ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

Model Exam (2022 – 23)
CLASS – XII - MATHEMATICS

Time Allowed : 3 Hrs | Maximum Marks : 90

PART – I

I. Answer ALL questions. **20x1 = 20**

1) The circle passing through $(1,-2)$ and touching the axis of x at $(3,0)$ passing through the point
(1) $(-5,2)$ (2) $(2,-5)$ (3) $(5,-2)$ (4) $(-2,5)$

2) For any value of $n\in\mathbb{Z}$, $\int_0^{\pi} e^{\cos^2 x}\cos^3[(2n+1)x]\,dx$ is
(1) $\frac{\pi}{2}$ (2) π (3) 0 (4) 2

3) According to the rational root theorem, which number is not possible rational zero of $4x^7+2x^4-10x^3-5$?
(1) -1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

4) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is
(1) 0.4 cu.cm (2) 0.45 cu.cm (3) 2 cu.cm (4) 4.8 cu.cm

5) The order of the differential equation of all circles with centre at (h, k) and radius 'a' is
(1) 2 (2) 3 (3) 4 (4) 1

6) In the set $\mathbb{Q}$ define $a\odot b=a+b+ab$. For what value of y, $3\odot(y\odot 5)=7$?
(1) $y=\frac{2}{3}$ (2) $y=\frac{-2}{3}$ (3) $y=\frac{-3}{2}$ (4) $y=4$

7) If $P=\begin{bmatrix}1 & x & 0\\ 1 & 3 & 0\\ 2 & 4 & -2\end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A|=4$, then x is
(1) 15 (2) 12 (3) 14 (4) 11

8) The principal argument of $\frac{3}{-1+i}$ is
(1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

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9) The minimum value of the function $|3-x|+9$ is

(1) 0 (2) 3 (3) 6 (4) 9

10) If $\vec{a}=2\hat{i}+3\hat{j}-k$, $\vec{b}=\hat{i}+2\hat{j}-5k$, $\vec{c}=3\hat{i}+5\hat{j}-k$, then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is

(1) $-17\hat{i}+21\hat{j}-97\hat{k}$ (2) $17\hat{i}+21\hat{j}-123\hat{k}$

(3) $-17\hat{i}-21\hat{j}+97\hat{k}$ (4) $-17\hat{i}-21\hat{j}-97\hat{k}$

11) Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X–3)$ is

(1) 0.24 b) 0.48 (3) 0.6 (4) 0.96

12) If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle, then the third angle is

(1) $\frac{\pi}{4}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

13) If the direction cosines of a line are $\frac{1}{c},\frac{1}{c},\frac{1}{c}$, then

(1) $c=\pm 3$ (2) $c=\pm\sqrt{3}$ (3) $c>0$ (4) $0<c<1$

14) The volume of solid of revolution of the region bounded by $y^2=x(a-x)$ about x-axis is

(1) πa^3 (2) $\frac{\pi a^3}{4}$ (3) $\frac{\pi a^3}{5}$ (4) $\frac{\pi a^3}{6}$

15) If A^TA^{-1} is symmetric, then $A^2=$

(1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$

16) The degree of the differential equation $y(x)=1+\frac{dy}{dx}+\frac{1}{1\cdot 2}\left(\frac{dy}{dx}\right)^2+\frac{1}{1\cdot 2\cdot 3}\left(\frac{dy}{dx}\right)^3+....$ is

(1) 2 (2) 3 (3) 1 (4) 4

17) Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ parallel to the straight line $2x-y=1$. One of the points of contact of tangents on the hyperbola is

(1) $\left(\frac{9}{2\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$ (2) $\left(\frac{-9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (3) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (4) $\left(3\sqrt{3},-2\sqrt{2}\right)$

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18) The area of the triangle formed by the complex numbers $z, iz,$ and $z+iz$ in the Argand's diagram is

(1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

19) If $w(x,y,z)=x^2(y-z)+y^2(z-x)+z^2(x-y)$, then $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}$ is

(1) $xy+yz+zx$ (2) $x(y+z)$ (3) $y(z+x)$ (4) 0

20) A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?

(1) $\frac{57}{20^3}$ (2) $\frac{57}{20^2}$ (3) $\frac{19^3}{20^3}$ (4) $\frac{57}{20}$

PART – II

II. Answer any SEVEN questions. Question 30 is compulsory **7x2 = 14**

21) The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function

$$f(x)=\begin{cases}3e^{-3x} & x>0\\ 0 & \text{elsewhere}\end{cases}.$$

Find the expected life of this electronic equipment.

22) Find the value of $\sin^{-1}\left[\sin\left(\frac{5\pi}{4}\right)\right]$.

23) Find the value of the following:

$\int_0^{\frac{\pi}{2}} \sin^5 x \cos^4 x \, dx$

24) Find the rank of the following matrix

$$\begin{bmatrix}2 & 0 & -7\\ 0 & 3 & 1\\ 0 & 0 & 1\end{bmatrix}$$

25) Find the equation of the ellipse with foci $(\pm 2,0)$, vertices $(\pm 3,0)$.

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26) Find the values in the interval $(1,2)$ of the mean value theorem satisfied by the function $f(x)=x-x^2$ for $1\le x\le 2$.

27) Find the coordinates of the point where the straight line $\vec{r}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+t\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ intersects the plane $x-y+z-5=0$.

28) Prove that, in an algebraic structure the identity element (if exists) is unique.

29) If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when $\theta=\frac{\pi}{2}$

30) Show that, if $x=r\cos\theta$, $y=r\sin\theta$, then $\frac{\partial r}{\partial x}$ is equal to $\cos\theta$.

PART – III

III. Answer any SEVEN questions. Question 40 is compulsory **7x3 = 21**

31) Solve : $(x-5)(x-7)(x+6)(x+4)=504$

32) Solve $\frac{dy}{dx}+2y=e^{-x}$.

33) The probability that Mr.Q hits a target at any trial is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time.

34) If $\frac{1+z}{1-z}=\cos 2\theta+i\sin 2\theta$, show that $z=i\tan\theta$.

35) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

36) Find the equations of tangent and normal to the ellipse $x^2+4y^2=32$ when $\theta=\frac{\pi}{4}$.

37) Given $A=\begin{bmatrix}1 & -1\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}3 & -2\\ 1 & 1\end{bmatrix}$ and $C=\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}$, find a matrix X such that $AXB=C$.

38) With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.

39) Evaluate the following:

$$\int_0^{\frac{1}{\sqrt{2}}}\frac{e^{\sin^{-1}x}\sin^{-1}x}{\sqrt{1-x^2}}\,dx$$

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40) Sketch the graph of cosine function and inverse cosine function in their principal domain.

PART – IV

IV. Answer ALL questions. **7x5 = 35**

41) a) Suppose that $f(x)$ given below represents a probability mass function,

x	1	2	3	4	5	6
$f(x)$	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

Find (i) the value of c (ii) Mean and variance.

OR

b) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

42) a) Find, by integration, the volume of the container which is in the shape of a right circular conical frustum as shown in the Fig 42.(a).

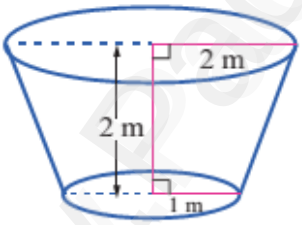

Fig. 42.(a)

OR

b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground $12m$ away from the starting point. Find the angle of projection.

43) a) Find the value of k for which the equations $kx-2y+z=1$, $x-2ky+z=-2$, $x-2y+kz=1$ have

(i) no solution (ii) unique solution (iii) infinitely many solution

OR

b) In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur? $\left[\log(2.43)=0.88789;\ \log(0.5)=-0.69315\right]$

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44) a) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.

OR

b) Suppose z_1, z_2, and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

45) a) Solve the equation $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$

OR

b) Find the area of the region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$.

46) a) Solve:

$2\tan^{-1} x = \cos^{-1}\frac{1-a^2}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2}$, $a > 0$, $b > 0$.

OR

b) Find the parametric form of vector equation of a straight line passing through the point of intersection of the straight lines $\vec{r} = (\hat{i} + 3\hat{j} - \hat{k}) + t(2\hat{i} + 3\hat{j} + 2\hat{k})$ and $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$, and perpendicular to both straight lines.

47) a) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

$$18x^2 + 12y^2 - 144x + 48y + 120 = 0$$

OR

b) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the commutative and associative properties satisfied by $*$ on M. Also, examine the existence of identity, existence of inverse properties for the operation $*$ on M.

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