ST. ANNE'S ACADEMY
(MATHS & PHYSICS TUITION CENTRE)
I FLOOR, JAFRO DENTAL CLINIC, HOLY CROSS COLLEGE ROAD, PUNNAI NAGAR, NAGERCOIL – 629004

Model Exam (2022 – 23)
CLASS – XII - MATHEMATICS

Time Allowed : 3 Hrs | Maximum Marks : 90

PART – I

I. Answer ALL questions. **20x1 = 20**

1) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(1) $0.3xdx\,m^3$ (2) $0.03x\,m^3$ (3) $0.03x^2m^3$ (4) $0.03x^3m^3$

2) If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to

(1) 1 (2) -1 (3) $\sqrt{3}i$ (4) $-\sqrt{3}i$

3) If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

(1) 3 (2) -1 (3) 1 (4) 9

4) If $\cot^{-1}x=\frac{2\pi}{5}$ for some $x\in R$, the value of $\tan^{-1}x$ is

(1) $-\frac{\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $-\frac{\pi}{5}$

5) If A^TA^{-1} is symmetric, then $A^2=$

(1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$

6) According to the rational root theorem, which number is not possible rational zero of $4x^7+2x^4-10x^3-5$?

(1) -1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

7) Which one of the following is incorrect? For any two propositions p and q, we have

(1) $\neg(p\vee q)\equiv\neg p\wedge\neg q$ (2) $\neg(p\wedge q)\equiv\neg p\vee\neg q$

(3) $\neg(p\vee q)\equiv\neg p\vee\neg q$ (4) $\neg(\neg p)\equiv p$

8) Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X-3)$ is

(1) 0.24 b) 0.48 (3) 0.6 (4) 0.96

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9) If $[\vec{a},\vec{b},\vec{c}]=1$, then the value of $\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}}+\frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b})\cdot\vec{c}}+\frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b})\cdot\vec{a}}$ is

(1) 1 (2) -1 (3) 2 (4) 3

10) The area of the triangle formed by the complex numbers z, iz, and $z+iz$ in the Argand's diagram is

(1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

11) If the function $f(x)=\frac{1}{12}$ for $a<x<b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

(1) 0 and 12 (2) 5 and 17 (3) 7 and 19 (4) 16 and 24

12) If the function $f(x)=\frac{1}{12}$ for $a<x<b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

(1) 0 and 12 (2) 5 and 17 (3) 7 and 19 (4) 16 and 24

13) Let $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$ and $4B=\begin{bmatrix}3&1&-1\\1&3&x\\-1&1&3\end{bmatrix}$. If B is the inverse of A, then the value of x is

(1) 2 (2) 4 (3) 3 (4) 1

14) If $\cot^{-1}x=\frac{2\pi}{5}$ for some $x\in R$, the value of $\tan^{-1}x$ is

(1) $-\frac{\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $-\frac{\pi}{5}$

15) In the last column of the truth table for $\neg(p\vee\neg q)$ the number of final outcomes of the truth value 'F' are

(1) 1 (2) 2 (3) 3 (4) 4

16) The circle passing through $(1,-2)$ and touching the axis of x at $(3,0)$ passing through the point

(1) $(-5,2)$ (2) $(2,-5)$ (3) $(5,-2)$ (4) $(-2,5)$

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17) The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

(1) $y = 0$ (2) $y = \pm\sqrt{3}$ (3) $y = \frac{1}{2}$ (4) $y = \pm 3$

18) If the planes $\vec{r}.(2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r}.(4\hat{i} + \hat{j} - \mu\hat{k}) = 5$ are parallel, then the value of λ and μ are

(1) $\frac{1}{2}, -2$ (2) $-\frac{1}{2}, 2$ (3) $-\frac{1}{2}, -2$ (4) $\frac{1}{2}, 2$

19) $\sin(\tan^{-1} x), |x| < 1$ is equal to

(1) $\frac{x}{\sqrt{1-x^2}}$ (2) $\frac{1}{\sqrt{1-x^2}}$ (3) $\frac{1}{\sqrt{1+x^2}}$ (4) $\frac{x}{\sqrt{1+x^2}}$

20) The maximum value of the product of two positive numbers, when their sum of the squares is 200, is

(1) 100 (2) $25\sqrt{7}$ (3) 28 (4) $24\sqrt{14}$

PART – II

II. Answer any SEVEN questions. Question 30 is compulsory **7x2 = 14**

21) Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number

22) Write in polar form of the following complex numbers

$$\frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}.$$

23) Find the equation of the hyperbola with vertices $(0, \pm 4)$ and foci $(0, \pm 6)$.

24) Find the value in the interval $\left(\frac{1}{2}, 2\right)$ satisfied by the Rolle's theorem for the function $f(x) = x + \frac{1}{x}, x \in \left[\frac{1}{2}, 2\right]$.

25) If the rank of the matrix $A = \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2, then λ = ?

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26) If α, β, and γ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\sum\frac{1}{\beta\gamma}$ in terms of the coefficients.

27) Construct the truth table for the following statement.

$$(p\vee q)\vee\neg q$$

28) Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$.

29) Evaluate : $\lim_{x\to 1} x^{\frac{1}{1-x}}$.

30) Sketch the graph of $y=\sin^{-1}x$ in [-1, 1].

PART – III

III. Answer any SEVEN questions. Question 40 is compulsory **7x3 = 21**

31) Evaluate $\int_0^1 x^5\left(1-x^2\right)^5 dx$.

32) Prove that the point of intersection of the tangents at 't_1' and 't_2' on the parabola $y^2=4ax$ is $\left[at_1t_2,\ a\left(t_1+t_2\right)\right]$.

33) Determine the values of λ for which the following system of equations
$(3\lambda-8)x+3y+3z=0,\ 3x+(3\lambda-8)y+3z=0,\ 3x+3y+(3\lambda-8)z=0$
has a non-trivial solution.

34) Check whether the statement $p\to(q\to p)$ is a tautology or a contradiction without using the truth table.

35) Solve the following equation

$$\sin^2 x-5\sin x+4=0$$

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36) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x,y,z<1$, show that

37) A random variable X has the following probability mass function.

x	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2\le X<5)$

38) Find the equation of the plane passing through the intersection of the planes $2x+3y-z+7=0$ and $x+y-2z+5=0$ and is perpendicular to the plane $x+y-3z-5=0$.

39) Solve $y^2+x^2\frac{dy}{dx}=xy\frac{dy}{dx}$.

40) Let $u(x,y)=e^{-2y}\cos(2x)$ for all $(x,y)\in\mathbb{R}^2$. Prove that u is a harmonic function in $\mathbb{R}^2$.

PART – IV

IV. Answer ALL questions. **7x5 = 35**

41) a) A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) the probability of getting at least one correct answer.

OR

b) Find the area of the region bounded between the curves $y=\sin x$ and $y=\cos x$ and the lines $x=0$ and $x=\pi$.

42) a) Find the local extrema for the following function using second derivative test :

$$f(x)=x^2e^{-2x}$$

OR

b) Find the centre, foci, and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$

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43) a) If $A=\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x+y+2z=1, 3x+2y+z=7, 2x+y+3z=2$.

OR

b) Solve $\left(1+x^3\right)\frac{dy}{dx}+6x^2y=1+x^2$.

44) a) If $z(x,y)=x\tan^{-1}(xy)$, $x=t^2$, $y=se^t$, $s,t\in\mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ at $s=t=1$.

OR

b) Solve the equation $z^3+8i=0$, where $z\in\mathbb{C}$.

45) a) Solve the following equation: $x^4-10x^3+26x^2-10x+1=0$.

OR

b) Prove that:

$$\int_0^a x f(x)\,dx=\frac{a}{2}\int_0^a f(x)\,dx \text{ if } f(a-x)=f(x).$$

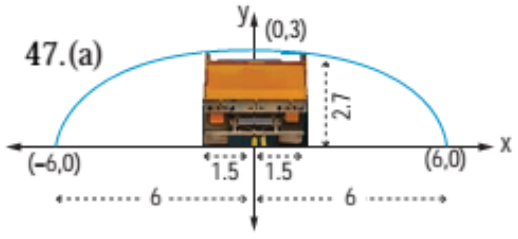
46) a) Find the domain of $\sin^{-1}\left(2-3x^2\right)$

OR

b) Show that the lines $\vec{r}=\left(-\hat{i}-3\hat{j}-5\hat{k}\right)+s\left(3\hat{i}+5\hat{j}+7\hat{k}\right)$ and $\vec{r}=\left(2\hat{i}+4\hat{j}+6\hat{k}\right)+t\left(\hat{i}+4\hat{j}+7\hat{k}\right)$ are coplanar. Also,find the non-parametric form of vector equation of the plane containing these lines.

47) a) A semielliptical archway over a one-way road has a height of $3m$ and a width of $12m$. The truck has a width of $3m$ and a height of $2.7m$. Will the truck clear the opening of the archway? (Fig. 47.(a))

47.(a)



OR

b) Prove de-Morgan's Laws using truth table.