MERCURY COACHING CENTRE
2-A, 2ND MAIN ROAD, BHARATHIDASAN NAGAR, MUDALIARPET, PUDUCHERRY – 4.
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M O D E L E X A M I

Std : XII

FULL PORTION 2023

Time: 3 hrs] **MATHEMATICS** **Total Marks: 90**

I CHOOSE THE CORRECT ANSWER: **(20 X 1 = 20)**

1. According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$?

   (1) -1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

2. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

   (1) $\frac{2}{\sqrt{3}}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{4}{3}$ (4) $\frac{3}{2}$

3. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is

   (1) 3 (2) -1 (3) 1 (4) 9

4. The area of the triangle formed by the complex numbers z, iz and z + iz in the Argand's diagram is (1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

5. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

   (1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 3

6. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

   (1) $t = 0$ (2) $t = 3$ (3) $t = 1$ (4) $t = \frac{1}{3}$

7. The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0,3]$ is

   (1) 1 (2) $\sqrt{2}$ (3) $\frac{3}{2}$ (4) 2

8. The order and degree of the differential equation $\sqrt{\sin x}\ (dx + dy) = \sqrt{\cos x}\ (dx - dy)$ is

   (1) 1, 2 (2) 2, 2 (3) 1, 1 (4) 2, 1

9. The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is

   (1) $y + \sin^{-1} x = c$ (2) $x + \sin^{-1} y = 0$ (3) $y^2 + 2\sin^{-1} x = c$ (4) $x^2 + 2\sin^{-1} y = 0$

10. If $A=\begin{bmatrix}3 & 5\\ 1 & 2\end{bmatrix}$, $B=adjA$ and $C=3A$, then $\frac{|adjB|}{|C|}=$

(1) $\frac{1}{3}$ (2) 1 (3) $\frac{1}{4}$ (4) $\frac{1}{9}$

11. If $adjA=\begin{bmatrix}2 & 3\\ 4 & -1\end{bmatrix}$ and $adjB=\begin{bmatrix}1 & -2\\ -3 & 1\end{bmatrix}$ then $adj(AB)$ is

(1) $\begin{bmatrix}-7 & 7\\ -1 & -9\end{bmatrix}$ (2) $\begin{bmatrix}-6 & 5\\ -2 & -10\end{bmatrix}$ (3) $\begin{bmatrix}-7 & -1\\ 7 & -9\end{bmatrix}$ (4) $\begin{bmatrix}-6 & -2\\ 5 & -10\end{bmatrix}$

12. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then is equal to $\vec{a}$ x($\vec{b}$ x $\vec{c}$)

1) $\vec{a}$ 2) $\vec{0}$ 3) $\vec{c}$ 4) $\vec{b}$

13. If f(x, y) = e ^{xy}, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

1) (1 + y) e ^{xy} 2) xy e^{xy} 3) (1+ x) e ^{xy} 4) (1 + xy) e ^{xy}

14. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x dx$ is

1) 0 2) $\frac{2}{3}$ 3) $\frac{1}{2}$ 4) $\frac{3}{2}$

15. If $\vec{a}$ x($\vec{b}$ x $\vec{c}$) = ($\vec{a}$ x $\vec{b}$) x $\vec{c}$, where $\vec{a}$, $\vec{b}$, $\vec{c}$ are any three vectors such that $\vec{b}$. $\vec{c}$ $\neq 0$, and $\vec{a}$. $\vec{b}$ $\neq 0$, then $\vec{a}$ and $\vec{c}$ are

1) perpendicular 2) inclined at an angle $\frac{\pi}{6}$

3) inclined at an angle $\frac{\pi}{3}$ 4) parallel

16. If f(x) is even then $\int_{-a}^{a} f(x)dx$ is

1) 0 2) $-2\int_0^a f(x)dx$ 3) $\int_0^a f(x)dx$ 4) $2\int_0^a f(x)dx$

17. Let X have a Bernoulli distribution with mean 0.4, then the variance of (2X – 3) is

1) 0.96 2) 0.24 3) 0.48 4) 0.6

18. The truth values of the following statements are

(i) Chennai is in India or $\sqrt{2}$ is an integer.

(ii) Chennai is in India or $\sqrt{2}$ is an irrational number.

(iii) Chennai is in China or $\sqrt{2}$ is an integer.

(iv) Chennai is in China or $\sqrt{2}$ is an irrational number.

1) TFTF 2) TFFT 3) FTFT 4) TTFT

19. ' – ' is binary operation on

1) Z 2) Q – {0} 3) R – {0} 4) N

20. $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)$ is equal to

1) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ 2) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ 3) 4) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

II Answer any seven questions (Q.NO. 30 COMPULOSORY) (7 x 2 = 14)

21. Find the rank of the matrix $\begin{bmatrix}1 & -2 & -1 & 0\\ 3 & -6 & -3 & 1\end{bmatrix}$

22. Find z^{-1}, if $z = (2 + 3i)(1 – i)$.

23. Find the value of $sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

24. Find the length of the latus rectum of the hyperbola $16y^2 – 9x^2 = 144$.

25. Find the intervals of monotonicity for the function $f(x) = x^2 – 4x + 4$.

26. Estimate the value of $\int_0^{0.5} x^2 dx$ using the Riemann sums corresponding to 5 subintervals of equal width and applying left-end rule.

27. Find the distance between the parallel planes $x + 2y – 2z + 1 = 0$ and $2x + 4y – 4z + 5 = 0$.

28. An urn contains 5mangoes and 4 apples. Three fruits are taken at random. If the number of apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.

29. Prove : In an algebraic structure the identity element must by unique.

30. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. Form a Differential equation involving the rate of change of the radius of the rain drop.

III Answer any seven questions: (Q. NO. 40 is compulsory) (7 x 3 = 21)

31. Find the adjoint of the matrix $A = \begin{bmatrix}1 & 3\\ 2 & -5\end{bmatrix}$ and verify that $A(\text{adj } A) = (\text{adj } A)\, A = |A|I$.

32. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}\right)$.

33. If α and β are the roots of the quadratic equation $2x^2 – 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

34. The line $3x + 4y – 12 = 0$ meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter.

35. Find two positive numbers whose product is 20 and their sum is minimum.

36. Find the points where the straight line passes through (6, 7, 4) and (8, 4, 9) cuts the xz and yz planes

37. If U = log ($x^3 + y^3 + z^3$), find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$.

38. Show that $y = a\cos bx$ is a solution of the differential equation $\frac{d^2y}{dx^2} + b^2y = 0$

39. Prove that $p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

40. Evaluate: $\int_0^{\frac{\pi}{2}} x^2 \cos 2x dx$

IV Answer the following: **(7 x 5 = 35)**

41. a) Investigate for what values of λ and μ the system of linear equations x + 2y + z = 7, x + y + λz = μ, x + 3y – 5z = 5 has (i) no solution (ii) a unique solution. **(OR)**

b) Solve the equation : $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

42. a) Solve the equation $z^3 + 8i = 0$, where $z \in C$. **(OR)**

b) Draw the curve sin x in the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $sin^{-1}x$ in [- 1, 1].

43. a) A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if *X* denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) the probability of getting at least one correct answer.

(OR)

b) Prove using vectors, sin (α – β) = sin α cos β – cos α sin β.

44. a) Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r} = (\vec{i} - \vec{j} + 3\vec{k}) + t(2\vec{i} - \vec{j} + 4\vec{k})$ and perpendicular to plane $\vec{r} \cdot (\vec{i} + 2\vec{j} + \vec{k}) = 8$. **(OR)**

b) Solve: ($x^3 + y^3$) dy – x^2 y dx = 0

45. a) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$. **(OR)**

b) If $u = sec^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$.

46. a) A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major-axis 20 cm and minor-axis 10 cm about its major-axis. Find its volume using integration. **(OR)**

b) Find the vertex, focus, equation of directrix and length of the latus rectum of $y^2 - 4y - 8x + 12 = 0$.

47. a) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let * be the matrix multiplication. Determine whether M is closed under *. If so, examine the associative property, existence of identity, existence of inverse properties for the operation * on M. **(OR)**

b) A tunnel through a mountain for a four lane highway is to have a elliptical opening . The total width of the highway is to be 16m and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be?