CENTUM ACHIEVERS' ACADEMY 56,KASTHURI BAI 4TH STREET,GANAPATHY, CBE-06.PH.NO.7667761819		
XII STD(MATHS)	FULL PORTION –7	TIME : 2 ½ Hrs MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : (20× 1 = 20)

1. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = \text{adj } A$ and $C = 3A$, then $\frac{|\text{adj } B|}{|C|} =$

   (1) $\frac{1}{3}$ (2) $\frac{1}{9}$ (3) $\frac{1}{4}$ (4) 1

2. If $A^T A^{-1}$ is symmetric, then $A^2 =$

   (1) A^{-1} (2) $(A^T)^2$ (3) A^τ (4) $(A^{-1})^2$

3. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

   (1) (1,0) (2) (−1,1) (3) (0,1) (4) (1,1)

4. If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to

   (1) 1 (2) −1 (3) $\sqrt{3}i$ (4) $-\sqrt{3}i$

5. The number of positive zeros of the polynomial $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$ is

   (1) 0 (2) n (3) $< n$ (4) r

6. If the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belongs to

   (1) $[-1,1]$ (2) $[\sqrt{2}, 2]$ (3) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ (4) $[-2, -\sqrt{2}]$

7. $\sin^{-1}\left(\tan \frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$. Then x is a root of the equation

   (1) $x^2 - x - 6 = 0$ (2) $x^2 - x - 12 = 0$ (3) $x^2 + x - 12 = 0$ (4) $x^2 + x - 6 = 0$

8. The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3).

   (1) $\frac{6}{5}$ (2) $\frac{5}{3}$ (3) $\frac{10}{3}$ (4) $\frac{3}{5}$

9. The angle between the line $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) + t(2\hat{\imath} + \hat{\jmath} - 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{\imath} + \hat{\jmath}) + 4 = 0$ is

   (1) 0° (2) 30° (3) 45° (4) 90°

10. If the planes $\vec{r} \cdot (2\hat{\imath} - \lambda\hat{\jmath} + \hat{k}) = 3$ and $\vec{r} \cdot (4\hat{\imath} + \hat{\jmath} - \mu\hat{k}) = 5$ are parallel, then the value of λ and μ are

    (1) $\frac{1}{2}, -2$ (2) $-\frac{1}{2}, 2$ (3) $-\frac{1}{2}, -2$ (4) $\frac{1}{2}, 2$

11. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

(1) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (2) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (3) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (4) $\left[0, \frac{\pi}{4}\right]$

12. The maximum slope of the tangent to the curve $y = e^x \sin x, x \in [0, 2\pi]$ is at

(1) $x = \frac{\pi}{4}$ (2) $x = \frac{\pi}{2}$ (3) $x = \pi$ (4) $x = \frac{3\pi}{2}$

13. If $f(x) = \frac{x}{x+1}$, then its differential is given by

(1) $\frac{-1}{(x+1)^2} dx$ (2) $\frac{1}{(x+1)^2} dx$ (3) $\frac{1}{x+1} dx$ (4) $\frac{-1}{x+1} dx$

14. If $u(x, y) = x^2 + 3xy + y - 2019$, then $\left.\frac{\partial u}{\partial x}\right|_{(4,-5)}$ is equal to

(1) -4 (2) -3 (3) -7 (4) 13

15. If $f(x) = \int_0^x t \cos t \, dt$, then $\frac{df}{dx} =$

(1) $\cos x - x \sin x$ (2) $\sin x + x \cos x$ (3) $x \cos x$ (4) $x \sin x$

16. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

(1) 10 (2) 5 (3) 8 (4) 9

17. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{\lambda}$ is

(1) $\frac{x}{e^{\lambda}}$ (2) $\frac{e^{\lambda}}{x}$ (3) λe^x (4) e^x

18. The solution of $\frac{dy}{dx} = 2^{y-x}$ is

(1) $2^x + 2^y = C$ (2) $2^x - 2^y = C$ (3) $\frac{1}{2^x} - \frac{1}{2^y} = C$ (4) $x + y = C$

19. Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X - 3)$ is

(1) 0.24 (2) 0.48 (3) 0.6 (4) 0.96

20. Which one is the contrapositive of the statement $(p \vee q) \rightarrow r$?

(1) $\neg r \rightarrow (\neg p \wedge \neg q)$ (2) $\neg r \rightarrow (p \vee q)$

(3) $r \rightarrow (p \wedge q)$ (4) $p \rightarrow (q \vee r)$

PART-II

(i) Answer any SEVEN questions. **($7 \times 2 = 14$)**

(ii) Qn.No.30 is compulsory

21. If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$.

22. Find the principal argument Arg z, when $z = \frac{-2}{1+i\sqrt{3}}$.

23. Examine for the rational roots of $x^8 - 3x + 1 = 0$

24. Find the value, if it exists. If not, give the reason for non-existence $\sin^{-1}[\sin 5]$.

25. Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to $2x + 2y + 3 = 0$.

26. If the straight lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{m^2}$ and $\frac{x-3}{1}=\frac{y-2}{m^2}=\frac{z-1}{2}$ are coplanar, find the distinct real values of m.

27. A particle moves so that the distance moved is according to the law $s(t)=\frac{t^3}{3}-t^2+3$. At what time the velocity and acceleration are zero.

28. If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

29. Find the volume of the solid formed by revolving the region bounded by the parabola $y=x^2$, x-axis, ordinates $x=0$ and $x=1$ about the x-axis.

30. Form the differential equation by eliminating the arbitrary constants A and B from $y=A\cos x+B\sin x$.

PART-III

(i) Answer any SEVEN questions. **$(7\times 3=21)$**

(ii) Qn.No.40 is compulsory

31. If $z=(\cos\theta+i\sin\theta)$, show that $z^n+\frac{1}{z^n}=2\cos n\theta$ and $z^n-\frac{1}{z^n}=2i\sin n\theta$..

32. Show that the equation $x^9-5x^5+4x^4+2x^2+1=0$ has atleast 6 imaginary solutions.

33. Prove that $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

34. The line $3x+4y-12=0$ meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter.

35. Expand $\sin x$ in ascending powers $x-\frac{\pi}{4}$ upto three non-zero terms.

36. A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10 cm. Use the differentials to find approximately how many cubic centimeters of paint is used to paint this cube. Also calculate the exact amount of paint used to paint this cube.

37. Evaluate the integrals as the limits of sums $\int_0^1(5x+4)dx$

38. Find the differential equation of the family of all ellipses having foci on the x-axis and centre at the origin.

39. If X is the random variable with distribution function $F(x)$ given by,

$$F(x)=\begin{cases}0, & -\infty<x<0\\ \frac{1}{2}(x^2+x) & 0\le x<1\\ 1, & 1\le x<\infty\end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.3\le X\le 0.6)$

40. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$.

PART-IV

Answer the following questions. **$(7\times 5=35)$**

41. a) If the system of equations $px+by+cz=0, ax+qy+cz=0, ax+by+rz=0$ has a non-trivial solution and $p\neq a, q\neq b, r\neq c$, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$. **(OR)**

b) Simplify (i) $(1+i)^{18}$ (ii) $(-\sqrt{3}+3i)^{31}$.

42. a) If the roots of $x^3 + px^2 + qx + r = 0$ are in H.P., prove that $9pqr = 27r^2 + 2q^3$.Assume $p, q, r \neq 0$ **(OR)**

b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

43. a) Find the value of(i) $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ (ii) $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$. **(OR)**

b) Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

44. a) Find the equation of the circle passing through the points $(1,1), (2,-1)$, and $(3,2)$. **(OR)**

b) Solve the Linear differential equations $(1 + x + xy^2)\frac{dy}{dx} + (y + y^3) = 0$

45. a) Find the area of the region bounded by the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$, **(OR)**

b) Find the parametric form of vector equation of the straight line passing through $(-1,2,1)$ and parallel to the straight line $\vec{r} = (2\hat{\imath} + 3\hat{\jmath} - \hat{k}) + t(\hat{\imath} - 2\hat{\jmath} + \hat{k})$ and hence find the shortest distance between the lines.

46. a) If $V(x,y) = e^x(x\cos y - y\sin y)$, then prove that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$.**(OR)**

b) The probability density function of X is given by $f(x) = \begin{cases} ke^{-\frac{x}{3}} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$

Find (i) the value of k (ii) the distribution function (iii) $P(X < 3)$
(iv) $P(5 \leq X)$ (v) $P(X \leq 4)$.

47. a) Prove that among all the rectangles of the given area square has the least perimeter. **(OR)**

b) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set.

$m * n = m + n - mn; m, n \in \mathbb{Z}$

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