www.Padasalai.Net - Public Exam 2023 - Model Question Paper

STD: XII **REG NUMBER :** **TIME: 3Hrs**

DATE: 03.02.2023 **MATHEMATICS** **MARKS: 90**

PART - I **20 X 1 = 20**

NOTE: (i) Answer all the questions.(ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer :

1. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|\text{adj}(AB)| =$

   1) – 40 2) –80 3) –60 4) –20

2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

   1) 17 2) 14 3) 19 4) 21

3. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

   1) $\frac{1}{2}$ 2) 1 3) 2 4) 3

4. The principal value of arg z lies in the interval

   1) $[0, \pi/2]$ 2) $(-\pi, \pi]$ 3) $[0, \pi]$ 4) $(-\pi, 0]$

5. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

   1) $a \geq 0$ 2) $a > 0$ 3) $a < 0$ 4) $a \leq 0$

6. $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for

   1) $-\pi \leq x \leq 0$ 2) $0 \leq x \leq \pi$ 3) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ 4) $-\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$

7. If $\cot^{-1} 2$ and $\cot^{-1} 3$ are two angles of a triangle, then the third angle is

   1) $\frac{\pi}{4}$ 2) $\frac{3\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{3}$

8. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

   1) 1 2) 3 3) $\sqrt{10}$ 4) $\sqrt{11}$

9. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

   1) $2ab$ 2) ab 3) $\sqrt{ab}$ 4) $\frac{a}{b}$

10. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

    1) $c = \pm 3$ 2) $c = \pm \sqrt{3}$ 3) $c > 0$ 4) $0 < c < 1$

11. If $m\hat{i} + 2\hat{j} + \hat{k}$ and $4\hat{i} - 9\hat{j} + 2\hat{k}$ are perpendicular then, m is

    1) -4 2) 8 3) 4 4) 12

12. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1,9]$ is

(1) 2 (2) 2.5 (3) 3 (4) 3.5

13. Equation of the tangent to the curve $y = x^3$ at (1,1) is

1) $y = 2x - 3$ 2) $y = 2x + 3$ 3) $y = 3x + 2$ 4) $y = 3x - 2$

14. If $v(x, y) = \log\left(e^x + e^y\right)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

1) $e^x + e^y$ 2) $\frac{1}{e^x + e^y}$ 3) 2 4) 1

15. The value of $\int_0^{\pi} \sin^4 x\, dx$ is

1) $\frac{3\pi}{10}$ 2) $\frac{3\pi}{8}$ 3) $\frac{3\pi}{4}$ 4) $\frac{3\pi}{2}$

16. $\int_0^a \sqrt{a^2 - x^2}\, dx =$

1) πa^2 2) $\frac{\pi a^2}{2}$ 3) $2\pi a$ 4) $\frac{\pi a^2}{4}$

17. If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is

1) $\log \sin x$ 2) $\cos x$ 3) $\tan x$ 4) $\cot x$

18. The degree of the differential equation $\sqrt{1 + \left(\frac{dy}{dx}\right)^{1/3}} = \frac{d^2y}{dx^2}$

1) 1 2) 2 3) 3 4) 6

19. If X is a binomial random variable with expected value 6 and variance 2.4, Then $P\{X = 5\}$ is

1) $\binom{10}{5}\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$ 2) $\binom{10}{5}\left(\frac{3}{5}\right)^5$ 3) $\binom{10}{5}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^6$ 4) $\binom{10}{5}\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$

20. The proposition $p \wedge (\neg p \vee q)$ is

(1) a tautology (2) a contradiction

(3) logically equivalent to $p \wedge q$ (4) logically equivalent to $p \vee q$

PART - II 7 X 2 = 14

Answer any seven of the following questions. Q.No.30 is compulsory.

21. If $adj\, A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

22. If $|z| = 1$, show that $2 \le |z^2 - 3| \le 4$.

23. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

24. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.

25. Find centre and radius of the circle $x^2 + (y+2)^2 = 0$.

26. Evaluate: $\lim_{x\to\infty} e^{-x}\sqrt{x}$.

27. Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by $y = 2x^2$, $y = 0$ and $x = 1$.

28. Solve : $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

29. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function $f(x) = \begin{cases} 3e^{-3x} & for\ x > 0 \\ 0 & elsewhere \end{cases}$. Find the expected life of this electronic equipment.

30. Find the angle between the line $\vec{r} = (\hat{i} + \hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} - \hat{k})$ and the plane $\vec{r}.(\hat{i} + \hat{j}) = 1$.

PART - III 7 X 3 = 21

Answer any seven of the following questions. Q.No.40 is compulsory.

31. Solve the system of linear equations by matrix inversion method: $2x - y = 8,\ 3x + 2y = -2$.

32. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ show that
(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$ and (ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$.

33. Identify the type of conic and find centre, foci, vertices, and directrices of $\frac{x^2}{3} + \frac{y^2}{10} = 1$.

34. Prove that $\left[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}\right] = \left[\vec{a}, \vec{b}, \vec{c}\right]^2$.

35. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.

36. Evaluate: $\int_0^{\frac{1}{\sqrt{2}}} \frac{e^{\sin^{-1}x}\sin^{-1}x}{\sqrt{1-x^2}}\,dx$.

37. Solve : $\frac{dy}{dx} = \frac{\sin^2 x}{1+x^3} - \frac{3x^2}{1+x^3}y$.

38. Show that $p \to q$ and $q \to p$ are not equivalent.

39. Find the mean and variance of a random variable X, whose probability density function is
$f(x) = \begin{cases} \lambda e^{-\lambda x} & for\ x \geq 0 \\ 0 & otherwise \end{cases}$

40. Find the intervals of concavity and the point of inflection of the function $f(x) = sin2x$ in $(0, \pi)$.

PART - IV 7 X 5 = 35

Answer all the questions.

41. *a*) Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9,\ 7x + 3y - 5z = 8,\ 2x + 3y + \lambda z = \mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

42. *a*) Let $z_1, z_2,$ and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$.

Prove that $\left|\frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3}\right| = r$.

(OR)

b) If $a_1, a_2, a_3, ...a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+...+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}.$$

43. *a*) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4*m* when it is 6*m* away from the point of projection. Finally it reaches the ground 12*m* away from the starting point. Find the angle of projection.

(OR)

b) Find the vector and Cartesian equation of the plane containing the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3}$ and $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z+1}{1}$.

44. *a*) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$.

(OR)

b) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

45. *a*) Father of a family wishes to divide his square field bounded by $x = 0, x = 4, y = 4$ and $y = 0$ along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them.

(OR)

b) Water at temperature $100^\circ C$ cools in 10 minutes to $80^\circ C$ in a room temperature of $25^\circ C$. Find
(i) The temperature of water after 20 minutes
(ii) The time when the temperature is $40^\circ C$ $\left[\log_e \frac{11}{15} = -0.3101;\ \log_e 5 = 1.6094\right]$

46. *a*) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function $f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & otherwise \end{cases}$

Find (i) the value of k (ii) the distribution function (iii) the probability that daily sales will fall between 300 litres and 500 litres ?

(OR)

b) Verify (i) closure property, (ii) commutative property, (iii) associative property,(iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on Z_5 using table corresponding to addition modulo 5.

47. *a*) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

(OR)

b) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent**.**

Prepared by

A.KANNAN M.Sc,B.Ed., PG ASST. IN MATHS, KMHSS, MOHANUR, NAMAKKAL. Mob: 9894019808

XII MATHS FULL TEST - II ANSWER KEY FEB - 2023

ONE MARKS

1. ② -80
2. ③ 19
3. ② 1
4. ② $(-\pi, \pi]$
5. ③ $a<0$
6. ② $0 \le x \le \pi$
7. ② $\frac{3\pi}{4}$
8. ③ $\sqrt{10}$
9. ① $2ab$
10. ② $c = \pm\sqrt{3}$
11. ③ 4
12. ③ 3
13. ④ $y = 3x-2$
14. ④ 1
15. ② $\frac{3\pi}{8}$
16. ④ $\frac{\pi a^2}{4}$
17. ④ $\cot x$
18. ④ 6
19. ④ $\binom{10}{5}\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$
20. ③ Logically equivalent to $P \wedge Q$

TWO MARKS

21. $|adj A| = \begin{vmatrix} -1 & 2 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = 9$

$A^{-1} = \pm \frac{1}{\sqrt{|adj A|}} adj A$

$= \pm \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

22. $|z^2 - 3| \ge 2$

$|z^2 - 3| \le 4$

$2 \le |z^2 - 3| \le 4$

23. $S \cdot R = P + Q = \frac{-n}{l}$

$P \cdot R = PQ = \frac{n}{l^2}$

$\sqrt{\frac{P}{Q}} + \sqrt{\frac{Q}{P}} + \sqrt{\frac{n}{l}} =$

$= \frac{P+Q}{\sqrt{PQ}} + \sqrt{\frac{n}{l}}$

$= -\sqrt{\frac{n}{l}} + \sqrt{\frac{n}{l}} = 0$

24. $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

$= \sin^{-1}\sin\left(\frac{5\pi}{9} + \frac{\pi}{9}\right)$

$= \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

$= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$

$= \sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)$

$= \frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

25. Centre $= (0, -2)$

radius $= 0$

26. $\lim_{x\to\infty} e^{-x}\sqrt{x} = \lim_{x\to\infty} \frac{\sqrt{x}}{e^x}$ $\left(\frac{\infty}{\infty}\right)$

$= \lim_{x\to\infty} \frac{1}{2\sqrt{x}e^x}$

$= 0$

27. $y = x^2$, $x = 1$

$V = \pi \int_0^1 y^2 dx$

$= \pi \int_0^1 x^4 dx$

$= \frac{4\pi}{5}$ cu. units

28. $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1} y = \sin^{-1} x + c$

29. Mean $= E(X) = \int_{-\infty}^{\infty} x f(x) dx$

$= 3\int_0^{\infty} x e^{-3x} dx$

$= 3\left(\frac{1!}{3^2}\right)$

$= \frac{1}{3}$

30. $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{n} = \hat{i} + \hat{j}$

$\theta = \sin^{-1}\left(\frac{|\vec{b} \times \vec{n}|}{|\vec{b}||\vec{n}|}\right)$

$\theta = \sin^{-1}\left(\frac{3}{\sqrt{6}\sqrt{2}}\right)$

$\theta = \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

THREE MARKS

31. $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ -2 \end{bmatrix}$

$x = 2, y = -4$

32. $a+b+c = 0$ then $a^3 + b^3 + c^3 = 3abc$

$(\cos\alpha)^3 + (\cos\beta)^3 + (\cos\gamma)^3$

$= \frac{1}{4}[\cos(\alpha+\beta+\gamma) + 3\cos(\alpha+\beta+\gamma)]$

Equating

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$

$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$

33. $a^2 = 25, b^2 = 9, e = \frac{4}{5}$

$ae = 4, \frac{a}{e} = \frac{25}{4}$

Centre: $(h, k) = (0, 0)$

Foci: $(\pm ae, 0) = (\pm 4, 0)$

Vertices: $(\pm a, 0) = (\pm 5, 0)$

Directrices: $x = \pm\frac{a}{e} = \pm\frac{25}{4}$

34. $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]$

$= (\vec{a}\times\vec{b}) \cdot [(\vec{b}\times\vec{c}) \times (\vec{c}\times\vec{a})]$

$= (\vec{a}\times\vec{b}) \cdot ([\vec{a}, \vec{b}, \vec{c}]\vec{c})$

$= [\vec{a}, \vec{b}, \vec{c}][\vec{a}, \vec{b}, \vec{c}]$

$= [\vec{a}, \vec{b}, \vec{c}]^2$

35. $T = 2\pi\sqrt{\frac{l}{g}}$

$\log T = \log 2\pi + \frac{1}{2}[\log l - \log g]$

$\frac{dT}{T} = \frac{1}{2}\frac{dl}{l} = 0.01$

% error $= \frac{\Delta T}{T} \times 100$

$\approx \frac{dT}{T} \times 100$

$= 0.01 \times 100 = 1\%$

36. $I = \int_0^{\pi/2} e^{\sin^{-1}x} \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx$

$t = \sin^{-1}x \Rightarrow dt = \frac{1}{\sqrt{1-x^2}} dx$

$I = \int_0^{\pi/4} te^t dt$

$= [te^t - e^t]_0^{\pi/4}$

$= e^{\frac{\pi}{4}}\left(\frac{\pi}{4} - 1\right) + 1$

37. $\frac{dy}{dx} + \frac{3x^2 y}{1+x^3} = \frac{\sin^2 x}{1+x^3}$

$P = \frac{3x^2}{1+x^3}$, $Q = \frac{\sin^2 x}{1+x^3}$

$I.F = 1 + x^3$

$y(I.F) = \int Q(I.F) dx + c$

$y(1+x^3) = \int \sin^2 x dx + c$

$y(1+x^3) = \frac{x}{2} - \frac{\sin 2x}{4} + c$

38.

P	Q	P → Q	Q → P
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

$P \to Q \not\equiv Q \to P$

39. $E(X) = \lambda \int_0^{\infty} x e^{-\lambda x} dx$

$= \frac{1}{\lambda}$

$E(X^2) = \lambda \int_0^{\infty} x^2 e^{-\lambda x} dx$

$= \frac{2}{\lambda^2}$

$Var(X) = E(X^2) - [E(X)]^2$

$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$

$\therefore E(X) = \frac{1}{\lambda}$

$Var(X) = \frac{1}{\lambda^2}$

40. $f(x) = \sin 2x$

$f'(x) = 2\cos 2x$

$f''(x) = -4\sin 2x$

$f''(x) = 0$

$-4\sin 2x = 0$

$2x = 0, \pi, 2\pi, \ldots$

$x = \frac{\pi}{2} \in (0, \pi)$

Interval	$f'(x)$	Concavity
$(0, \frac{\pi}{2})$	−ve	downward
$(\frac{\pi}{2}, \pi)$	+ve	upward

Point of inflection $(\frac{\pi}{2}, 0)$

FIVE MARKS

41. ⑤ $[A|B] = \begin{bmatrix} 2 & 3 & 5 & 9 \\ 1 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{bmatrix}$

$\sim \begin{bmatrix} 2 & 3 & 5 & 9 \\ 0 & -15 & -45 & -17 \\ 0 & 0 & \lambda-5 & \mu-9 \end{bmatrix}$

$R_2 \to 2R_2 - 7R_1$, $R_3 \to R_3 - R_2$

(i) No solution

when $\lambda = 5$, $\mu \neq 9$

$\rho(A) = 2$, $\rho([A|B]) = 3$

(ii) Unique solution

when $\lambda \neq 5$, $\mu \in R$

$\rho(A) = \rho([A|B]) = 3$

(iii) Infinite solution

when $\lambda = 5$, $\mu = 9$

$\rho(A) = \rho([A|B]) = 2$

42. ⑤ $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$

$\frac{1}{3}$ is a solution

3 is another root

$\frac{1}{3}$	6	−5	−38	−5	6
	0	2	−1	−13	−6
3	6	−3	−39	−18	0
	0	18	45	18	
	6	15	6	0	

$6x^2 + 15x + 6 = 0$

$\div 3$, $2x^2 + 5x + 2 = 0$

$x = -\frac{1}{2}$, $x = -2$

The roots are $\frac{1}{3}, 3, -\frac{1}{2}, -2$.

42 (a) $z_1=\frac{r^2}{z_1}, z_2=\frac{r^2}{z_2}, z_3=\frac{r^2}{z_3}$

$|z_1+z_2+z_3| = \left|\frac{r^2}{z_1}+\frac{r^2}{z_2}+\frac{r^2}{z_3}\right|$

$= r^2\left|\frac{z_2z_3+z_1z_3+z_1z_2}{z_1z_2z_3}\right|$

$= r^2\frac{|z_1z_2+z_2z_3+z_3z_1|}{|z_1z_2z_3|}$

$= r^2\frac{|z_1z_2+z_2z_3+z_3z_1|}{r^3}$

$r = \left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|$

(b) $\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)=\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)$

$=\tan^{-1}a_2-\tan^{-1}a_1$

$\tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)=\tan^{-1}a_3-\tan^{-1}a_2$

Continuing

$\tan^{-1}\left(\frac{a_n-a_{n-1}}{1+a_{n-1}a_n}\right)=\tan^{-1}a_n-\tan^{-1}a_{n-1}$

Adding vertically

$=\tan^{-1}a_n-\tan^{-1}a_1$

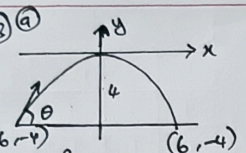
$=\tan\left(\tan^{-1}a_n-\tan^{-1}a_1\right)$

$=\tan\left[\tan^{-1}\left(\frac{a_n-a_1}{1+a_1a_n}\right)\right]$

$=\frac{a_n-a_1}{1+a_1a_n}$

LHS = RHS

43 (a)



$x^2=-4ay$

It passes $(6,-4)$

$36=-4a(-4)$

$4a=9$

$x^2=-9y$

$\frac{dy}{dx}=-\frac{2x}{9}$

at $(-6,-4)$ $\frac{dy}{dx}=\frac{-2(-6)}{9}$

$\tan\theta=\frac{4}{3}\Rightarrow\theta=\tan^{-1}\left(\frac{4}{3}\right)$

(b) $\vec{a}=2\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}=2\hat{i}+3\hat{j}+3\hat{k}$

$\vec{c}=3\hat{i}+2\hat{j}+\hat{k}$

Vector equation

$\vec{r}=\vec{a}+s\vec{b}+t\vec{c}$

$\vec{r}=(2\hat{i}+2\hat{j}+\hat{k})+s(2\hat{i}+3\hat{j}+3\hat{k})+t(3\hat{i}+2\hat{j}+\hat{k})$

Cartesian equation

$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$

$\begin{vmatrix} x-2 & y-2 & z-1 \\ 2 & 3 & 3 \\ 3 & 2 & 1 \end{vmatrix}=0$

$3x-7y+5z+3=0$

44 (a) $ax_0^2+by_0^2=1$

$cx_0^2+dy_0^2=1$

$(a-c)x_0^2+(b-d)y_0^2=0$

$ax^2+by^2=1\Rightarrow\frac{dy}{dx}=-\frac{ax}{by}$

$cx^2+dy^2=1\Rightarrow\frac{dy}{dx}=-\frac{cx}{dy}$

$m_1m_2=-1$

$\left(-\frac{ax_0}{by_0}\right)\times\left(-\frac{cx_0}{dy_0}\right)=-1$

$acx_0^2+bdy_0^2=0$

$\frac{a-c}{ac}=\frac{b-d}{bd}$

$\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$

(b) $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$

u is not a homogeneous

$\tan u=\frac{x^3+y^3}{x-y}=f(x,y)$

$f(tx,ty)=t^2f(x,y)$

f is a homogeneous with degree 2.

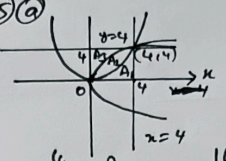
$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=2f$

$x\frac{\partial}{\partial x}(\tan u)+y\frac{\partial}{\partial y}(\tan u)=2\tan u$

$x\sec^2u\frac{\partial u}{\partial x}+y\sec^2u\frac{\partial u}{\partial y}=2\tan u$

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

45 (a)



$A_1=\int_0^4\frac{x^2}{4}dx=\frac{16}{3}$

$A_2=\int_0^4\left(\sqrt{4y}-\frac{x^2}{4}\right)dx$

$=\left[\frac{4}{3}x^{\frac{3}{2}}-\frac{x^3}{12}\right]_0^4$

$=\frac{16}{3}$

$A_3=\int_0^4\frac{y^2}{4}dx=\frac{16}{3}$

(b) $\frac{dT}{dt}\propto(T-S)$

$\frac{dT}{dt}=k(T-S)$

$T-S=ce^{kt}$

$T=25+ce^{kt}$

when $t=0$, $T=100$

$c=75$

$T=25+75e^{kt}$

when $t=10$, $T=80$

$e^{10k}=\frac{11}{15}\Rightarrow k=\frac{1}{10}\log\left(\frac{11}{15}\right)$

(i) when $t=20$

$T=25+75e^{20k}$

$=25+75\times\left(\frac{11}{15}\right)^2$

$T=65.33^\circ C$

(ii) when $T=40$

$t=\frac{1}{k}\log\left(\frac{1}{5}\right)=51.9$ min (approx)

46 (a) (i) $\int_{200}^{600}k\,dx=1$

$k(600-200)=1$

$k=\frac{1}{400}$

(ii) when $x\le200$

$F(x)=\int_{-\infty}^{x}0\,dx=0$

when $x\le600$

$F(x)=\int_{200}^{x}\frac{1}{400}dx=\frac{x}{400}-\frac{1}{2}$

when $x>600$

$F(x)=\int_{200}^{600}\frac{1}{400}dx=1$

$$F(x)=\begin{cases}0, & x<200\\ \frac{x}{400}-\frac{1}{2}, & 200\le x\le600\\ 1, & x>600\end{cases}$$

(iii) $P(300\le x\le500)$

$=F(500)-F(300)=\frac{1}{2}$

(b)

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Closure: Each box has an unique element of Z_5.

Commutative: The entries are symmetrical about the main diagonal.

Associative: From the table associative is true

Identity: Identity element is 0

Inverse: Inverse of 0 is 0, 1 is 4, 2 is 3, 3 is 4

47 (a) $\frac{b}{a}=\frac{h}{a-r}$

$h=(a-r)\frac{b}{a}$

$V=\pi r^2h=\frac{\pi b}{a}(ar^2-r^3)$

$V'=\frac{\pi b}{a}(2ar-3r^2)$

$V''=\frac{\pi b}{a}(2a-6r)$

$V'=0\Rightarrow r=\frac{2a}{3}$

when $r=\frac{2a}{3}$, $V''<0$

Volume is maximum

$V=\frac{\pi b}{a}\left(\frac{4a^2}{9}\right)\left(a-\frac{2a}{3}\right)$

$=\frac{4}{9}\left(\frac{1}{3}\pi a^2b\right)$

$=\frac{4}{9}$(Volume of cone)

(b) $\overrightarrow{AD}\perp\overrightarrow{BC}$

$\overrightarrow{OA}\cdot\overrightarrow{BC}=0$

$\vec{a}\cdot(\vec{c}-\vec{b})=0$

$\vec{a}\cdot\vec{c}-\vec{a}\cdot\vec{b}=0$ ... (1)

$\overrightarrow{BE}\perp\overrightarrow{CA}$

$\overrightarrow{OB}\cdot\overrightarrow{CA}=0$

$\vec{b}\cdot(\vec{a}-\vec{c})=0$

$\vec{a}\cdot\vec{b}-\vec{b}\cdot\vec{c}=0$ ... (2)

(1)+(2) $\Rightarrow\vec{a}\cdot\vec{c}-\vec{b}\cdot\vec{c}=0$

$\vec{c}\cdot(\vec{a}-\vec{b})=0$

$\overrightarrow{OC}\cdot\overrightarrow{BA}=0$

$\overrightarrow{OC}\perp\overrightarrow{AB}$

$\therefore$ Altitude of a triangles are concurrent.

A.KANNAN M.Sc, B.Ed
PG ASST IN MATHS
KMHSS, MOHANUR,
NAMAKKAL.
Mobile: 9894019808