XII MATHEMATICS

PUB. DATE : 27/03/23

PUB. TIME : 10:45 P.M

MARCH - 2023 PUBLIC EXAMINATION

ANSWER KEYS

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QUESTION NO	TYPE - A	TYPE - B
1	b) $P(A) = n$	a) $1/e^2$
2	d) 1	b) $x \in [1/2, 1]$
3	b) $x \in [1/2, 1]$	a) 3
4	a) $y = kx$	a) $8/3$
5	a) 3	b) z
6	c) 0	b) $\log 2$
7	b) 2	d) 2
8	d) 2	d) 1
9	a) $1/e^2$	a) $-\pi/6$
10	d) z	d) $-A$
11	a) $8/3$	d) n
12	a) $\pi/2$	c) 5
13	c) 5	c) 2
14	c) 2	a) $y = kx$
15	b) z	b) $2ab$
16	d) $-A$	a) $\pi/2$
17	b) $\log 2$	b) $P(A) = n$
18	d) n	b) 2
19	a) $-\pi/6$	d) z
20	b) $2ab$	c) 0

PART – B

QUESTION NO	ANSWER	MARKS
21	Let $\|z_1\| = \|z\| = 2$ $\|z_2\| = \|3+4i\| = 5$ We know that $\|\|z_1\| - \|z_2\|\| \le \|z_1+z_2\| \le \|z_1\|+\|z_2\|$ $\|2-5\| \le \|z+3+4i\| \le 2+5$ $3 \le \|z+3+4i\| \le 7$	2
22	$lx^2+nx+n=0$ $p+q = -n/l$ and $pq = n/l$ $\sqrt{p/q}+\sqrt{q/p}+\sqrt{n/l} = \frac{-n/l\sqrt{n/l}}{\sqrt{-n/l}} + \frac{\sqrt{n}}{\sqrt{l}} = 0$	1 1
23	Condition for line tangent to the circle is $c^2 = a^2(1+m^2)$ $c^2 = 9(17)$ $\boxed{c = \pm 3\sqrt{17}}$	1 1
24	Given $r = 10$ cm $dr = 9.9 - 10 = -0.1$ Volume of sphere $\boxed{V = 4/3\,\pi r^3}$ $dv = 4/3 \cdot \pi 3r^2 dr$ $= -40\pi$ cm^3	1 1
25	$\int_b^\infty \frac{1}{a^2+x^2}dx = \left[\frac{1}{a}\tan^{-1}(x/a)\right]_b^\infty$ $= 1/a\left[\tan^{-1}(\infty) - \tan^{-1}(a/b)\right]$ $= 1/a\left[\pi/2 - \tan^{-1}\left(\frac{b}{a}\right)\right]$	1 1

QUESTION NO	ANSWER	MARKS
26	Given $p = 7$ $\vec{d} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ $\hat{d} = \frac{\vec{d}}{\lvert\vec{d}\rvert} = \frac{1}{5\sqrt{2}}(3\hat{i} - 4\hat{j} + 5\hat{k})$ The vector equation of the plane is $\vec{r} \cdot \hat{d} = p$ $\vec{r} \cdot \frac{1}{5\sqrt{2}}(3\hat{i} - 4\hat{j} + 5\hat{k}) = 7$	1 1
27	$A \vee B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $A \wedge B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$	1 1
28	$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ $AA^T = A^TA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ A is Orthogonal $[\because AA^T = A^TA = I]$	1 1
29	$y = x^2 + 3x - 2$ $dy/dx = 2x + 3$ at $(1,2)$ $m = 5$ Equation of tangent : $5x - y - 3 = 0$	1 1
30	Let $y = e^{(\cos\theta + i\sin\theta)}$ Take log on both side $\log y = \log e^{(\cos\theta + i\sin\theta)} = \cos\theta + i\sin\theta$ $= \cos\theta + i\sin\theta = a + ib$ form (OR)	2

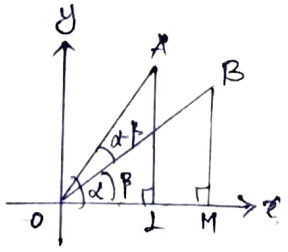
QUESTION NO	ANSWER	MARKS
30	$e^{\cos\theta + i\sin\theta} = e^{\cos\theta}\, e^{i\sin\theta}$	1
	$= e^{\cos\theta}\cos(\sin\theta) + i e^{\cos\theta}\sin(\sin\theta)$	1
	PART - III	
31	The equation of Parabola is $(x+1)^2 = 4a(y+2)$	1
	It Passes $(3,6) \Rightarrow a = 1/2$	1
	$(x+1)^2 = 2(y+2)$ or $x^2 + 2x - 2y - 3 = 0$	1
32	The maximum distance $= a + ae = 152 \times 10^6$	1
	The minimum distance $= a - ae = 94.5 \times 10^6$	
	$(1) - (2) \Rightarrow 2ae = (152 - 94.5) \times 10^6$	
	$= 57.5 \times 10^6$	1
	$= 575 \times 10^5$	
	The distance from the Sun to the other focus $= 575 \times 10^5$	1
33	The Domain is $-1 < 3x - 1 < 0$	1
	$0 < 3x < 1$	
	$0 < x < 1/3$ (or) $x \in (0, 1/3)$	2
34	$\vec{b} = 2\hat{i} + 2\hat{j} - \hat{k}$	
	$\lvert\vec{b}\rvert = \sqrt{4+4+1} = \sqrt{9} = 3$	
	The direction Cosines of the straight line are $2/3, 2/3, -1/3$	1

QUESTION NO	ANSWER	MARKS
	$\cos\alpha = 2/3 \cdot \cos\beta = 2/3 \cdot \cos\gamma = -1/3$ $\alpha = \cos^{-1}(2/3) \cdot \beta = \cos^{-1}(2/3) \cdot \gamma = \cos^{-1}(-1/3)$	2
35	Let $f(x) = (x)^{2/3}$, $x_0 = 125$ $\Delta x = -2$	1
	$f'(x) = 2/3\, x^{-1/3} = \frac{2}{3x^{1/3}}$	1
	Let $f(x_0) + f'(x_0)(x - x_0) = L(x)$ $(123)^{2/3} \simeq 24.73$	1
36	$\cos y\, dy = \frac{e^x(x\log x + 1)}{x}\, dx$ $= e^x\left(\log x + 1/x\right) dx$	1
	Integrating we get $\sin y = e^x \log x + c$	2
37	$\lvert F(\alpha)\rvert = 1$ $\text{adj}\,[F(\alpha)] = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$	1
	$[F(\alpha)]^{-1} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$ $= F(-\alpha)$	2
38		2
	Question / Answer	1

P	q	P→q	q→P
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

QUESTION NO	ANSWER	MARKS
39	$z = (2+3i)(1-i) = 5+i$	
	$z^{-1} = 1/z = \frac{1}{5+i} = \frac{10-2i}{52}$	2
	$z^{-1} = \frac{5}{26} - \frac{i}{26}$	1
40	$a+b+c=0$	
	$b+c=-a$, $c+a=-b$, $a+b=-c$	
	$(b+c-a)x^2 + (c+a-b)x + (a+b-c) = 0$	
	$(-2a)x^2 + (-2b)x + (-2c) = 0$	
	$-2ax^2 - 2bx - 2c = 0$	2
	$\div -2 \Rightarrow ax^2 + bx + c = 0$	
	$\Delta = b^2 - 4ac = (c-a)^2 > 0$	1
	The Roots are Rational	

PART - IV

QUESTION NO.	ANSWER	MARKS
41 (a)	$z^3 + 8i = 0 \quad z = [2^3(-i)]^{1/3}$ $= 2[\cos(-\pi/2) + i\sin(-\pi/2)]^{1/3}$ $= 2[\cos(2k\pi - \pi/2) + i\sin(2k\pi - \pi/2)]^{1/3}$ $= 2[\cos(4k-1)\pi/6 + i\sin(4k-1)\pi/6]$ $\Rightarrow k = 0,1,2$	1 2
	Put $k = 0,1,2 \Rightarrow$ $z = \sqrt{3} - i,\ 2i,\ -\sqrt{3} - i$ The values of z are $\sqrt{3} - i, 2i, -\sqrt{3} - i$	2
41 (b)	$(1 + x + xy^2)\frac{dy}{dx} = -(y + y^3)$ $\frac{dx}{dy} + \frac{1}{y}x = -\frac{1}{y(1+y^2)}$ $I.F = y$	1
	The General solution is $x(I.F) = \int Q(I.F)\,dy + C$ $xy = \int \frac{-1}{y(1+y^2)}\, y\,dy + C$ $xy = \tan^{-1}y + C$	2 2
42 (a)	 Let $\vec{OA} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$ $\vec{OB} = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$	1

QUESTION NO	ANSWER	MARKS
	$\vec{OA}\cdot\vec{OB} = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ —— (1)	1
	By defn, $\vec{OA}\cdot\vec{OB} = \cos(\alpha-\beta)$ —— (2)	2
	From (1) & (2) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$	1
A2 (b)	(i) $k = 1/400$	1
	(ii) $F(x) = \begin{cases} 0 & \text{for } x<200 \\ \frac{x}{400} - \frac{1}{2} & \text{for } 200 \le x \le 600 \\ 1 & \text{for } x>600 \end{cases}$	2
	(iii) $P(300 < x < 500) = 1/2$	2
A3 (a)	$18x^2 + 12y^2 - 144x + 48y + 120 = 0$	
	$\frac{(x-4)^2}{12} + \frac{(y+2)^2}{18} = 1$	1
	$\frac{x^2}{12} + \frac{y^2}{18} = 1$	
	$a^2 = 18 \Rightarrow a = 3\sqrt{2}$ $b^2 = 12 \Rightarrow b = 2\sqrt{3}$	1
	$(ae)^2 = a^2 - b^2 = ae = \sqrt{6}$ and $e = 1/\sqrt{3}$ $a/e = 3\sqrt{6}$	1
	Center : $(4,-2)$ Vertex : $A(4, 3\sqrt{2}-2), A'(4, -3\sqrt{2}-2)$ Focus : $S(4, \sqrt{6}-2)$ $S'(4, -\sqrt{6}-2)$ Equation of directrices $y = \pm a/e \Rightarrow y = -2 \pm 3\sqrt{6}$	2

QUESTION NO	ANSWER	MARKS
43 (b)	$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ $\alpha + \beta + \cos^{-1}z = \pi$ $\alpha + \beta = \pi - \cos^{-1}z$ — (1)	1
	$\cos(\alpha+\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ $\cos(\pi - \cos^{-1}z) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$ $-\cos(\cos^{-1}z) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$ $z + xy = \sqrt{1-x^2}\sqrt{1-y^2}$	2
	Squaring on both sides $(z+xy)^2 = (1-x^2)(1-y^2)$ $z^2 + x^2y^2 + 2xyz = 1 - y^2 - x^2 + x^2y^2$ $x^2 + y^2 + z^2 + 2xyz = 1$	2
44 (a)	$36a - 6b + c = 8$ — (1) $4a - 2b + c = -12$ — (2) $9a + 3b + c = 8$ — (3)	1
	$[A\|B] = \begin{bmatrix} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{bmatrix}$	1
	$\sim \begin{bmatrix} 4 & -2 & 1 & -12 \\ 36 & -6 & 1 & 8 \\ 9 & 3 & 1 & 8 \end{bmatrix}$ $R_2 \leftrightarrow R_1$	
	$\sim \begin{bmatrix} 1 & -1/2 & 1/4 & -3 \\ 0 & 12 & -8 & 116 \\ 0 & 0 & 6 & -60 \end{bmatrix}$ $R_3 \to 12R_3 - R_2$	
	$a = 1 \quad b = 3 \quad c = -10$	1
	$y = x^2 + 3x - 10$	1
	$60 = 60$ yes, he meet his friend	1

QUESTION NO	ANSWER	MARKS
44 (b)	$x^2+4y^2=8$ —— (1) $x^2-2y^2=4$ —— (2) $y^2=1/6$ and $x^2=16/3$ $m_1=-\frac{x}{4y}$ and $m_2=-\frac{x}{2y}$ $m_1 \times m_2=\left(\frac{-x}{4y}\right)\left(\frac{-x}{2y}\right)=\frac{x^2}{8y^2}$ $m_1 \times m_2=\frac{-16/3}{8(1/6)}=-1$ So, the given curves cut orthogonally	1 2 1 1
45 (a)	Let $\vec{a}=\hat{i}-\hat{j}+3\vec{k}$ $\vec{u}=2\hat{i}-\hat{j}+4\vec{k}$ $\vec{v}=\hat{i}+2\hat{j}+\hat{k}$ Parametric form: $\vec{r}=(\hat{i}-\hat{j}+3\vec{k})+s(2\hat{i}-\hat{j}+4\vec{k})+t(\hat{i}+2\hat{j}+\hat{k})$ Cartesian form: $\begin{vmatrix} x-1 & y+1 & z-3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix}=0$ $9x-2y-5z+4=0$	1 2 2

QUESTION NO	ANSWER	MARKS
45 (b)	Given $\frac{1}{3}$ is a solution 2 is a solution	1

$\frac{1}{3}$	6	−5	−38	−5	6
	0	2	−1	−13	−6
3	6	−3	−39	−18	0
	0	18	45	18	0
	6	15	6	0	

2

$6x^2 + 15x + 6 = 0$ is a factor

$2x^2 + 5x + 2 = 0$ 1

$x = -\frac{1}{2}, x = -2$

∴ The solutions are $-2, -\frac{1}{2}, 3, \frac{1}{3}$ 1

46 (a) $P \rightarrow (\neg q \vee r) \equiv \neg P \vee (\neg q \vee r)$

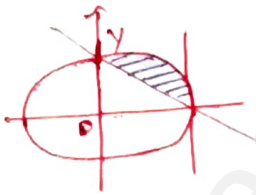
P	q	r	¬P	¬q	P→(¬q∨r)	¬P∨(¬q∨r)
T	T	T	F	F	T	T
T	T	F	F	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

4

Last two Columns are Identical

$P \rightarrow (\neg P \vee r) \equiv \neg P \vee (\neg q \vee r)$ 1

QUESTION NO	ANSWER	MARKS
A6 (B)	Let A be the amount at time t $A = Ce^{kt}$ $A = Ce^{0.05t}$	1
	$t=0 \Rightarrow A = 10{,}000$ $C = 10{,}000$	2
	$t = 1.5 \Rightarrow A = 10{,}000\, e^{0.075}$	2
A7 (a)	$y = \frac{\log x}{x}$ $\frac{dy}{dx} = \frac{1 - \log x}{x^2}$ $\frac{d^2y}{dx^2} = \frac{-3 + 2\log x}{x^3}$ $dy/dx = 0 \Rightarrow x = e$	2
	If $x = e \Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{e^3} < 0$ (It has local maximum)	1
	when $x = e \Rightarrow y = 1/e$	1
	The local maximum value is : $1/e$	

QUESTION NO	ANSWER	MARKS
47 (b)	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ — (1) $\frac{x}{a}+\frac{y}{b}=1$ — (2)	1
	Point of Intersections (0, b) and (a, 0)	
	(1) $\Rightarrow$ $y=\frac{b}{a}\sqrt{a^2-x^2}$ (2) $\Rightarrow$ $y=\frac{b}{a}(a-x)$	1
	Required Area $= A = \int_0^a \left[\frac{b}{a}\sqrt{a^2-x^2}-\frac{b}{a}(a-x)\right]dx$ $= \frac{ab}{4}(\pi-2)$ sq units.	2
	 	1

அரசு வழங்கும் விடைக்குறிப்பிற்கும் நான் உருவாக்கியுள்ள விடைக்குறிப்பிற்கும் மதிப்பெண் பங்கீடுதலில் மாற்றங்கள் உள்ளன.

விடைக்குறிப்பில் பிழைகள் இருப்பின் மன்னிக்கவும்

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31/3/22
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