

## **BUSINESS MATHEMATICS AND STATISTICS**

### **KEY POINTS**

### **1.MATRICES AND DETERMINANTS**

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1. Singular  $|A| = 0$
2.  $\text{adj}(A) = [\text{co factor } A]^T$
3.  $A^{-1} = \frac{1}{|A|} \text{adj}A$

5. To find A from  $(A^{-1})^{-1} = A$

#### **4. The Hawkins – Simon conditions**

- (i) the main diagonal elements in  $I - B$  must be positive and
- (ii)  $|I - B|$  must be positive.

6.  $|A^{-1}| = \frac{1}{|A|}$

### **2.ALGEBRA**

1.  $\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

2.  $\frac{x^2-3}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$

3.  $\frac{1}{(x^2+4)(x+1)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+1}$

4.  $n! = n \times n - 1 \times n - 2 \dots \dots$

5.  $0! = 1$

6. Independent jobs = m+n ways

7. No of arrangement  $nP_r = \frac{n!}{(n-r)!}$

8. No of permutation repeat allowed =  $n^r$

9. Distinct words from letter  $= \frac{n!}{p_1!p_2! \dots}$

10. Circular permutation of different item  $= (n - 1)!$

11. Circular permutation of identical item  $= \frac{(n-1)!}{2}$

12.  $nC_r = \frac{n!}{r!(n-r)!}$  (without permutation)

13. If  $nCx = nCy \rightarrow x = y$  or  $x + y = n$

14. General term :  $T_{r+1} = nC_r x^{n-r} a^r$

15. Binomial theorem :  $(x + a)^n = nc_0 x^n a^0 + nc_1 x^{n-1} a^1 \dots \dots$

16. No of terms : n+1

17. Sum of co efficient :  $2^n$

18. Middle Term : n is even  $\rightarrow t_{\frac{n}{2}+1}$  n is odd :  $t_{\frac{n-1}{2}+1}$  and  $t_{\frac{n+1}{2}+1}$

19. Independent of x means - find constant term

### **3.ANALYTICAL GEOMETRY**

1. Equal distance  $\rightarrow PA = PB$

2. Angle between slope  $\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

3. Concurrent of three lines

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

4. Distance of the point from line  $= \left| \frac{ax_1 + bx_2 + c}{\sqrt{a^2 + b^2}} \right|$

5. Pair of straight lines  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

i) condition for pair of straight lines  $\rightarrow \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$  ii) represent a circle  $\rightarrow a = b$  and  $h = 0$

iii) Angle between the lines  $\theta = \tan^{-1} \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$  iv)  $m_1 + m_2 = \frac{-2h}{b}$ ,  $m_1 \times m_2 = \frac{a}{b}$

6. Equation of circle  $(x - h)^2 + (y - k)^2 = r^2$

7. Equation of circle with end points  $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

8. General eqn of circle  $x^2 + y^2 + 2gx + 2fy + c = 0$ , centre  $= (-g, -f)$  radius  $= \sqrt{g^2 + f^2 - c}$

9. Parametric form of circle  $x = r\cos\theta$ ,  $y = r\sin\theta$

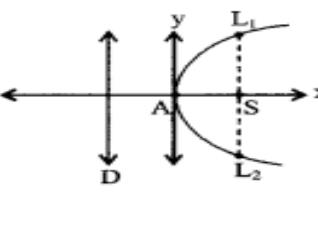
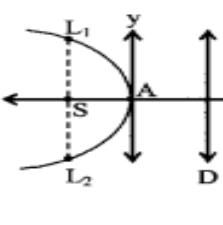
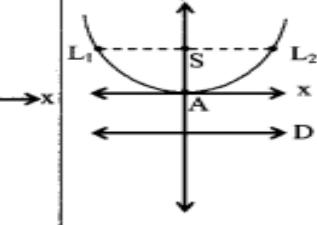
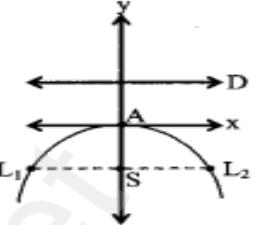
10. Equation of tangent :  $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$$(x^2 \rightarrow xx_1, y^2 \rightarrow yy_1, x \rightarrow \frac{x+x_1}{2}, y = \frac{y+y_1}{2})$$

$$11. \text{length of tangent} = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

12. condition for straight line be a tangent  $y = mx + c$  to  $x^2 + y^2 = a^2 \rightarrow c^2 = a^2(1 + m^2)$

13.

				
Equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Coordinates of Vertex (A)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Axis of Parabola	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Coordinates of focus (S)	(a, 0)	(-a, 0)	(0, a)	(0, -a)
Equation of directrix (D)	$x = -a$	$x = a$	$y = -a$	$y = a$
Length of latus rectum $L_1L_2$	4a	4a	4a	4a

#### 4.TRIGONOMETRY

#### TRIGONOMETRIC IDENTITIES

##### RECIPROCAL IDENTITIES

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

##### PYTHAGOREAN IDENTITIES

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

##### DOUBLE-ANGLE IDENTITIES

$$\sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$$

##### SUM AND DIFFERENCE IDENTITIES

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

##### HALF-ANGLE IDENTITIES

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

A , B ----- A+B , A - B	
2SC = S + S	
2CS = S - S	
2CC = C + C	
-2SS = C - C	
$\frac{C+D}{2}, \frac{C-D}{2}$ ----- C , D	

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$	
$\cosec^{-1}(-x) = -\cosec^{-1}x$	$\sin^{-1}(x) + \sin^{-1}(y)$
$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$	$= \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$
$\cosec^{-1}x + \sec^{-1}x = \frac{\pi}{2}$	
$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$	

## 5.Differential Calculus

$\lim_{x \rightarrow a} \left( \frac{x^n - a^n}{x - a} \right) = na^{n-1}$	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(x^n) = nx^{n-1}$
$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$	$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(x) = 1$
$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$	$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(k) = 0, k \text{ is a constant}$
$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$	$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(kx) = k, k \text{ is a constant}$
$\lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} = \log_a e$	$\frac{d}{dx}(\cot x) = -\cosec^2 x$	$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$
$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$	$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$	$\frac{d}{dx}(\cosec x) = -\cosec x \cot x$	$\frac{d}{dx}(e^x) = e^x$
$\lim_{x \rightarrow 0} \cos x = 1$	$\frac{d}{dx}(a^x) = a^x \log_e a$	$\frac{d}{dx}(e^{ax}) = ae^{ax}$
$\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(e^{ax+b}) = ae^{ax+b}$
$\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = 1$	$\frac{d}{dx}(\log_e x) = \frac{1}{x}$	$\frac{d}{dx}(e^{-x^2}) = -2x e^{-x^2}$
$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$	$\frac{d}{dx}(a^x) = a^x \log_e a$
$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$		

## 6.Applications of Differentiation

### Some standard results

1. Total cost:  $C(x) = f(x) + k$

2. Average cost:  $AC = \frac{f(x) + k}{x} = \frac{c(x)}{x}$

3. Average variable cost:  $AVC = \frac{f(x)}{x}$

4. Average fixed cost:  $AFC = \frac{k}{x}$

5. Marginal cost:  $MC = \frac{dC}{dx}$

6. Marginal Average cost:  $MAC = \frac{d}{dx}(AC)$

7. Total cost:  $C(x) = AC \times x$

8. Revenue:  $R = px$

9. Average Revenue:  $AR = \frac{R}{x} = p$

10. Marginal Revenue:  $MR = \frac{dR}{dx}$

11. Profit:  $P(x) = R(x) - C(x)$

12. Elasticity:  $\eta = \frac{x}{y} \cdot \frac{dy}{dx}$

13. Elasticity of demand:  $\eta_d = -\frac{p}{x} \cdot \frac{dx}{dp}$

14. Elasticity of supply:  $\eta_s = \frac{p}{x} \cdot \frac{dx}{dp}$

15. Relationship between  $MR$ ,  $AR$  and  $\eta_d$ :

$$MR = AR \left[ 1 - \frac{1}{\eta_d} \right] \text{ (or) } \eta_d = \frac{AR}{AR - MR}$$

16. Marginal function of  $y$  with respect to  $x$  (or) Instantaneous rate of change of  $y$  with respect to  $x$  is  $\frac{dy}{dx}$

17. Average cost [AC] is minimum when  $MC = AC$

18. Total revenue [TR] is maximum when  $MR = 0$

19. Profit [ $P(x)$ ] is maximum when  $MR = MC$

20. In price elasticity of a function,

(a) If  $|\eta| > 1$ , then the function is elastic

(b) If  $|\eta| = 1$ , then the function is unit elastic

(c) If  $|\eta| < 1$ , then the function is inelastic.

21.  $EOQ = q_0 = Rt_0 = \sqrt{\frac{2C_3 R}{C_1}}$

22. Optimum number of orders per year

$$n_0 = \frac{\text{demand}}{\text{EOQ}} = R \sqrt{\frac{C_1}{2C_3 R}} = \sqrt{\frac{RC_1}{2C_3}} = \frac{1}{t_0}$$

23. Minimum inventory cost per unit time,

$$C_0 = \sqrt{2C_1 C_3 R}$$

24. Carrying cost =  $\frac{q_0}{2} \times C_1$  and

$$\text{ordering cost} = \frac{R}{q_0} \times C_3$$

25. At  $EOQ$ , Ordering cost = Carrying cost

26. If  $u(x,y)$  is a continuous function of  $x$

$$\text{and } y \text{ then, } \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$$

● If  $u = f(x, y)$  is a homogeneous function of degree  $n$ , then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu$

● The partial elasticity of demand  $q$  with respect to  $p_1$  is defined to be

$$\eta_{qp_1} = \frac{Eq}{Ep_1} = \frac{-p_1}{q} \frac{\partial q}{\partial p_1}$$

● The partial elasticity of demand  $q$  with respect to  $p_2$  is defined to be

$$\eta_{qp_2} = \frac{Eq}{Ep_2} = \frac{-p_2}{q} \frac{\partial q}{\partial p_2}$$

## 7. FINANCIAL MATHEMATICS

Annuity Due (Payments are made at the beginning of each payment period)

$$A = \frac{a}{i} (1+i) \left[ (1+i)^n - 1 \right]$$

Immediate Annuity (or) Ordinary Annuity (Payment are made at the end of each payment period)

$$A = \frac{a}{i} \left[ (1+i)^n - 1 \right]$$

Present worth

$$P = \frac{a}{i} (1+i) \left[ 1 - (1+i)^{-n} \right]$$

Present worth

$$P = \frac{a}{i} \left[ 1 - (1+i)^{-n} \right]$$

Present worth

$$P = \frac{a}{i}$$

- Annual income of a shareholders =  $\frac{n \times r \times F.V}{100}$   
Where  $n$  = number of shares with the shareholders  
 $r$  = rate of dividend,
- Annual Return =  $\frac{\text{Annual income}}{\text{investment in shares}} \times 100\%$
- Number of Shares held =  $\frac{\text{investment}}{\text{M.V(or)F.V.of one share(as the type of investment)}}$   
(or)  
 $= \frac{\text{Annual income}}{\text{income from one share}} \text{ (or)} = \frac{\text{Total F.V}}{\text{F.V.of one share}}$

## 8.DESCRIPTIVE STATISTICS AND PROBABILITY

To find Q ,D, P --- Arrange the data either in ascending or descending order      AM  $\geq$  GM  $\geq$  HM

**Geometric mean :** Increasing rate , population growth    **Harmonic mean :** profit , speed , per litre , per kilo

**Arithmetic mean:** average of both

	Ungrouped data	Grouped data (discrete case)	Grouped data (Continuous case)
Quartiles	$Q_3 = \text{Size of } \left(\frac{3(n+1)}{4}\right) \text{ th value}$	Size of $\left(\frac{3(N+1)}{4}\right)$ th value	$L + \left(\frac{\frac{3N}{4} - pcf}{f}\right) \times c$
Deciles	$D_2 = \text{Size of } \left(\frac{2(n+1)}{10}\right) \text{ th value}$	Size of $\left(\frac{2(N+1)}{10}\right)$ th value	$L + \left(\frac{\frac{2N}{10} - pcf}{f}\right) \times c$
Percentiles	$P_{60} = \text{Size of } \left(\frac{60(n+1)}{100}\right) \text{ th value}$	Size of $\left(\frac{60(N+1)}{100}\right)$ th value	$L + \left(\frac{\frac{60N}{100} - pcf}{f}\right) \times c$
Geometric mean	$\text{Antilog}\left(\frac{\sum \log x}{n}\right)$	$\text{Antilog}\left(\frac{\sum f \log x}{N}\right)$	$\text{Antilog}\left(\frac{\sum f \log m}{N}\right)$
Harmonic mean	$\frac{n}{\sum \frac{1}{x}}$	$\frac{N}{\sum \frac{f}{x}}$	$\frac{N}{\sum \frac{f}{m}}$
Arithmetic mean	$\frac{\sum x}{n}$	$\frac{\sum fx}{N}$	$\frac{\sum fm}{N}$
Quartile Deviation	$\frac{Q_3 - Q_1}{2}$	Relative Measure / Coefficient of MD about mean	$\frac{\text{mean deviation about mean}}{\text{mean}}$
Relative measures for QD/ Coefficient of QD	$\frac{Q_3 - Q_1}{Q_3 + Q_1}$	Relative Measure / Coefficient of MD about median	$\frac{\text{mean deviation about median}}{\text{mean}}$
MD about Mean	$\frac{\sum  X - \bar{X} }{n}$	$\frac{\sum f  X - \bar{X} }{N}$	$\frac{\sum f  M - \bar{X} }{N}$
MD about Median	$\frac{\sum  X - \text{median} }{n}$	$\frac{\sum f  X - \text{median} }{N}$	$\frac{\sum f  M - \text{median} }{N}$

Median	Size of $\left(\frac{(n+1)}{2}\right)$ th value	Size of $\left(\frac{(N+1)}{2}\right)$ th value	$L + \left(\frac{\frac{N}{2} - pcf}{f}\right) \times c$
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If A and B are two independent events then  $P(A \text{ and } B) = P(A \cap B) = P(A) P(B)$

$$P(A \text{ and } B \text{ and } C) = P(A \cap B \cap C) = P(A) P(B) P(C) ,$$

**Dependent:**  $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(A)}$

**Baye's Theorem :**  $P\left(\frac{G_2}{B}\right) = \frac{P(G_2)P(B/G_2)}{P(G_1)P(B/G_1) + P(G_2)P(B/G_2)}$

## 9.CORRELATION AND REGRESSION ANALYSIS

Karl Pearson's Correlation Coefficient :

$$1. r = \frac{COV(X,Y)}{\sigma_x \sigma_y}$$

	Mean method	Original data	Assumed mean method
r	$\frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}}$	$\frac{N \sum XY - \sum X \sum Y}{\sqrt{N \sum X^2 - (\sum X)^2 \times N \sum Y^2 - (\sum Y)^2}}$	$\frac{N \sum xy - \sum x \sum y}{\sqrt{N \sum x^2 - (\sum x)^2 \times N \sum y^2 - (\sum y)^2}}$

$$2. \text{Spearman's Rank Correlation Coefficient : } \rho = 1 - \frac{6 \sum d^2}{N(N^2-1)}, \quad d = R_x - R_y \text{ and}$$

Rank coefficient of correlation value lies between -1 and +1.

$$3. \text{Regression Equation of X on Y: } X - \bar{X} = b_{xy}(Y - \bar{Y}), \quad b_{xy} = \frac{N \sum XY - \sum X \sum Y}{\sqrt{N \sum Y^2 - (\sum Y)^2}} \quad \text{or} \quad \frac{\sum xy}{\sum y^2} \quad \text{or} \quad b_{xy} = r \frac{\sigma_x}{\sigma_y}$$

$$4. \text{Regression Equation of Y on X : } Y - \bar{Y} = b_{yx}(X - \bar{X}), \quad b_{yx} = \frac{N \sum XY - \sum X \sum Y}{\sqrt{N \sum X^2 - (\sum X)^2}} \quad \text{or} \quad \frac{\sum xy}{\sum x^2} \quad \text{or} \quad b_{yx} = r \frac{\sigma_y}{\sigma_x}$$

5.  $r = \pm \sqrt{b_{xy} \times b_{yx}}$     If one of the regression coefficients is greater than unity, the other must be less than unity.    Both the regression coefficients are of same sign.

## 10. OPERATIONS RESEARCH

$$1. EFT = EST + t_{ij} \quad 2. LST = LFT - t_{ij}$$