

## CENTUM ACHIEVERS' ACADEMY

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XII STANDARD	VOLUME -2	TIME : 2 ½ Hrs
		MARKS : 100

PART - I

Choose the best answer :

[ 20 × 1 = 20 ]

- A stone is thrown up vertically. The height it reaches at time  $t$  seconds is given by  $x = 80t - 16t^2$ . The stone reaches the maximum height in time  $t$  seconds is given by  
(1) 2                      (2) 2.5                      (3) 3                      (4) 3.5
- The tangent to the curve  $y^2 - xy + 9 = 0$  is vertical when  
(1)  $y = 0$                       (2)  $y = \pm\sqrt{3}$                       (3)  $y = \frac{1}{2}$                       (4)  $y = \pm 3$
- One of the closest points on the curve  $x^2 - y^2 = 4$  to the point  $(6,0)$  is  
(1)  $(2,0)$                       (2)  $(\sqrt{5}, 1)$                       (3)  $(3, \sqrt{5})$                       (4)  $(\sqrt{13}, -\sqrt{3})$
- The point of inflection of the curve  $y = (x - 1)^3$  is  
(1)  $(0,0)$                       (2)  $(0,1)$                       (3)  $(1,0)$                       (4)  $(1,1)$
- If  $u(x, y) = e^{x^2+y^2}$ , then  $\frac{\partial u}{\partial x}$  is equal to  
(1)  $e^{x^2+y^2}$                       (2)  $2xu$                       (3)  $x^2u$                       (4)  $y^2u$
- If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is  
(1) 0.4 cu.cm                      (2) 0.45 cu.cm                      (3) 2 cu.cm                      (4) 4.8 cu.cm
- If  $f(x, y, z) = xy + yz + zx$ , then  $f_x - f_z$  is equal to  
(1)  $z - x$                       (2)  $y - z$                       (3)  $x - z$                       (4)  $y - x$
- The value of  $\int_0^{\frac{\pi}{6}} \cos^3 3x \, dx$  is  
(1)  $\frac{2}{3}$                       (2)  $\frac{2}{9}$                       (3)  $\frac{1}{9}$                       (4)  $\frac{1}{3}$
- The area between  $y^2 = 4x$  and its latus rectum is  
(1)  $\frac{2}{3}$                       (2)  $\frac{4}{3}$                       (3)  $\frac{8}{3}$                       (4)  $\frac{5}{3}$
- The volume of solid of revolution of the region bounded by  $y^2 = x(a - x)$  about x-axis is  
(1)  $\pi a^3$                       (2)  $\frac{\pi a^3}{4}$                       (3)  $\frac{\pi a^3}{5}$                       (4)  $\frac{\pi a^3}{6}$
- The general solution of the differential equation  $\frac{dy}{dx} = \frac{y}{x}$  is  
(1)  $xy = k$                       (2)  $y = k \log x$                       (3)  $y = kx$                       (4)  $\log y = kx$

12. The integrating factor of the differential equation  $\frac{dy}{dx} + y = \frac{1+y}{\lambda}$  is  
 (1)  $\frac{x}{e^{\lambda}}$       (2)  $\frac{e^{\lambda}}{x}$       (3)  $\lambda e^x$       (4)  $e^x$
13. The solution of  $\frac{dy}{dx} = 2^{y-x}$  is  
 (1)  $2^x + 2^y = C$       (2)  $2^x - 2^y = C$       (3)  $\frac{1}{2^x} - \frac{1}{2^y} = C$       (4)  $x + y = C$
14.  $P$  is the amount of certain substance left in after time  $t$ . If the rate of evaporation of the substance is proportional to the amount remaining, then  
 (1)  $P = Ce^{kt}$       (2)  $P = Ce^{-kt}$       (3)  $P = Ckt$       (4)  $Pt = C$
15. A random variable  $X$  has binomial distribution with  $n = 25$  and  $p = 0.8$  then standard deviation of  $X$  is  
 (1) 6      (2) 4      (3) 3      (4) 2
16. If  $P(X = 0) = 1 - P(X = 1)$ . If  $E(X) = 3 \text{Var}(X)$ , then  $P(X = 0)$  is  
 (1)  $\frac{2}{3}$       (2)  $\frac{2}{5}$       (3)  $\frac{1}{5}$       (4)  $\frac{1}{3}$
17. If in 6 trials,  $X$  is a binomial variable which follows the relation  $9P(X = 4) = P(X = 2)$ , then the probability of success is  
 (1) 0.125      (2) 0.25      (3) 0.375      (4) 0.75
18. If a compound statement involves 3 simple statements, then the number of rows in the truth table is  
 (1) 9      (2) 8      (3) 6      (4) 3
19. The proposition  $p \wedge (\neg p \vee q)$  is  
 (1) a tautology      (2) a contradiction  
 (3) logically equivalent to  $p \wedge q$       (4) logically equivalent to  $p \vee q$
20. Subtraction is not a binary operation in  
 (1)  $\mathbb{R}$       (2)  $\mathbb{Z}$       (3)  $\mathbb{N}$       (4)  $\mathbb{Q}$

**PART - II**

[ 7 × 2 = 14 ]

**Answer the following questions:**

21. A particle moves so that the distance moved is according to the law  $s(t) = \frac{t^3}{3} - t^2 + 3$ . At what time the velocity and acceleration are zero.
22. Suppose  $f(x)$  is a differentiable function for all  $x$  with  $f'(x) \leq 29$  and  $f(2) = 17$ . What is the maximum value of  $f(7)$  ?
23. A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8 cm. Find approximations for the change in the volume.
24. Evaluate:  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x \, dx$ .
25. Show that  $y = ae^{-3x} + b$ , where  $a$  and  $b$  are arbitrary constants, is a solution of the differential equation  $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$
26. Using binomial distribution find the mean and variance of  $X$  for a fair coin is tossed 100 times, and  $X$  denote the number of heads.

27. Check whether the statement  $p \rightarrow (q \rightarrow p)$  is a tautology or a contradiction without using the truth table

**PART - III**

[7 × 3 = 21 ]

**Answer the following questions:**

28. Find the equations of the tangents to the curve  $y = 1 + x^3$  for which the tangent is orthogonal with the line  $x + 12y = 12$ .

29. Assuming  $\log_{10} e = 0.4343$ , find an approximate value of  $\log_{10} 1003$ .

30. Evaluate  $\int_0^{\frac{\pi}{2}} \frac{dx}{4\sin^2 x + 5\cos^2 x}$

31. Find the differential equation of the family of all ellipses having foci on the  $x$ -axis and centre at the origin.

32. Solve  $\frac{dy}{dx} + 2y = e^{-x}$ .

33. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win ₹ 15 for each red ball selected and we lose ₹ 10 for each black ball selected. If  $X$  denotes the winning amount, find the values of  $X$  and number of points in its inverse images.

34. Construct the truth table for  $(p \vee q) \wedge (p \vee \neg q)$ .

**PART - IV**

[9 × 5 = 45 ]

**Answer the following questions:**

35. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall,  
(i) how fast is the top of the ladder moving down the wall?  
(ii) at what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

36. A hollow cone with base radius  $a$  cm and height  $b$  cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is  $\frac{4}{9}$  times volume of the cone.

37. Show that  $f_{xy} = f_{yx}$  where,  $f(x, y) = \tan^{-1} \left( \frac{x}{y} \right)$ .

38. Evaluate  $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$ .

39. The curve  $y = (x - 2)^2 + 1$  has a minimum point at  $P$ . A point  $Q$  on the curve is such that the slope of  $PQ$  is 2. Find the area bounded by the curve and the chord  $PQ$ .

40. Solve  $(1 + 2e^{x/y})dx + 2e^{x/y} \left(1 - \frac{x}{y}\right)dy = 0$ .

41. A pot of boiling water at  $100^\circ\text{C}$  is removed from a stove at time  $t = 0$  and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to  $80^\circ\text{C}$ , and another 5 minutes later it has dropped to  $65^\circ\text{C}$ . Determine the temperature of the kitchen.

42. On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and  $X$  denotes the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective.

43. Let  $A$  be  $\mathbb{Q} \setminus \{1\}$ . Define  $*$  on  $A$  by  $x * y = x + y - xy$ . Is  $*$  binary on  $A$ ? If so, examine the commutative, associative, the existence of identity, existence of inverse properties for the operation  $*$  on  $A$