

**COMMON PUBLIC EXAMINATION -MARCH -2023**  
**XII - MATHEMATICS**      **TENTATIVE ANSWER KEY**

**PART – I**

Q.No	CODE – A	CODE – B	MARKS
1.	(b) $\rho(A) = n$	(a) $\frac{1}{e^2}$	1
2.	(d) 1	(b) $x \in \left[\frac{1}{2}, 1\right]$	1
3.	(b) $x \in \left[\frac{1}{2}, 1\right]$	(a) 3	1
4.	(a) $y=kx$	(a) $\frac{8}{3}$	1
5.	(a) 3	(b) z	1
6.	(c) 0	(b) $\log 2$	1
7.	(b) 2	(d) 2	1
8.	(d) 2	(d) 1	1
9.	(a) $\frac{1}{e^2}$	(a) $\frac{-\pi}{6}$	1
10.	(d) z	(d) - 4	1
11.	(a) $\frac{8}{3}$	(d) n	1
12.	(a) $\frac{\pi}{2}$	(c) 5	1
13.	(c) 5	(c) 2	1
14.	(c) 2	(a) $y=kx$	1
15.	(b) z	(b) $2ab$	1
16.	(d) - 4	(a) $\frac{\pi}{2}$	1
17.	(b) $\log 2$	(b) $\rho(A) = n$	1
18.	(d) n	(b) 2	1
19.	(a) $\frac{-\pi}{6}$	(d) z	1
20.	(b) $2ab$	(c) 0	1

**PART – II**

21.	Let $ z_1 = z =2$ and $ z_2 = 3+4i =5$ We know that $  z_1 - z_2   \leq  z_1+z_2  \leq  z_1 + z_2 $ $ 2-5  \leq  z+3+4i  \leq 2+5$ $3 \leq  z+3+4i  \leq 7$	2
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22.	$lx^2 + nx + n = 0 \Rightarrow p+q = \frac{-n}{l}$ and $pq = \frac{n}{l}$ $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = \frac{-\frac{n}{l}\sqrt{\frac{n}{l}}}{\sqrt{\frac{-n}{l}}} + \frac{\sqrt{n}}{\sqrt{l}} = 0$	1 1
23.	Condition for line tangent to the circle is $c^2 = a^2(1+m^2)$ $c^2 = 9 \quad (17)$ $c = \pm 3\sqrt{17}$	2
24.	Given: $r = 10\text{cm}$ and $dr = 9.9-10=-0.1$ volume of a sphere $V = \frac{4}{3}\pi r^3$ $dV = \frac{4}{3}\pi 3r^2 dr$ $= -40\pi cm^3$	1 1
25.	$\int_b^\infty \frac{1}{a^2+x^2} dx = \left[ \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) \right]_b^\infty$ $= \frac{1}{a} \left[ \tan^{-1}(\infty) - \tan^{-1} \left( \frac{a}{b} \right) \right]$ $= \frac{1}{a} \left[ \frac{\pi}{2} - \tan^{-1} \left( \frac{b}{a} \right) \right]$	1 1
26.	Given $p = 7$ and $\vec{d} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ $\hat{d} = \frac{\vec{d}}{ \vec{d} } = \frac{1}{5\sqrt{2}} (3\hat{i} - 4\hat{j} + 5\hat{k})$ The vector equation of the plane is $\hat{r} \cdot \hat{d} = p$ $\hat{r} \cdot \frac{1}{5\sqrt{2}} (3\hat{i} - 4\hat{j} + 5\hat{k}) = 7$	2
27.	$A \vee B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $A \wedge B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$	1 1
28.	Let $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ ; $A^T = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ $AA^T = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$ similarly $A^T A = I_2$ $\therefore AA^T = A^T A = I_2$ $\therefore A$ is orthogonal	1
29.	$y = x^2 + 3x - 2$ $\frac{dy}{dx} = 2x + 3 \text{ at } (1,2) \Rightarrow m = 5$ Equation of the tangent: $5x - y - 3 = 0$	1 1
30.	$e^{\cos \theta + i \sin \theta} = e^{\cos \theta} e^{i \sin \theta}$ $= e^{\cos \theta} [\cos(\sin \theta) + i \sin(\sin \theta)] \text{ (or)}$ $= e^{\cos \theta} \cos(\sin \theta) + i e^{\cos \theta} \sin(\sin \theta)$	2

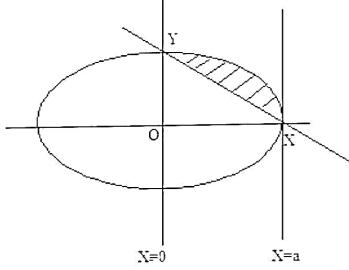
**PART – III**

31.	<p>The equation of parabola is <math>(x+1)^2 = 4a(y+2)</math>          It passes <math>(3,6) \Rightarrow a = \frac{1}{2}</math>  <math>(x+1)^2 = 2(y+2)</math> (or) <math>x^2 + 2x - 2y - 3 = 0</math></p>	1 1 1
32.	<p>The maximum distance = <math>a+ae=152 \times 10^6</math>----- (1)          The minimum distance = <math>a-ae= 94.5 \times 10^6</math>----- (2)  <math>(1)-(2) \quad 2ae = (152- 94.5) \times 10^6</math>  <math>= 57.5 \times 10^6</math>  <math>= 575 \times 10^5</math>          The distance from the Sun to the other focus = <math>575 \times 10^5</math></p>	1 2
33.	<p>The Domain is <math>-1 &lt; 3x - 1 &lt; 0</math>  <math>0 &lt; 3x &lt; 1</math>  <math>0 &lt; x &lt; \frac{1}{3}</math> (or) <math>x \in (0, \frac{1}{3})</math></p>	1 2
34.	<p><math>\hat{b} = 2\hat{i} + 2\hat{j} - \hat{k}</math> and <math>\ \hat{b}\  = \sqrt{4+4+1} = \sqrt{9} = 3</math>          The direction cosines of the straight line are <math>\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}</math>  <math>\cos \alpha = \frac{2}{3}, \cos \beta = \frac{2}{3}</math> and <math>\cos \gamma = \frac{-1}{3}</math>  <math>\alpha = \cos^{-1}\left(\frac{2}{3}\right), \beta = \cos^{-1}\left(\frac{2}{3}\right)</math> and <math>\gamma = \cos^{-1}\left(\frac{-1}{3}\right)</math></p>	1 2
35.	<p>Let <math>f(x) = (x)^{\frac{2}{3}}, x_0 = 125</math> and <math>\Delta x = -2</math>  <math>f'(x) = \frac{2}{3}x^{\frac{-1}{3}} \Rightarrow f'(x) = \frac{2}{3x^{\frac{1}{3}}}</math>  <math>L(x) = f(x_0) + f'(x_0)(x - x_0)</math>  <math>L(x) = (125)^{\frac{2}{3}} + \frac{2}{3(125)^{\frac{1}{3}}} (x - 125)</math>  <math>(123)^{\frac{2}{3}} \approx 24.73</math></p>	1 1 1 1
36.	$\begin{aligned} \cos y dy &= \frac{e^x(x \log x + 1)}{x} dx \\ &= e^x \left( \log x + \frac{1}{x} \right) dx \end{aligned}$ <p>Integrating we get,  <math>\sin y = e^x \log x + c</math></p>	1 2
37.	<p><math> F(\alpha)  = 1</math>  <math>\text{adj}[F(\alpha)] = \begin{vmatrix} \cos \alpha &amp; 0 &amp; -\sin \alpha \\ 0 &amp; 1 &amp; 0 \\ \sin \alpha &amp; 0 &amp; \cos \alpha \end{vmatrix}</math>  <math>[F(\alpha)]^{-1} = \begin{vmatrix} \cos \alpha &amp; 0 &amp; -\sin \alpha \\ 0 &amp; 1 &amp; 0 \\ \sin \alpha &amp; 0 &amp; \cos \alpha \end{vmatrix} = [F(-\alpha)]</math></p>	1 1 2

38. <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tr><td><math>p</math></td><td><math>q</math></td><td><math>p \rightarrow q</math></td><td><math>q \rightarrow p</math></td></tr> <tr><td>T</td><td>T</td><td>T</td><td>T</td></tr> <tr><td>T</td><td>F</td><td>F</td><td>T</td></tr> <tr><td>F</td><td>T</td><td>T</td><td>F</td></tr> <tr><td>F</td><td>F</td><td>T</td><td>T</td></tr> </table>	$p$	$q$	$p \rightarrow q$	$q \rightarrow p$	T	T	T	T	T	F	F	T	F	T	T	F	F	F	T	T	Last two columns are not equal $p \rightarrow q \neq q \rightarrow p$	2   1
$p$	$q$	$p \rightarrow q$	$q \rightarrow p$																			
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T	F	F	T																			
F	T	T	F																			
F	F	T	T																			
39. $Z = (2+3i)(1-i) = 5+i$ $Z^{-1} = \frac{1}{z} = \frac{1}{5+i} = \frac{10-24i}{52}$ $Z^{-1} = \frac{5}{26} - \frac{1}{26}i$		2  1																				
40. $a+b+c=0$ and $b+c=-a, c+a=-b, a+b=-c$ $(b+c-a)x^2 + (c+a-b)x + (a+b-c) = 0$ $(-2a)x^2 + (-2bx)x + (-2c) = 0$ $ax^2 + bx + c = 0$ $\Delta = b^2 - 4ac = (c-a)^2 > 0$ The roots are rational		2  1																				
<b>PART – IV</b>																						
41.(a) $z^3 + 8i = 0 \Rightarrow z = [2^3(-i)]^{\frac{1}{3}}$ $= 2 \left[ \cos\left(\frac{-\pi}{2}\right) + i \sin\left(\frac{-\pi}{2}\right) \right]^{\frac{1}{3}}$ $= 2 \left[ \cos\left(2k\pi - \frac{\pi}{2}\right) + i \sin\left(2k\pi - \frac{\pi}{2}\right) \right]^{\frac{1}{3}}$ $= 2 \left[ \cos(4k-1)\frac{\pi}{2} + i \sin(4k-1)\frac{\pi}{2} \right]^{\frac{1}{3}}$ $z = 2 \left[ \cos(4k-1)\frac{\pi}{6} + i \sin(4k-1)\frac{\pi}{6} \right] \quad k = 0, 1, 2$	1  1  1  1																					
$\therefore k = 0, 1, 2 \Rightarrow z = \sqrt{3}-i, 2i, -\sqrt{3}-i$ $\therefore$ The values of $z$ are $\sqrt{3}-i, 2i$ and $-\sqrt{3}-i$	2																					
<b>Note:</b> Any other method																						
41.(b) $(1+x+xy^2) \frac{dy}{dx} = -(y+y^3)$ $\frac{dx}{dy} + \frac{1}{y}x = -\frac{1}{y(1+y^2)}$ $I.F = y$ $\therefore$ The general solution is $x(I.F) = \int Q(I.F) dy + C$ $xy = \int \frac{-1}{y(1+y^2)} y dy + C$ $\therefore xy = -\tan^{-1}y + C$	1  1  1  1  2																					

42.(a)	<p>Diagram</p> <p><math>\overrightarrow{OA} = \cos \alpha \vec{i} + \sin \alpha \vec{j}; \overrightarrow{OB} = \cos \beta \vec{j} + \sin \beta \vec{j}</math></p> <p><math>\overrightarrow{OA} \cdot \overrightarrow{OB} = \cos \alpha \cos \beta + \sin \alpha \sin \beta \dots \dots (1)</math></p> <p>By definition</p> <p><math>\overrightarrow{OA} \cdot \overrightarrow{OB} =  \overrightarrow{OA}   \overrightarrow{OB}  \cos (\alpha - \beta) = \cos (\alpha - \beta) \dots \dots (1)</math></p> <p>From (1) and (2)</p> <p><math>\cos (\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta</math></p>	1
		1
42.(b)	<p>i) <math>k = \frac{1}{400}</math></p> <p>ii) <math>F(x) = \begin{cases} 0 &amp; \text{for } x &lt; 200 \\ \frac{x}{400} - \frac{1}{2} &amp; \text{for } 200 \leq x \leq 600 \\ 1 &amp; \text{for } x &gt; 600 \end{cases}</math></p> <p>iii) <math>P(300 &lt; x &lt; 500) = \frac{1}{2}</math></p>	<p>1</p> <p>3</p> <p>1</p>
43.(a)	<p><math>18x^2 + 12y^2 - 144x + 48y + 120 = 0</math></p> <p><math>\frac{(x-4)^2}{12} + \frac{(y+2)^2}{18} = 1 \Rightarrow \frac{x^2}{12} + \frac{y^2}{18} = 1</math></p> <p><math>a^2 = 18 \Rightarrow a = 3\sqrt{2}; b^2 = 12 \Rightarrow b = 2\sqrt{3}</math></p> <p><math>(ae)^2 = a^2 - b^2 = ae = \sqrt{6} \text{ and } e = \frac{1}{\sqrt{3}}; \frac{a}{e} = 3\sqrt{6}</math></p> <p>Center : (4, -2), Vertex : A(4, <math>3\sqrt{2}</math> - 2), A<sup>1</sup>(4, <math>-3\sqrt{2}</math> - 2)</p> <p>Focus : S(4, <math>\sqrt{6}</math> - 2), S<sup>1</sup>(4, -<math>\sqrt{6}</math> - 2)</p> <p>Equation of diectrices <math>Y = \pm \frac{a}{e} \Rightarrow y = -2 \pm 3\sqrt{6}</math></p>	<p>1</p> <p>2</p> <p>1</p> <p>1</p>
43.(b)	<p><math>\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi</math></p> <p><math>\alpha + \beta + \cos^{-1} z = \pi \Rightarrow \alpha + \beta = \pi - \cos^{-1} z \dots \dots (1)</math></p> <p><math>\cos(\alpha + \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta</math></p> <p><math>\cos(\pi - \cos^{-1} z) = xy - \sqrt{1 - x^2} \sqrt{1 - y^2}</math></p> <p><math>-\cos(\cos^{-1} z) = xy - \sqrt{1 - x^2} \sqrt{1 - y^2}</math></p> <p><math>z + xy = \sqrt{1 - x^2} \sqrt{1 - y^2}</math></p> <p>Squaring on bothsides, we get,</p> <p><math>(z + xy)^2 = (1 - x^2)(1 - y^2)</math></p> <p><math>z^2 + x^2 y^2 + 2xyz = 1 - y^2 - x^2 + x^2 y^2</math></p> <p><math>x^2 + y^2 + z^2 + 2xyz = 1</math></p>	<p>1</p> <p>2</p> <p>2</p> <p>2</p>
44.(a)	<p><math>36a - 6b + c = 8 \dots \dots (1)</math>, <math>4a - 2b + c = -12 \dots \dots (2)</math>, <math>9a + 3b + c = 8 \dots \dots (3)</math></p> <p><math>[A B] = \left[ \begin{array}{ccc c} 36 &amp; -6 &amp; 1 &amp; 8 \\ 4 &amp; -2 &amp; 1 &amp; -12 \\ 9 &amp; 3 &amp; 1 &amp; 8 \end{array} \right] \sim \left[ \begin{array}{ccc c} 4 &amp; -2 &amp; 1 &amp; -12 \\ 36 &amp; -6 &amp; 1 &amp; 8 \\ 9 &amp; 3 &amp; 1 &amp; 8 \end{array} \right] R_2 \leftrightarrow R_2</math></p> <p><math>\sim \left[ \begin{array}{ccc c} 1 &amp; \frac{-1}{2} &amp; \frac{1}{4} &amp; -3 \\ 0 &amp; 12 &amp; -8 &amp; 116 \\ 0 &amp; 0 &amp; 6 &amp; -60 \end{array} \right] R_3 \rightarrow 12R_3 - R_2</math></p> <p><math>\therefore a = 1, b = 3 \text{ and } c = -10</math></p>	<p>1</p> <p>2</p>

	$y = x^2 + 3x - 10$ $60 = 60$ Yes, he meet his friend	2																																																																								
44.(b)	$x^2 + 4y^2 = 8 \dots\dots\dots(1)$ and $x^2 - 2y^2 = 4 \dots\dots\dots(2)$ $y^2 = \frac{4}{8}$ and $x^2 = \frac{16}{3}$ $m_1 = -\frac{x}{4y}$ and $m_2 = -\frac{x}{2y}$ $m_1 \times m_2 = \left(-\frac{x}{4y}\right) \times \left(\frac{x}{2y}\right) = \frac{x^2}{8y^2}$ $m_1 \times m_2 = -\frac{\frac{16}{3}}{8\left(\frac{2}{3}\right)} = -1$ So, the given curves cut orthogonally.	1 2 1 1																																																																								
45.(a)	$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ ; $\vec{u} = 2\hat{i} - \hat{j} + 4\hat{k}$ ; $\vec{v} = \hat{i} + 2\hat{j} + \hat{k}$ <b>Parametric form</b> $\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + s(2\hat{i} - \hat{j} + 4\hat{k}) + t(\hat{i} + 2\hat{j} + \hat{k})$ <b>Cartesian form</b> $\begin{vmatrix} x - 1 & y + 1 & z - 3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$ $9x - 2y - 5z + 4 = 0$	2 1 2																																																																								
45.(b)	Given $\frac{1}{3}$ is a solution. $\therefore 3$ is an other root. $6x^2 + 15x + 6 = 0$ is another factor. $2x^2 + 5x + 2 = 0$ $x = \frac{-1}{2}, x = -2$ $\therefore$ The solutions of the given equation are $\frac{1}{3}, 3, \frac{-1}{2}, -2$	$\begin{array}{ ccccc } \hline & & & & \\ \hline 6 & -5 & -38 & -5 & 6 \\ 0 & 2 & -1 & -13 & -6 \\ \hline 6 & -3 & -39 & -18 & 0 \\ 0 & 18 & 45 & 18 & \\ \hline 6 & 15 & 6 & 0 & \\ \hline \end{array}$ 1 1 2 1																																																																								
46.(a)	$p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>p</th><th>q</th><th>r</th><th><math>\neg p</math></th><th><math>\neg q</math></th><th><math>(\neg q \vee r)</math></th><th><math>p \rightarrow (\neg q \vee r)</math></th><th><math>\neg p \vee (\neg q \vee r)</math></th> </tr> </thead> <tbody> <tr><td>T</td><td>T</td><td>T</td><td>F</td><td>F</td><td>T</td><td>T</td><td>T</td></tr> <tr><td>T</td><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td></tr> <tr><td>T</td><td>F</td><td>T</td><td>F</td><td>T</td><td>T</td><td>T</td><td>T</td></tr> <tr><td>T</td><td>F</td><td>F</td><td>F</td><td>T</td><td>T</td><td>T</td><td>T</td></tr> <tr><td>F</td><td>T</td><td>T</td><td>T</td><td>F</td><td>T</td><td>T</td><td>T</td></tr> <tr><td>F</td><td>T</td><td>F</td><td>T</td><td>F</td><td>T</td><td>T</td><td>T</td></tr> <tr><td>F</td><td>F</td><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td></tr> <tr><td>F</td><td>F</td><td>F</td><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td></tr> </tbody> </table> Last two columns are identical. $\therefore p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$	p	q	r	$\neg p$	$\neg q$	$(\neg q \vee r)$	$p \rightarrow (\neg q \vee r)$	$\neg p \vee (\neg q \vee r)$	T	T	T	F	F	T	T	T	T	T	F	F	F	F	F	F	T	F	T	F	T	T	T	T	T	F	F	F	T	T	T	T	F	T	T	T	F	T	T	T	F	T	F	T	F	T	T	T	F	F	T	T	T	T	T	T	F	F	F	T	T	T	T	T	5
p	q	r	$\neg p$	$\neg q$	$(\neg q \vee r)$	$p \rightarrow (\neg q \vee r)$	$\neg p \vee (\neg q \vee r)$																																																																			
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46.(b) Let A be amount at time t $\frac{dA}{dt} \propto A \Rightarrow \frac{dA}{dt} = kA$ $\Rightarrow \frac{dA}{dt} = 0.05A \Rightarrow A = Ce^{0.05t}$ When $t = 0, A = 10,000 \Rightarrow C=10,000$ When $t = 1.5 \Rightarrow A = 10,000 e^{0.075}$	1 1 1 2
47.(a) $y = \frac{\log x}{x}$ $\frac{dy}{dx} = \frac{1-\log x}{x^2}$ and $\frac{d^2y}{dx^2} = \frac{-3+2\log x}{x^3}$ $\frac{dy}{dx} = 0 \Rightarrow x = e$ If $x = e \Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{e^3} < 0$ (It has local maximum) When $x = e \Rightarrow y = \frac{1}{e}$ The local max value is: $\frac{1}{e}$	2 1 1
47.(b) Diagram $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \dots \dots \dots (1)$ $\frac{x}{a} + \frac{y}{b} = 1 \quad \dots \dots \dots (2)$ Point of intersection is: $(0,b)$ and $(a,0)$ $(1) \Rightarrow y = \frac{b}{a} \sqrt{a^2 - x^2}$ $(2) \Rightarrow y = \frac{b}{a}(a-x)$ Required area $A = \int_0^a \left[ \frac{b}{a} \sqrt{a^2 - x^2} - \frac{b}{a}(a-x) \right] dx$ $= \frac{ab}{4} (\pi - 2)$ sq units.	1  1 2 1