Show that $y = mx + \frac{7}{m}, m \neq 0$ is a solution of the differential equation $xy' + 7\frac{1}{y'} - y = 0$.

29. Write the converse, inverse, and contrapositive of the following implication.
    If x and y are numbers such that $x = y$, then $x^2 = y^2$

30. Find the asymptotes of $f(x) = \frac{x^2+6x-4}{3x-6}$

PART-III

(i) **Answer any SEVEN questions.** $(7\times 3 = 21)$

(ii) **Qn.No.40 is compulsory**

31. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

32. Find the square roots of $-5 - 12i$.

33. Solve the cubic equation : $2x^3 - x^2 - 18x + 9 = 0$. if sum of two of its roots vanishes

34. Simplify $\sin^{-1}[\sin\ 10]$

35. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval: $f(x) = x^3 - 3x + 2, x \in [-2,2]$

36. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.

37. Evaluate $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$

38. Solve the differential equations $\frac{dy}{dx} = \tan^2(x + y)$

39. Define an operation $*$ on $\mathbb{Q}$ as follows: $a * b = \left(\frac{a+b}{2}\right)$; $a, b \in \mathbb{Q}$. Examine the closure, commutative, and associative properties satisfied by $*$ on $\mathbb{Q}$.

40. An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.

PART-IV

Answer the following questions. **$(7\times 5 = 35)$**

41. a) Find the value of k for which the equations $kx - 2y + z = 1,\ x - 2ky + z = -2,\ x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution **(OR)**

b) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$.

42. a) Solve: $(2x-1)(x+3)(x-2)(2x+3) + 20 = 0$ **(OR)**

b) Find the centre, foci, and eccentricity of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$

43. a) Solve $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$. **(OR)**

b) If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$

44. a) Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$. **(OR)**

b) For function $f(x,y) = \frac{3x}{y+\sin x}$ show that $f_{xy} = f_{yx}$

45. a) Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane $x + 2y + 3z = 2$. **(OR)**

b) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?

46. a) Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance. **(OR)**

b) Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally.

47. a) Solve $(x^2 + y^2)dy = xy\,dx$. It is given that $y(1) = 1$ and $y(x_0) = e$. Find the value of x_0. **(OR)**

b) Using the equivalence property, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.

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