LONDON KRISHNAMOORTHY MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

+2

2023 MODEL FULL PORTION QUESTION PAPERS - 1

MATHEMATICS

MARKS: 100 | **TIME: 3 HOURS + 15 MIN.**

I CHOOSE THE CORRECT ANSWERS: $\mathbf{20 \times 1 = 20}$

1. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 4 (d) 3

2. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then $adj(adjA)$ is

$(a)\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ $(b)\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ $(c)\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ $(d)\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

3. If $\omega = \text{cis}\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

$(a)1$ $(b)2$ $(c)3$ $(d)4$

4. If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is

$(a)\ \frac{1}{2}$ (b) 1 (c) 2 (d) 3

5. The value of $\sum_{i=1}^{13}(i^n + i^{n-1})$ is

$(a) 1 + i$ $(b) i$ $(c) 1$ $(d) 0$

6. A zero of $x^3 + 64$ is

$(a)0$ $(b)4$ $(c)4i$ $(d) - 4$

7. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to

$(a)\ \frac{1}{2}$ $(b)\ \frac{1}{\sqrt{5}}$ $(c)\ \frac{2}{\sqrt{5}}$ $(d)\ \frac{\sqrt{3}}{2}$

8. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ then $\cos^{-1}x + \cos^{-1}y$ is equal to

$(a)\ \frac{2\pi}{3}$ $(b)\ \frac{\pi}{3}$ $(c)\ \frac{\pi}{6}$ $(d)\ \pi$

9. The equation of the normal to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ which is parallel to the line $2x + 4y = 3$ is

$(a)\ x + 2y = 3$ $(b)\ x + 2y + 3 = 0$ $(c) 2x + 4y + 3 = 0$ $(d) x - 2y + 3 = 0$

10. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$. One of the points of contact of tangents on the hyperbola is

$(a)\ \left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ $(b)\ \left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $(c)\ \left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $(d)(3\sqrt{3}, 2\sqrt{2})$

11. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$, then (α, β) is

$(a)\ (-5,5)$ $(b)\ (-6,7)$ $(c)\ (5,-5)$ $(d)\ (6,-7)$

12. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

$(a)\ \frac{\pi}{6}$ $(b)\ \frac{\pi}{4}$ $(c)\ \frac{\pi}{3}$ $(d)\ \frac{\pi}{2}$

13. What is the value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

(a) 0 (b) 1 (c) 2 $(d)\infty$

14. The minimum value of the function $|3-x|+9$is

$(a)\ 0$ $(b)\ 3$ $(c)\ 6$ $(d)\ 9$

15. If $f(x,y)=e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$is equal to

$(a)\ xye^{xy}$ $(b)\ (1+xy)e^{xy}$ $(c)\ (1+y)e^{xy}$ $(d)\ (1+x)e^{xy}$

16. The value of $\int_{-1}^{2}|x|\,dx$is

$(a)\frac{1}{2}$ $(b)\frac{3}{2}$ $(c)\frac{5}{2}$ $(d)\frac{7}{2}$

17. The value of $\int_0^1(\sin^{-1}x)^2\,dx$ is

$(a)\frac{\pi^2}{4}-1$ $(b)\frac{\pi^2}{4}+2$ $(c)\frac{\pi^2}{4}+1$ $(d)\frac{\pi^2}{4}-2$

18. The solution of the differential equation $\frac{dy}{dx}+\frac{1}{\sqrt{1-x^2}}=0$ is

$(a)\ y+\sin^{-1}x=c$ $(b)\ x+\sin^{-1}y=0$ $(c) y^2+2\sin^{-1}x=C$ $(d)\ x^2+2\sin^{-1}y=0$

19. buses carry, respectively, 42, 36, 34, and 48 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on that bus. Then $E[X]$ and $E[Y]$ respectively are

$(a)\ 50,40$ $(b)\ 40,50$ $(c)\ 40.75,40$ $(d)\ 41,41$

20. The probability function of a random variable is defined as:

x	-2	-1	0	1	2
$f(x)$	k	$2k$	$3k$	$4k$	$5k$

Then $E(X)$ is equal to:

$(a)\frac{1}{15}$ $(b)\frac{1}{10}$ $(c)\frac{1}{3}$ $(d)\frac{2}{3}$

II ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER32 IS COMPULSARY. **$8\times2=16$**

21. Verify the property $(A^T)^{-1}=(A^{-1})^T$, with $A=\begin{bmatrix}2 & 9\\1 & 7\end{bmatrix}$.**(EG 1.8)**

22. Simplify the following $\sum_{n=1}^{102}i^n$**(EG 2.1)**

23. Solve the equation :$x^4-14x^2+45=0$.**(EX 3.3 - 7)**

24. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.**(EG 4.5)**

25. Find the general equation of a circle with centre $(-3,-4)$and radius 3 units. **(EG 5.1)**

26. If $\vec{a}=-3\vec{i}-\vec{j}+5\vec{k},\ \vec{b}=\vec{i}-2\vec{j}+\vec{k},\ \vec{c}=4\vec{j}-5\vec{k}$, find $\vec{a}.(\vec{b}\times\vec{c})$.**(EG 6.12)**

27. Evaluate: $\lim_{x\to\infty}\left(\frac{e^x}{x^m}\right), m\in\mathbb{N}$.**(EG 7.42)**

28. Find differential dy for each of the following functions: $y=\frac{(1-2x)^3}{3-4x}$**(EX 8.2 – 1)**

29. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x\cos x\,dx$.**(EG 9.24)**

30. Find the differential equation of the family of (i) all non-vertical lines in aplane (ii) all nonhorizontal lines in a plane. **(EX 10.3 – 1)**

31. Construct the truth table for the following statements. $\neg p\wedge\neg q$ **(EX 12.2 – 6)**

32. The mean and standard deviation of a binomial variate$X$ are respectively 6 and 2. Find the probability mass function **(EX 11.5 – 7)**

III ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER44 IS COMPULSARY. **$8\times3=24$**

33. If $=\begin{bmatrix}4 & 3\\2 & 5\end{bmatrix}$, find x and $y$such that $A^2+xA+yI_2=0_2$.Hence, find A^{-1}.**(EG 1.10)**

34. The complex numbers u, v and $w$are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$.**(EX 2.4 - 4)**
35. If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid. **(EX 3.1 - 1)**
36. Find the equation of the parabola whose vertex is $(5, -2)$and focus $(2, -2)$.**(EG 5.15)**
37. Using vector method, prove that if the diagonals of a parallelogram are equal, then it is a rectangle. **(EX 6.1 - 5)**
38. A particle is fired straight up from the ground to reach a height of $s$feet in $t$seconds, where $s(t) = 128t - 16t^2$. (a)Compute the maximum height of the particle reached.(b)What is the velocity when the particle hits the ground? **(EG 7.5)**
39. Assuming $\log_{10} e = 0.4343$, find an approximate value of$\log_{10} 1003$.**(EX 8.2 – 4)**
40. Prove that $\int_0^\infty e^{-x} x^n dx = n!$, where n is a positive integer. **(EG 9.43)**
41. Form the differential equation by eliminating the arbitrary constants$A$ and B from $y = A \cos x + B \sin x$.**(EG 10.3)**
42. If $X \sim B(n, p)$ such that $4P(X = 4) = P(x = 2)$and $n = 6$. Find the distribution, mean and standard deviation. **(EX 11.5 – 8)**
43. Using the equivalence property, S.T. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.**(EG 12.19)**
44. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$,if $6x^2 < 1$.**(EG 4.27)**

IV ANSWERS THE FOLLOWING QUESTIONS: $\mathbf{8 \times 5 = 40}$

45. $(\boldsymbol{a})$In a $T20$ match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy −coordinate system in the vertical plane and the ball traversed through the points $(10,8), (20,16), (30,18)$ can you conclude that Chennai Super Kings won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is $(70,0)$).**(EG 1.26)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Determine $k$and solve the equation $2x^3 - 6x^2 + 3x + k = 0$if one of its roots is twice the sum of the other two roots. **(EX 3.3 - 4)**

46. $(\boldsymbol{a})$A bridge has a parabolic arch that is $10m$high in the centre and $30m$wide at the bottom. Find the height of the arch $6m$from the centre, on either sides. **(EX 5.5 - 1)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$If $z = x + iy$and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$,then show that $x^2 + y^2 + 3x - 3y + 2 = 0$.**(EX 2.7 - 6)**

47. $(\boldsymbol{a})$Find the non-parametric form of vector equation, and cartesian equation of the plane passing through the point $(2,3,6)$ and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$.**(EX 6.7 – 1)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Show that the line $x - y + 4 = 0$is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact. **(EX 5.4 - 3)**

48. $(\boldsymbol{a})$If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.**(EG 8.22)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours? **(EX 10.8 – 1)**

49. $(\boldsymbol{a})$If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights (i)exactly 10 will have a useful life of at least 600 hours; (ii)at least 11 will have a useful life of at least 600 hours; (iii)at least 2 will not have a useful life of at least 600 hours. **(EX 11.5 – 6)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$If we blow air into a balloon of spherical shape at a rate of 1000 cm^3 per second. At what rate the radius of the balloon changes when the radius is $7cm$? Also compute the rate at which the surface area changes. **(EG 7.7)**

50. $(\boldsymbol{a})$We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume? **(EG 7.62)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})(i)$ Define an operation$*$ on $\mathbb{Q}$as follows: $(a * b) = \frac{a+b}{2}, a, b \in \mathbb{Q}$. Examine the closure, commutative, and associative properties satisfied by $*$ on $\mathbb{Q}$. (ii) Define an operation$*$ on $\mathbb{Q}$as follows: $(a * b) = \frac{a+b}{2}, a, b \in \mathbb{Q}$. Examine the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$.**(EX 12.1 – 5)**

51. $(\boldsymbol{a})$Simplify: $\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$.**(EX 4.5 – 8**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Find the inverse of each of the following by Gauss-Jordan method: $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$**(EX 1.2 - 3)**

52. $(\boldsymbol{a})$Find the value of (iii) $cos\left(sin^{-1}\left(\frac{4}{5}\right) - tan^{-1}\left(\frac{3}{4}\right)\right)$**(EX 4.3 - 4)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Using integration, find the area of the region which is bounded by x −axis, the tangent and normal to the circle $x^2 + y^2 = 4$ drawn at $(1, \sqrt{3})$.**(EG 9.61)**

LONDON KRISHNAMOORTHY MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

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2023 MODEL FULL PORTION QUESTION PAPERS - 2

MATHEMATICS

MARKS: 100 TIME: 3 HOURS + 15 MIN.

I CHOOSE THE CORRECT ANSWERS: $20 \times 1 = 20$

1. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ then $(A^T)^{-1}$
   $(a) \begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ $(b) \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ $(c) \begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ $(d) \begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$
2. If $A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then $A =$
   $(a) \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ $(b) \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ $(c) \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ $(d) \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$
3. The product of all four values of $\left(\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is
   $(a) -2$ $(b) -1$ $(c) 1$ $(d) 2$
4. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is
   $(a)\ \frac{1}{i+2}$ $(b)\ \frac{-1}{i+2}$ $(c)\ \frac{-1}{i-2}$ $(d)\ \frac{1}{i-2}$
5. A polynomial equation in x of degree n always has
   (a) n distinct roots (b) n real roots (c) n imaginary roots (d) at most one root.
6. If $|x| \le 1$, then $2\tan^{-1}x - \sin^{-1}\frac{2x}{1+x^2}$ is equal to
   $(a)\ \tan^{-1}x$ $(b)\ \sin^{-1}x$ $(c)\ 0$ $(d)\ \pi$
7. $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for
   $(a) -\pi \le x \le 0$ $(b) 0 \le x \le \pi$ $(c) \frac{-\pi}{2} \le x \le \frac{\pi}{2}$ $(d) \frac{-\pi}{4} \le x \le \frac{3\pi}{4}$
8. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
   $(a)\ 2ab$ $(b)\ ab$ $(c)\ \sqrt{ab}$ $(d)\ \frac{a}{b}$
9. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if
   $(a)\ 15 < m < 65$ $(b)\ 35 < m < 85$ $(c) -85 < m < -35$ $(d) -35 < m < -15$
10. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is
    $(a)\ \frac{\pi}{6}$ $(b)\ \frac{\pi}{4}$ $(c)\ \frac{\pi}{3}$ $(d)\ \frac{\pi}{2}$
11. The angle between the line $\vec{r} = (\vec{i} + 2\vec{j} - 3\vec{k}) + t(2\vec{i} + \vec{j} - 2\vec{k})$ and the plane $\vec{r}.(\vec{i} + \vec{j}) + 4 = 0$ is $(a)\ 0°$ $(b)\ 30°$ $(c)\ 45°$ $(d)\ 90°$
12. The number given by the Rolle's Theorem for the function $x^3 - 3x^2, x \in [0,3]$ is
    $(a)\ 1$ $(b)\ \sqrt{2}$ $(c)\ \frac{3}{2}$ $(d)\ 2$
13. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is
    $(a) -4\sqrt{3}$ $(b)\ -4$ $(c) \frac{\sqrt{3}}{12}$ $(d) 4\sqrt{3}$
14. A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is
    $(a)\ 0.2\%$ $(b)\ 0.4\%$ $(c)\ 0.04\%$ $(d)\ 0.08\%$

15. The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x −axis is

$(a)\ \pi a^3$ $(b)\ \frac{\pi a^3}{4}$ $(c)\ \frac{\pi a^3}{5}$ $(d)\ \frac{\pi a^3}{6}$

16. If $\int_0^a \frac{1}{4+x^2}dx$ then a is

(a) 4 (b) 1 (c) 3 (d) 2

17. The differential equation representing the family of curves $y = A\cos(x+B)$, where A and B are parameters, is

$(a)\ \frac{d^2y}{dx^2} - y = 0$ $(b)\ \frac{d^2y}{dx^2} + y = 0$ $(c)\ \frac{d^2y}{dx^2} = 0$ $(d)\ \frac{d^2x}{dy^2} = 0$

18. If p and q are the order and degree of the differential equation $y\frac{dy}{dx} + x^3\frac{d^2y}{dx^2} + xy = \cos x$,when

$(a)\ p < q$ $(b)\ p = q$ $(c)\ p > q$ (d) $p$exists, q does not exist

19. If $f(x) = \begin{cases} 2x, 0 \le x \le a \\ 0,\ Otherwise \end{cases}$ is a probability density function of a random variable, then the value of a is

(a) 1 (b) 2 (c) 3 (d) 4

20. Which one of the following statements has the truth value$T$?

(a) sin $x$is an even function. (b)Every square matrix is non-singular

(c)The product of complex number and its conjugate is purely imaginary

(d) 5 is an irrational number

II ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER32 IS COMPULSARY. $\mathbf{8 \times 2 = 16}$

21. If $A$is symmetric, prove that then $adj\ A$ is also symmetric. **(EG 1.7)**

22. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$.**(EG 2.4)**

23. Find the monic polynomial equation of minimum degree with real coefficients having $2-\sqrt{3}i$as a root. **(EG 3.8)**

24. For what value of $x$does $\sin x = \sin^{-1} x$?**(EX 4.1 - 5)**

25. Find the equation of the circle described on the chord $3x + y + 5 = 0$of the circle $x^2 + y^2 = 16$as diameter. **(EG 5.2)**

26. If $\vec{a} = \vec{i} - \vec{k},\ \vec{b} = x\vec{i} + \vec{j} + (1-x)\vec{k},\ \vec{c} = y\vec{i} + x\vec{j} + (1+x-y)\vec{k}$show that $[\vec{a}, \vec{b}, \vec{c}]$depends on neither $x$nor y.**(EX 6.2 - 8)**

27. If the volume of a cube of side length $x$is $v = x^3$. Find the rate of change of the volume with respect to $x$when $x = 5$ units. **(EX 7.1 – 4)**

28. Evaluate: $\int_0^\infty e^{-ax}x^n dx$,where $a > 0$.**(EG 9.44)**

29. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images. **(EX 11.1 – 1)**

30. Compute $P(X = k)$ for the binomial distribution, $B(n,p)$ where $n = 10, p = \frac{1}{5}, k = 4$ **(EX 11.5 – 1)**

31. Construct the truth table for the following statements. $\neg(p \wedge \neg q)$**(EX 12.2 – 6)**

32. Let $g(x,y) = 2y + x^2, x = 2r - s, y = r^2 + 2s,\ r,s \in \mathbb{R}$. Find $\frac{\partial g}{\partial r}$.**(EG 8.20)**

III ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER44 IS COMPULSARY. $\mathbf{8 \times 3 = 24}$

33. Find a matrix A if $adjA = \begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$.**(EG 1.5)**

34. If $|z| = 2$ show that $3 \le |z+3+4i| \le 7$.**(EG 2.13)**

35. If the roots of $x^3 + px^2 + qx + r = 0$,are in $H.P.$, prove that$9pqr = 27r^3 + 2p$.**(EG 3.21)**

36. Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$. **(EG 4.28)**

37. Find the angle between the lines $\vec{r} = (\vec{\imath} + 2\vec{\jmath} + 4\vec{k}) + t(2\vec{\imath} + 2\vec{\jmath} + \vec{k})$and the straight line passing through the points $(5,1,4)$ and $(9,2,12)$.**(EG 6.29)**

38. Show that there lies a point on the curve $f(x) = x(x+3)e^{\frac{-\pi}{2}}, -3 \le x \le 0$where tangent drawn is parallel to the x −axis. **(EX 7.3 – 9)**

39. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$.**(EX 8.7 – 4)**

40. Find the area of the region bounded by $2x - y + 1 = 0$, $y = -1$, $y = 3$ and y −axis. **(EX 9.8 – 2)**

41. Solve the following differential equations: $\sin\frac{dy}{dx} = a, y(0) = 1$**(EX 10.5 – 4)**

42. A six sided die is marked ‘2’on one face, ‘3’on two of its faces, and ‘4’on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images. **(EX 11.1 – 5)**

43. Let $*$ be defined on $\mathbb{R}$ by $(a * b) = a + b + ab - 7$. Is$*$ binary on $\mathbb{R}$? If so, find $3 * \left(\frac{-7}{15}\right)$.**(EX 12.1 – 3)**

44. If the normal at the pointt_1 on the parabola $y^2 = 4ax$meets the parabola again at the point t_2 then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.**(EX 5.4 - 8)**

IV ANSWERS THE FOLLOWING QUESTIONS: **$8 \times 5 = 40$**

45. $(\boldsymbol{a})$Solve the following system of linear equations by matrix inversion method:
$x + y + z - 2 = 0, 6x - 4y + 5z - 31 = 0, 5x + 2y + 2z = 13$**(EX 1.3 - 1)**
$(\boldsymbol{OR})$
$(\boldsymbol{b})$Solve the equation $z^3 + 8i = 0$, where $z \in C$.**(EG 2.34)**

46. $(\boldsymbol{a})$Solve $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left(\cot^{-1}\left(\frac{3}{4}\right)\right)$.**(EG 4.29)**
$(\boldsymbol{OR})$
$(\boldsymbol{b})$Prove by vector method that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.**(EG 6.5)**

47. $(\boldsymbol{a})$Find the equation of the circle passing through the points$(1,1), (2,-1)$ and $(3,2)$.**(EG 5.10)**
$(\boldsymbol{OR})$
$(\boldsymbol{b})$Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression. **(EX 3.3 - 3)**

48. $(\boldsymbol{a})$If $v(x,y) = \log\left(\frac{x^2+y^2}{\sqrt{x+y}}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$.**(EX 8.7 – 5)**
$(\boldsymbol{OR})$
$(\boldsymbol{b})$Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years? **(EX 10.8 – 6)**

49. $(\boldsymbol{a})$Using integration, find the area of the region which is bounded by x −axis, the tangent and normal to the circle $x^2 + y^2 = 4$ drawn at $(1, \sqrt{3})$.**(EG 9.61)**
$(\boldsymbol{OR})$
$(\boldsymbol{b})$On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the

probability that (i)two products are defective (ii)at most one product is defective (iii)at least two products are defective. **(EG 11.22)**

50. $(\boldsymbol{a})$Points $A$and $B$are 10 kmapart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 kmcloser to Athan B . Show that the location of the explosion is restricted to a particular curve and find an equation of it. **(EX 5.5 - 10)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$as a root. **(EX 3.2 - 4)**

51. $(\boldsymbol{a})$Show that (i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (ii) $\neg(p \rightarrow q) \equiv p \wedge \neg q$.**(EX 12.2 - 8)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$A particle moves along a horizontal line such that its position at any time $t \geq 0$ is given by $s(t)=t^3-6t^2+9t+1$, where $s$is measured in metres and $t$in seconds? (a) At what time the particle is at rest? (b) At what time the particle changes direction? (c) Find the total distance travelled by the particle in the first 2 seconds. **(EG 7.6)**

52. $(\boldsymbol{a})$For the function $f(x)=4x^3+3x^2-6x+1$find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection. **(EX 7.7 – 3)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Prove that (ii) $sin^{-1}\frac{3}{5}+cos^{-1}\frac{12}{13}=sin^{-1}\frac{16}{65}$**(EX 4.5 - 4)**

LONDON KRISHNAMOORTHY MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

M-3 **+2**

2023 MODEL FULL PORTION QUESTION PAPERS - 3

MATHEMATICS

MARKS: 100 **TIME: 3 HOURS + 15 MIN.**

I CHOOSE THE CORRECT ANSWERS: $20 \times 1 = 20$

1. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{4}{5} \end{bmatrix}$ and $A^T = A^{-1}$ then the value of x is

$(a)\ \frac{-4}{5}$ $(b)\ \frac{-3}{5}$ $(c)\ \frac{3}{5}$ $(d)\ \frac{4}{5}$

2. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

$(a)15$ $(b)12$ $(c)14$ $(d)11$

3. If $\left|z - \frac{3}{z}\right| = 2$, then the least value of $|z|$ is

$(a)\ 1$ $(b)\ 2$ $(c)\ 3$ $(d)\ 5$

4. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

$(a)\ z$ $(b)\ \bar{z}$ $(c)\ \frac{1}{z}$ $(d)\ 1$

5. According to the rational root theorem, which number is not possible rational root of $4x^7 + 2x^4 - 10x^3 - 5$?

$(a)\ -1$ $(b)\ \frac{5}{4}$ $(c)\ \frac{4}{5}$ $(d)\ 5$

6. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

$(a)\ \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ $(b)\ \frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ $(c)\ \frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ $(d)\ \cos^{-1}\left(\frac{1}{2}\right)$

7. If the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belongs to

$(a)\ [-1,1]$ $(b)\ [\sqrt{2}, 2]$ $(c)\ [-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ $(d)\ [-2, -\sqrt{2}] \cap [\sqrt{2}, 2]$

8. The radius of the circle passing through the point $(6,2)$ two of whose diameter are $x + y = 6$ and $x + 2y = 4$ is

$(a)\ 10$ $(b)\ 2\sqrt{5}$ $(c)\ 6$ $(d)\ 4$

9. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is

$(a)\ 2$ $(b)\ 3$ $(c)1$ $(d)\ 4$

10. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

$(a)\ 0$ $(b)\ 1$ $(c)\ 2$ $(d)\ 3$

11. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to

$(a)\ \vec{a}$ $(b)\ \vec{b}$ $(c)\ \vec{c}$ $(d)\vec{0}$

12. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1,9]$ is

$(a)\ 2$ $(b)\ 2.5$ $(c)\ 3$ $(d)\ 3.5$

13. The maximum slope of the tangent to the curve $y = e^x \sin x, x \in [0, 2\pi]$ is at

$(a) x = \frac{\pi}{4}$ $(b) x = \frac{\pi}{2}$ $(c) x = \pi$ $(d) x = \frac{3\pi}{2}$

14. If $f(x) = \frac{x}{x+1}$, then its differential is given by

$(a) \frac{-1}{(x+1)^2} dx$ $(b) \frac{1}{(x+1)^2} dx$ $(c) \frac{1}{x+1} dx$ $(d) \frac{-1}{x+1} dx$

15. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

(a) 10 (b) 5 (c) 8 (d) 9

16. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$ is

$(a) \frac{3}{2}$ $(b) \frac{1}{2}$ (c) 0 $(d) \frac{2}{3}$

17. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{\lambda}$ is

$(a) \frac{x}{e^{\lambda}}$ $(b) \frac{e^{\lambda}}{x}$ $(c) \lambda e^x$ $(d) e^x$

18. The integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is x, then $P(x)$

(a) x $(b) \frac{x^2}{2}$ $(c) \frac{1}{x}$ $(d) \frac{1}{x^2}$

19. Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with Probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of $E[X]$ is

(a) 0.11 (b) 1.1 (c) 11 (d) 1

20. If $P(X = 0) = 1 - P(X = 1)$. If $E[X] = 3Var(X)$, then $P(X = 0)$.

$(a) \frac{2}{3}$ $(b) \frac{2}{5}$ $(c) \frac{1}{5}$ $(d) \frac{1}{3}$

II ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER32 IS COMPULSARY. $\mathbf{8 \times 2 = 16}$

21. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal. **(EG 1.11)**

22. Which one of the points $i, -2 + i$ and 3 is farthest from the origin? **(EG 2.11)**

23. Construct a cubic equation with roots 1, 2and3.**(EX 3.1 - 2)**

24. Find the principal value of $\sin^{-1}(2)$, if it exists. **(EG 4.2)**

25. Determine whether $x + y - 1 = 0$ is the equation of a diameter of the circle $x^2 + y^2 - 6x + 4y + c = 0$, for all possible values of c.**(EG 5.3)**

26. Show that the lines $\frac{x-1}{4} = \frac{2-y}{6} = \frac{z-4}{12}$ and $\frac{x-3}{-2} = \frac{y-3}{3} = \frac{5-z}{6}$ are parallel. **(EG 6.32)**

27. Let $f, g: (a, b) \to \mathbb{R}$ be differentiable functions. Show that $d(fg) = f dg + g df$.**(EG 8.5)**

28. Evaluate $\int_0^{\pi} x^2 \cos nx \, dx$ where n is a positive integer. **(EG 9.31)**

29. Express each of the following physical statements in the form of differential equation.(ii) The population P of a city increases at a rate proportional to the product of population and to the difference between 5,00,000 and the population. **(EX 10.2 – 1(ii))**

30. The probability density function of X is given by $f(x) = \begin{cases} ke^{\frac{-x}{3}}, & for\ x > 0 \\ 0, & for\ x \le 0 \end{cases}$. Find (i) the value of k.**(EX 11.3 – 4)**

31. How many rows are needed for following statement formulae? (i) $p \vee \neg t \wedge (p \vee \neg s)$ (ii) $((p \wedge q) \vee (\neg r \vee \neg s)) \wedge (\neg t \wedge v)$ **(EG 12.13)**

32. Evaluate the limit $\lim_{x \to 0} \left(\frac{\sin mx}{x}\right)$.**(EG 7.35)**

III ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER44 IS COMPULSARY. $8 \times 3 = 24$

33. If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}. **(EG 1.12)**

34. For any two complex numbers z_1 and z_2, such that $|z_1| = |z_2| = 1$ and $z_1 z_2 \neq -1$, then show that $\frac{z_1+z_2}{1+z_1 z_2}$ is a real number. **(EX 2.5 - 2)**

35. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$, if the product of two roots is 1. **(EX 3.1 - 4)**

36. Show that $\cot(\sin^{-1} x) = \frac{\sqrt{1-x^2}}{x}, -1 \leq x \leq 1$ and $x \neq 0$. **(EG 4.26)**

37. The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex **(EG 5.33)**

38. Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(EG 5.15)**

39. Let $F(x, y) = x^3 y + y^2 x + 7$ for all $(x, y) \in \mathbb{R}^2$. Calculate $\frac{\partial F}{\partial x}(-1,3)$ and $\frac{\partial F}{\partial y}(-2,1)$. **(EG 8.12)**

40. Evaluate the following definite integrals: $(ii) \int_{-1}^{1} \frac{dx}{x^2+2x+5}$ **(EX 9.3 – 1)**

41. Find the differential equation of the family of all the parabolas with latus rectum $4a$ and whose axes are parallel to the x $-$axis. **(EX 10.3 – 4)**

42. For the random variable X with the given probability mass function as below, find the mean and variance. (iv) $f(x) = \begin{cases} \frac{1}{2} e^{\frac{-x}{2}}, & for\ x > 0 \\ 0, & \text{otherwise} \end{cases}$ **(EX 11.4 – 1)**

43. Verify (i) closure property (ii) commutative property, and (iii) associative property of the following operation on the given set. $(a * b) = a^b; \forall\, a, b \in \mathbb{N}$.(exponentiation property) **(EG 12.6)**

44. Expand $\log(1 + x)$ as a Maclaurin's series upto 4 non-zero terms for $-1 < x \leq 1$. **(EG 7.30)**

IV ANSWERS THE FOLLOWING QUESTIONS: $8 \times 5 = 40$

45. $(\boldsymbol{a})$ Show that the straight lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ are skew and hence find the shortest distance between them. **(EX 6.5 - 5)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$ Find the value of k for which the equations $kx - 2y + z = 1, x - 2ky + z = -2, x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution **(EX 1.6 - 2)**

46. $(\boldsymbol{a})$ By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$. **(EG 6.3)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$ Identify the type of conic and find centre, foci, vertices, and directrices of each of the following: (v) $18x^2 + 12y^2 - 144x + 48y + 120 = 0$. **(EX 5.2 - 8)**

47. $(\boldsymbol{a})$ Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros. **(EX 3.3 - 5)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$ Prove that the values of $\sqrt[4]{-1}$ are $\pm \frac{1}{\sqrt{2}}(1 \pm i)$. **(EX 2.8 - 10)**

48. $(\boldsymbol{a})$ Solve: $(ii) 2\tan^{-1} x = \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right), a > 0, b > 0$. **(EX 4.5 - 9)**

$(\boldsymbol{OR})$

($\boldsymbol{b}$)Find the equations of tangent and normal to the ellipse $x^2 + 4y^2 = 32$when $\theta = \frac{\pi}{4}$.**(EG 5.28)**

49. ($\boldsymbol{a}$)Find the acute angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection $(0,0), (1,1)$.**(EG 7.15)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)If $w(x, y, z) = \log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$, find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$.**(EX 8.7 – 6)**

50. ($\boldsymbol{a}$)Find the area of the region bounded between the parabolas $y^2 = 4x$and $x^2 = 4y$.**(EG 9.54)**(

$\boldsymbol{OR}$)

($\boldsymbol{b}$)Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance. **(EG 11.17)**

51. ($\boldsymbol{a}$)At 10.00 $A.M.$ a woman took a cup of hot instant coffee from her microwave oven and placed it on a nearby Kitchen counter to cool. At this instant the temperature of the coffee was $180°F$, and 10 minutes later it was $160°F$. Assume that constant temperature of the kitchen was $70°F$. (i)What was the temperature of the coffee at 10.15 $A.M.$? (ii) The woman likes to drink coffee when its temperature is between $130°F$and $140°F$between what times should she have drunk the coffee? **(EX 10.8 – 8)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Solve the following system of linear equations, by Gaussian elimination method $4x + 3y + 6z = 25, x + 5y + 7z = 13, 2x + 9y + z = 1$.**(EG 1.27)**

52. ($\boldsymbol{a}$)Construct the truth table for the following statements. $(\neg p \to r) \wedge (p \leftrightarrow q)$**(EX 12.2 – 6)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 $cubic\ m/min$, how fast is the depth of the water increases when the water is 8 metres deep? **(EX 7.1 – 8)**

LONDON KRISHNAMOORTHY MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

M-4 **+2**

2023 MODEL FULL PORTION QUESTION PAPERS - 4

MATHEMATICS

MARKS: 100 TIME: 3 HOURS + 15 MIN.

I CHOOSE THE CORRECT ANSWERS: $20 \times 1 = 20$

1. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$ then B^{-1}
(a) $\begin{bmatrix} 2 & -5 \\ 3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

2. If $A^T A^{-1}$ is symmetric, then $A^2 =$
(a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$

3. The principal argument of $(\sin 40° + i\cos 40°)$ is
(a) $-110°$ (b) $-70°$ (c) $70°$ (d) $110°$

4. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to
(a) 0 (b) 1 (c) 2 (d) 3

5. The number of positive roots of the polynomial $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$ is
(a) 0 (b) n (c) $< n$ (d) r

6. The value of $\sin^{-1}(\cos x), 0 \le x \le \pi$ is
(a) $\pi - x$ (b) $x - \frac{\pi}{2}$ (c) $\frac{\pi}{2} - x$ (d) $x - \pi$

7. $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - 2\sin^2 x) =$
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

8. The locus of a point whose distance from $(-2,0)$ is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is
(a) a parabola (b) a hyperbola (c) an ellipse (d) a circle

9. The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a+b)x - 4 = 0$ then the value of $(a+b)$ is
(a) 2 (b) 4 (c) 0 (d) -2

10. The coordinates of the point where the line $\vec{r} = (6\vec{i} - \vec{j} + 3\vec{k}) + t(-\vec{i} + 4\vec{k})$ meets the plane $\vec{r}.(\vec{i} + \vec{j} - \vec{k}) = 3$ are
(a) $(2,1,0)$ (b) $(7,-1,-7)$ (c) $(1,2,-6)$ (d) $(5,-1,1)$

11. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is
(a) $\frac{\sqrt{7}}{2\sqrt{2}}$ (b) $\frac{7}{2}$ (c) $\frac{\sqrt{7}}{2}$ (d) $\frac{7}{2\sqrt{2}}$

12. The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is -0.25 ?
(a) -8 (b) -4 (c) -2 (d) 0

13. The volume of a sphere is increasing in volume at the rate of $3\pi\ cm^3/sec$ .The rate of change of its radius when radius is $\frac{1}{2} cm$
(a) $3\ cm/s$ (b) $2\ cm/s$ (c) $1\ cm/s$ (d) $12 cm/s$

14. If $v(x,y) = \log(e^x + e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to
(a) $e^x + e^y$ (b) $\frac{1}{e^x + e^y}$ (c) 2 (d) 1

15. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$is equal to
$(a) e^{x^2+y^2}$ $(b)\ 2xu$ $(c) x^2u$ $(d) y^2u$

16. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the value of $f(1)$ is
$(a)\frac{1}{2}$ $(b)\ 2$ $(c)\ 1$ $(d)\ \frac{3}{4}$

17. The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through $(-1,1)$. Then the equation of the curve is
$(a)\ y = x^3 + 2$ $(b)\ y = 3x^2 + 4$ $(c)\ y = 3x^3 + 4$ $(d)\ y = x^3 + 5$

18. A rod of length $2l$ is broken into two pieces at random. The probability density function of the shorter of the two pieces is $f(x) = \begin{cases} \frac{1}{l}, & 0 < x < l \\ 0, & l \le x \le 2l \end{cases}$.The mean and variance of the shorter of the two pieces are respectively
$(a)\frac{l}{2}, \frac{l^2}{3}$ $(b)\frac{l}{2}, \frac{l^2}{6}$ $(c)\ l, \frac{l^2}{12}$ $(d)\frac{l}{2}, \frac{l^2}{12}$

19. If in 6 trials, X is a binomial variate which follows the relation $9P(X = 4) = P(X = 2)$, then the probability of success is
$(a)\ 0.125$ $(b)\ 0.25$ $(c)\ 0.375$ $(d)\ 0.75$

20. In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value $'F'$ are
$(a)\ 1$ $(b)\ 2$ $(c)\ 3$ $(d)\ 4$

II ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER32 IS COMPULSARY. **$8 \times 2 = 16$**

21. Solve the following system of linear equations, using matrix inversion method: $5x + 2y = 3, 3x + 2y = 5$.**(EG 1.22)**

22. If $z = x + iy$,find the following in rectangular form. $Im(3z + 4\bar{z} - 4i)$ **(EX 2.4 - 2)**

23. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$as a root. **(EX 3.2 - 2)**

24. State the reason for $cos^{-1}\left(\cos\left(\frac{-\pi}{6}\right)\right) \neq \frac{-\pi}{6}$**(EX 4.2 - 2)**

25. Find the general equation of the circle whose diameter is the line segment joining the points $(-4,-2)$and $(1,1)$. **(EG 5.4)**

26. Show that the vectors $\vec{i} + 2\vec{j} - 3\vec{k},\ 2\vec{i} - \vec{j} + 2\vec{k}$ and $3\vec{i} + \vec{j} - \vec{k}$ are coplanar. **(EG 6.14)**

27. If $v(x,y) = x^2 - xy + \frac{1}{4}y^2 + 7, x, y \in \mathbb{R}$, find the differential dv.**(EX 8.5 – 3)**

28. Evaluate the following: $\int_0^{\frac{\pi}{2}} \sin^{10} x\, dx$**(EX 9.6 – 1)**

29. Using binomial distribution find the mean and variance of X for the following experiments (i) A fair coin is tossed 100 times, and X denote the number of heads. (ii) A fair die is tossed 240 times, and X denote the number of times that four appeared. **(EX 11.5 – 3)**

30. Fill in the following table so that the binary operation $*$ on $A = \{a, b, c\}$ is commutative.

$*$	a	b	c
a	b		
b	c	b	a
c	a		c

(EX 12.1 – 6)

31. Identify the type of conic section of the equations. $3x^2 + 3y^2 - 4x + 3y + 10 = 0$**(EX 5.3 - 2)**

32. Find the absolute extrema of the following functions on the given closed interval. $f(x) = x^2 - 12x + 10;\ [1, 2]$**(EX 7.6 – 1)**

III ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER44 IS COMPULSARY. $\mathbf{8 \times 3 = 24}$

33. Find the adjoint of the following: $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$**(EX 1.1 - 1)**
34. If $\left|z - \frac{2}{z}\right| = 2$, show that the greatest and least value of $|z|$are $\sqrt{3} + 1$and $\sqrt{3} - 1$respectively. **(EX 2.5 - 6)**
35. Solve the equation $(x-2)(x-7)(x-3)(x+2) + 19 = 0$.**(EG 3.23)**
36. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$.**(EX 4.5 - 6)**
37. Prove that the point of intersection of the tangents at t_1 and$t_2$on the parabola$y^2 = 4ax$is $[at_1t_2, a(t_1 + t_2)]$.**(EX 5.4 - 7)**
38. Determine the intervals of concavity of the curve $f(x) = (x-1)^3.(x-5), x \in \mathbb{R}$and, points of inflection if any. **(EG 7.57)**
39. In a newly developed city, it is estimated that the voting population (in thousands) will increase according to $V(t) = 30 + 12t^2 - t^3\ , 0 \le t \le 8$ where t is the time in years. Find the approximate change in voters for the time change from 4 to $4\frac{1}{6}$ year. **(EX 8.2 – 8)**
40. Find, by integration, the volume of the solid generated by revolving about the x –axis, the region enclosed by $y = e^{-2x}, y = 0, x = 0$ and $x = 1$.**(EX 9.9 – 2)**
41. The slope of the tangent to the curve at any point is the reciprocal of fourtimes the ordinate at that point. The curve passes through $(2,5)$. Find theequation of the curve. **(EX 10.4 – 3)**
42. Find the mean and variance of a random variable X , whose probability density function is $f(x) = \begin{cases} \lambda e^{-\lambda x}, & for\ x \ge 0 \\ 0, & Otherwise \end{cases}$**(EG 11.18)**
43. Verify whether the following compound propositions are tautologies or contradictions or contingency $(iii)(p \to q) \leftrightarrow (\neg p \to q)$**(EX 12.2 – 7)**
44. Find the points where the straight line passes through $(6,7,4)$ and $(8,4,9)$ cuts the xz andyzplanes. **(EX 6.4 - 3)**

IV ANSWERS THE FOLLOWING QUESTIONS: $\mathbf{8 \times 5 = 40}$

45. $(\boldsymbol{a})$Test for consistency and if possible, solve the following systems of equations by rank method. (ii) $3x + y + z = 2, x - 3y + 2z = 1, 7x - y + 4z = 5$.**(EX 1.6 - 1)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$If the straight lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines. **(EX 6.8 – 4)**

46. $(\boldsymbol{a})$Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $(v) y^2 - 4y - 8x + 12 = 0$**(EX 5.2 - 4)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$If $a_1, a_2, a_3, \ldots, a_n$is an arithmetic progression with common difference d,P.T. $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + .. + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$. **(EG 4.23)**

47. $(\boldsymbol{a})$If $\omega \ne 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$are $-1, 1 - 2\omega, 1 - 2\omega^2$.**(EX 2.8 - 6)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Prove that the ellipse $x^2 + 4y^2 = 8$and the hyperbola $x^2 - 2y^2 = 4$intersect orthogonally. **(EG 7.18)**

48. $(\boldsymbol{a})$Solve :$(2x-1)(x+3)(x-2)(2x+3) + 20 = 0$.**(EX 3.4 - 2)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 $m$when it is 6 $m$away from the point of projection. Finally it reaches the ground 12 $m$away from the starting point. Find the angle of projection. **(EX 5.5 - 9)**

49. $(\boldsymbol{a})$Solve the following differential equations: $(x)\frac{dy}{dx} = \tan^2(x+y)$**(EX 10.5 – 4)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Verify (i) closure property, (ii) commutative property,(iii) associativeproperty, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5.**(EG 12.9)**

50. $(\boldsymbol{a})$The probability density function of the random variable X is given by $f(x) = \begin{cases} 16xe^{-4x}, & for\ x > 0 \\ 0, & for\ x \leq 0 \end{cases}$.For find the mean and variance of X.**(EX 11.4 – 7)**

$(\boldsymbol{OR})$

$(\boldsymbol{a})$Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.**(EX 6.1 - 9)**

51. $(\boldsymbol{a})$The region enclosed between the graphs of $y = x$ and $y = x^2$ is denoted by R, Find the volume generated when R is rotated through $360°$ about x −axis. **(EX 9.9 – 4)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$The mean and standard deviation of a binomial variate$X$ are respectively 6 and 2. Find (i) the probability mass function (ii) $P(X = 3)$$(iii)$ $P(X \geq 2)$.**(EX 11.5 – 7)**

52. $(\boldsymbol{a})$A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire?**(EX 7.8 – 4)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Let $z(x,y) = x^3 - 3x^2y^3$, where $x = se^t, y = se^{-t}$, $s, t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.**(EX 8.6 – 8)**

LONDON KRISHNAMOORTHY MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

M-5 **+2**

2023 MODEL FULL PORTION QUESTION PAPERS - 5
MATHEMATICS

MARKS: 100 TIME: 3 HOURS + 15 MIN.

I CHOOSE THE CORRECT ANSWERS: $20 \times 1 = 20$

1. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I - A =$
   $(a) A^{-1}$ $(b) \frac{A^{-1}}{2}$ $(c) 3A^{-1}$ $(d) 2A^{-1}$
2. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is
   $(a) 0$ $(b) -2$ $(c) -3$ $(d) -1$
3. If $z = x + iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is
   (a) real axis (b) imaginary axis (c) ellipse (d) circle
4. If $|z - 2 + i| \leq 2$, then the greatest value of $|z|$ is
   $(a) \sqrt{3} - 2$ $(b) \sqrt{3} + 2$ $(c) \sqrt{5} - 2$ $(d) \sqrt{5} + 2$
5. If α, β and γ are the roots of $x^3 + px^2 + qx + r = 0$, then $\sum \frac{1}{\alpha}$ is
   $(a) \frac{-q}{r}$ $(b) \frac{-p}{r}$ $(c) \frac{q}{r}$ $(d) \frac{-q}{p}$
6. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1} x$ is
   $(a) \frac{-\pi}{10}$ $(b) \frac{\pi}{5}$ $(c) \frac{\pi}{10}$ $(d) \frac{-\pi}{5}$
7. If $x = \frac{1}{5}$, the value of $\cos(\cos^{-1} x + 2\sin^{-1} x)$ is
   $(a) -\sqrt{\frac{24}{25}}$ $(b) \sqrt{\frac{24}{25}}$ $(c) \frac{1}{5}$ $(d) \frac{-1}{5}$
8. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are $(11, 2)$ the coordinates of the other end are
   $(a) (-5, 2)$ $(b) (2, -5)$ $(c) (5, -2)$ $(d) (-2, 5)$
9. Consider an ellipse whose centre is of the origin and its major axis is along x −axis. If itseccentrcity is 35and the distance between its foci is 6, then the area of the quadrilateral inscribed in the ellipse with diagonals as major and minor axis of the ellipse is
   (a) 8 (b) 32 (c) 80 (d) 40
10. If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1, \lambda > 0$ is $\frac{1}{5}$, then the value of λ is
    $(a) 2\sqrt{3}$ $(b) 3\sqrt{2}$ $(c) 0$ $(d) 1$
11. Find the point on the curve $6y = x^3 + 2$ at which y −coordinate changes 8 times as fast as x −coordinate is
    $(a) (4, 11)$ $(b) (4, -11)$ $(c) (-4, 11)$ $(d) (-4, -11)$
12. Angle between $y^2 = x$ and $x^2 = y$ at the origin is
    $(a) \tan^{-1}\frac{3}{4}$ $(b) \tan^{-1}\frac{4}{3}$ $(c) \frac{\pi}{2}$ $(d) \frac{\pi}{4}$
13. If $(x, y) = x^y, x > 0,$, then $\frac{\partial w}{\partial x}$ is equal to

$(a) x^y \log x$ $(b)\ y \log x$ $(c) yx^{y-1}$ $(d)\ x \log y$

14. If $f(x,y,z) = xy + yz + zx$, then $f_x - f_z$ is equal to
$(a)\ z - x$ $(b)\ y - z$ $(c)\ x - z$ $(d)\ y - x$

15. The area between $y^2 = 4x$ and its latus rectum is
$(a) \frac{2}{3}$ $(b) \frac{4}{3}$ $(c) \frac{8}{3}$ $(d) \frac{5}{3}$

16. If $f(x) = \int_1^x \frac{e^{\sin u}}{u} du, x > 1$ and $\int_1^3 \frac{e^{\sin x^2}}{x} dx = \frac{1}{2}[f(a) - f(1)]$, then one of the possible value of a is
$(a)\ 3$ $(b)\ 6$ $(c)\ 9$ $(d)\ 5$

17. The differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitrary constants is
$(a) \frac{d^2y}{dx^2} + y = 0$ $(b) \frac{d^2y}{dx^2} - y = 0$ $(c) \frac{dy}{dx} + y = 0$ $(d)\ \frac{dy}{dx} + y = 0$

18. P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount remaining, then
$(a)\ P = Ce^{kt}$ $(b)\ P = Ce^{-kt}$ $(c)\ P = Ckt$ $(d)\ P = C$

19. Let X be random variable with probability density function $f(x) = \begin{cases} \frac{2}{x^3}, & x \geq 1 \\ 0, & x < 1 \end{cases}$.Which of the following statement is correct
(a)both mean and variance exist (b) mean exists but variance does not exist
(c)both mean and variance do not exist (d) variance exists but Mean does not exist.

20. Which one is the contra positive of the statement $(p \vee q) \rightarrow r$?
$(a)\ \neg r \rightarrow (\neg p \wedge \neg q)$ $(b)\ \neg r \rightarrow (p \vee q)$ $(c)\ r \rightarrow (p \wedge q)$ $(d)\ p \rightarrow (q \vee r)$

II ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER32 IS COMPULSARY. $\mathbf{8 \times 2 = 16}$

21. Solve the following systems of linear equations by Cramer's rule: $5x - 2y + 16 = 0, x + 3y - 7 = 0$**(EX 1.4 - 1)**
22. If $x^2 + 2(k+2)x + 9k = 0$has equal roots, find k.**(EG 3.12)**
23. Find the value of $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$.**(EX 4.2 - 5)**
24. Examine the position of the point (2,3)with respect to the circle $x^2 + y^2 - 6x - 8y + 12 = 0$.**(EG 5.5)**
25. If the vectors $\vec{a} = a\vec{i} + a\vec{j} + a\vec{k},\ \vec{i} + \vec{k},\ c\vec{i} + c\vec{j} + b\vec{k}$ are coplanar, prove that $c$is the geometric mean of $a$and b.**(EX 6.2 - 9)**
26. Find the local extremum of the function $f(x) = x^4 + 32x$.**(EG 7.59)**
27. If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease? **(EG 8.7)**
28. Evaluate the following$\int_0^\infty x^5 e^{-3x} dx$**(EX 9.7 – 1)**
29. Consider the binary operation $*$ defined on the set $A = \{a, b, c, d\}$ by the following table: Is it commutative and associative?

$*$	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	a	a	c

(EX 12.1 – 7)

30. For each of the following differential equations, determine its order, degree (if exists) $(x)\ x = e^{xy\left(\frac{dy}{dx}\right)}$ **(EX 10.1 – 1)**

31. Find the square root of $6 - 8i$.**(EG 2.17)**

32. The probability density function of random variable X is given by $f(x) = \begin{cases} k, & 1 \leq x \leq 5 \\ 0, & Otherwise \end{cases}$. Find the value of k.**(EG 11.14)**

III ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER44 IS COMPULSARY. **$8 \times 3 = 24$**

33. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.**(EX 1.1 -3)**

34. Find the fourth roots of unity. **(EG 2.33)**

35. If α , β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, form a cubic equation whose roots are 2α , 2β , 2γ **(EX 3.1 - 3)**

36. Prove that$\tan^{-1} x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{3x-x^3}{1-3x^2}, |x| < \frac{1}{\sqrt{3}}$.**(EX 4.5 - 7)**

37. The maximum and minimum distances of the Earth from the Sun respectively are $152 \times 10^6 km$ and $94.5 \times 10^6 km$.The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus. **(EG 5.31)**

38. Prove by vector method that the area of the quadrilateral $ABCD$having diagonals $AC$and $BD$is $\frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right|$.**(EX 6.1 - 6)**

39. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high? **(EG 7.9)**

40. Let for all $w(x,y) = xy + \frac{e^y}{y^2+1}$. for all $(x,y) \in \mathbb{R}^2$.Calculate $\frac{\partial^2 w}{\partial y \partial x}$ and $\frac{\partial^2 w}{\partial x \partial y}$.**(EG 8.14)**

41. Solve the following differential equations: $\tan y \frac{dy}{dx} = \cos(x+y) + \cos(x-y)$**(EX 10.5 – 4)**

42. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the distribution. **(EX 11.5 – 9)**

43. Verify the (i) closure property, (ii) commutative property,(iii) associativeproperty (iv) existence of identity and(v) existence of inverse for thearithmetic operation $+$ on $\mathbb{Z}$.**(EG 12.2)**

44. Prove that $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)\,dx = \frac{\pi}{8}\log 2$.**(EG 9.27)**

IV ANSWERS THE FOLLOWING QUESTIONS: **$8 \times 5 = 40$**

45. $(\boldsymbol{a})$A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8)$, $(-2,-12)$ and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.) **(EX 1.5 - 4)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Solve: $(x^3 + y^3)dy - x^2 y dx = 0$**(EX 10.6 – 2)**

46. $(\boldsymbol{a})$Solve the equation $z^3 + 27 = 0$.**(EX 2.8 - 5)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$A rectangular page is to contain 24 cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is1 cm. What should be the dimensions of the page so that the area of the paper used is minimum. **(EX 7.8 – 5)**

47. ($\boldsymbol{a}$)If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$and $0 < x, y, z < 1$,show that $x^2 + y^2 + z^2 + 2xyz = 1$.**(EG 4.22)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Find the equation of the plane passing through the line of intersection of the planes $x + 2y + 3z = 2$and $x - y + z + 11 = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$.**(EX 6.9 – 2)**

48. ($\boldsymbol{a}$)A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major-axis 20 cm and minor-axis 10 cm about its major-axis. Find its volume using integration. **(EX 9.9 – 6)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Let $w(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}, (x, y, z) \neq (0,0,0)$. Show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$.**(EX 8.4 – 6)**

49. ($\boldsymbol{a}$)Suppose that $f(x)$ given below represents a probability mass function,

x	1	2	3	4	5	6
$f(x)$	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

Find (i) the value of c (ii) Mean and variance. **(EG 11.16)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Find the angle between the rectangular hyperbola $xy = 2$ and the parabola $x^2 + 4y = 0$.**(EX 7.2 – 9)**

50. ($\boldsymbol{a}$)Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $(iv)x^2 - 2x + 8 + 17 = 0y$**(EX 5.2 - 4)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)If the straight lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines. **(EX 6.8 – 4)**

51. ($\boldsymbol{a}$)If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$,prove that $A^{-1} = A^T$.**(EX 1.1 -5)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table. **(EX 12.2 – 15)**

52. ($\boldsymbol{a}$)Find the equations of the tangents to the curve $y = 1 + x^3$ for which the tangent is orthogonal with the line $x + 12y = 12$.**(EX 7.2 – 6)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Solve: $\left(y - e^{\sin^{-1} x}\right)\frac{dx}{dy} + \sqrt{1 - x^2} = 0$**(EX 10.7 – 7)**