HIGHER SECONDARY

FIRST YEAR

இயற்பியல்

PHYSICS

NUMERICAL PROBLEMS

2022 – 2023

PREPARED BY

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“ thoehs; KOtJk; xtnthU kzpj;JspAk; NeHikaha;, czikaha; ciofFdwtHfspd; fuq;fNs J}a;ikahd fuq;fs;.”

UNIT – 1 (NATURE OF PHYSICAL WORLD AND MEASUREMENT)

1. From a point on the ground, the top of a tree is seen to have an angle of elevation 60^0. The distance between the tree and a point is 50 m. Calculate the height of the tree?

Solution

$\theta = 60^0$, The distance between the tree and a point $x = 50$ m,
Height of the tree (h)=?

For triangulation method $\tan\theta = \frac{h}{x}$

h = x tan θ ; = 50 × tan 60^0 ; = 50 × 1.732

h = 86.6 m; **The height of the tree is 86.6 m.**

2. A RADAR signal is beamed towards a planet and its echo is received 7 minutes later. If the distance between the planet and the Earth is 6.3×10^{10} m. Calculate the speed of the signal?

Solution

The distance of the planet from the Earth
$d = 6.3 \times 10^{10}$ m
Time t = 7 minutes = 7 × 60 s.
the speed of signal V =?

The speed of signal $V = \frac{2d}{t} = \frac{2x6.3x10^{10}}{7x60}$;

$V = 3x10^8\ ms^{-1}$

No.	Log
12.6	1.1004
420	2.6232
(–)	$\bar{2}$.4772
Antilog	3.000×10^8

3. Two resistances $R_1 = (100 \pm 3)\ \Omega$, $R_2 = (150 \pm 2)\ \Omega$, are connected in series. What is their equivalent resistance?

Solution

$R_1 = 100 \pm 3\,\Omega$, $R_2 = 150 \pm 2\Omega$
Equivalent resistance R =?
Equivalent resistance R = R_1+R_2
$= (100 \pm 3) + (150 \pm 2)$; $= (100 + 150) \pm (3 + 2)$
$R = (250 \pm 5)\ \Omega$

4. The temperatures of two bodies measured by a thermometer are $t_1 = (20 + 0.5)°C$, $t_2 = (50 \pm 0.5)°C$. Calculate the temperature difference and the error therein.

Solution

$t_1 = (20 \pm 0.5)°C$ $t_2 = (50 \pm 0.5)°C$ ‘ temperature difference t=?
$t = t_2 - t_1$; $= (50 \pm 0.5) - (20 \pm 0.5)°C$
$= (50 - 20) \pm (0.5+0.5)$; **$t = (30 \pm 1)°C$**

5. A physical quantity x is given by $x = \frac{a^2b^2}{c\sqrt{d}}$ .If the percentage errors of measurement in a, b, c and d are 4%, 2%, 3% and 1% respectively, then calculate the percentage error in the calculation of x.

Solution

Given $x = \frac{a^2b^2}{c\sqrt{d}}$;

The percentage error in x is given by

$$\frac{\Delta x}{x} \times 100 = 2\frac{\Delta a}{a} \times 100 + 3\frac{\Delta b}{b} \times 100 + \frac{\Delta c}{c} \times 100 + \frac{1}{2}\frac{\Delta d}{d} \times 100$$

= (2 × 4%) + (3 × 2%) + (1 × 3%) + (½ ×1%); = 8% +6%+3% +0.5%

The percentage error is x = 17.5%

6. State the number of significant figures in the following

i) 600800 - Four
ii) 400 - One
iii) 0.007 - One
iv) 5213.0 - Five
v) 2.65 × 10^{24}m - Three
vi) 0.0006032 – Four

7. Round off the following numbers as indicated

i) 18.35 up to 3 digits	:	1.84
ii) 19.45 up to 3 digits	:	19.4
iii) 101.55 × 10^6 up to 4 digits	:	101.6 x 106
iv) 248337 up to digits 3 digits	:	248000
v) 12.653 up to 3 digits	:	12.7

8. Convert 76 cm of mercury pressure into Nm^{-2} using the method of dimensions.

Solution

In cgs system 76 cm of mercury pressure = 76 × 13.6 × 980 dyne cm^{-2}

The dimensional formula of pressure P is $[ML^{-1}T^{-2}]$

$$P_1[M_1^a\ L_1^b\ T_1^c] = P_2[M_2^a\ L_2^b\ T_2^c]\ ;\ P_2 = P_1\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

M_1 = 1g, M_2 = 1kg; L_1 = 1 cm, L_2 = 1m; T_1 = 1 s, T_2 = 1s

As a =1, b= -1, and c = -2

$$\text{Then } P_2 = 76 \times 13.6 \times 980 \left[\frac{1kg}{1kg}\right]^1 \left[\frac{1\text{cm}}{1\text{m}}\right]^{-1} \left[\frac{1\text{s}}{1\text{s}}\right]^{-2}$$

$$= 76 \times 13.6 \times 980 \left[\frac{10^{-3}kg}{1\text{kg}}\right]^1 \left[\frac{10^{-2}\text{m}}{1\text{m}}\right]^{-1} \left[\frac{1\text{s}}{1\text{s}}\right]^{-2}$$

= 76 × 13.6 × 980 × $[10^{-3}]$ × 10^2

P_2 = 1.01 × 10^5 Nm^{-2}

9. If the value of universal gravitational constant in SI is 6.6×10^{-11} Nm² kg⁻², then find its value in CGS System?

Solution

Let G_{SI} be the gravitational constant in the SI system and G_{cgs} in the cgs system. Then $G_{SI} = 6.6 \times 10^{-11} Nm^2 kg^{-2}$

G_{cgs} =?

$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$; $G_{cgs} = G_{SI}\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

M_1 = 1 kg, L_1 = 1m, T_1 = 1s ; M_2 = 1 g, L_2 = 1 cm, T_2 = 1s

The dimensional formula for G is $M^{-1}L^3T^{-2}$; a = -1, b = 3 and c = -2

$G_{cgs} = 6.6 \times 10^{-11}\left[\frac{1kg}{1g}\right]^{-1} \left[\frac{1m}{1cm}\right]^3 \left[\frac{1s}{1s}\right]^{-2}$

$= 6.6 \times 10^{-11}\left[\frac{1kg}{10^{-3}kg}\right]^{-1} \left[\frac{1m}{10^{-2}m}\right]^3 \left[\frac{1s}{1s}\right]^{-2}$

$= 6.6 \times 10^{-11} \times 10^{-3} \times 10^{6} \times 1$; **$G_{cgs} = 6.6 \times 10^{-8}$ dyne cm²g⁻²**

10. Check the correctness of the equation ½ mv² = mgh using dimensional analysis method.

Solution

Dimensional formula for ½ mv² = $[M][LT^{-1}]^2 = [ML^2T^{-2}]$

Dimensional formula for mgh = $[M][LT^{-2}][L] = [ML^2T^{-2}]$

$[ML^2T^{-2}] = [ML^2T^{-2}]$

Both sides are dimensionally the same, hence the equations ½ mv² = mgh is dimensionally correct.

11. Obtain an expression for the time period T of a simple pendulum. The time period T depends on (i) mass 'm' of the bob (ii) length 'l' of the pendulum and (iii) acceleration due to gravity g at the place where the pendulum is suspended. (Constant k = 2π)

Solution

$T \propto m^a l^b g^c$; $T = k\, m^a l^b g^c$

Here k is the dimensionless constant. Rewriting the above equation with dimensions

$[T^1] = [M^a]\,[L^b]\,[LT^{-2}]^c$ $[M^0L^0T^1] = [M^a\, L^{b+c}\, T^{-2c}]$

Comparing the powers of M, L and T on both sides, a=0, b+c=0, -2c=1

Solving for a,b and c a = 0, b = 1/2, and c = −1/2

From the above equation $T = k.\, m^0\, l^{1/2}\, g^{1/2}$

$T = k\left(\frac{l}{g}\right)^{1/2}$; $k\sqrt{l/g}$; **Experimentally k = 2π, hence $T = 2\pi\sqrt{l/g}$**

EXERCISE PROBLEM

12. In a submarine equipped with sonar, the time delay between the generation of a pulse and its echo after reflection from an enemy submarine is observed to be 80 s. If the speed of sound in water is 1460 ms^{-1}. What is the distance of enemy submarine?

Solution

v = 80s , v = 1460 ms^{-1}, D = ?

$D = \frac{v\,t}{2} = \frac{1460 \times 80}{2}$; = 1460 x 40 ; 58400m

D = 58.4km

13. The radius of the circle is 3.12 m. Calculate the area of the circle with regard to significant figures.

Solution

r = 31.2 m ; A = ?

$A = \pi r^2$; = 3.14 x 3.12 x 3.12 ; = 30.57m^2

A = 30.6 m^2 (rounding off with significant figure3)

No.	Log
3.14	0.4969
3.12	0.4942
3.12	0.4942
(+)	1.4853
Antilog	3.057×10^1

14. Jupiter is at a distance of 824.7 million km from the Earth. Its angular diameter is measured to be 35.72˝. Calculate the diameter of Jupiter

Solution

X = 824.7 million km = 82
4.7 x 10^6 x 10^3 m

θ = 35.72'' = 35.72 x 4.85 x 10^{-6} rad; b = ?

$x = \frac{b}{\theta}$; $b = x\theta$;

= 824.7 x 10^9 x 35.72 x 4.85 x 10^{-6}

= 1.428 x 10^5 x 10^3 m ;

b = 1.428 x 10^5km

No.	Log
824.7	2.9163
35.72	1.5529
4.85	0.6857
(+)	5.1549
Antilog	1.428×10^5

UNIT – 2 (KINEMATICS)

15. Two vectors $\vec{A}$ and $\vec{B}$ of magnitude 5 units and 7 units respectively make an angle 60^0 with each other. Find the magnitude of the resultant vector and its direction with respect to the vector $\vec{A}$

Solution

The magnitude of the resultant vector $\vec{R}$ is given by

$R = |\vec{R}| = \sqrt{5^2 + 7^2 + 2x5x7\cos 60^0}$; = $\sqrt{25 + 49 + \frac{70x1}{2}}$;

$\mathbf{R} = \sqrt{\mathbf{109}}$ units

The angle α between $\vec{R}$ and $\vec{A}$ is given by $\tan\alpha = \frac{\mathbf{B\ Sin\ \theta}}{\mathbf{A+B\ Cos\theta}}$

$= \frac{\mathbf{7\ x\ sin60^0}}{\mathbf{5+7\ cos60^0}}$; $= \frac{\mathbf{7\ x\ \sqrt{3}}}{\mathbf{10+7}}$ $= \frac{\mathbf{7\ x\ \sqrt{3}}}{\mathbf{17}}$; $\cong 0.713$; $\boldsymbol{\alpha} \cong 35^0$

16. Two vectors $\vec{A}$ and $\vec{B}$ of magnitude 5 units and 7 units make an angle 60^0 with each other . Find the magnitude of the difference vector $\vec{A} - \vec{B}$ and its direction with respect to the vector $\vec{A}$

Solution

The magnitude of the difference vector $\vec{A} - \vec{B}$ is given by

$|\vec{A} - \vec{B}| = \sqrt{5^2 + 7^2 - 2x5x7\cos 60^0}$; = $\sqrt{25 + 49 - 35}$;

$= \sqrt{\mathbf{39}}$ units

The angle that between $\vec{A} - \vec{B}$ makes the vector $\vec{A}$ given by

$\tan\alpha = \frac{\mathrm{B\ Sin\ \theta}}{\mathrm{A+B\ Cos\theta}}$

$= \frac{7\ x\ \sin 60^0}{5-7\ \cos 60^0}$; $= \frac{7\ x\ \sqrt{3}}{10-7}$ $= \frac{7}{\sqrt{3}}$; =4.041 ; $\alpha = \tan^{-1}(4.041)$; $\alpha \cong 76^0$

17. Two vectors $\vec{A}$ and $\vec{B}$ are given in the component form as $\vec{A} = 5\vec{i} + 7\vec{j} - 4\vec{k}$ and $\vec{B} = 6\vec{i} + 3\vec{j} + 2\vec{k}$. Find $\vec{A} + \vec{B}$, $\vec{B} + \vec{A}$, $\vec{A} - \vec{B}$, $\vec{B} - \vec{A}$.

Solution

$\vec{A} + \vec{B} = (5\vec{i} + 7\vec{j} - 4\vec{k}) + (6\vec{i} + 3\vec{j} + 2\vec{k})$; $= \mathbf{11\vec{i} + 10\vec{j} - 2\vec{k}}$.

$\vec{B} + \vec{A} = (6\vec{i} + 3\vec{j} + 2\vec{k}) + (5\vec{i} + 7\vec{j} - 4\vec{k})$; $= \mathbf{11\vec{i} + 10\vec{j} - 2\vec{k}}$.

$\vec{A} - \vec{B} = (5\vec{i} + 7\vec{j} - 4\vec{k}) - (6\vec{i} + 3\vec{j} + 2\vec{k})$; $= \mathbf{-\vec{i} + 4\vec{j} - 6\vec{k}}$.

$\vec{B} - \vec{A} = (6\vec{i} + 3\vec{j} + 2\vec{k}) - (5\vec{i} + 7\vec{j} - 4\vec{k})$; $= \mathbf{\vec{i} - 4\vec{j} + 6\vec{k}}$.

Note that the vectors $\vec{A} + \vec{B}$ and $\vec{B} + \vec{A}$ are same and the vectors $\vec{A} - \vec{B}$ and $\vec{B} - \vec{A}$ are opposite to each other.

18. Given the vector $\vec{A} = 2\vec{i} + 3\vec{j}$, what is $3\vec{A}$?

Solution

$3\vec{A} = 3(2\vec{i} + 3\vec{j}) = 6\vec{i} + 9\vec{j}$. The vector $3\vec{A}$ is given as in the same direction as vector $\vec{A}$

19. Given two vectors $\vec{A} = 2\vec{i} + 4\vec{j} + 5\vec{k}$ and $\vec{B} = \vec{i} + 3\vec{j} + 6\vec{k}$. Find the product $\vec{A}\,.\,\vec{B}$, and the magnitudes of $\vec{A}$ and $\vec{B}$. What is the angle between them?

Solution

$\vec{A}\,.\,\vec{B}$ = 2+12+30 = 44

Magnitude A = $\sqrt{4 + 16 + 25}$; = $\sqrt{45}$ units

Magnitude B = $\sqrt{1 + 9 + 36}$; = $\sqrt{46}$ units

The angle between the two vectors is given by

$\theta = \cos^{-1}\left(\frac{\vec{A}\,.\,\vec{B}}{AB}\right)$;

= $\cos^{-1}\left(\frac{44}{\sqrt{45 \times 46}}\right)$; = $\cos^{-1}\left(\frac{44}{45.49}\right)$; = $\cos^{-1}(0.967)$ $\therefore$ $\boldsymbol{\theta \cong 15^0}$

No.	Log
44	1.6435
½ x 2070	1.6580
(–)	$\bar{1}$.9855
Antilog	9.672 x 10^{-1}

20. Check whether the following vectors are orthogonal.
i) $\vec{A} = 2\vec{i} + 3\vec{j}$ and $\vec{B} = 4\vec{i} - 5\vec{j}$ ii) $\vec{C} = 5\vec{i} + 2\vec{j}$ and $\vec{D} = 2\vec{i} - 5\vec{j}$

Solution

$\vec{A}.\vec{B}$ = 8 -15 = -7≠0 . Here $\vec{A}$ and $\vec{B}$ are not orthogonal to each other.

$\vec{C}.\vec{D}$ = 10 -10 = 0 . Here $\vec{C}$ and $\vec{D}$ are orthogonal to each other.

21. Two vectors are given as $\vec{r} = 2\vec{i} + 3\vec{j} + 5\vec{k}$ and $\vec{F} = 3\vec{i} - 2\vec{j} + 4\vec{k}$. Find the resultant vector $\vec{\tau} = \vec{r} \times \vec{F}$

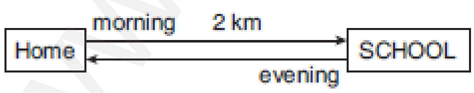
Solution

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 5 \\ 3 & -2 & 4 \end{vmatrix}$$

= $(12-(-10))\,\hat{\imath} + (15-8)\,\hat{\jmath} + (-4-9)\,\hat{k}$; $\boldsymbol{\vec{\tau} = 22\hat{\imath} + 7\hat{\jmath} - 13\hat{k}}$

22. Assume your school is located 2 km away from your home. In the morning you are going to school and in the evening you come back home. In this entire trip what is the distance travelled and the displacement covered?

Solution



The displacement covered is zero. It is because your initial and final positions are the same. But the distance travelled is 4 km.

23. An athlete covers 3 rounds on a circular track of radius 50 m. Calculate the total distance and displacement travelled by him.

Solution

The total distance the athlete covered =3x circumference of track

Distance = 3 × 2π ×50 m ; = 300π m(or)

Distance ≈ 300 x 3.14 ≈ 942 m

The **displacement is zero**, since the athlete reaches the same point A after three rounds from where he started.

24. The position vector of a particle is given $\vec{r} = 2t\vec{i} + 3t^2\vec{j} - 5\vec{k}$.
a) Calculate the velocity and speed of the particle at any instant *t*
b) Calculate the velocity and speed of the particle at time *t* = 2 s
Solution

The velocity $\bar{\bar{v}} = \frac{d\bar{\bar{r}}}{dt} = 2\vec{i} + 6t\vec{j}$;
The speed $v(t) = \sqrt{2^2 + (6t)^2}$ ms^{-1}
The velocity of the particle at *t* = 2 s ;
$\vec{v}(2\text{sec}) = 2\vec{i} + 12\vec{j}$
The speed of the particle at *t* = 2 s ;
$v(2s) = \sqrt{2^2 + 12^2} = \sqrt{4 + 144}$
$= \sqrt{148}$; ≈ **12.16 ms^{-1}**

No.	Log
½ x 148	½ x 2.1703 1.0851
Antilog	1.216×10^1

25. The velocity of three particles A, B, C are given below. Which particle travels at the greatest speed?

$$\bar{v}_A = 3\vec{i} - 5\vec{j} + 2\vec{k}\ ;\ \bar{v}_B = \vec{i} + 2\vec{j} + 3\vec{k}\ ;\ \bar{v}_C = 5\vec{i} + 3\vec{j} + 4\vec{k}$$

Solution

Speed of A = $|\bar{v}_A| = \sqrt{(3)^2 + (-5)^2 + (2)^2}$; $= \sqrt{9 + 25 + 4}$; ; $= \sqrt{38}$ ms^{-1}
Speed of B = $|\bar{v}_B| = \sqrt{(1)^2 + (2)^2 + (3)^2}$; $= \sqrt{1 + 4 + 9}$; ; $= \sqrt{14}$ ms^{-1}
Speed of C = $|\bar{v}_C| = \sqrt{(5)^2 + (3)^2 + (4)^2}$; $= \sqrt{25 + 9 + 16}$; ; $= \sqrt{50}$ ms^{-1}
The particle C has the greatest speed $\sqrt{50} > \sqrt{38} > \sqrt{14}$

26. Consider two masses of 10 g and 1 kg moving with the same speed 10 m s^{-1}. Calculate the magnitude of the momentum.
Solution

p = mv
For the mass of 10 g, m = 0.01 kg; p = 0.01 × 10 = 0.1 kg ms^{-1}
For the mass of 1 kg; p = 1 × 10 = 10 kg ms^{-1}
Thus even though **both the masses have the same speed**, the momentum of the heavier mass is **100 times greater than that of the lighter mass.**

27. A particle moves along the x-axis in such a way that its coordinates *x* varies with time 't' according to the equation *x* = 2 - 5t + 6t^2. What is the initial velocity of the particle?
Solution

X = 2 – 5t + 6t^2
Velocity $v = \frac{dx}{dt} = \frac{d}{dt}(2 - 5t + 6t^2)$ or v = -5 + 12t
For initial velocity, t = 0. Initial velocity = –5 ms^{-1}

28. Suppose two trains A and B are moving with uniform velocities along parallel tracks but in opposite directions. Let the velocity of train A be 40 km h^{-1} due east and that of train B be 40 km h^{-1} due west. Calculate the relative velocities of the trains

Solution

Relative velocity of A with respect to B V_{AB} = 80 km h^{-1} due east Thus to a passenger in train B, the train A will appear to move east with a velocity of 80 km h^{-1}

The relative velocity of B with respect to A, V_{BA} = 80 km h^{-1} due west
To a passenger in train A, the train B will appear to move westwards with a velocity of 80 km h^{-1}

29. Consider two trains A and B moving along parallel tracks with the same velocity in the same direction. Let the velocity of each train be 50 km h^{-1} due east. Calculate the relative velocities of the trains.

Solution

Relative velocity of B with respect to A, $v_{BA} = v_B - v_A$
= 50 km h^{-1} + (−50) km h^{-1} ; = 0 km h^{-1}
Similarly, relative velocity of A with respect to B i.e., v_{AB} is also zero.
Thus each train will appear to be at rest with respect to the other.

30. How long will a boy sitting near the window of a train travelling at 36 km h^{-1} see a train passing by in the opposite direction with a speed of 18 km h^{-1}. The length of the slow-moving train is 90 m.

Solution

The relative velocity of the slow-moving train with respect to the boy is
= (36 + 18) km h^{-1} = 54 km h^{-1} = $54 \times \frac{5}{18}$ ms^{-1} = 15 ms^{-1}
Since the boy will watch the full length of the other train, to find the time taken to watch the full train: $15 = \frac{90}{t}$ or $t = \frac{90}{15}$; **t = 6s**

31. A swimmer's speed in the direction of flow of a river is 12 km h^{-1}. Against the direction of flow of the river the swimmer's speed is 6 km h^{-1}. Calculate the swimmer's speed in still water and the velocity of the river flow.

Solution

Let v_s and v_r, represent the velocities of the swimmer and river respectively with respect to ground.
$v_s + v_r = 12$ (1) and $v_s - v_r = 6$ (2)
Adding the both equations (1) and (2)
$2v_s = 12 + 6 = 18$ km h^{-1} or vs = 9 km h^{-1}
From Equation (1),
$9 + v_r = 12$ or $v_r = 3$ km h^{-1}
When the river flow and swimmer move in the same direction, the net velocity of swimmer is 12 km h^{-1}.

32. An iron ball and a feather are both falling from a height of 10 m.
a) What are the time taken by the iron ball and feather to reach the ground?
b) What are the velocities of iron ball and feather when they reach the ground? (Ignore air resistance and take g = 10 m s^{-2})

Solution

Since kinematic equations are independent of mass of the object.
The time taken by both iron ball and feather to reach the ground are the same. This is given by T = $\sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \text{ x } 10}{10}} = \sqrt{2}s \approx$ **1.414s**

Thus, both feather and iron ball reach ground at the same time.

$v = \sqrt{2gh} = \sqrt{2 \text{ x } 10 \text{ x } 10} = \sqrt{200}$ ms^{-1} $\approx$ **14.14 ms^{-1}**

33. A train was moving at the rate of 54 km h^{-1} when brakes were applied. It came to rest within a distance of 225 m. Calculate the retardation produced in the train.

Solution

The final velocity of the particle v = 0

The initial velocity of the particle u = 54 x $\frac{5}{18}$ ms^{-1} = 15 ms^{-1} ; s = 225 m

Retardation is always against the velocity of the particle.

$v^2 = u^2 - 2aS$; $0 = (15)^2 - 2a(225)$; $450a = 225$

a = $\frac{225}{450}$ ms^{-2} ; =0.5 ms^{-2} ; **Retardation = =0.5 ms^{-2}**

34. In the cricket game, a batsman strikes the ball such that it moves with the speed 30 m s^{-1} at an angle 30^0 with the horizontal. The boundary line of the cricket ground is located at a distance of 75 m from the batsman? Will the ball go for a six? (Neglect the air resistance and take acceleration due to gravity g = 10 m s^{-2}).

Solution

The motion of the cricket ball in air is essentially a projectile motion. As we have already seen, the range (horizontal distance) of the projectile motion is given by

R = $\frac{u^2 sin2\theta}{g}$; The initial speed u= 30m s^{-1}

The projection angle θ = 30^0

The horizontal distance travelled by the cricket ball

No.	Log
45	1.6532
1.732	0.2385
(–)	1.8917
Antilog	7.793 x 10^1

R = $\frac{(30)^2 \text{x } sin60^0}{10} = \frac{900 \text{ x } \frac{\sqrt{3}}{2}}{10}$; = 77.94 m

This distance is greater than the distance of the boundary line. Hence the ball will cross this line and go for a six.

EXERCISE PROBLEM

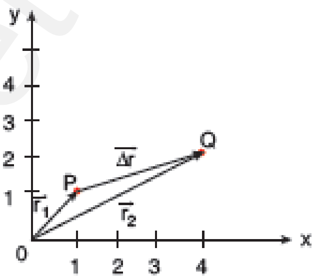
35. The position vectors particle has length 1m and makes 30^0 with the *x*-axis. What are the lengths of the x and y components of the position vector?

Solution

l = 1m , θ= 30^0 ; Length of x – component $l_x = l\cos\theta = 1 \text{ x } \cos 30^0 = \frac{\sqrt{3}}{2}$

Length of y – component $l_y = l\sin\theta = 1 \text{ x } \sin 30^0 = \frac{1}{2} = 0.5$

36. A particle has its position moved from $\vec{r_1} = 3\hat{\imath} + 4\hat{\jmath}$ to $\vec{r_2} = \hat{\imath} + 2\hat{\jmath}$ Calculate the displacement vector $(\Delta\vec{r})$ and draw the $\vec{r_1}$, $\vec{r_2}$ and $\Delta\vec{r}$ vector in a two dimensional Cartesian coordinate system.



$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$; ; = $(\hat{\imath} + 2\hat{\jmath}) - (3\hat{\imath} + 4\hat{\jmath})$; = $\hat{\imath} + 2\hat{\jmath} - 3\hat{\imath} - 4\hat{\jmath}$

$\Delta\vec{r} = -2\hat{\imath} - 2\hat{\jmath}$

37. Calculate the average velocity of the particle whose position vector changes from $\vec{r_1} = 3\hat{\imath} + 6\hat{\jmath}$ to $\vec{r_2} = \widehat{2\imath} + 3\hat{\jmath}$ in a time 5 second.

Solution

The average velocity $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t}$; = $\frac{(\widehat{2\imath}+3\hat{\jmath})-(3\hat{\imath}+6\hat{\jmath})}{5}$

= $\frac{2\hat{\imath}+3\hat{\jmath}-5\hat{\imath}-6\hat{\jmath}}{5}$; = $\frac{-3\hat{\jmath}-3\hat{\jmath}}{5}$; $\vec{v}_{avg} = -\frac{3}{5}(\hat{\imath}+\hat{\jmath})$

38. Convert the vector $\hat{r} = 3\hat{\imath} + 2\hat{\jmath}$ into a unit vector.

Solution

The magnitude of the vector $\hat{r} = 3\hat{\imath} + 2\hat{\jmath}$

$|\hat{r}| = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$; $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{3\hat{\imath}+2\hat{\jmath}}{\sqrt{13}}$

39. What are the resultants of the vector product of two given vectors given by $\vec{A} = 4\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{B} = 5\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

Solution

$$\vec{A} \text{ x } \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & 1 \\ 5 & 3 & -4 \end{vmatrix}$$

= $(8 - 3)\hat{\imath} + (5+16)\hat{\jmath} + (12+10)\hat{k}$;

$\vec{A} \text{ x } \vec{B} = 5\hat{\imath} + 21\hat{\jmath} + 22\hat{k}$

40. An object at an angle such that the horizontal range is 4 times of the maximum height. What is the angle of projection of the object?
Solution

Horizontal Range R = $\frac{u^2 \sin 2\theta}{g}$; maximum height h_{max} = $\frac{u^2 \sin^2\theta}{2g}$;

R = $4h_{max}$; $\frac{u^2 \sin 2\theta}{g} = 4 \times \frac{u^2 \sin^2\theta}{2g}$

$\frac{u^2 2 \sin\theta \cos\theta}{g} = 4 \times \frac{u^2 \sin^2\theta}{2g}$; $\cos\theta = \sin\theta$; $\frac{\sin\theta}{\cos\theta} = 1$; $\tan\theta = 1$

$\boldsymbol{\theta = tan^{-1}(1) = 45^0}$

41. Calculate the area of the triangle for which two of its sides are given by the vectors $\vec{A} = 5\hat{\imath} - 3\hat{\jmath}$ and $\vec{B} = 4\hat{\imath} + 6\hat{\jmath}$
Solution

$\Delta = \frac{1}{2}|\vec{A} \times \vec{B}|$ -----------1

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & -3 & 0 \\ 4 & 6 & 0 \end{vmatrix}$

= (0 – 0) $\hat{\imath}$ + (0 – 0) $\hat{\jmath}$ + (30 + 12) $\hat{k}$; $|\vec{A} \times \vec{B}|$ = 42$\hat{k}$

$|\vec{A} \times \vec{B}|$ = 42 put this in equation 1 $\Delta = \frac{1}{2}$ x 42 ; $\boldsymbol{\Delta}$ = 21m²

42. If Earth completes one revolution in 24 hours, what is the angular displacement made by Earth in one hour. Express your answer in both radian and degree.
Solution

Angular displacement for one complete revolution (i.e.) for 24 hours = 360^0

Hence, angular displacement for one hour, $\theta = \frac{360^0}{24}$; = 15^0 or

$\theta = \frac{\pi}{180^0} \times 15^0$; = $\frac{\pi}{12}$ rad [$180^0 = \pi$ rad]

43. A object is thrown with initial speed 5 ms⁻¹ with an angle of projection 30⁰. What is the height and range reached by the particle?
Solution

No.	Log
25	1.3979
78.4	1.8943
(–)	$\bar{1}$.5036
Antilog	3.188 x 10⁻¹

i) maximum height of the projectile, $h_{max} = \frac{u^2 \sin^2\theta}{2g}$

$h_{max} = \frac{5^2 \sin 30^0 \sin 30^0}{2 \times 9.8}$; $= \frac{25 \times \left[\frac{1}{2}\right] \times \left[\frac{1}{2}\right]}{2 \times 9.8}$; $= \frac{25}{8 \times 9.8}$; $= \frac{25}{78.4}$; h_{max} = 0.3188m

ii) Horizontal Range R = $\frac{u^2 \sin 2\theta}{g}$; = $\frac{u^2 2 \sin\theta \cos\theta}{g}$; = $\frac{5^2 \times 2 \sin 30^0 \cos 30^0}{9.8}$

$= \frac{25 \times 2 \left[\frac{1}{2}\right] \times \left[\frac{\sqrt{3}}{2}\right]}{9.8}$; $= \frac{25 \times 1.732}{2 \times 9.8} = \frac{43.300}{19.6}$; R = 2.21m

No.	Log
43.3	1.6365
19.6	1.2922
(−)	0.3443
Antilog	2.209×10^0

44. A foot-ball player hits the ball with speed 20 ms^{-1} with angle 30^0 with respect to horizontal direction. The goal post is at distance of 40 m from him. Find out whether ball reaches the goal post?

Solution

Here the reaches ball is considering as a projectile. Its range

Horizontal Range R = $\frac{u^2 \sin 2\theta}{g}$; = $\frac{u^2 2 \sin\theta \cos\theta}{g}$; = $\frac{20^2 \times 2 \sin 30^0 \cos 30^0}{9.8}$

$= \frac{400 \times 2 \left[\frac{1}{2}\right] \times \left[\frac{\sqrt{3}}{2}\right]}{9.8}$; $= \frac{400 \times 1.732}{2 \times 9.8} = \frac{692.800}{19.6}$; R = 35.35m

Thus the range of the reaches ball is 35.35 m. But the goal post is at a distance of 40 m from him. So ball will not reach the goal post.

45. If an object is thrown horizontally with an initial speed 10 m s^{-1} from the top of a building of height 100 m. what is the horizontal distance covered by the particle?

Solution

Horizontal range of the object projected horizontally,

$R = u\sqrt{\frac{2h}{g}}$; $= 10\sqrt{\frac{2 \times 100}{9.8}} = 10\sqrt{\frac{200}{9.8}}$; $= \sqrt{\frac{200 \times 100}{9.8}}$; $= \sqrt{\frac{20000}{9.8}}$;

= 45.18m ≅ 45m

No.	Log
20000	4.3010
9.8	0.9912
(−)	3.3098
½ x 3.3098 = 1.6549	
Antilog	4.518×10^1

46. An object is executing uniform circular motion with an angular speed of $\frac{\pi}{12}$ radian per second. At t = 0 the object starts at an angle θ = 0. What is the angular displacement of the particle after 4 s ?

Solution

$\omega = \frac{\theta}{t}$ or $\theta = \omega t$; $\theta = \frac{\pi}{12} \times 4$; $= \frac{\pi}{3}$ rad ; $= 60^0$

47. The Moon is orbiting the Earth approximately once in 27 days, what is the angle transverse by the Moon per day?

Solution

Angle traversed by the Moon for one complete rotation

(i.e.) for 27days = 360^0 = $2\pi rad$

Angle traversed by the Moon for one day,

$\theta = \frac{2\pi}{27}$; $= \frac{2 \times 3.14}{27}$; $= \frac{6.28}{27}$

θ =0.2362 rad = 13.33^0 [1 rad = 57.27^0]

No.	Log
6.28	0.7980
27	1.4314
(−)	$\bar{1}.3666$
Antilog	2.326×10^{-1}

48. An object of mass m has angular acceleration α = 0.2 rads^{-1} . What is the angular displacement covered by the object after 3 second? (Assume that the object started with angle zero with zero angular velocity).

Solution

From equation for uniform circular motion, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ [$\omega_0 = 0$]

$\theta = 0 + \frac{1}{2} \text{ x } 0.2 \text{ x } 3^2$; $\theta = \frac{1}{2} \text{ x } 0.2 \text{ x } 9$; = 0.1 x 9

$\boldsymbol{\theta = 0.9}$ **rad ≅ 51^0** [1 rad = 57.27^0]

நான் வாசிக்காத எந்த ஒரு புத்தகத்தைப் பிறர் எனக்குத் தருகிறானோ அவனே எனக்கு நல்ல நண்பன் என்கிற ஆபிரகாம் லிங்கன். இந்தப் புத்தகம் உங்களது தேர்விலும் வெற்றி பெற படிக்கற்களாக அமைந்தால் அது எனக்குக் கிடைத்த வெற்றி.

“உலகில் பிறந்த ஒவ்வொருவரும், மற்றவர்
செய்ய முடியாத ஒன்றைச் செய்து முடிக்கும்
தனிச்சிறப்பு மிக்க ஆற்றலைப்
பெற்றிருக்கிறார்கள். அந்தச்செயல் எது எனக்
கண்டு கொள்ளுங்கள். அதை மட்டுமே
வளர்த்துக் கொண்டு வந்தால், உங்கள்
வழியில் நீங்களும் சாதனை படைத்து
வெற்றி காண இயலும்.”

UNIT – III (LAWS OF MOTION)

49. **A book of mass m is at rest on the table. (1) What are the forces acting on the book? (2) What are the forces exerted by the book? (3) Draw the free body diagram for the book.**

Solution

(1) There are two forces acting on the book.

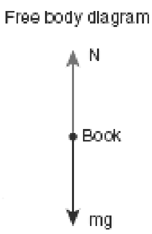
(i) Gravitational force (mg) acting downwards on the book

(ii) Normal contact force (N) exerted by the surface of the table on the book. It acts upwards as shown in the figure.

(2) According to Newton's third law, there are two reaction forces exerted by the book.

(i) The book exerts an equal and opposite force (mg) on the Earth which acts upwards.

(ii) The book exerts a force which is equal and opposite to normal force on the surface of the table (N) acting downwards.



50. **If two objects of masses 2.5 kg and 100 kg experience the same force 5 N, what is the acceleration experienced by each of them?**

Solution

For the object of mass 2.5 kg, the acceleration is a = $\frac{F}{m} = \frac{5}{2.5}$; = 2ms^{-2}

For the object of mass 100 kg, the acceleration is a = $\frac{F}{m} = \frac{5}{100}$; = 0.05ms^{-2}

51. **A person rides a bike with a constant velocity $\vec{v}$ with respect to ground and another biker accelerates with acceleration $\vec{a}$ with respect to ground. Who can apply Newton's second law with respect to a stationary observer on the ground?**

Solution

Second biker cannot apply Newton's second law, because he is moving with acceleration $\vec{a}$ with respect to Earth (he is not in inertial frame). But the first biker can apply Newton's second law because he is moving at constant velocity with respect to Earth (he is in inertial frame).

52. **The position vector of a particle is given by $\vec{r} = 3t\vec{i} + 5t^2\vec{j} + 7\vec{k}$. Find the direction in which the particle experiences net force?**

Solution

Velocity of the particle, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t)\vec{i} + \frac{d}{dt}(5t^2)\vec{j} + \frac{d}{dt}(7)\vec{k}$

$\frac{d\vec{r}}{dt} = 3\vec{i} + 10t\vec{j}$; Acceleration of the particle $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = 10\vec{j}$

Here, the particle has acceleration only along positive y direction. According to Newton's second law, net force must also act along positive y direction. In addition, the particle has constant velocity in positive x direction and no velocity in z direction. Hence, there are no net force along x or z direction.

53. A particle of mass 2 kg experiences two forces, $\vec{F}_1 = 5\vec{\imath} + 8\vec{\jmath} + 7\vec{k}$ and , $\vec{F}_2 = 3\vec{\imath} - 4\vec{\jmath} + 3\vec{k}$. What is the acceleration of the particle?

Solution

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$. Acceleration is $\vec{a} = \frac{\vec{F}_{net}}{m}$

$\vec{F}_{net} = (5+3)\vec{\imath} + (8-4)\vec{\jmath} + (7+3)\vec{k}$; $\vec{F}_{net} = 8\vec{\imath} + 4\vec{\jmath} + 10\vec{k}$

$\vec{a} = \left(\frac{8}{2}\right)\vec{\imath} + \left(\frac{4}{2}\right)\vec{\jmath} + \left(\frac{10}{2}\right)\vec{k}$; $\vec{\mathbf{a}} = \mathbf{4}\vec{\boldsymbol{\imath}} + \mathbf{2}\vec{\boldsymbol{\jmath}} + \mathbf{5}\vec{\mathbf{k}}$

54. An object of mass 10 kg moving with a speed of 15 m s^{-1} hits the wall and comes to rest within a) 0.03 second b) 10 second .Calculate the impulse and average force acting on the object in both the cases.

Solution

Initial momentum of the object P_I = 10x15 = 150 $kgms^{-1}$

final momentum of the object P_f = 0 ; Δp = 150 – 0 = 150 $kgms^{-1}$

(a) Impulse J = Δp = 150 Ns ; (b) Impulse J = Δp = 150 Ns

(a) Average force $F_{avg} = \frac{\Delta p}{\Delta t} = \frac{150}{0.03}$; = 5000N ;

(b) Average force $F_{avg} = \frac{\Delta p}{\Delta t} = \frac{150}{10}$; = 15N

impulse is the same in both cases, but the average force is different.

55. Consider an object of mass 2 kg resting on the floor. The coefficient of static friction between the object and the floor is μ_s =0.8. What force must be applied on the object to move it?

Solution

Since the object is at rest, the gravitational force experienced by an object is balanced by normal force exerted by floor.

N = mg

The maximum static frictional force $f_s^{max} = \mu_s N = \mu_s mg$

f_s^{max} = 0.8 x 2 x 9.8 = 15.68N

Therefore, to move the object the external force should be greater than maximum static friction F_{ext} > 15.68 N

56. Consider an object of mass 50 kg at rest on the floor. A Force of 5 N is applied on the object but it does not move. What is the frictional force that acts on the object?

Solution

When the object is at rest, the external force and the static frictional force are equal and opposite.

The magnitudes of these two forces are equal, $f_s = F_{ext}$

Therefore, the static frictional force acting on the object is fs = 5N

The direction of this frictional force is opposite to the direction of F_{ext} .

57. If a stone of mass 0.25 kg tied to a string executes uniform circular motion with a speed of 2 ms^{-1} of radius 3 m, what is the magnitude of tensional force acting on the stone?

Solution : $F_{cp} = \frac{mv^2}{r}$; $\frac{\frac{1}{4} \text{ x } (2)^2}{3}$ = 0.333N

58. Consider a circular leveled road of radius 10 m having coefficient of static friction 0.81. Three cars (A, B and C) are travelling with speed 7 m s^{-1}, 8 m s^{-1} and 10 ms^{-1} respectively. Which car will skid when it moves in the circular level road? (g =10 m s^{-2})

Solution

From the safe turn condition, the speed of the vehicle (v) must be less than or equal $\sqrt{\mu_s rg}$; $v \leq \sqrt{\mu_s rg}$; $\sqrt{\mu_s rg} = \sqrt{0.81 \text{ x } 10 \text{ x } 10}$ = 9 ms^{-1}

For car C, $\sqrt{\mu_s rg}$ is less than v

The speed of car A, B and C are 7 ms^{-1}, 8 ms^{-1} and 10 ms^{-1} respectively. The **cars A and B will have safe turns.** But the **car C has speed 10 ms^{-1}** while it turns which exceeds the safe turning speed. Hence, **the car C will skid.**

59. Consider a circular road of radius 20 meter banked at an angle of 15 degrees. With what speed a car has to move on the turn so that it will have safe turn?

Solution

$v = \sqrt{(rg \tan\theta)} = \sqrt{20 \text{ x } 9.8 \text{ x } \tan 15^0}$

$= \sqrt{20 \text{ x } 9.8 \text{ x } 0.26}$ = 7.1 ms^{-1}

The safe speed for the car on this road is 7.1 ms^{-1}

No.	Log
20	1.3010
9.8	0.9912
0.26	$\bar{1}$.4150
(+)	1.7072 / 2 0.8536
Antilog	7.139 x 10^0

60. Calculate the centrifugal force experienced by a man of 60 kg standing at Chennai? (Given: Latitude of Chennai is 13^0)

Solution

The centrifugal force is given by $F_c = m\omega^2 R\cos\theta$

The angular velocity of Earth = $\frac{2\pi}{T}$,

where T is time period of the Earth (24 hours)

$\omega = \frac{2\pi}{24 \text{ x } 60 \text{ x } 60} = \frac{2\pi}{86400}$ = 7.268 x 10^{-5} radsec^{-1}

The radius of the Earth R = 6400 Km = 6400 × 10^3 m

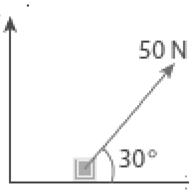
Latitude of Chennai is 13^0

F_{cf} = 60 x $(7.268 \text{ x } 10^{-5})^2$ x 6400 x 10^3 x cos (13^0)

F_{cf} = 1.9678 N A 60 kg man experiences centrifugal force of approximately 2 Newton. But due to Earth's gravity a man of 60 kg experiences a **force=mg=60×9.8=588N.** This force is very much larger than the centrifugal force.

EXERCISE PROBLEM

61. A force of 50N act on the object of mass 20 kg. shown in the figure. Calculate the acceleration of the object in x and y directions.



Solution

From Newton's second law; $F=m\ a$

Hence the acceleration ; a = $\frac{F}{m} = \frac{50}{20}$; = 2.5 ms^{-2}

The acceleration in x-axis; a_x = acos θ; = 2.5 x $cos30^0$; = 2.5 x $\frac{\sqrt{3}}{2}$

= 1.25 x 1.732; **a_x = 1.165 ms^{-2}**

The acceleration in y-axis; a_y = asin θ; = 2.5 x $sin30^0$; = 2.5 x $\frac{1}{2}$;

a_y= 1.25 ms^{-2}

62. A spider of mass 50 g is hanging on a string of a cob web. What is the tension in the string?

Solution

Here two forces acting on the spider.

(1) Downward gravitational force (mg) (2) Upward tension (T)

Hence, $\boldsymbol{T}= \boldsymbol{mg}$; = 50 x 10^{-3} x 9.8 =490 X 10^{-3} ; **T = 0.49 N**

63. A bob attached to the string oscillates back and forth. Resolve the forces acting on the bob in to components. What is the acceleration experience by the bob at an angle θ .

Solution

In the arc path, the restoring force acting along the tangential direction gives the tangential acceleration. Hence from Newton's second law,

$F_{\text{res force}} = mg\sin\theta$; $m\ a_T = mg\sin\theta$ ∴ $\boldsymbol{a_T = g\sin\theta}$

The tension acting along the string gives centripetal acceleration. Hence form Newton's second law, $T - mg\cos\theta = m\ a_{cp}$

$$a_{cp} = \frac{T - mg\cos\theta}{m}$$

64. Calculate the acceleration of the bicycle of mass 25 kg as shown in Figures 1 and 2.

Solution

Apply Newton's second law in figure (1)

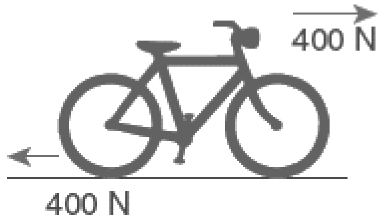
500 − 400 = $m\ a$

$a = \frac{F}{m}$; $= \frac{500-400}{25} = \frac{100}{25}$; a = 4$ms^{-2}$

Apply Newton's second law in figure (2)

400 − 400 = $m\ a$

$a = \frac{F}{m}$; $= \frac{400-400}{25} = \frac{0}{25}$; a = 0$ms^{-2}$



65. A stone of mass 2 kg is attached to a string of length 1 meter. The string can withstand maximum tension 200 N. What is the maximum speed that stone can have during the whirling motion?

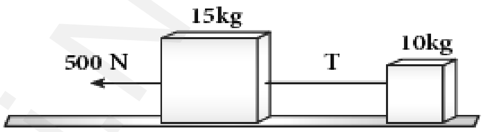
Solution

During whirling motion of the stone, the tension acting along the string provides necessary centripetal force.

If tension becomes maximum, then the centripetal force also be maximum. Hence $T_{max} = (F_{cp})_{max} = \frac{mv_{max}^2}{r}$; $v_{max}^2 = \frac{T_{max}\, r}{m}$;

$= \frac{200 \times 1}{2}$; $= 100$; $v_{max} = 10\ ms^{-1}$

66. Two bodies of masses 15 kg and 10 kg are connected with light string kept on a smooth surface. A horizontal force F=500 N is applied to a 15 kg as shown in the figure. Calculate the tension acting in the string.



Solution

Here motion is along horizontal direction only.

Consider the motion of mass m1 ; $F - T = m_1 a$ (or) $500 - T = 15\,a$

(or) T = 500 – 15 a – – – – (1)

Consider the motion of mass m_2 ; T = m_2 a = 10 a – – – – (2)

From equation (1) and (2)

$500 - 15\,a = 10\,a$;

$25\,a = 500$; $\boldsymbol{a} = \frac{500}{25}$; $= \mathbf{20}\ \boldsymbol{m}s^{-2}$

Put this in equation (2), we get T = **10 a** = **10** x **20** = **200**N

67. People often say "For every action there is an equivalent opposite reaction". Here they meant 'action of a human'. Is it correct to apply Newton's third law to human actions? What is mean by 'action' in Newton third law? Give your arguments based on Newton's laws.

Ans: Newton's third law is applicable to only human's actions which involves physical force. Third law is not applicable to human's psychological actions or thoughts.

68. A car takes a turn with velocity 50 ms⁻¹ on the circular road of radius of curvature 10 m. calculate the centrifugal force experienced by a person of mass 60kg inside the car?

Solution

Centrifugal force is given by, $F_{cf} = \frac{mv^2}{r}$; $= \frac{60 \times 50 \times 50}{10}$; = 6 x 2500

F_{cf} =15000 N

69. A long stick rests on the surface. A person standing 10 m away from the stick. With what minimum speed an object of mass 0.5 kg should he thrown so that it hits the stick. (Assume the coefficient of kinetic friction is 0.7).

Solution

When the stone moves towards the stick, it experiences kinetic friction.

According to Newton's second law,

$f_k = -ma$; $\mu_k N = -ma$; $\mu_k mg = -ma$; $a = -\mu_k g$

From equations of motion, $v^2 = u^2 + 2as$

When the stone hits the stick, it comes to rest. So

$v = 0$; $0 = u^2 + 2as$; $u^2 = -2as$;

$u^2 = -2(-\mu_k g)s$; $u^2 = 2\mu_k gs$

$u = \sqrt{2\mu_k gs}$; $= \sqrt{2 \times 0.7 \times 9.8 \times 10}$;

$= \sqrt{137.2}$; **u = 11.71ms^{-1}**

No.	Log
137.2	2.137 x ½ 1.0687
Antilog	1.171 x 10^{1}

எத்தனையோ மாணவர்கள் பள்ளிக்கு எழுதப்படாத பலகைகளாக வந்து பின்னர் கிறுக்கப்பட்ட தாள்களாகக் கசங்கிப் போவதைப் பார்க்கலாம். நீ பெற்றோருக்கு உண்மையாக இரு. எந்தப் பழக்கத்தையும் அவர்களுக்குத் தெரியவா போகிறது என, நீ மேற்கொள்ளும் நடவடிக்கைகள் உனக்கு நீயே தோண்டிக்கொள்ளும் புதைக்குழி.

உன் பெற்றோரிடம் பேசு. அவர்களோடு நடையயில். விடுமுறை நாட்களில் அம்மாவை சோற்றை உருட்டி கைகளில் போடச் சொல்லி உண். உன் தந்தையோடு மனம் விட்டுப்பேசு. உன் இனிய வார்த்தைகளே அவர்கள் அயர்வைப் போக்கும் விசிறிக்காற்று. பள்ளியில் நடப்பவற்றைப் பகிர்ந்துகொள். ஒரு வயதிற்குப் பிறகு அவர்களும் உன்னை தோழனாக நடத்தவே விரும்புவார்கள். அதற்கான தகுதியை வளர்த்துக்கொள்.

எனக்குத் தெரிந்து சில மாணவர்கள் பெற்றோரின் புகைப்படத்தை வைத்திருப்பார்கள். அவர்கள் எல்லோரும் நல்ல நிலையில் இருக்கிறார்கள்.

உனக்கு அவர்கள் மீது கோபம் வருவது இயற்கை. இந்தப்பருவம் அப்படி. நீ நினைத்ததை எல்லாம் அடைய வேண்டும் என்கிற எண்ணத்திற்கு முட்டுக்கட்டை போட்டால், உனக்குக் கோபம் பொத்துக் கொண்டு வரும். அப்போதெல்லாம் சின்ன வயதில் உனக்கு நடக்கவும், பேசவும், சோறூட்டவும் அவர்கள் குழந்தையாக மாறிய காட்சியை நினைத்துக் கொள். அன்பு மேலிடும். இப்போதும் சொல்கிறேன்.

நல்ல மகனாக இருப்பதைக் காட்டிலும் மகத்தாக சாதனை வேறு ஒன்றுமில்லை.

UNIT – IV (WORK, ENERGY AND POWER)

70. A box is pulled with a force of 25 N to produce a displacement of 15 m. If the angle between the force and displacement is 30^0, find the work done by the force.

Solution

Force, F = 25 N; Displacement, dr = 15 m;
Angle between F and dr, $\theta = 30^0$
Work done, W = Fdr cos θ ; W = 25 x 15 x $\cos 30^0$
$= 25 \times 15 \times \frac{\sqrt{3}}{2}$; **W = 324.76 J**

No.	Log
12.5	1.0969
15	1.1761
1.732	0.2385
(+)	2.5115
Antilog	3.247×10^2

71. A variable force F = k x^2 acts on a particle which is initially at rest. Calculate the work done by the force during the displacement of the particle from x = 0 m to x = 4 m. (Assume the constant k =1 N m^{-2})

Solution

Work done, $W = \int_{x_i}^{x_f} F(x)dx = k\int_0^4 x^2 dx$; $\frac{64}{3}$ Nm

72. Two objects of masses 2 kg and 4 kg are moving with the same momentum of 20 kg m s^{-1}.
(a) Will they have same kinetic energy?
(b) Will they have same speed?

Solution

(a) The kinetic energy of the mass is given by $KE = \frac{P^2}{2m}$

For the object of mass 2kg, kinetic energy is $KE_1 = \frac{(20)^2}{2x2} = \frac{400}{4} = 100J$

For the object of mass 4kg, kinetic energy is $KE_2 = \frac{(20)^2}{2x4} = \frac{400}{8} = 50J$

the kinetic energy of **both masses is not the same.** The kinetic energy of the **heavier object has lesser kinetic energy than smaller mass.**

(b) As the momentum, p = mv, the two objects **will not have same speed.**

73. An object of mass 2 kg is taken to a height 5 m from the ground $g=10ms^{-2}$.
(a) Calculate the potential energy stored in the object.
(b) Where does this potential energy come from?
(c) What external force must act to bring the mass to that height?
(d) What is the net force that acts on the object while the object is taken to the height 'h'?

Solution

(a) The potential energy U=mgh =2×10×5 = 100 J. Here the positive sign implies that the energy is stored on the mass.

(b) This potential energy is transferred from external agency which applies the force on the mass.

(c) The external applied force $\vec{F}_a$ which takes the object to the height 5 m is $\vec{F}_a = -\vec{F}_g$. $\vec{F}_a = -(-mg\vec{j})\,; = mg\vec{j}$, where $\vec{j}$ represents unit vector along vertical upward direction.

(d) From the definition of potential energy, the object must be moved at constant velocity. So the net force acting on the object is zero

$\vec{F}_g + \vec{F}_a = 0$

74. Let the two springs A and B be such that $k_A > k_B$. On which spring will more work have to be done if they are stretched by the same force?

Solution

$F = k_A x_A = k_B x_B$; $x_A = \frac{F}{k_A}$; $x_B = \frac{F}{k_B}$

The work done on the springs are stored as potential energy in the springs.

$U_A = \frac{1}{2} k_A x_A^2$; $U_B = \frac{1}{2} k_B x_B^2$

$\frac{U_A}{U_B} = \frac{k_A x_A^2}{k_B x_B^2} = \frac{k_A\left(\frac{F}{k_A}\right)^2}{k_B\left(\frac{F}{k_B}\right)^2}$; $= \frac{\frac{1}{k_A}}{\frac{1}{k_B}}$; $\frac{U_A}{U_B} = \frac{k_B}{k_A}$

$k_A > k_B$ implies that $U_B > U_A$. thus, more work is done on B than A.

75. Consider an object of mass 2 kg moved by an external force 20 N in a surface having coefficient of kinetic friction 0.9 to a distance 10 m. What is the work done by the external force and kinetic friction? Comment on the result. (Assume g = 10 ms^{-2})

Solution

m = 2 kg, d = 10 m, F_{ext} = 20 N, μk = 0.9. When an object is in motion on the horizontal surface, it experiences two forces.

(a) External force, F_{ext} = 20 N

(b) Kinetic friction $f_k = \mu k$ mg = 0.9 x (2) x 10 **= 18N**

The work done by the external force W_{ext} = Fd = 20 x10 = 200J

The work done by the force of kinetic friction $W_k = f_k$ d= (-18)×10 = -180 J.

Here the negative sign implies that the force of kinetic friction is opposite to the direction of displacement.

The total work done on the object $W_{total} = W_{ext} + W_k$; = 200 J -180 J = 20 J .

Since the friction is a non-conservative force, out of 200 J given by the external force, the 180 J is lost and it cannot be recovered.

76. An object of mass 1 kg is falling from the height h = 10 m. Calculate

(a) The total energy of an object at h =10 m

(b) Potential energy of the object when it is at h = 4 m

(c) Kinetic energy of the object when it is at h = 4 m

(d) What will be the speed of the object when it hits the ground? (Assume g =10 ms^{-2})

Solution

(a) The gravitational force is a conservative force. So the total energy remains constant throughout the motion. At h =10 m, the total energy *E* is entirely potential energy.

E = U = mgh = 1 x 10 x 10 = 100J

(b) The potential energy of the object at h = 4 m is
$U = mgh = 1 \times 10 \times 4 = 10J$

(c) Since the total energy is constant throughout the motion, the kinetic energy at h = 4m must be KE = E – U = 100 – 40 = 60J
Alternatively, the kinetic energy could also be found from velocity of the object at 4 m. At the height 4 m, the object has fallen through a height of 6 m.
The velocity after falling 6 m is calculated from the equation of motion,
$V = \sqrt{2gh} = \sqrt{2 \times 10 \times 6} = \sqrt{120}$ ms^{-1} ; $v^2 = 120$
The kinetic energy is KE = ½ mv² = ½ x 1 x 120 = 60J

(d) When the object is just about to hit the ground, the total energy is completely kinetic and the potential energy, U = 0
$E = KE = \frac{1}{2} mv^2 = 100J$; $v = \sqrt{\frac{2}{m} KE} = \sqrt{\frac{2}{1} \times 100} = \sqrt{200} \approx 14.12\ ms^{-1}$

77. Water in a bucket tied with rope is whirled around in a vertical circle of radius 0.5 m. Calculate the minimum velocity at the lowest point so that the water does not spill from it in the course of motion. ($g = 10\ ms^{-2}$)

Solution

Radius of circle r = 0.5m

The required speed at the highest point $v_2 = \sqrt{gr} = \sqrt{10 \times 0.5} = \sqrt{5}\ ms^{-1}$

The speed at the lowest point $v_1 = \sqrt{5gr} = \sqrt{5} \times \sqrt{gr} = \sqrt{5} \times \sqrt{5} = 5ms^{-1}$

78. Calculate the energy consumed in electrical units when a 75 W fan is used for 8 hours daily for one month (30 days).

Solution

Power, P = 75 W
Time of usage, t = 8 hour × 30 days = 240 hours
Electrical energy consumed is the product of power and time of usage.
Electrical energy = power × time of usage = P × t
=75 watt x 240 hour ; =18000 watt hour
=18 kilowatt hour = 18kWh
1 electrical unit = 1kWh ; Electrical energy =18 unit

79. Show that the ratio of velocities of equal masses in an inelastic collision when one of the masses is stationary is $\frac{v_1}{v_2} = \frac{1-e}{1+e}$

Solution

$e = \frac{\text{Velocity of seperation (after collision)}}{\text{Velocity of approach (before collision)}}$; $= \frac{(V_2 - V_1)}{(u_1 - u_2)} = \frac{(V_2 - V_1)}{(u_1 - 0)} = \frac{(V_2 - V_1)}{u_1}$; $v_2 - v_1 = eu_1$ -------1

From the law of conservation of linear momentum, $mu_1 = mv_1 + mv_2$; $u_1 = v_1 + v_2$ ---------2

using the equation 2 for u_1 in 1, we get $v_2 - v_1 = e(v_1 + v_2)$

on simplification, we get $\frac{v_1}{v_2} = \frac{1-e}{1+e}$

EXERCISE PROBLEM

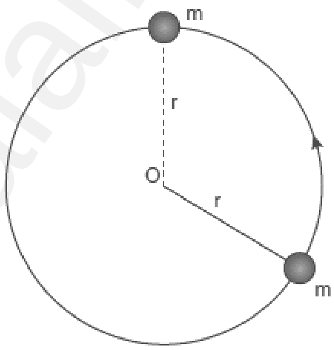
80. Calculate the work done by a force of 30 N in lifting a load of 2kg to a height of 10m ($g = 10\ m\ s^{-2}$)

Solution

Work done by gravitational force: W= FS=30 x 10= $\mathbf{300\ J}$

81. A bob of mass m is attached to one end of the rod of negligible mass and length r, the other end of which is pivoted freely at a fixed centre O as shown in the figure. What initial speed must be given to the object to reach the top of the circle? (Hint: Use law of conservation of energy). Is this speed less or greater than speed obtained in the theory?

Solution



At point A, Potential energy $=mg\ (0)=0$

Kinetic energy = $\frac{1}{2}mv_A^2$

Total energy =0 + = $\boldsymbol{\frac{1}{2}mv_A^2}$ **= =** $\boldsymbol{\frac{1}{2}mv_A^2}$

At point B, Potential energy $=mgh=mg\ (2r)$

Kinetic energy= = $\frac{1}{2}mv_B^2$ = = $\frac{1}{2}m(0)$ = 0

Total energy$=mg\ (2r)+ 0 = mg\ (2\ r)$

According to the law of conservation of energy, the total energy is always constant. Hence, $\frac{1}{2}mv_A^2 = m\ g\ (2\ r)$; $v_A^2 = 4\,g\,r$; $\boldsymbol{v_A = \sqrt{4gr}}$

This speed is less than the speed obtained in the theory, because the bob must have a speed at point A , $\boldsymbol{v_A} \geq \sqrt{\mathbf{5gr}}$ to stay in the circular path

UNIT – V (MOTION OF SYSTEM OF PARTICLES AND RIGID BODIES)

82. The position vectors of two point masses 10 kg and 5 kg are $(-3\vec{\imath} + 2\vec{\jmath} + 4\vec{k})$m and $(3\vec{\imath} + 6\vec{\jmath} + 5\vec{k})$m respectively. Locate the position of centre of mass.

Solution

m_1= 10kg, m_2 = 5kg ; $\vec{r}_1 =(-3\vec{\imath} + 2\vec{\jmath} + 4\vec{k})$m ; $\vec{r}_2 = (3\vec{\imath} + 6\vec{\jmath} + 5\vec{k})$m

$$\vec{r} = \frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2} \;;\; \vec{r} = \frac{10(-3\vec{\imath}+2\vec{\jmath}+4\vec{k})+5(3\vec{\imath}+6\vec{\jmath}+5\vec{k})}{10+5} \;;$$

$$= \frac{30\vec{\imath}+2\vec{\jmath}+40\vec{k}+15\vec{\imath}+30\vec{\jmath}+25\vec{k}}{10+5} \;;\; = \frac{-15\vec{\imath}+50\vec{\jmath}+65\vec{k}}{15} \;;\; = \left(-\vec{\imath} + \frac{10}{3}\vec{\jmath} + \frac{13}{3}\vec{k}\right)\text{m}$$

The centre of mass is located at position $\vec{r}$

83. A force of $(4\vec{\imath} - 3\vec{\jmath} + 5\vec{k})$N is applied at a point whose position vector is $(7\vec{\imath} + 4\vec{\jmath} - 2\vec{k})$m. find the torque of force about the origin.

Solution

$$\vec{\tau} = \vec{r} \text{x} \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 7 & 4 & -2 \\ 4 & -3 & 5 \end{vmatrix}$$

= (20 –6) $\hat{\imath}$ – (35 +8) $\hat{\jmath}$ + (–21–16) $\hat{k}$;

= (14$\hat{\imath}$ – 43$\hat{\jmath}$ – 37 $\hat{k}$) Nm

84. A cyclist while negotiating a circular path with speed 20 m s^{-1} is found to bend an angle by 30o with vertical. What is the radius of the circular path? (given, g = 10 ms^{-2})

Solution

Speed of the cyclist, v = 20 ms^{-1} ;
Angle of bending with vertical, θ = 30^0

Equation for angle of bending, $\tan\theta = \frac{v^2}{rg}$

Rewriting the above equation for radius $r = \frac{v^2}{\tan\theta g}$

$$r = \frac{(20)^2}{\tan 30^0 \text{ x } 10} \;;\; = \frac{20 \text{ x } 20}{(\tan 30^0) \text{ x } 10} \;;\; = \frac{400}{\left(\frac{1}{\sqrt{3}}\right) \text{x } 10} \;;\; = (\sqrt{3}) \text{ x } 40 \;;$$

= 1.732 x 40 ; **r = 69.28m**

85. Find the rotational kinetic energy of a ring of mass 9 kg and radius 3 m rotating with 240 rpm about an axis passing through its centre and perpendicular to its plane. (rpm is a unit of speed of rotation which means revolutions per minute)

Solution

The rotational kinetic energy is , KE = $\frac{1}{2}I\omega^2$.

The moment of Inertia of the ring is, I = MR^2

I = 9 x 3^2 ; = 9 x 9; = 81 kgm^2

The angular speed of the ring is, ω = 240 rpm ; = $\frac{240 \text{ x } 2\pi}{60}$rads^{-1}

KE = $\frac{1}{2}$ x 81 x $\left(\frac{240 \text{ x } 2\pi}{60}\right)^2$. ; = $\frac{1}{2}$ x 81 x $(8\pi)^2$;

KE = $\frac{1}{2}$ x 81 x 64 $(\pi)^2$;

= 2592 x $(\pi)^2$; KE ≈ 25920J

KE = 25.920 kJ [$(\pi)^2$ ≈ 10]

86. A rolling wheel has velocity of its centre of mass as 5 ms^{-1}. If its radius is 1.5 m and angular velocity is 3 rads^{-1}, then check whether it is in pure rolling or not.

Solution

Translational velocity (v_{TRANS}) or velocity of centre of mass,

v_{CM} = 5 m s^{-1}

The radius is, R = 1.5 m and the angular velocity is, ω = 3 rads^{-1}

Rotational velocity, v_{ROT} = Rω

v_{ROT} = 1.5×3; v_{ROT} = 4.5 ms^{-1}

As v_{CM} > Rω (or) v_{TRANS} > Rω, It is not in pure rolling, but sliding.

87. A solid sphere is undergoing pure rolling. What is the ratio of its translational kinetic energy to rotational kinetic energy?

Solution

The expression for total kinetic energy in pure rolling is,

KE = KE $_{TRANS}$ + KE$_{ROT}$

For any object the total kinetic energy **KE = $\frac{1}{2}MV_{CM}^2 + \frac{1}{2}MV_{CM}^2\left(\frac{K^2}{R^2}\right)$**

KE = $\frac{1}{2}MV_{CM}^2\left(1+\frac{K^2}{R^2}\right)$ then, $\frac{1}{2}MV_{CM}^2\left(1+\frac{K^2}{R^2}\right)$; = $\frac{1}{2}MV_{CM}^2 + \frac{1}{2}MV_{CM}^2\left(\frac{K^2}{R^2}\right)$

The above equation suggests that in pure rolling the ratio of total kinetic energy, translational kinetic energy and rotational kinetic energy is given as, KE : KE $_{TRANS}$: KE $_{ROT}$ :: $\left(1+\frac{K^2}{R^2}\right)$: 1 : $\left(\frac{K^2}{R^2}\right)$

Now, KE $_{TRANS}$: KE $_{ROT}$ :: 1 : $\left(\frac{K^2}{R^2}\right)$; For a solid sphere , $\frac{K^2}{R^2} = \frac{2}{5}$

Then, KE $_{TRANS}$: KE $_{ROT}$ :: 1 : $\frac{2}{5}$ or **KE $_{TRANS}$: KE $_{ROT}$ :: 5 : 2**

88. Four round objects namely a ring, a disc, a hollow sphere and a solid sphere with same radius R start to roll down an incline at the same time. Find out which object will reach the bottom first.

Solution

For all the four objects namely the ring, disc, hollow sphere and solid sphere, the radii of gyration K are R, $\sqrt{\frac{1}{2}}$ R, $\sqrt{\frac{2}{3}}$ R, $\sqrt{\frac{2}{5}}$ R With numerical values the radius of gyration K are 1R, 0.707R, 0.816R, 0.632R respectively. The expression for time taken for rolling has the radius of gyration K.

$$t = \sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{g\sin^2\theta}}.$$

The one with least value of radius of gyration K will take the shortest time to reach the bottom of the inclined plane. The order of objects reaching the bottom is first, solid sphere; second, disc; third, hollow sphere and last, ring.

EXERCISE PROBLEM

89. Two particles P and Q of mass 1kg and 3 kg respectively start moving towards each other from rest under mutual attraction. What is the velocity of their centre of mass?

Solution

Since they are at rest initially, the velocity of centre of mass of the system is zero. (i.e.) Initially, v_{CM}=0

There is no external force and their translational motion are only due to the internal forces. It doecs not change the position of the centre of mass. So the velocity of the centre of mass v_{CM}=0

90. Find the moment of inertia of a hydrogen molecule about an axis passing through its centre of mass and perpendicular to the inter-atomic axis. Given: mass of hydrogen atom 1.7×10^{-27} kg and inter atomic distance is equal to 4×10^{-10}m

Solution

The moment of inertia of a hydrogen molecule about an axis passing through its centre of mass and perpendicular to the inter-atomic axis,

$I_{CM}= 2m_H d^2 + m_H d^2 = 2m_H d^2$

$I_{CM} = 2 \times 1.7 \times 10^{-27} \times 2 \times 10^{10} \times 2 \times 10^{10}$

$I_{CM} = 13.6 \times 10^{-47}$

$\mathbf{I_{CM} = 1.36 \times 10^{-46}}$ **kgm**

UNIT – VI (GRAVITATION)

91. Calculate the value of g in the following two cases:

(a) If a mango of mass ½ kg falls from a tree from a height of 15 meters, what is the acceleration due to gravity when it begins to fall?

Solution

$g' = g\left(1 - 2\frac{h}{h_e}\right)$; $g' = 9.8\left(1 - \frac{2 \times 15}{6400 \times 10^3}\right)$; $g' = 9.8(1 - 0.469 \times 10^5)$

But 1 – 0.00000469 ≅ 1 ; Therefore $g' = g$

(b) Consider a satellite orbiting the Earth in a circular orbit of radius 1600 km above the surface of the Earth. What is the acceleration experienced by the satellite due to Earth's gravitational force?

Solution

$g' = g\left(1 - 2\frac{h}{h_e}\right)$; $g' = g\left(1 - \frac{2 \times 1600 \times 10^3}{6400 \times 10^3}\right)$; $g' = g\left(1 - \frac{2}{4}\right)$; $g' = g\left(1 - \frac{1}{2}\right)$

$= \frac{g}{2}$ The above two examples show that the acceleration due to gravity is a constant near the surface of the Earth.

92. Find out the value of $'g$ in your school laboratory?

Solution

Calculate the latitude of the city or village where the school is located. The information is available in Google search. For example, the latitude of Chennai is approximately 13 degrees. $g' = g = \omega^2 R\cos^2\lambda$

Here $\omega^2 R$ = (2 x 3.14 / 86400)^2 x (6400 x 10^3) = 3.4 x 10^{-2} ms^{-2}.

It is to be noted that the value of λ should be in radian and not in degree. 13 degrees is equivalent to 0.2268 rad.

$g' = 9.8 - (3.4 \times 10^{-2}) \times (\cos 0.2268)^2$

$g' = 9.7677\ ms^{-2}$

93. Calculate the energy of the Moon orbiting the Earth.

Solution

Assuming the orbit of the Moon to be circular, the energy of Moon is given by,

$E_m = -\frac{GM_E M_m}{2R_m}$; where M_E is the mass of Earth 6.02 ×10^{24} kg; M_m is the mass of Moon 7.35 ×10^{22} kg; and R_m is the distance between the Moon and the center of the Earth 3.84 ×10^5km

G = 6.67 x 10^{-11} Nm^2kg^{-2}

$E_m = -\frac{6.67 \times 10^{-11} \times 6.02 \times 10^{24} \times 7.35 \times 10^{22}}{2 \times 3.84 \times 10^5 \times 10^3}$; $E_m = -38.42 \times 10^{-19} \times 10^{46}$

$E_m = -38.42 \times 10^{27}$ Joule

The negative energy implies that the Moon is bound to the Earth.

Same method can be used to prove that the energy of the Earth is also negative.

EXERCISE PROBLEM

94. An unknown planet orbits the Sun with distance twice the semi major axis distance of the Earth's orbit. If the Earth's time period is T_1, what is the time period of this unknown planet?

Solution

$a_2 = 2a_1$. From Kepler's third law $T_1^2 \propto a_1^3$; $T_2^2 \propto a_2^3$

$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$; $\frac{T_1^2}{T_2^2} = \frac{a_1^3}{(2a_1)^3}$; $= \frac{a_1^3}{(8a_1)^3}$; $= \frac{1}{8}$; $T_2^2 = 8T_1^2$

$T_2 = \sqrt{8}T_1$; $= 2\sqrt{2}\ T_1$

95. Assume that you are in another solar system and provided with the set of data given below consisting of the planets' semi major axes and time periods. Can you infer the relation connecting semi major axis and time period?

Planet (imaginary)	Time period(T) (in year)	Semi major axis (a) (in AU)
Kurinji	2	8
Mullai	3	18
Marutham	4	32
Neithal	5	50
Paalai	6	72

Solution

1) For Kurunji ; T = 2 years, a = 8AU = 2 x 4 = 2 $(2)^2$ = $2T^2$
2) For Mullai ; = 3 years, a = 18AU = 2 x 9 = 2 $(3)^2$ = 2 T^2
3) For Marutham ; 4 years, a = 32AU = 2 x 16 = 2 $(4)^2$ = 2 T^2
4) For Neithal ; 5 years, a = 50AU = 2 x 25 = 2 $(5)^2$ = 2 T^2
5) For Paalai ; 6 years, a = 72AU = 2 x 36 = 2 $(6)^2$ = 2 T^2

Hence the relation connecting semi major axis and time period; a= $2T^2$

96. If the masses and mutual distance between the two objects are doubled, what is the change in the gravitational force between them?

Solution

By Newton's law of gravitation, $F = \frac{Gm_1m_2}{r^2}$

If $m_1 \rightarrow 2m_1$, $m_2 \rightarrow 2m_2$ and r = 2r

$= \frac{G2m_12m_2}{(2r)^2}$; $= \frac{4Gm_1m_2}{4r^2}$; $= \frac{Gm_1m_2}{r^2}$; $=$ F ; There is no change in the force.

97. If the angular momentum of a planet is given by $\vec{L}$=5 $t^2\hat{i}$− 6 $t\,\hat{j}$+ 3 $\hat{k}$. What is the torque experienced by the planet? Will the torque be in the same direction as that of the angular momentum?

Solution

Torque is given by, $\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d}{dt}(5\,t^2\hat{\imath} - 6t\hat{\jmath} + 3\hat{k})$; $\vec{\tau} = \mathbf{10}t\,\hat{\imath} - \mathbf{6}\hat{\jmath}$

Here the torque produced will be in the direction of angular momentum.

98. Suppose unknowingly you wrote the universal gravitational constant value as G' =6.67 x 10^{11} .instead of the correct value G =6.67 x 10^{-11}, what is the acceleration due to gravity g' for this incorrect G? According to this new acceleration due to gravity, what will be your weight W'?

Solution

The acceleration due to gravity for the value G =6.67x 10^{-11} is,

$$g = \frac{GM}{R^2}; = \frac{6.67 \text{ x } 10^{-11} \text{ x M}}{R^2} \text{ ------------ } 1$$

The acceleration due to gravity for the value G' =6.67x 10^{11} is,

$$g' = \frac{G'M}{R^2}; = \frac{6.67 \text{ x } 10^{11} \text{ x M}}{R^2} \text{ ------------ } 1$$

Divide (2) by (1)

$$\frac{g'}{g} = \frac{\left(\frac{G'M}{R^2}\right)}{\left(\frac{GM}{R^2}\right)}; = \frac{G'}{G}; = \frac{6.67 \text{ x } 10^{11}}{6.67 \text{ x } 10^{-11}}; 10^{22}; \boldsymbol{g'} = \mathbf{10^{22}g}$$

The equivalent weight is , $w' = mg' = mg10^{22};\ 10^{22}w$

99. What is the gravitational potential energy of the Earth and Sun? The Earth to Sun distance is around 150 million km. The mass of the Earth is 5.9 × 10^{24} kg and mass of the Sun is 1.9×10^{30} kg.

Solution

R_E=150 x 10^6 km → Distance between Sun and Earth

M_e=5.9 x 10^{24} kg → Mass of the Earth

M_s=1.9 x10^{30} kg → Mass of the Sun

G =6.67 x 10^{-11} Nm^2kg^{-2} →Gravitational constant

$$U(r) = -\frac{GM_SM_E}{R_E}; = -\frac{6.67 \text{ x } 10^{-11} \text{ x } 1.9 \text{ x } 10^{30} \text{x } 5.9 \text{ x } 10^{24}}{150 \text{ x } 10^6 \text{x } 10^3}$$

$$= -\frac{6.67 \text{ x } 1.9 \text{ x } 5.9 \text{ x } 10^{43}}{150 \text{ x } 10^9}; = -\frac{\mathbf{6.67 \text{ x } 1.9 \text{ x } 5.9 \text{ x } 10^{34}}}{\mathbf{150}}$$

$$U(r) = -4.985 \text{ x } 10^{-1} \text{x } 10^{34}; = -49.85 \text{ x } 10^{-2} \text{x } 10^{34}$$

$$\mathbf{U(r)} = \mathbf{-49.85 \text{ x} 10^{32}} \text{ joule}$$

No.	Log
6.67	0.8241
1.9	0.2788
5.9	0.7709
(+) 150	1.8738 2.1761
(−)	$\bar{1}$.6977
Antilog	4.985 x 10^{-1}

100. Earth revolves around the Sun at 30 km s^{-1}. Calculate the kinetic energy of the Earth. If the calculated the potential energy of the Earth is – 49.85 x 10^{32} joule. then what is the total energy of the Earth in that case? Is the total energy positive? Give reasons.

Solution

v=30 kms^{-1}=30 x 10^3 ms^{-1}

The kinetic energy of the Earth KE = ½ M_ev^2

KE = ½ x 5.9 x 10^{24} x 30 x 10^3 x 30 x 10^3

KE = 2.95 x 10^{24} x 30 x 10^3 x 30 x 10^3

KE = 2.95 x 10^{24} x 9 x 10^8

KE = 26.55 **x 10^{32}** Joule

Hence the total energy, E = KE + UE

= 26.55 x 10^{32} + (-49.85 x 10^{32})

E = (26.55 – 49.85) x 10^{32} ; E = **−23.2 x 10^{32}** Joule

The negative sign implies that Earth is bounded with Sun.

UNIT – VII (PROPERTIES OF MATTER)

101. A wire 10 m long has a cross-sectional area 1.25×10^{-4} m². It is subjected to a load of 5 kg. If Young's modulus of the material is 4×10^{10} Nm^{-2}, calculate the elongation produced in the wire. Take g = 10 ms^{-2}.

Solution

$$\frac{F}{A} = Y \times \frac{\Delta L}{L} \; ; \Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right) \; ; \left(\frac{150}{1.25 \times 10^{-4}}\right)\left(\frac{10}{4 \times 10^{10}}\right) \; ; = 10^{-4}\ \text{m}$$

102. A metallic cube of side 100 cm is subjected to a uniform force acting normal to the whole surface of the cube. The pressure is 10^6 pascal. If the volume changes by 1.5×10^{-5} m³, calculate the bulk modulus of the material.

Solution

$$K = \frac{\frac{F}{A}}{\frac{\Delta V}{V}} = P\frac{V}{\Delta V} \; ; K = \frac{10^6 \times 1}{1.5 \times 10^{-5}} \; ; = 6.67 \times 10^{10}\ \text{Nm}^{-2}$$

No.	Log
10^{11}	11.0000
1.5	0.1761
(−)	10.8239
Antilog	6.666×10^{10}

103. A metal cube of side 0.20 m is subjected to a shearing force of 4000 N. The top surface is displaced through 0.50 cm with respect to the bottom. Calculate the shear modulus of elasticity of the metal.

Solution

Here, L = 0.20m, F=4000N, x=0.50cm ; =0.005m and Area A = L² = 0.04 m²

$$\text{Therefore,} \eta_R = \left(\frac{F}{A}\right) \times \left(\frac{L}{x}\right) \; ; = \left(\frac{4000}{0.04}\right) \times \left(\frac{0.20}{0.005}\right) \; ; = 4 \times 10^6\ \text{Nm}^{-2}$$

104. A wire of length 2 m with the area of cross-section 10^{-6} m² is used to suspend a load of 980 N. Calculate i) the stress developed in the wire ii) the strain and iii) the energy stored. Given: Y=12 × 10^{10}N m^{-2}

Solution

i) Stress $= \frac{F}{A} = \frac{980}{10^{-6}}$; = 98 x 10^7 Nm^{-2}

ii) Strain $= \frac{\text{Stress}}{Y}$; $\frac{98 \times 10^7}{12 \times 10^{10}}$; =8.17 x 10^{-3}

iii) Volume = 2 x 10^{-6} m³

Energy = ½ (Stress x Strain) x Volume

= ½ (98x10^7) x8.17x10^{-3}) x2x 10^{-6} = 8J

No.	Log
980	2.9912
81.66	1.9120
(+)	4.9032
Antilog	8.002×10^4

105. A solid sphere has a radius of 1.5 cm and a mass of 0.038 kg. Calculate the specific gravity or relative density of the sphere.

Solution

Radius of the sphere R = 1.5 cm; mass m = 0.038 kg

Volume of the sphere V= $\frac{4}{3}\pi R^3$; = $\frac{4}{3}(3.14) \times (1.5 \times 10^{-2})^3$; 1.413 x 10^{-5}m³

Density $\rho = \frac{m}{V} = \frac{0.038\ \text{kg}}{1.413 \times 10^{-5}\text{m}^3}$; = 2690 kgm^{-3}

Hence, the specific gravity of the sphere = $\frac{2690}{1000}$ = 2.69

106. Two pistons of a hydraulic lift have diameters of 60 cm and 5 cm. What is the force exerted by the larger piston when 50 N is placed on the smaller piston?

Solution

Since, the diameter of the pistons are given, we can calculate the radius of the piston r = $\frac{D}{2}$

Area of smaller piston , $A_1 = \pi\left(\frac{5}{2}\right)^2 = \pi(2.5)^2$

Area of larger piston , $A_2 = \pi\left(\frac{600}{2}\right)^2 = \pi(30)^2$

$F_2 = \frac{A_2}{A_1} \times F_1 = (50N) \times \left(\frac{30}{2.5}\right)^2$; 7200 N

This means that with the force of 50 N, the force of 7200 N can be lifted

107. A cube of wood floating in water supports a 300 g mass at the centre of its top face. When the mass is removed, the cube rises by 3 cm. Determine the volume of the cube.

Solution

Let each side of the cube be l. The volume occupied by 3 cm depth of cube,

V = (3cm) x l^2; $= 3l^2$cm

According to the principle of floatation, we have $V_{\rho g} = mg \Rightarrow V_\rho = m$

ρ is density of water = 1000 kgm^{-3}

$\Rightarrow (3l^2 \times 10^{-2}m) \times (1000\ kgm^{-3}) = 300 \times 10^{-3}kg$

$l^2 = \frac{300 \times 10^{-3}}{3 \times 10^{-2} \times 1000} m^2 ; \Rightarrow l^2 = 100 \times 10^{-4}\ m^2$

$l = 10 \times 10^{-2}$ m = 10cm ; **volume of cube V = l^3 = 1000 cm^3**

108. A metal plate of area 2.5×10^{-4} m^2 is placed on a 0.25×10^{-3} m thick layer of castor oil. If a force of 2.5 N is needed to move the plate with a velocity 3×10^{-2} m s^{-1}, calculate the coefficient of viscosity of castor oil.

Given: A=2.5×10^{-4} m^2, dx = 0.25×10^{-3}m, F=2.5 N and dv = 3×10^{-2} ms^{-1}

Solution

$F = -\eta A\frac{dv}{dx}$; $\eta = \frac{F}{A}\frac{dx}{dv}$; $= \frac{(2.5\ N)\ (0.25 \times 10^{-3}m)}{(2.5 \times 10^{-4}m^2)(3 \times 10^{-2}\ ms^{-1})}$;

= 0.083 x 10^3 N m^{-2}s

109. Let 2.4×10^{-4} J of work is done to increase the area of a fi lm of soap bubble from 50 cm^2 to 100 cm^2. Calculate the value of surface tension of soap solution.

Solution

A soap bubble has two free surfaces, therefore increase in surface area

$\Delta A = A_2 - A_1 = 2(100-50) \times 10^{-4}m^2 = 100 \times 10^{-4}m^2$.

Since, work done W = T x ΔA ; $T = \frac{W}{\Delta A}$; $= \frac{2.4 \times 10^{-4}J}{100 \times 10^{-4}m^2}$;

= 2.4 x 10^{-2} Nm^{-1}

110. If excess pressure is balanced by a column of oil (with specific gravity 0.8) 4 mm high, where R = 2.0 cm, find the surface tension of the soap bubble.

Solution

The excess of pressure inside the soap bubble is $\Delta p = P_2 - P_1 = \frac{4T}{R}$

$\Delta p = P_2 - P_1 = \rho gh$ $[\rho gh = \frac{4T}{R}]$

Surface tension, T = $\frac{\rho ghR}{4}$; = $\frac{(800)(9.8)(4 \text{ x } 10^{-3})(2 \text{ x } 10^{-2})}{4}$;T=$15.68 \text{x} 10^{-2}$ Nm^{-1}

111. Mercury has an angle of contact equal to 140^0 with soda lime glass. A narrow tube of radius 2 mm, made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury T=0.456 Nm^{-1}; Density of mercury ρ = 13.6×10^3 kgm^{-3}

Solution

Capillary descent, cos140 = cos(90+50) –sin50 = –0.7660

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \text{ x } (0.456 \text{ Nm}^{-1})\,(\cos 140^0)}{(2 \text{ x } 10^{-3} \text{ m})\,(13.6 \text{ x } 10^3)\,(9.8 \text{ ms}^{-2})} ; = \frac{2 \text{ x } 0.456 \text{ x } (-0.7660)}{2 \text{ x } 13.6 \text{ x } 9.8}$$

$$= \frac{-0.6986}{266.56} ; = -2.62 \text{ x } 10^{-3} \text{ m}$$

where, negative sign indicates that there is fall of mercury (mercury is depressed) in glass tube.

112. In a normal adult, the average speed of the blood through the aorta (radius r = 0.8 cm) is 0.33 ms^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.4 cm. Calculate the speed of the blood through the arteries.

No.	Log
1.32	0.1206
30	1.4771
(–)	$\bar{2}$.6435
Antilog	4.400 x 10^{-2}

Solution

$a_1v_1 = 30a_2v_2$; $\pi r_1^2 v_1 = 30\,\pi r_2^2 v_2$; $V_2 = \frac{1}{30}\left(\frac{r_1}{r_2}\right)^2 v_1$

$V_2 = \frac{1}{30}\left(\frac{0.8 \text{ x } 10^{-2}\text{m}}{0.4 \text{ x } 10^{-2}\text{ m}}\right)^2$ x (0.33 ms^{-1}) ; V_2 =0.044 ms^{-1}

EXERCISE PROBLEM

113. A capillary of diameter d mm is dipped in water such that the water rises to a height of 30*mm*. If the radius of the capillary is made 2/3 of its previous value, then compute the height up to which water will rise in the new capillary?

Solution

Surface tension by capillary rise method is, $T = \frac{\rho r h g}{2\cos\theta}$; $h = \frac{2T\cos\theta}{\rho r g}$

$h \propto \frac{1}{r}$ (or) $hr = \text{constant}$

$h_1 r_1 = h_2 r_2$; $h_2 = \frac{h_1 r_1}{r_2}$; $= \frac{30 \times 10^{-3} \times r}{\left(\frac{2r}{3}\right)}$; $= \frac{3 \times 30 \times 10^{-3} \times r}{2r}$

$h_2 = 45 \times 10^{-3}$ m = 45mm

114. The reading of pressure meter attached with a closed pipe is 5×10^5 N m^{-2}. On opening the valve of the pipe, the reading of the pressure meter is 4.5×10^5 Nm^{-2}. Calculate the speed of the water flowing in the pipe.

Solution

Under closed state, velocity of water; v_1=0

Under open state, velocity of water; $v_2 = v$

Density of water; ρ =1000 $kg\ m^{-3}$

According to Bernoulli's theorem, for horizontal pipe

$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2$; $\frac{P_1}{\rho} - \frac{P_2}{\rho} = \frac{1}{2}v_2^2 - \frac{1}{2}v_1^2$

$\frac{P_1 - P_2}{\rho} = \frac{1}{2}(v_2^2 - v_1^2)$; $v_2^2 - v_1^2 = \frac{2}{\rho}(P_1 - P_2)$

$v^2 - 0 = \frac{2}{1000}(5 \times 10^5 - 4.5 \times 10^5)$; $v^2 = \frac{2}{1000}(5 - 4.5 \times 10^5)$

$v^2 = \frac{2}{1000} \times 0.5 \times 10^5$; $= \frac{10^5}{10^3}$; $= 10^2$; v = 10ms^{-1}

UNIT – VIII (HEAT AND THERMODYNAMICS)

115. Eiffel tower is made up of iron and its height is roughly 300 m. During winter season (January) in France the temperature is 2^0 C and in hot summer its average temperature 25^0 C. Calculate the change in height of Eiffel tower between summer and winter. The linear thermal expansion coefficient for iron $\alpha = 10 \times 10^{-6}$ per °C

Solution

$\frac{\Delta L}{L_0} = a_L \Delta T$; $\Delta L = a_L L_0 \Delta T$;

$\Delta L = 10\times10^{-6}\times300\times23 = 0.069$ m=69 mm

116. A person does 30 kJ work on 2 kg of water by stirring using a paddle wheel. While stirring, around 5 kcal of heat is released from water through its container to the surface and surroundings by thermal conduction and radiation. What is the change in internal energy of the system?

Solution

Work done on the system (by the person while stirring), W = −30 kJ = −30,000J

Heat flowing out of the system, Q = −5 kcal = −5×4184 J =−20920 J

Using First law of thermodynamics, ΔU = Q−W

ΔU = −20,920 J− (−30,000) J

ΔU = −20,920 J+30,000 J = 9080 J

Here, the heat lost is less than the work done on the system, so the change in internal energy is positive.

117. Jogging every day is good for health. Assume that when you jog a work of 500 kJ is done and 230 kJ of heat is given off. What is the change in internal energy of your body?

Solution

Work done by the system (body), W = +500 kJ

Heat released from the system (body),Q = −230 kJ

The change in internal energy of a body = ΔU =−230 kJ−500 kJ = −730 kJ

118. 500 g of water is heated from 30^0 C to 60^0C. Ignoring the slight expansion of water, calculate the change in internal energy of the water? (specific heat of water 4184 J/kg.K)

Solution

When the water is heated from 30^0C to 60^0C, there is only a slight change in its volume. So we can treat this process as isochoric. In an isochoric process the work done by the system is zero. The given heat supplied is used to increase only the internal energy. $\Delta U = Q = m s_v \Delta T$

The mass of water = 500 g =0.5 kg;

The change in temperature = 30K

The heat Q = 0.5×4184×30 = 62.76 kJ

119. During a cyclic process, a heat engine absorbs 500 J of heat from a hot reservoir, does work and ejects an amount of heat 300 J into the surroundings (cold reservoir). Calculate the efficiency of the heat engine?

Solution

The efficiency of heat engine is given by $\eta = 1 - \frac{Q_L}{Q_H}$; $\eta = 1 - \frac{300}{500}$;
$= 1 - \frac{3}{5}$; $\eta = 1 - 0.6$; 0.4

The heat engine has 40% efficiency, implying that this heat engine converts only 40% of the input heat into work.

120. There are two Carnot engines A and B operating in two different temperature regions. For Engine A, the temperatures of the two reservoirs are 150⁰C and 100⁰C. For engine B the temperatures of the reservoirs are 350⁰C and 300⁰C. Which engine has lesser efficiency?

Solution

The efficiency for engine A =$1 - \frac{373}{623}$;
= 0.11. Engine A has 11% efficiency

No.	Log
373	2.5717
423	2.6263
(−)	$\bar{1}$.9454
Antilog	8.819×10^{-1}

The efficiency for engine B =$1 - \frac{573}{623}$; = 0.08.

Engine B has 8% efficiency

No.	Log
573	2.7582
623	2.7945
(−)	$\bar{1}$.9637
Antilog	9.198×10^{-1}

121. A refrigerator has COP of 3. How much work must be supplied to the refrigerator in order to remove 200 J of heat from its interion?

Solution COP = $\beta = \frac{Q_L}{W}$; W = $\frac{Q_L}{COP}$; = $\frac{200}{3}$; = 66.67J

EXERCISE PROBLEM

122. Calculate the number of moles of air is in the inflated balloon at room temperature The radius of the balloon is 10 cm, and pressure inside the balloon is 180 kPa.

Solution

From ideal gas equation PV = μRT or $\mu = \frac{PV}{RT}$ --------1

Volume of spherical shaped balloon, V = $\frac{4}{3}\pi r^3$

= $\frac{4}{3} \times 3.14 \times (10 \times 10^{-2})^3$; $4 \times 1.046 \times 10^{-3}$;

V = 4.184×10^{-3}

From equation 1,= $\frac{180 \times 10^3 \times 4.184 \times 10^{-3}}{8.31 \times (27+273)}$;

$= \frac{180 \times 4.184}{8.31 \times 300}$;

$= \frac{180 \times 4.184}{2493}$; = 0.3021 ; **μ ≅ 0.3 moles**

No.	Log
180	2.2553
4.184	0.6216
(+) 2493	2.8769 3.3967
(−)	$\bar{1}$.4802
Antilog	3.021×10^{-1}

123. In the planet Mars, the average temperature is around -53°C and atmospheric pressure is 0.9 kPa. Calculate the number of moles of the molecules in unit volume in the planet Mars? Is this greater than that in earth?

Solution

Number of molecules per unit volume in Mars planet is $\mu_{mars} = \frac{PV}{RT}$

$= \frac{0.9 \times 1000 \times 1}{8.31 \times (-53+273)}$; $= \frac{900}{8.31 \times 220}$; $= \frac{90}{8.31 \times 22}$; $= \frac{90}{182.82}$;

μ_{mars} = 0.4922 moles

Number of molecules per unit volume in Earth planet is $\mu_{earth} = \frac{PV}{RT}$

$= \frac{101.3 \times 1000 \times 1}{8.31 \times (27+273)}$; $= \frac{101300}{8.31 \times 300}$; $= \frac{1013}{8.31 \times 3}$; $= \frac{1013}{24.93}$;

μ_{earth} = 40.46 moles

124. A man starts bicycling in the morning at a temperature around 25⁰ C, he checked the pressure of tire which is equal to be 500 kPa. Afternoon he found that the absolute pressure in the tyre is increased to 520 kPa. By assuming the expansion of tyre is negligible, what is the temperature of tyre at afternoon?

Solution

Let this is considered as isochoric process, then P = $\left[\frac{\mu R}{V}\right] T$

In morning , $P_1 = \left[\frac{\mu R}{V}\right] T_1$ ----------1 In afternoon , $P_2 = \left[\frac{\mu R}{V}\right] T_2$ ------------2

Divide equation (1) by (2)

$\frac{P_1}{P_2} = \frac{T_1}{T_2}$; $T_2 = \frac{P_1}{P_2} T_1$; $T_2 = \frac{520 \times 1000}{500 \times 1000}$ (25+273) ; $= \frac{52}{50} \times 298$

= 1.04 x 298; = 309.92 k, T_2 = 309.92 k or 36.92 ⁰c

125. For a given ideal gas 6 × 105*J* heat energy is supplied and the volume of gas is increased from 4 m3 to 6 m3 at atmospheric pressure. Calculate (a) the work done by the gas (b) change in internal energy of the gas (c) graph this process in PV and TV diagram.

Solution

(a) Work done by the gas:

Work done by the gas during isobaric expansion, W = P ΔV = P [$V_f - V_i$] ;

W = 101.3 x 10^3 x (6 – 4); = 101300 x (2) = 202600 J; **W = 202.6kJ**

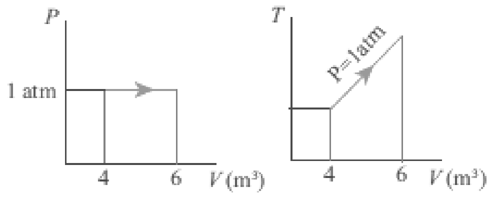
(b) Change in internal energy of the gas

From second law of thermodynamics Q = ΔU + W (or) ΔU = Q – W

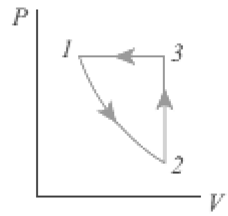
= 6 x 10^5 – 202600; ΔU = 6 x 10^5 − 2.026 x10^5;

ΔU = (6 − 2.026) x 10^5 ; = 3.974 x 10^5 = 397.4 x 10^3 J ; **ΔU = 397.4kJ**

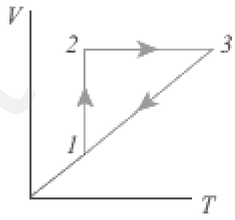
(c) PV diagram and TV diagram



126. Consider the following cyclic process consist of isotherm, isochoric and isobar which is given in the figure.
Draw the same cyclic process qualitatively in the V-T diagram where T is taken along x direction and V is taken along y-direction. Analyze the nature of heat exchange in each process.



Solution



Process 1 to 2 = increase in volume. So heat must be added.
Process 2 to 3 = Volume remains constant. Increase in temperature. The given heat is used to increase the internal energy.
Process 3 to 1: Pressure remains constant. Volume and Temperature are reduced. Heat flows out of the system. It is an isobaric compression where the work is done on the system.

127. Suppose a person wants to increase the efficiency of the reversible heat engine that is operating between 100°C and 300°C. He had two ways to increase the efficiency. (a) By decreasing the cold reservoir temperature from 100°C to 50°C and keeping the hot reservoir temperature constant (b) by increasing the temperature of the hot reservoir from 300°C to 350°C by keeping the cold reservoir temperature constant. Which is the suitable method?
Solution

Temperature of hot source; T_H=(300+273)= 573 K
Temperature of cold sink; T_L=(100+273)= 373 K

Hence efficiency of heat engine $\eta = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H}$;

$$\eta = \frac{573 - 373}{573}$$

$\eta = \frac{200}{573}$; $= 0.349$; **η = 34.9%**

No.	Log
200	2.3010
573	2.7582
(−)	$\bar{1}.5428$
Antilog	3.490×10^{-1}

In case (a) T_H = (300 + 273) = 573 K &

T_L = (50 + 273) = 323 K

$\eta = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H}$; $\eta = \frac{573-323}{573}$

$\eta = \frac{250}{573}$; $= 0.4362$; **η = 43.6%**

No.	Log
250	2.3979
573	2.7582
(−)	$\bar{1}.6397$
Antilog	4.362×10^{-1}

In case (b) T_H = (350 + 273) = 623 K &

T_L = (100 + 273) = 373 K

$\eta = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H}$; $\eta = \frac{623-373}{623}$

$\eta = \frac{250}{623}$; $= 0.4012$; **η = 40.1%**

No.	Log
250	2.3979
623	2.7945
(−)	$\bar{1}.6034$
Antilog	4.012×10^{-1}

128. A Carnot engine whose efficiency is 45% takes heat from a source maintained at a temperature of 327°C. To have an engine of efficiency 60% what must be the intake temperature for the same exhaust (sink) temperature?

Solution

Efficiency when , T_H = (327+ 273) = 600 K & η = 45% = 0.45

$\eta = 1 - \frac{T_L}{T_H}$; $0.45 = 1 - \frac{T_L}{600}$; $\frac{T_L}{600} = 1 - 0.45$; = 0.55

T_L = 0.55 x 600 = 330 K ; **T_L = 330 K or 57⁰C**

Efficiency when , T_H = (57+ 273) = 330 K & η = 60% = 0.60

$\eta = 1 - \frac{T_L}{T_H}$; $0.60 = 1 - \frac{330}{T_H}$; $\frac{330}{T_H} = 1 - 0.60$; = 0.40

$T_H = \frac{330}{0.4} = \frac{3300}{4}$; = 825 K ; **T_H = 825 K or 552⁰C**

129. An ideal refrigerator keeps its content at 0°C while the room temperature is 27°C. Calculate its coefficient of performance.

Solution

Coefficient of performance (COP) of refrigerator, when

T_H = (27 + 273) = 300 K & T_L = (0 +273) = 273 K

COP = $\beta = \frac{T_L}{T_H - T_L}$; $\beta = \frac{273}{300-273}$; $= \frac{273}{27}$;

β =10.11

No.	Log
273	2.4362
27	1.4314
(−)	1.0048
Antilog	1.011×10^1

UNIT – IX (KINETIC THEORY OF GASES)

130. A football at 27^{0}C has 0.5 mole of air molecules. Calculate the internal energy of air in the ball.

Solution

The internal energy of ideal gas = $\frac{3}{2}$ NkT

The number of air molecules is given in terms of number of moles so, rewrite the expression as U = $\frac{3}{2}$ μRT; Nk = μR. Here μ is number of moles.

Gas constant R = 8.31 $\frac{J}{mol\ k}$; Temperature T =273+27=300K

U = $\frac{3}{2}$ x 0.5 x 8.31 x 300 = 1869.75 J.

This is approximately equivalent to the kinetic energy of a man of 57 kg running with a speed of 8 ms^{-1}.

131. A room contains oxygen and hydrogen molecules in the ratio 3:1. The temperature of the room is 27°C. The molar mass of 02 is 32 g mol-1 and of H_2 is 2 g mol^{-1}. The value of gas constant R is 8.32 J mol^{-1} K^{-1}.

(a) Calculate rms speed of oxygen and hydrogen molecule.

(b) Average kinetic energy per oxygen molecule and per hydrogen molecule

(c) Ratio of average kinetic energy of oxygen molecules and hydrogen molecules.

Solution

(a) Absolute Temperature T=27°C =27+273=300 K.

Gas constant R=8.32 J mol^{-1} k^{-1}

For Oxygen molecule: Molar mass M=32 g =32 x 10^{-3} kg mol^{-1}

rms speed (V_{rms}) = $\sqrt{\frac{3RT}{M}}$; = $\sqrt{\frac{3 \times 8.32 \times 300}{32 \times 10^{-3}}}$; 483.73 ms^{-1} ≈ 484 ms^{-1}

For Hydrogen molecule: Molar mass M= 2 x 10^{-3} kg mol^{-1}

rms speed (V_{rms}) = $\sqrt{\frac{3RT}{M}}$; = $\sqrt{\frac{3 \times 8.32 \times 300}{2 \times 10^{-3}}}$; 1934 ms^{-1} ;=1.93k ms^{-1}

The rms speed of hydrogen is 4 times greater than rms speed of oxygen at the same temperature $\frac{1934}{484}$ ≈ 4.

(b) The average kinetic energy per molecule is $\frac{3}{2}$ kT

$\frac{3}{2}$ kT = $\frac{3}{2}$ x 1.38 x 10^{-23} x 300 = 6.21 x 10^{-21} J

(c) Average kinetic energy of total oxygen molecules = $\frac{3}{2}$ N_O kT

Average kinetic energy of total hydrogen molecules = $\frac{3}{2}$ N_H kT

The ratio of average kinetic energy of oxygen molecules with average kinetic energy of hydrogen molecules is 3:1.

132. Ten particles are moving at the speed of 2, 3, 4, 5, 5, 5, 6, 6, 7 and 9 m s^{-1}. Calculate rms speed, average speed and most probable speed.

Solution

The average speed $\bar{v} = \frac{2+3+4+5+5+5+6+6+7+9}{10}$; = 5.2 ms^{-1}

To find the rms speed, first calculate the mean square speed

$\overline{v^2} = \frac{2^2+3^2+4^2+5^2+5^2+5^2+6^2+6^2+7^2+9^2}{10}$; = 30.6 m^{2}s^{-2}

$V_{max} = \sqrt{\overline{v^2}} = \sqrt{30.6}$; = 5.53 ms^{-1}

The most probable speed is 5 ms^{-1} because three of the particles have that speed.

133. Find the adiabatic exponent γ for mixture of μ_1 moles of monoatomic gas and μ_2 moles of a diatomic gas at normal temperature (27^{0}C) .

Solution

The specific heat of one mole of a monoatomic gas $C_V = \frac{3}{2}R$

For μ_1 mole, $C_V = \frac{3}{2}\mu_1 R$, $C_P = \frac{5}{2}\mu_1 R$

The specific heat of one mole of a diatomic gas , $C_V = \frac{5}{2}R$

For μ_2 mole, $C_V = \frac{5}{2}\mu_2 R$, $C_P = \frac{7}{2}\mu_2 R$

specific heat of the mixture at constant volume $C_V = \frac{3}{2}\mu_1 R + \frac{5}{2}\mu_2 R$

The specific heat of the mixture at constant pressure

$C_P = \frac{5}{2}\mu_1 R + \frac{7}{2}\mu_2 R$

The adiabatic exponent $\gamma = \frac{C_P}{C_V} = \frac{5\mu_1 + 7\mu_2}{3\mu_1 + 5\mu_2}$

134. An oxygen molecule is travelling in air at 300 K and 1 atm, and the diameter of oxygen molecule is 1.2×10^{-10}m. Calculate the mean free path of oxygen molecule.

Solution

$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$.

We have to find the number density n By using ideal gas law

$n = \frac{N}{V} = \frac{P}{kT} = \frac{101.3 \times 10^3}{1.381 \times 10^{-23} \times 300}$; = 2.499 x 10^{25} molecules / m^3

$$\lambda = \frac{1}{\sqrt{2} \times \pi \times 2.449 \times 10^{25} \times (1.2 \times 10^{-10})^2}$$

$= \frac{1}{15.65 \times 10^5}$; $\lambda = 0.63 \times 10^{-6}$ m

EXERCISE PROBLEM

135. A fresh air is composed of nitrogen N_2(78%) and oxygen O_2(21%). Find the rms speed of N_2 and O_2 at 20°C.

Solution

N_A = Avogadro Number and R = 8.314 J mol^{-1} K^{-1}

1) **Nitrogen molecule (N_2)**

Atomic mass of Nitrogen = 14,

Then One Nitrogen molecule = 2×14=28

Thus 28 *g* nitrogen gas contains N_A number of nitrogen molecules Hence, Molecular mass of one mole of nitrogen molecule, M = 28 g/mol = 0.028 kg/mol

RMS speed of nitrogen molecule,

$$(v_{rms}) = \sqrt{\frac{3RT}{M}} ;= \sqrt{\frac{3 \times 8.314 \times (20+273)}{0.028}}$$

$$= \sqrt{\frac{3 \times 8.314 \times 293}{0.028}} ;= 510.9 \text{ ms}^{-1} ; (v_{rms}) = \mathbf{511 \text{ ms}^{-1}}$$

No.	Log
3	0.4771
8.314	0.9198
293	2.4669
(+)	3.8638
0.028	$\bar{2}$.4472
(−)	5.4166 x ½ = 2.7083
Antilog	5.109 x 10^2

2) Oxygen molecule (O_2)

Atomic mass of Oxygen = 16. Then One Oxygen molecule =2×16=32

Thus 32 *g* Oxygen contains NA number of oxygen molecules

Hence, Molecular mass of one mole of oxygen molecule M = 32 g/mol = 0.032kg/mol

RMS speed of oxygen molecule, (v_{rms})

$$= \sqrt{\frac{3RT}{M}} ;= \sqrt{\frac{3 \times 8.314 \times (20+273)}{0.032}}$$

$$= \sqrt{\frac{3 \times 8.314 \times 293}{0.032}} ;= 477.9 \text{ ms}^{-1} ; (v_{rms}) = \mathbf{478 \text{ ms}^{-1}}$$

No.	Log
3	0.4771
8.314	0.9198
293	2.4669
(+)	3.8638
0.032	$\bar{2}$.5051
(−)	5.3587 x ½ = 2.6793
Antilog	4.779 x 10^2

136. If the rms speed of methane gas in the Jupiter's atmosphere is 471.8 ms^{-1}, show that the surface temperature of Jupiter is sub-zero.

Solution

$$(v_{rms}) = \sqrt{\frac{3RT}{M}} \text{ or } v_{rms}^2 = \frac{3RT}{M} \text{ or } T = \frac{v_{rms}^2 M}{3R}$$

$$T = \frac{(471.8)^2 x\ 0.016}{3\ x\ 8.314} ; \frac{471.8\ x\ 471.8\ x\ 0.016}{24.942} ;$$

T = 142.8 K ≈143 K

or T = 143 – 273 ; = – 130^{0}C

Thus surface temperature of Jupiter planet is less than 0°C

No.	Log
471.0	2.6738
471.0	2.6738
0.016	$\bar{2}$.2041
(+)	3.5517
24.942	1.3969
(−)	2.1548
Antilog	1.428 x 10^2

137. Calculate the temperature at which the rms velocity of a gas triples its value at S.T.P. (standard temperature T_1 = 273 K)

Solution

At standard temperature and pressure (STP) ; $v_{rms\,1} = v$ & T_1 = 273 K

At new temperature and pressure ; $v_{rms\,2} = 3v$ & $T_2 = ?$

By definition, $v_{rms\,1} = \sqrt{\frac{3RT_1}{M}}$ -----------1 ; $v_{rms\,2} = \sqrt{\frac{3RT_2}{M}}$ -----------2

Divide equation 2 by 1

$\frac{v_{rms\,2}}{v_{rms\,1}} = \frac{\sqrt{\frac{3RT_2}{M}}}{\sqrt{\frac{3RT_1}{M}}} = \sqrt{\frac{T_2}{T_1}}$; $\left[\frac{v_{rms\,2}}{v_{rms\,1}}\right]^2 = \frac{T_2}{T_1}$; $T_2 = T_1 \left[\frac{v_{rms\,2}}{v_{rms\,1}}\right]^2$; 273 x $\left[\frac{3v}{v}\right]^2$

= 273 x 9 ; **T_2 = 2557 K**

138. A gas is at temperature 80°C and pressure 5×10^{-10}N m^{-2}. What is the number of molecules per m3 if Boltzmann's constant is 1.38×10^{-23} J K^{-1}

Solution

Ideal gas constant PV = nkT;

$n = \frac{PV}{kT} = \frac{5 \text{ x } 10^{-10} \text{x } 1}{1.38 \text{ x } 10^{-23} \text{x}(80+273)}$; $= \frac{5 \text{ x } 10^{13}}{1.38 \text{ x } 353}$

n = 1.026 x 10^{-2} x 10^{13}; **n = 1.026 x 10^{11}**

139. Calculate the mean free path of air molecules at STP. The diameter of N_2 and O_2 is about 3 x 10^{-10} m.

Solution

At STP P = 1.013 x 10^5 Nm^{-2} & T = 273K

By ideal gas equation, PV = NKT or $\frac{N}{V} = \frac{P}{kT}$ or n = $\frac{P}{kT}$

$n = \frac{P}{kT}$; $= \frac{1.013 \text{ x } 10^5}{1.38 \text{ x } 10^{-23} \text{ x } 273}$; $n = \frac{1.013 \text{ x } 10^{28}}{1.38 \text{ x } 273}$;

n = 2.688 x 10^{-3} x 10^{28}

n = 2.688 x 10^{25} molecules /m^3

The mean free path $\lambda = \frac{1}{\sqrt{2}\pi n d^2}$

$\lambda = \frac{1}{3.14 \text{ x } (3 \text{ x } 10^{-10})^2 \text{x } 2.688 \text{ x } 10^{25} \text{x } 1.414}$; $= \frac{1}{3.14 \text{ x } 9 \text{ x } 2.688 \text{ x } 1.414 \text{ x } 10^5}$

= 9.313 x 10^{-3} x 10^{-5} ; **λ = 9.313 x 10^{-8} m**

140. A gas made of a mixture of 2 moles of oxygen and 4 moles of argon at temperature T. Calculate the energy of the gas in terms of RT. Neglect the vibrational modes.

Solution

Oxygen (O_2) is a di atomic molecule. Its number of degrees of freedom f = 5

Argon (Ar) is a mono atomic molecule. Its number of degrees of freedom f =3

For mono atomic molecule, total energy of μ mole of gas,

$U_1 = \mu_1 \text{ x } \frac{3}{2} N_A kT; = \mu_1 \text{ x } \frac{3}{2} RT$

For di atomic molecule, total energy of μ mole of gas,

$U_2 = \mu_2 \text{ x } \frac{5}{2} N_A kT; = \mu_2 \text{ x } \frac{5}{2} RT$

Total energy of gas mixture $U = U_1 + U_2$

$U = \mu_1 \text{ x } \frac{3}{2} RT + \mu_2 \text{ x } \frac{5}{2} RT$; $U = 4 \text{ x } \frac{3}{2} RT + 2 \text{ x } \frac{5}{2} RT$

U = 6 RT + 5 RT; **U = 11RT**

141. Estimate the total number of air molecules in a room of capacity 25m³ at a temperature of 27°C.

Solution

Ideal gas constant PV = NkT or $N = \frac{PV}{kT}$;

$$= \frac{1.013 \text{ x } 10^5 \text{x } 25}{1.38 \text{ x } 10^{-23} \text{ x } (27 + 273)}$$

$$= \frac{1.013 \text{ x } 10^5 \text{x } 25}{1.38 \text{ x } 10^{-23} \text{ x } 300} ; = \frac{1.013 \text{ x } 25 \text{ x } 10^{28}}{414}$$

$N = 6.116 \text{ x } 10^{-3} \text{ x } 10^{28}$;

$N = 6.116 \text{ x } 10^{25}$ molecules

No.	Log
1.013 25	0.0056 1.3979
(+) 414	1.4035 2.6170
(−)	$\bar{2}.7865$
Antilog	$6.116 \text{ x } 10^{-2}$

UNIT – X (OSCILLATIONS)

142. Consider two springs whose force constants are 1 Nm^{-1} and 2 Nm^{-1} which are connected in series. Calculate the effective spring constant (k_s) and comment on k_s .

Solution

$k_1 = 1$ N m^{-1}, $k_2 = 2$ Nm^{-1} ; $k_s = \frac{k_1 k_2}{k_1+k_2}$ Nm^{-1} ; $k_s = \frac{1 \times 2}{1+2} = \frac{2}{3}$ Nm^{-1}

$k_s < k_1$ and $k_s < k_2$

Therefore, the effective spring constant is lesser than both k_1 and k_2

143. Consider two springs with force constants 1 Nm^{-1} and 2 Nm^{-1} connected in parallel. Calculate the effective spring constant (k_p) and comment on k_p.

Solution

$k_1 = 1$ N m^{-1}, $k_2 = 2$ Nm^{-1}; $k_p = k_1 + k_2$ Nm^{-1}

$k_p = 1 + 2 = 3$ Nm^{-1} ; $k_p > k_1$ and $k_p > k_2$

Therefore, the effective spring constant is greater than both k_1 and k_2.

144. If the length of the simple pendulum is increased by 44% from its original length, calculate the percentage increase in time period of the pendulum.

Solution

$T \propto \sqrt{l}$; T = constant $\sqrt{l}$

$\frac{T_f}{T_i} = \sqrt{\frac{1+\frac{44}{100}l}{l}}$; $\sqrt{1.44}$ = 1.2 ;

Therefore, $T_f = 1.2\, T_i = T_i + 20\%\, T_i$

EXERCISE PROBLEM

145. Consider a simple pendulum of length l = 0.9 m which is properly placed on a trolley rolling down on a inclined plane which is at θ = 45° with the horizontal. Assuming that the inclined plane is frictionless, calculate the time period of oscillation of the simple pendulum.

Solution

The effective value of acceleration due to gravity will be equal to the component of g normal to the inclined plane which is $g' = g\ cos\ \theta$

Then the time period is given by T = $2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{g\ cos\theta}}$

T = $2\pi\sqrt{\frac{0.9}{9.8 \text{ x } cos45^0}}; = 2\pi\sqrt{\frac{0.9}{9.8 \text{ x } \frac{1}{\sqrt{2}}}}; ; = 2 \text{ x } 3.14\sqrt{\frac{0.9}{9.8 \text{ x } 0.707}}$

T = $6.28\sqrt{\frac{0.9}{6.9286}}; = 6.28\sqrt{0.1290}$; = 6.28 x 0.3604 ; **T= 2.263s**

146. A piece of wood of mass m is floating erect in a liquid whose density is ρ. If it is slightly pressed down and released, then executes simple harmonic motion. Show that its time period of oscillation is T = $2\pi\sqrt{\frac{m}{A\,g\rho}}$

Solution

Let the wood piece of mass 'm' and area 'A' floating in liquid of density 'ρ' is pressed down by a distance 'x' and released, so that it executes SHM.

The restoring force is given by, $F = k\ x$ (or) mg = kx (or) k = $\frac{mg}{x}$;

$= \frac{(\rho v)g}{x}; = \frac{(\rho Ax)g}{x}; = \rho Ag$

The time period of vertical oscillation is, T = $2\pi\sqrt{\frac{m}{k}}; = 2\pi\sqrt{\frac{m}{A\,g\rho}}$

UNIT – XI (WAVES)

147. The average range of frequencies at which human beings can hear sound waves varies from 20 Hz to 20 kHz. Calculate the wavelength of the sound wave in these limits. (Assume the speed of sound to be 340 ms^{-1}.

Solution

$\lambda_1 = \frac{v}{f_1} = \frac{340}{20}$; = 17 m ; $\lambda_2 = \frac{v}{f_2} = \frac{340}{20 \times 10^3}$; = 0.017m

Therefore, the audible wavelength region is from 0.017 m to 17 m when the velocity of sound in that region is 340 ms^{-1}.

148. A man saw a toy duck on a wave in an ocean. He noticed that the duck moved up and down 15 times per minute. He roughly measured the wavelength of the ocean wave as 1.2 m. Calculate the time taken by the toy duck for going one time up and down and also the velocity of the ocean wave.

Solution

Given that the number of times the toy duck moves up and down is 15 times per minute. This information gives us frequency (the number of times the toy duck moves up and down)

$f = \frac{\text{15 times toy duck moves up and down}}{\text{one minute}}$

But one minute is 60 second, therefore, expressing time in terms of second.

$f = \frac{15}{60} = \frac{1}{4}$; = 0.25 Hz

The time taken by the toy duck for going one time up and down is time period which is inverse of frequency $T = \frac{1}{f} = \frac{1}{0.25}$; = 4s

The velocity of ocean wave is $v = \lambda f$ = 1.2 x 0.25 = 0.3ms^{-1}

149. Calculate the speed of sound in a steel rod whose Young's modulus Y = 2 × 10^{11} Nm^{-2} and ρ = 7800 kg m^{-3}.

Solution

No.	Log
10^{11} 3900	11.0000 3.5911
(−)	7.4089 x ½ 3.7044
Antilog	5.062×10^3

$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{7800}}$; $\sqrt{0.2564 \times 10^8}$;

= 0.506 x 10^4 ms^{-1} ; **= 5 x 10^3 ms^{-1}**

Therefore, longitudinal waves travel faster in a solid than in a liquid or a gas. Now you may understand why a shepherd checks before crossing railway track by keeping his ears on the rails to safeguard his cattle.

150. An increase in pressure of 100 kPa causes a certain volume of water to decrease by 0.005% of its original volume. (a) Calculate the bulk modulus of water? (b) Compute the speed of sound (compressional waves) in water?

Solution

(a) Bulk modulus $B = V\left|\frac{\Delta P}{\Delta V}\right| = \frac{100 \times 10^3}{0.005 \times 10^{-2}}$; $= \frac{100 \times 10^3}{5 \times 10^{-5}}$; = 2000Mpa, where Mpa is mega pascal

(b) Speed of sound in water is $v = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2000 \times 10^6}{1000}}$ = 1414 ms^{-1}

151. Suppose a man stands at a distance from a cliff and claps his hands. He receives an echo from the cliff after 4 second. Calculate the distance between the man and the cliff. Assume the speed of sound to be 343 ms^{-1}.

Solution

The time taken by the sound to come back as echo is $2t = 4 \Rightarrow t = 2$ s

$\therefore$The distance is $d = vt = (343 \text{ m s}^{-1})(2 \text{ s}) = 686$ m.

152. A mobile phone tower transmits a wave signal of frequency 900MHz. Calculate the length of the waves transmitted from the mobile phone tower.

Solution

Frequency, f = 900 MHz; = 900×10^6 Hz

The speed of wave is $c = 3 \times 10^8 \text{ms}^{-1}$

$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{900 \times 10^6}$; = 0.33m

153. Two vibrating tuning forks produce waves whose equation is given by $y_1 = 5 \sin(240\pi t)$ and $y_2 = 4 \sin(244\pi t)$. Compute the number of beats per second.

Solution

Given $y_1 = 5 \sin(240\pi t)$ and $y_2 = 4 \sin(244\pi t)$

Comparing with $y = A \sin(2\pi f_1 t)$, we get

$2\pi f_1 = 240\pi \Rightarrow f_1 = 120$Hz ; $2\pi f_2 = 244\pi \Rightarrow f_2 = 122$Hz

The number of beats produced is $|f_1 - f_2| = |120 - 122| = |-2|$

=2 beats per sec

154. A baby cries on seeing a dog and the cry is detected at a distance of 3.0 m such that the intensity of sound at this distance is 10^{-2} Wm^{-2}. Calculate the intensity of the baby's cry at a distance 6.0 m.

Solution

I_1 is the intensity of sound detected at a distance 3.0 m and it is given as 10^{-2} W m^{-2}.

Let I_2 be the intensity of sound detected at a distance 6.0 m. Then,

$r_1 = 3.0$ m, $r_2 = 6.0$ m and since, $I \propto \frac{1}{r^2}$

the power output does not depend on the observer and depends on the baby. Therefore, $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$; $I_2 = I_1 \frac{r_1^2}{r_2^2}$; $I_2 = 0.25 \times 10^{-2}$ Wm^{-2}

155. The sound level from a musical instrument playing is 50 dB. If three identical musical instruments are played together then compute the total intensity. The intensity of the sound from each instrument is 10^{-12} W m^{-2}

Solution

$\Delta L = 10\log_{10}\left[\frac{I_1}{I_0}\right]$ = 50 dB ; $\log_{10}\left[\frac{I_1}{I_0}\right]$ = 5 dB

$\frac{I_1}{I_0} = 10^5$; $I_1 = 10^5 I_0$; $= 10^5 \times 10^{-12}$ Wm^{-2}; $I_1 = 10^{-7}$ Wm^{-2}

Since three musical instruments are played, therefore $I_{total} = 3I_1 = 3 \times 10^{-7}$ Wm^{-2}

156. If a flute sounds a note with 450Hz, what are the frequencies of the second, third, and fourth harmonics of this pitch? If the clarinet sounds with a same note as 450Hz, then what are the frequencies of the lowest three harmonics produced.

Solution

For a flute which is an open pipe, we have
Second harmonics $f_2 = 2\ f_1 = 900$ Hz
Third harmonics $f_3 = 3\ f_1 = 1350$ Hz
Fourth harmonics $f_4 = 4\ f_1 = 1800$ Hz
For a clarinet which is a closed pipe, we have
Second harmonics $f_2 = 3\ f_1 = 1350$ Hz
Third harmonics $f_3 = 5\ f_1 = 2250$ Hz
Fourth harmonics $f_4 = 7\ f_1 = 3150$ Hz

157. If the third harmonics of a closed organ pipe is equal to the fundamental frequency of an open organ pipe, compute the length of the open organ pipe if the length of the closed organ pipe is 30 cm.

Solution

Let l_2 be the length of the open organ pipe, with l_1=30 cm the length of the closed organ pipe.
It is given that the third harmonic of closed organ pipe is equal to the fundamental frequency of open organ pipe.

The third harmonic of a closed organ pipe $f_2 = \frac{v}{\lambda_2} = \frac{3v}{4l_1}$; $= 3f_1$

The fundamental frequency of open organ pipe is $f_1 = \frac{v}{\lambda_1} = \frac{v}{2l_2}$;

Therefore $\frac{v}{2l_2} = \frac{3v}{4l_1}$; $l_2 = \frac{2l_1}{3}$ = 20 cm

158. A frequency generator with fixed frequency of 343 Hz is allowed to vibrate above a 1.0 m high tube. A pump is switched on to fill the water slowly in the tube. In order to get resonance, what must be the minimum height of the water? (speed of sound in air is 343 m s^{-1})

Solution

The wavelength , $\lambda = \frac{C}{f}$; $\lambda = \frac{343\ \text{ms}^{-1}}{343\ \text{Hz}}$; = 1.0m
Let the length of the resonant columns be L_1, L_2 and L_3.

The first resonance occurs at length L_1. $L_1 = \frac{\lambda}{4} = \frac{1}{4}$ = 0.25 m

The second resonance occurs at length L_2. $L_2 = \frac{3\lambda}{4} = \frac{3}{4}$ = 0.75 m

The third resonance occurs at length L_3. $L_3 = \frac{5\lambda}{4} = \frac{5}{4}$ = 1.25 m and so on.

Since total length of the tube is 1.0 m the third and other higher resonances do not occur. Therefore, the minimum height of water H_{min} for resonance is, H_{min} = 1.0 m – 0.75 m = 0.25 m

159. A student performed an experiment to determine the speed of sound in air using the resonance column method. The length of the air column that resonates in the fundamental mode with a tuning fork is 0.2 m. If the length is varied such that the same tuning fork resonates with the first overtone at 0.7 m. Calculate the end correction.

Solution

End correction e= $\frac{L_2 - 3L_1}{2}$; = $\frac{0.7-3\,(0.2)}{2}$; = 0.05 m

160. Consider a tuning fork which is used to produce resonance in an air column. A resonance air column is a glass tube whose length can be adjusted by a variable piston. At room temperature, the two successive resonances observed are at 20 cm and 85 cm of the column length. If the frequency of the length is 256 Hz, compute the velocity of the sound in air at room temperature.

Solution

Two successive length (resonance) to be L_1 = 20 cm and L_2 = 85 cm

The frequency is f = 256 Hz; v = f λ = 2f ΔL = 2f ($L_2 - L_1$)

= 2 × 256 × (85 − 20) × 10^{-2} m s^{-1}; v = 332.8 m s^{-1}

161. A sound of frequency 1500 Hz is emitted by a source which moves away from an observer and moves towards a cliff at a speed of 6 ms^{-1}.

(a) Calculate the frequency of the sound which is coming directly from the source.

(b) Compute the frequency of sound heard by the observer reflected off the cliff. Assume the speed of sound in air is 330 ms^{-1}.

Solution

(a) Source is moving away and observer is stationary, therefore, the frequency of sound heard directly from source is

$f' = \left[\frac{v}{v+v_s}\right] f = \left[\frac{330}{330+6}\right]$ x 1500 = 1473 Hz

(b) Sound is reflected from the cliff and reaches observer, therefore,

$f' = \left[\frac{v}{v-v_s}\right] f = \left[\frac{330}{330-6}\right]$ x 1500 = 1528 Hz

162. An observer observes two moving trains, one reaching the station and other leaving the station with equal speeds of 8 m s−1. If each train sounds its whistles with frequency 240 Hz, then calculate the number of beats heard by the observer.

Solution

Observer is stationary

(i) Source (train) is moving towards an observer:

The observed frequency due to train arriving station is

$f_{in} = \left[\frac{v}{v-v_s}\right] f = \left[\frac{330}{330-8}\right]$ x 240 = 246 Hz

(ii) Source (train) is moving away from an observer:
The observed frequency due to train leaving station is

$f_{out} = \left[\frac{v}{v+v_s}\right] f = \left[\frac{330}{330+8}\right]$ x 240 = 234 Hz

So the number of beats = $|f_{in} - f_{out}|$ = (246 – 234) = 12

EXERCISE PROBLEM

163. The speed of a wave in a certain medium is 900 m/s. If 3000 waves passes over a certain point of the medium in 2 minutes, then compute its wavelength?

Solution

Since 3000 waves passes over in 2 minutes (120 s), the number of waves passes per second is, $f = \frac{3000}{120}$; = 25 per second.

The wavelength $\lambda = \frac{v}{f} = \frac{900}{25}$; λ =36m

164. A ship in a sea sends SONAR waves straight down into the seawater from the bottom of the ship. The signal reflects from the deep bottom bed rock and returns to the ship after 3.5 s. After the ship moves to 100 km it sends another signal which returns back after 2s. Calculate the depth of the sea in each case and also compute the difference in height between two cases.

Solution

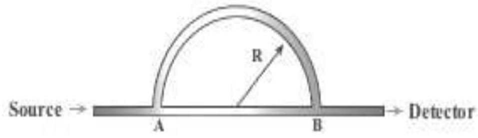
Depth at first place, $d_1 = \frac{v \times t_1}{2}$; $= \frac{1533 \times 3.5}{2}$; $= \frac{5365.5}{2}$; d_1 =2682.75m

Depth at second place, $d_2 = \frac{v \times t_2}{2}$; $= \frac{1533 \times 2}{2}$; $= \frac{5365.5}{2}$; d_1 =1533m

The difference in height between two cases $\Delta d = d_1 - d_2$

= 2682.75 – 1533 ; Δd = 1149.75m

165. A sound wave is transmitted into a tube as shown in figure. The sound waves splits into two waves at the point A which recombine at point B. Let R be the radius of the semi-circle which is varied until the first minimum. Calculate the radius of the semi-circle if the wavelength of the sound is 50.0 m.



Solution

The length of semi-circle from A to B; $r_1 = \pi R$ = 3.14R

The length of straight line from A to B; r_2 = 2R

Hence path difference, $\Delta r = r_1 - r_2$ = 3.14R – 2R

= R (3.14 − 2) = 1.14R

From the condition of minimum intensity,

$\Delta r = n \frac{\lambda}{2}$ (Here, n = 1, 3, 5,.......)

Hence condition for first minimum, $\Delta r = n \frac{\lambda}{2}$

(or) 1.14R = $\frac{\lambda}{2}$

$R = \frac{\lambda}{2 \times 1.14}$; $R = \frac{50}{2 \times 1.14}$; $= \frac{50}{2.28}$ R = 21.93m

No.	Log
50	1.6990
2.28	0.3579
(−)	1.3411
Antilog	2.193×10^1

166. N tuning forks are arranged in order of increasing frequency and any two successive tuning forks give n beats per second when sounded together. If the last fork gives double the frequency of the first (called as octave), Show that the frequency of the first tuning fork is f = (N−1)n.

Solution

Number of tuning forks = N

Let f be the frequency of first tuning fork and 'n' be the number beats per second when two successive forks are sounding together, then the frequencies of N tuning forks in ascending order,

f, f$\pm$n, f$\pm$2n, f$\pm$3n, .. , 2 f

This is similar to the AP having 'n' number of terms

(i.e.) a, a+d, a+2d, a+3d,......

We know the nth term in AP is, $t_n = a + (n-1)d$

Here, the frequency of last tuning fork is 2f, $then$

2 f = f + (N − 1)n ; 2 f − f = (N − 1) n ; **f = (N − 1) $\boldsymbol{n}$**

167. Consider two organ pipes of same length in which one organ pipe is closed and another organ pipe is open. If the fundamental frequency of closed pipe is 250 Hz. Calculate the fundamental frequency of the open pipe.

Solution

The third harmonic of a closed organ pipe $f_c = \frac{v}{4L}$ -----------1

The fundamental frequency of open organ pipe is $f_0 = \frac{v}{2L}$ ----------2

Divide equation 2 by 1 $\frac{f_0}{f_c} = \frac{\left[\frac{v}{2L}\right]}{\left[\frac{v}{4L}\right]}$ = 2 $f_0 = 2f_c$ = 2 x 250 = 500Hz

168. A police in a siren car moving with a velocity 20 ms^{-1} chases a thief who is moving in a car with a velocity V_0 ms^{-1}. The police car sounds at frequency 300Hz, and both of them move towards a stationary siren of frequency 400Hz. Calculate the speed in which thief is moving. (Assume the thief does not observe any beat)

Solution

Velocity of sound v =330 m/s

Velocity of a police siren car v_s=20 m/s

Frequency of a police siren car f =300Hz

Frequency of police siren heard by thief is

$$f_1 = \left[\frac{v - v_0}{v - v_s}\right] f; = \left[\frac{330 - v_0}{330 - 20}\right] x\ 300\ ; = \left[\frac{330 - v_0}{310}\right] x\ 300$$

Frequency of stationary siren f =400Hz

Frequency of stationary siren heard by thief

$$f_2 = \left[\frac{v + v_0}{v}\right] f; = \left[\frac{330 + v_0}{330}\right] x\ 400$$

It there are no beats then $f_1 = f_2$

$$\left[\frac{330 - v_0}{310}\right] x\ 300 = \left[\frac{330 + v_0}{330}\right] x\ 400$$

$$(330 - v_0)\ x\ 0.9677 = (330 + v_0)\ x\ 1.2121$$

$319.341 - 0.9677v_0 = 399.993 + 1.2121v_0$

$1.2121v_0 + 0.9677v_0 = 319.341 - 399.993$

$2.1798v_t = -80.652\ ;\ v_0 = \frac{80.652}{2.1798}\ ; = 36.99$ $\boldsymbol{v_0}$**= 37m/s**

“NeH i kahd K awrpapy; fp i l j;j
n t wwpapd; %y khf f; fp i l f;Fk; k fpo r;rpapd;
rpfuj; i j ahUk; m s f;fNt K bahJ.”