Delva Model Question Paper - 06

Part - I

1 marks:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
c	c	d	c	d	a	b	a	b	b	d	c	a	b

Part - II

2 marks:

15. $R = y^2$

$x=1,\ y^2 = 1^2 \Rightarrow 1$

$x=2,\ y^2 = 2^2 \Rightarrow 4$

$x=3,\ y^2 = 3^2 \Rightarrow 9$

$x=4,\ y^2 = 4^2 \Rightarrow 16$

$x=5,\ y^2 = 5^2 \Rightarrow 25$

$x=6,\ y^2 = 6^2 \Rightarrow 36$

$x=7,\ y^2 = 7^2 \Rightarrow 49 \notin A$

Domain $= \{1, 2, 3, 4, 5\}$

Range $= \{1, 4, 9, 16, 25, 36\}$

16. $fof = f(f(k))$

$5 = f(2k-1)$

$5 = 2(2k-1)-1$

$5 = 4k-2-1$

$5 = 4k-3$

$5+3 = 4k$

$8 = 4k$

$\frac{8}{4} = k$

$\boxed{2 = k}$

17.

2	1, 2, 3, 4, 5, 6, 7, 8, 9, 10
2	1, 1, 3, 2, 5, 3, 7, 4, 9, 5
2	1, 1, 3, 1, 5, 3, 7, 2, 9, 5
3	1, 1, 3, 1, 5, 3, 7, 1, 9, 5
3	1, 1, 1, 1, 5, 1, 7, 1, 3, 5
5	1, 1, 1, 1, 5, 1, 7, 1, 1, 5
7	1, 1, 1, 1, 1, 1, 7, 1, 1, 1
	1, 1, 1, 1, 1, 1, 1, 1, 1, 1

LCM $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$= 8 \times 9 \times 35$

$= 72 \times 35$

$\boxed{\text{LCM} = 2520}$

18. $a = 8 \quad S_\infty = \frac{32}{3}$

$$S_\infty = \frac{a}{1-r}$$

$$\frac{32}{3} = \frac{8}{1-r}$$

$$32(1-r) = 3 \times 8$$

$$32 - 32r = 24$$

$$32 - 24 = 32r$$

$$8 = 32r$$

$$\frac{8}{32} = r$$

$$\boxed{\frac{1}{4} = r}$$

19. $$\frac{x^2-16}{x^2+8x+16} = \frac{x^2-4^2}{(x+4)(x+4)}$$

$$= \frac{(x+4)(x-4)}{(x+4)(x+4)}$$

$$= \frac{x-4}{x+4}$$

16 → 4, 4

20. $d = t^2 - 0.75t$

$$t^2 - 0.75t = 11.25$$

$$t^2 - 0.75t - 11.25 = 0$$

$$(t+3)(t-3.75) = 0$$

$t+3=0$ | $t-3.75=0$

$t=-3$ | $t=3.76$

$-3 \notin N$

11.25 → 3, −3.75

Answer : 3.75 seconds

21. $$\sqrt{\frac{144\,a^8 b^{12} c^{16}}{81\,t^{12} g^4 h^{14}}} = \frac{12}{9}\left|\frac{a^4 b^6 c^8}{t^6 g^2 h^7}\right|$$

$$= \frac{4}{3}\left|\frac{a^4 b^6 c^8}{t^6 g^2 h^7}\right|$$

22. $4A - 3B = 4\begin{bmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{bmatrix} - 3\begin{bmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{1}{2} & 3 \\ 5 & -6 & 9 \end{bmatrix}$

$= \begin{bmatrix} 20 & 16 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{bmatrix} + \begin{bmatrix} 21 & -12 & 9 \\ -\frac{3}{4} & -\frac{21}{2} & -9 \\ -15 & 18 & -27 \end{bmatrix}$

$= \begin{bmatrix} 41 & 4 & 1 \\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9 \\ -11 & 54 & -11 \end{bmatrix}$

22. $\Delta ABC \sim \Delta PQR$

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{36}{24}$

$\frac{AB}{PQ} = \frac{36}{24} \Rightarrow \frac{AB}{10} = \frac{36}{24}$ (cm)

$AB = \frac{36 \times 10}{24}$

$\boxed{AB = 15 \text{ cm}}$

24. $\theta = 30°, c = -3$

$m = \tan\theta$

$m = \tan 30°$

$m = \frac{1}{\sqrt{3}}$

$y = mx + c$

$y = \frac{1}{\sqrt{3}}x - \frac{3}{1}$

$y = \frac{x - 3\sqrt{3}}{\sqrt{3}}$ (cm)

$y\sqrt{3} = x - 3\sqrt{3}$

$\boxed{x - y\sqrt{3} - 3\sqrt{3} = 0}$

25. L.H.S $\dfrac{\sin A(1-\cos A)+\sin A(1+\cos A)}{(1+\cos A)(1-\cos A)}$

$\dfrac{\sin A-\sin A\cos A+\sin A+\sin A\cos A}{1-\cos^2 A}$

$= \dfrac{2\sin A}{\sin^2 A}$

$= 2\times\dfrac{1}{\sin A}$

$= 2\operatorname{cosec} A \rightarrow$ R.H.S

Hence Proved

26. $l = 5$ cm, $R = 4$ cm, $r = 1$ cm

C.S.A of frustum $= \pi(R+r)l$ sq. units

$= \dfrac{22}{7}(4+1)\times 5$

$= \dfrac{22}{7}\times 5\times 5$

$= \dfrac{550}{7}$

C.S.A $= 78.57$ cm^2.

27. Range $R = L - S$

$13.67 = 70.08 - S$

$S = 70.08 - 13.67$

$\boxed{S = 56.41}$

28. $r_1 = 12$ $r_2 = 16$

$\dfrac{r_1}{r_2} = \dfrac{12}{16} \rightarrow \dfrac{3}{4}$

C.S.A of balloons $= \dfrac{4\pi r_1^2}{4\pi r_2^2}$

$= \left(\dfrac{r_1}{r_2}\right)^2 \rightarrow \left(\dfrac{3}{4}\right)^2$

C.S.A $= \dfrac{9}{16}$

Ratio of C.S.A of balloons is $\boxed{9:16}$

Part - III

29. $A = \{2, 3\}$ $B = \{0, 1\}$ $C = \{1, 2\}$

L.H.S $B \cap C = \{0, 1\} \cap \{1, 2\}$

$= \{1\}$

$A \times (B \cap C) = \{2, 3\} \times \{1\}$

$= \{(2, 1), (3, 1)\}$ —①

R.H.S $A \times B = \{2, 3\} \times \{0, 1\}$

$= \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\}$

$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cap (A \times C) = \{(2, 1), (3, 1)\}$ —②

① = ②

$\therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$

30. (i) $f(0) = 2(0) - 3$

$= 0 - 3$

$f(0) = -3$

$f(1) = 2(1) - 3$

$= 2 - 3$

$f(1) = -1$

$\frac{f(0) + f(1)}{2} = \frac{-3 - 1}{2} \rightarrow \frac{-4}{2} \rightarrow \boxed{-2}$

(ii) $f(x) = 0$

$f(x) = 2x - 3$

$0 = 2x - 3$

$3 = 2x$

$\boxed{\frac{3}{2} = x}$

(iii) $f(x) = 2x - 3$

$x = 2x - 3$

$x - 2x = -3$

$-x = -3$

$\boxed{x = 3}$

(iv) $f(x) = f(1-x)$

$f(x) = 2x - 3$

$f(1-x) = 2(1-x) - 3$

$= 2 - 2x - 3$

$f(1-x) = -2x - 1$

$f(x) = f(1-x)$

$(2x - 3) = -2x - 1$

$2x + 2x = -1 + 3$

$4x = 2$

$x = \frac{2}{4}$

$\boxed{x = \frac{1}{2}}$

31. $a-3d,\ a-d,\ a+d,\ a+3d$

$a - 3d + a - d + a + d + a + 3d = 28$

$4a = 28$

$a = \frac{28}{4}$

$\boxed{a = 7}$

$(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 276$

$a^2 - 2(a)(3d) + (3d)^2 + a^2 - 2ad + d^2 + \}$

$a^2 + 2ad + d^2 + a^2 + 2(a)(3d) + (3d)^2 \} = 276$

$a^2 + 9d^2 + a^2 + d^2 + a^2 + d^2 + a^2 + 9d^2 = 276$

$4a^2 + 20d^2 = 276$

$4(7)^2 + 20d^2 = 276$

$4(49) + 20d^2 = 276$

$196 + 20d^2 = 276$

$20d^2 = 276 - 196$

$20d^2 = 80$

$d^2 = \frac{80}{20}$

$d^2 = 4$

$d = \sqrt{4}$

$d = \pm 2$

$a-3d,\ a-d,\ a+d,\ a+3d$

If $a=7,\ d=2$

$7-3(2),\ 7-2,\ 7+2,\ 7+3(2)$

$7-6,\ 5,\ 9,\ 7+6$

$\boxed{1, 5, 9, 13}$

If $a=7,\ d=-2$

$7-3(-2),\ 7-(-2),\ 7+(-2),\ 7+3(-2)$

$7+6,\ 7+2,\ 7-2,\ 7-6$

$\boxed{13, 9, 5, 1}$

32. $x^2y^{-2} \rightarrow x^2 \times \frac{1}{y^2} \rightarrow \frac{x^2}{y^2} \rightarrow x^2 \div y^2$

$$x^2 \div y^2 = \left(\frac{a^2+3a-4}{3a^2-3}\right)^2 \div \left(\frac{a^2+2a-8}{2a^2-2a-4}\right)^2$$

$$= \left(\frac{(a-1)(a+4)}{3(a^2-1)}\right)^2 \div \left(\frac{(a-2)(a+4)}{2(a^2-a-2)}\right)^2$$

$$= \left(\frac{(a-1)(a+4)}{3(a+1)(a-1)}\right)^2 \div \left(\frac{(a-2)(a+4)}{2(a+1)(a-2)}\right)^2$$

$$= \frac{(a-1)^2(a+4)^2}{3^2(a+1)^2(a-1)^2} \div \frac{(a-2)^2(a+4)^2}{2^2(a+1)^2(a-2)^2}$$

$$= \frac{(a-1)^2(a+4)^2}{9(a+1)^2(a-1)^2} \times \frac{4(a+1)^2(a-2)^2}{(a-2)^2(a+4)^2}$$

$x^2y^{-2} = \boxed{\frac{4}{9}}$

33. $AB + BC + CA = 81$

$x + BC + 25 = 81$

$BC = 81 - 25 - x$

$BC = 31 - x$

By Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$25^2 = x^2 + (31-x)^2$

$625 = x^2 + (31)^2 - 2(31x) \cdot 2(\quad)^1$

$625 = 2x^2 - 62x + 961$

$2x^2 - 62x + 961 - 625 = 0$

$2x^2 - 62x + 336 = 0$

$\div 2.$ $x^2 - 31x + 168 = 0$

$(x-24)(x-7) = 0$

$x - 24 = 0$ | $x - 7 = 0$

$x = 24$ | $x = 7$ $\quad 24 > 7$

The smallest value is 7

34. LHS $AB = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2-2+0 & -1+8+2 \\ 4+1+0 & -2-4+2 \end{bmatrix} = \begin{bmatrix} 0 & 9 \\ 5 & -4 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} 0 & 5 \\ 9 & -4 \end{bmatrix} \quad -①$

RHS $B^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{bmatrix}$

$B^T \times A^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2-2+0 & 4+1+0 \\ -1+8+2 & -2-4+2 \end{bmatrix}$$

$$B^TA^T = \begin{bmatrix} 0 & 5 \\ 9 & -4 \end{bmatrix} \quad \text{—②}$$

① = ②

$(AB)^T = B^TA^T$

Hence Proved.

35. Alternate Segment Theorem Book Pg : 190, 191

36. $TR = y$ OT is perpendicular bisector of PQ.

$PR = QR = 4$ cm

In $\triangle ORP$, $OP^2 = OR^2 + PR^2$

$OR^2 = OP^2 - PR^2$

$OR^2 = 5^2 - 4^2 \rightarrow 25 - 16 = 9$

$OR = \sqrt{9} = 3$ cm.

$OT = OR + RT = 3 + y$ — ①

In $\triangle PRT$, $TP^2 = TR^2 + PR^2$ ——— ②

and $\triangle OPT$ we have, $OT^2 = TP^2 + OP^2$

$OT^2 = (TR^2 + PR^2) + OP^2$ (substitute for TP^2 from ②)

$(3+y)^2 = y^2 + 4^2 + 5^2$ (substitute for OT from ①)

$9 + 6y + y^2 = y^2 + 16 + 25$

$6y = 41 - 9$, we get $y = \frac{16}{3}$

From ②, $TP^2 = TR^2 + PR^2$

$TP^2 = \left(\frac{16}{3}\right)^2 + 4^2 = \frac{256}{9} + 16 = \frac{400}{9}$

$TP = \frac{20}{3}$ cm.

37. Area of quadrilateral $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$

$= \frac{1}{2}\begin{Bmatrix} 8 & 5 & -5 & -4 & 8 \\ 6 & 11 & 12 & 3 & 6 \end{Bmatrix}$

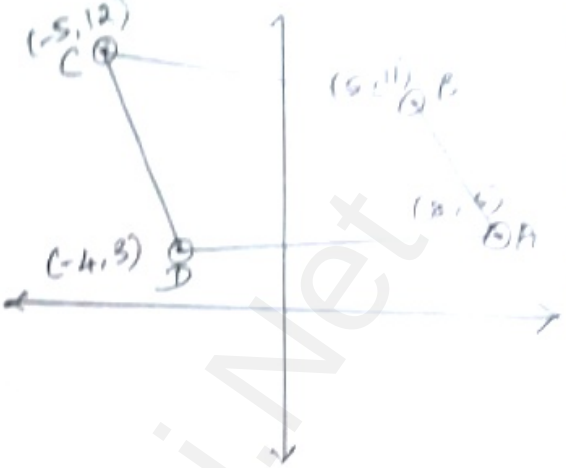
$= \frac{1}{2}\{(88+60-15-24)-(30-55-48+24)\}$

$= \frac{1}{2}\{109-(-49)\}$

$= \frac{1}{2}\{109+49\}$

$= \frac{1}{2}\{158\}$

$= 79$ sq. units



28. ΔBAC, $\tan 30^\circ = \frac{AB}{AC}$

$\frac{1}{\sqrt{3}} = \frac{200}{AC}$

$AC = 200\sqrt{3}$

ΔBAD, $\tan 45^\circ = \frac{AB}{AD}$

$1 = \frac{200}{AD}$ (cm)

$AD = 200$

$CD = AC + AD$

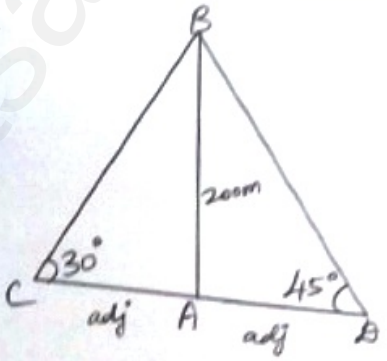
$CD = 200\sqrt{3} + 200$

$CD = 200(\sqrt{3}+1)$

$CD = 200(1.732+1)$

$CD = 200(2.732)$

$\boxed{CD = 546.4\text{m}}$



AB = lighthouse

CD = AC + AD

∴ Distance between two ships is 546.4m.

39 Volume of frustum = $\frac{1}{3}\pi h(R^2 + r^2 + Rr)$

$h = 14$,
$R = 20$,
$r = 8$

$= \frac{1}{3} \times \frac{22}{7} \times 14 [20^2 + 8^2 + 20 \times 8]$

$= \frac{22 \times 14}{7 \times 3} [400 + 64 + 160]$

$= \frac{22 \times 14}{21} (624)$

$= \frac{22 \times 14 \times 208}{7}$

$= \frac{10456.48}{1000} \rightarrow$ Litre

$= 10456 \times 40$

$= 418.24$

40. $\bar{x} = 45$, $\frac{\Sigma x^2}{n} = \frac{16364}{8}$

$\frac{\Sigma x^2}{n} = 2045.5$

$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{2045.5 - 45^2}$

$= \sqrt{20.6}$

$= 4.5277$

$C.V. = \frac{\sigma}{\bar{x}} \times 100\%$

$= \frac{4.5277}{45} \times 100\%$

$= 0.1006 \times 100\%$

C.V. = 10.06%

x	$d = x - \bar{x}$	d^2
38	$38 - 45 = -7$	49
40	$40 - 45 = -5$	25
47	$47 - 45 = 2$	4
44	$44 - 45 = -1$	1
46	$46 - 45 = 1$	1
43	$43 - 45 = -2$	4
49	$49 - 45 = 4$	16
53	$53 - 45 = 8$	64
		$\Sigma = 164$

Alternate: $\sigma = \sqrt{\frac{\Sigma d^2}{n}}$

$= \sqrt{\frac{164}{8}}$

$\sigma = 4.53$

41. $S = \{HHH, HTH, HHT, THH, TTH, THT, HTT, TTT\}$

$n(S) = 8$

$A = \{HHH, HTH, THH\}$

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$.

$B = \{HTH, HHT, THH, TTH, THT, HTT, TTT\}$

$n(B) = 7$

$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$.

$C = \{HHT, THH, HHH\}$

$n(C) = 3$ $\quad P(C) = \frac{n(C)}{n(S)} = \frac{3}{8}$

$A \cap B = \{HHT, HTH, THH\}$

$n(A \cap B) = 3$ $\quad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{8}$

$B \cap C = \{HHT, THH\}$

$n(B \cap C) = 2$ $\quad P(B \cap C) = \frac{n(B \cap C)}{n(S)} = \frac{2}{8} \rightarrow \frac{1}{4}$

$C \cap A = \{HHT, THH\}$

$n(C \cap A) = 2$ $\quad P(C \cap A) = \frac{n(C \cap A)}{n(S)} = \frac{2}{8} \rightarrow \frac{1}{4}$.

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8}$

$= \frac{8}{8}$

$P(A \cup B \cup C) = \boxed{1}$

42.

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \rightarrow (1)$$

$a+b = 5$ (given)

$b = 5-a$.

$$\frac{x}{a} + \frac{y}{5-a} = 1. \Rightarrow \frac{x(5-a)+ay}{a(5-a)} = \frac{1}{1}(CM)$$

$x(5-a) + ay = a(5-a)$.

$(6, -2)$ lies on the point.
$x \quad y$

$6(5-a) + a(-2) = 5a - a^2$

$30 - 6a - 2a - 5a + a^2 = 0$

$a^2 - 13a + 30 = 0$

30
$-10 \quad -3$

$(a-3)(a-10) = 0$

$a-3=0$ or $a-10=0$

$a=3$, or $b=10$.

At $a=3$.

$a+b=5$

$b=5-a$

$b=5-3$

$\Rightarrow b=2$

$$\frac{x}{3} + \frac{y}{2} = 1.$$

$$\frac{2x+3y}{6} = \frac{1}{1}(CM)$$

$$\boxed{2x+3y=6}$$

At $a=10$. $b=5-10$

$b=-5$.

$$\frac{x}{10} + \frac{y}{-2} = 1$$

$$\frac{x}{10} - \frac{y}{2} = 1$$

$$\boxed{x-2y=10}$$

$$\frac{x-2y}{10} = \frac{1}{1}(CM)$$