LONDON KRISHNAMOORTHY MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

M-6 +2

2023 MODEL FULL PORTION QUESTION PAPERS - 6

MATHEMATICS

MARKS: 100 TIME: 3 HOURS + 15 MIN.

I CHOOSE THE CORRECT ANSWERS: $20 \times 1 = 20$

1. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

   (a) 17 (b) 14 (c) 19 (d) 21

2. If $adjA = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $adjB = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$, then $adj(AB)$ is

   (a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

3. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

   (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$

4. If z is a complex number such that $z \in C\backslash R$ and $z + \frac{1}{z} \in R$, then $|z|$ is

   (a) 0 (b) 1 (c) 2 (d) 3

5. If $\sin^{-1}\frac{x}{5} + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value of x is

   (a) 4 (b) 5 (c) 2 (d) 3

6. $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \sec^{-1}\frac{5}{3} - \text{cosec}^{-1}\frac{13}{12}$ is equal to

   (a) 2π (b) π (c) 0 (d) $\tan^{-1}\frac{12}{65}$

7. If $P(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then $PF_1 + PF_2$ is

   (a) 8 (b) 6 (c) 10 (d) 12

8. The equation of the circle passing through (1,5) and (4,1) and touching y –axis is $x^2 + y^2 - 5x - 6y + 9 + \lambda(4x + 3y - 19) = 0$ where λ is equal to

   (a) $0, \frac{-40}{9}$ (b) 0 (c) $\frac{40}{9}$ (d) $\frac{-40}{9}$

9. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + \vec{j}$, $\vec{c} = \vec{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} \times \mu\vec{b}$, then the value of $\lambda + \mu$ is

   (a) 0 (b) 1 (c) 6 (d) 3

10. If $[\vec{a}, \vec{b}, \vec{c}] = 1$, then the value of $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}} + \frac{\vec{b}.(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b}).\vec{c}} + \frac{\vec{c}.(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b}).\vec{a}}$ is

    (a) 1 (b) -1 (c) 2 (d) 3

11. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

    (a) $t = 0$ (b) $t = 1$ (c) $t = 1$ (d) $t = 3$

12. A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

    (a) $\frac{3}{25}$ $radians/sec$ (b) $\frac{4}{25}$ $radians/sec$ (c) $\frac{1}{5}$ $radians/sec$ (d) $\frac{1}{3}$ $radians/sec$

13. If $g(x, y) = 3x^2 - 5y + 2y^2$, $x(t) = e^t$ and $y(t) = \cos t$, then $\frac{dg}{dt}$ is equal to

    (a) $6e^{2t} + 5\sin t - 4\cos t \sin t$ (b) $6e^{2t} - 5\sin t + 4\cos t \sin t$

(c) $3e^{2t} + 5\sin t + 4\cos t \sin t$ (d) $3e^{2t} - 5\sin t + 4\cos t \sin t$

14. The value of $\int_0^1 \frac{dx}{1+5^{\cos x}}$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

15. If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is

(a) $\log \sin x$ (b) $\cos x$ (c) $\tan x$ (d) $\cot x$

16. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents

(a) straight lines (b) circles (c) parabola (d) ellipse

17. If the function $f(x) = \frac{1}{12}$ $for\ a < x < b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

(a) 0and12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24

18. Four buses carrying 160 students from the same school arrive at a football stadium. The Consider a game where the player tosses a six-sided fair die. If the face that comes up is 6, the player wins ` 36, otherwise he loses ` $k2$, where k is the face that comes up $k = \{1, 2, 3, 4, 5\}$. The expected amount to win at this game in ` is

(a) $\frac{19}{6}$ (b) $\frac{-19}{6}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

19. Which one of the following is incorrect? For any two propositions p and q , we have

(a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ (b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

(c) $\neg(p \vee q) \equiv \neg p \vee \neg q$ (d) $\neg(\neg p) \equiv p$

20. If a compound statement involves 3 simple statements, then the number of rows in the truth table is

(a) 9 (b) 8 (c) 6 (d) 3

II ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER32 IS COMPULSARY. $\mathbf{8 \times 2 = 16}$

21. Find the matrix $A$for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.**(EX 1.1 -12)**

22. Show that $|3z - 5 + i|$represents a circle, and, find its centre and radius. **(EG 2.19)**

23. Formalate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6 . **(EX 3.1 - 11)**

24. Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$for $x > 0$.**(EG 4.24)**

25. If $y = 4x + c$is a tangent to the circle $x^2 + y^2 = 9$, find c.**(EG 5.12)**

26. If $2\vec{\imath} - \vec{\jmath} + 3\vec{k},\ 3\vec{\imath} + 2\vec{\jmath} + \vec{k}, \vec{\imath} + m\vec{\jmath} + 4\vec{k}$are coplanar, find the value of m.**(EG 6.15)**

27. If the mass $m(x)$ (in kilograms) of a thin rod of length x(in metres) is given by, $m(x) = \sqrt{3}x$then what is the rate of change of mass with respect to the length when it is $x = 3$ and $x =$ 27metres.**(EX 7.1 – 5)**

28. Evaluate: $\int_0^{\frac{\pi}{2}} \begin{vmatrix} \cos^4 x & 7 \\ \sin^5 x & 3 \end{vmatrix} dx$.**(EG 9.38)**

29. Compute $P(X = k)$ for binomial distribution, $B(n,p)$ where (i) $n = 6, p = \frac{1}{3}, k = 3$ **(EX 11.5 – 1)**

30. Write the statements in words corresponding to $\neg p, p \wedge q$, $p \vee q$ and $q \vee \neg p$,where p is 'It is cold' and q is 'It is raining.' **(EG 12.12)**

31. Solve the following differential equations: (ii) $ydx + (1 + x^2)\tan^{-1} x\, dy = 0$**(EX 10.5 – 4)**

32. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number. **(EX 8.1 – 7)**

III ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER44 IS COMPULSARY. $8 \times 3 = 24$

33. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = 0_2$.Hence find A^{-1}.**(EX 1.1 -4)**
34. If $\frac{1+z}{1-z} = \cos 2\theta + i \sin 2\theta$,show that $z = i \tan \theta$.**(EX 2.7 - 4)**
35. Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$,are in geometric progression. Assume $a, b, c, d \neq 0$**(EG 3.20)**
36. Find the number of solution of the equation $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1} 3x$.**(EX 4.5 - 10)**
37. If the equation$3x^2 + (3-p)xy + qy^2 - 2px = 8pq$represents a circle, Find$p$and q .Also determine the centre and radius of the circle. **(EX 5.1 - 12)**
38. Let $\vec{a} = \vec{i} + \vec{j} + \vec{k},\ \vec{b} = \vec{i}$and $\vec{c} = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$. If $c_1 = 1$ and $c_2 = 2$,find c_3 such that $\vec{a}, \vec{b}$and $\vec{c}$are coplanar. **(EX 6.2 - 7)**
39. Prove that among all the rectangles of the given area square has the least perimeter. **(EG 7.65)**
40. If $w(x, y, z) = x^2 + y^2 + z^2, x = e^t, y = e^t \sin t$ and $y = e^t \cos t$, find $\frac{dw}{dt}$.**(EX 8.6 – 3)**
41. Evaluate: $\int_0^{2\pi} x^2 \sin nx\, dx$, where n is a positive integer. **(EG 9.33)**
42. Show that $y = 2(x^2 - 1) + Ce^{-x^2}$ is a solution of the differential equation $\frac{dy}{dx} + 2xy - 4x^3 = 3$.**(EG 10.9)**
43. Establish the equivalence property connecting the bi-conditional with conditional: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$.**(EG 12.18)**
44. An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of inverse images. **(EG 11.3)**

IV ANSWERS THE FOLLOWING QUESTIONS: $8 \times 5 = 40$

45. ($\boldsymbol{a}$)Solve :$8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$.**(EX 3.5 - 3)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Two coast guard stations are located 600 km apart at points $A(0,0)$and $B(0,600)$.A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station Athan it is from station B . Determine the equation of hyperbola that passes through the location of the ship. **(EG 5.39)**

46. ($\boldsymbol{a}$)Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$.**(EX 6.7 – 2)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Solve the equation $z^3 + 27 = 0$.**(EX 2.8 - 5)**

47. ($\boldsymbol{a}$)Evaluate $\sin\left[sin^{-1}\left(\frac{3}{5}\right) + sec^{-1}\left(\frac{5}{4}\right)\right]$.**(EG 4.20)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Test for consistency of the following system of linear equations and if possible solve: $x - y + z = -9, 2x - 2y + 2z = -18, 3x - 3y + 3z + 27 = 0$.**(EG 1.31)**

48. ($\boldsymbol{a}$)Solve: $(1 + x^3)\frac{dy}{dx} + 6x^2y = 1 + x^2$.**(EG 10.25)**

($\boldsymbol{OR}$)

$(\boldsymbol{b})$A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 $sq.mtrs$ in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material? **(EX 7.8 – 6)**

49. $(\boldsymbol{a})$For each of the following functions find the $f_x, f_y$and show that $f_{xy} = f_{yx}$. (i) $f(x,y) = \frac{3x}{y+\sin x}$**(EX 8.4 – 2)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function $f(x) = \begin{cases} k, & 200 \le x \le 600 \\ 0, & \text{Otherwise} \end{cases}$ Find (i) the value of k (ii) the distribution function (iii) the probability that daily sales will fall between 300 litres and 500 litres? **(EX 11.3 – 3)**

50. $(\boldsymbol{a})$Identify the type of conic and find centre, foci, vertices, and directrices of each of the following: (vi) $9x^2 - y^2 - 36x - 6y + 18 = 0$**(EX 5.2 - 8)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k},\ \vec{b} = 3\vec{i} + 5\vec{j} + 2\vec{k},\ \vec{c} = -\vec{i} - 2\vec{j} + 3\vec{k}$, verify that $(ii)\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$.**(EX 6.3 - 4)**

51. $(\boldsymbol{a})$Verify (i) closure property, (ii) commutative property,(iii) associative property, (iv) existence of identity, and(v) existence of inverse for the operation X_{11} on a subset $A = \{1,3,4,5,9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$.**(EG 12.10)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Find the points on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is horizontal. **(EX 7.2 – 4)**

52. $(\boldsymbol{a})$If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$,show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.**(EX 1.1 -14)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Find the volume of the spherical cap of height h cut of from a sphere of radius r.**(EG 9.64)**

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M-7 **+2**

2023 MODEL FULL PORTION QUESTION PAPERS - 7
MATHEMATICS

MARKS: 100 **TIME: 3 HOURS + 15 MIN.**

I CHOOSE THE CORRECT ANSWERS: $20 \times 1 = 20$

1. If $\rho(A) = \rho([A|B])$,then the system $AX = B$of linear equations is
(a)Consistent and has a unique solution (b)consistent
(c)Consistent and has infinitely many solution (d)inconsistent

2. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$then $|adj\ (AB)| =$
(a) -40 (b) -80 (c) -60 (d) -20

3. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B)equals
(a) (1,0) (b) (−1,1) (c) (0,1) (d) (1,1)

4. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is
(a) $cis\frac{2\pi}{3}$ (b) $cis\frac{4\pi}{3}$ (c) $-cis\frac{2\pi}{3}$ (d) $-cis\frac{4\pi}{3}$

5. The polynomial $x^3 + 2x + 3$has
(a)one negative and two real roots (b) one positive and two imaginary roots
(c)three real roots (d) no solution

6. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$is
(a) 0 (b) 1 (c) 2 (d) 3

7. If the two tangents drawn from a point $P$to the parabola $y^2 = 4x$are at right angles then the locus of $P$is
(a) $2x + 1 = 0$ (b) $x = -1$ (c)$2x - 1 = 0$ (d)$x = 1$

8. If $x + y = k$is a normal to the parabola $y^2 = 12x$, then the value of $k$is
(a) 3 (b) -1 (c) 1 (d) 9

9. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$,then the value of $[\vec{a}, \vec{b}, \vec{c}]$is
(a) $|\vec{a}||\vec{b}||\vec{c}|$ (b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (c) 1 (d) -1

10. If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$,where $\vec{a}, \vec{b}, \vec{c}$are any three vectors such that $\vec{b}.\vec{c} \neq 0$and $\vec{a}.\vec{b} \neq 0$, then $\vec{a}$and $\vec{c}$are
(a)perpendicular (b) parallel (c)inclined at an angle $\frac{\pi}{3}$ (d) inclined at an angle $\frac{\pi}{6}$

11. The maximum product of two positive numbers, when their sum of the squares is 200, is
(a)100 (b)$25\sqrt{7}$ (c) 28 (d) $24\sqrt{14}$

12. The change in the surface area $S = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$is
(a) $12x_0 + dx$ (b) $12x_0dx$ (c) $6x_0dx$ (d) $6\ x_0 + dx$

13. The value of $\int_0^\infty e^{-3x}x^2 dx$ is
(a) $\frac{7}{27}$ (b) $\frac{5}{27}$ (c) $\frac{4}{27}$ (d) $\frac{2}{27}$

14. The value of $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) π

15. The solution of $\frac{dy}{dx} = 2^{y-x}$ is
(a) $2^x + 2^y = C$ (b) $2^x - 2^y = C$ (c) $\frac{1}{2^x} - \frac{1}{2^y} = C$ (d) $x + y = C$

16. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^{\frac{1}{4}} = 0$ are respectively
(a) 2, 3 (b) 3, 3 (c) 2, 6 (d) 2, 4

17. Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X-3)$ is
(a) 0.24 (b) 0.48 (c) 0.6 (d) 0.96

18. Which of the following is a discrete random variable? I. The number of cars crossing a particular signal in a day. II. The number of customers in a queue to buy train tickets at a moment. III. The time taken to complete a telephone call.
(a) I and II (b) II only (c) III only (d) II and III

19. Which one of the following is a binary operation on $\mathbb{N}$?
(a) Subtraction (b) Multiplication (c) Division (d) All the above

20. Which one of the following statements has truth value F ?
(a) Chennai is in India or 2 is an integer (b) Chennai is in India or 2 is an irrational number
(c) Chennai is in China or 2 is an integer (d) Chennai is in China or 2 is an irrational number

II ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER32 IS COMPULSARY. **$8 \times 2 = 16$**

21. If A is a non-singular matrix of odd order, prove that $|adj\ A|$ is positive. **(EG 1.4)**
22. Represent the complex number $(i)\ -1 - i$ $(ii)\ 1 + i\sqrt{3}$ in polar form. **(EG 2.23)**
23. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients. **(EG 3.3)**
24. Find the period and amplitude of $(i)\ y = \sin 7x$ $(ii)\ y = -\sin\left(\frac{1}{3}x\right)$ $(iii) y = 4\sin(-2x)$. **(EX 4.1 - 2)**
25. Obtain the equation of the circles with radius 5 cm and touching x –axis at the origin in general form. **(EX 5.1 - 1)**
26. Show that the points $(2,3,4), (-1,4,5)$ –and $(8,1,2)$ are collinear. **(EX 6.4 - 9)**
27. Suppose $f(x)$ is a differentiable function for all x with $f'(x) \le 29$ and $f(b) = 17$. What the maximum value is of $f(7)$? **(EG 7.27)**
28. If $w(x,y,z) = x^2y + y^2z + z^2x,\ x,y,z \in \mathbb{R}$, find the differential dw. **(EG 8.16)**
29. For each of the following differential equations, determine its order, degree (if exists) $y\frac{dy}{dx} = \frac{x}{\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3}$ **(EX 10.1 – 1)**
30. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred. **(EX 11.2 – 1)**
31. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$. **(EG 12.8)**
32. Evaluate: $\int_0^1 [2x]dx$ where $[\cdot]$ is the greatest integer function. **(EG 9.7)**

III ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER44 IS COMPULSARY. **$8 \times 3 = 24$**

33. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adj\ A) = (adj\ A)A = |A|I_2$. **(EX 1.1 -6)**

34. Find the principal argument $Arg\ z$,when $z = \frac{-2}{1+i\sqrt{3}}$. **(EG 2.24)**

35. If $p$and $q$are the roots of the equation $lx^2 + nx + n = 0$,show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$. **(EX 3.1 - 9)**

36. Find the domain of $\sin^{-1}(2 - 3x^2)$**(EG 4.4)**

37. Find the centre and radius of the circle $3x^2 + (a + 1)y^2 + 6x - 9y + a + 4 = 0$.**(EG 5.9)**

38. Prove by vector method that an angle in a semi-circle is a right angle. **(EX 6.1 - 3)**

39. Find the equations of tangent and normal to the curve $y = x^2 + 3x - 2$at the point $(1, 2)$.**(EG 7.11)**

40. Let $u(x, y) = e^{-2y}\cos(2x)$ for all $(x, y) \in \mathbb{R}^2$.Prove that u is a harmonic function in $\mathbb{R}^2$.**(EG 8.15)**

41. Find an approximate value of $\int_1^{1.5} x dx$ by applying the left-end rule with the partition $\{1.1,\ 1.2,\ 1.3,\ 1.4,\ 1.5\}$.**(EX 9.1 – 1)**

42. The probability that $Mr. Q$ hits a target at any trial is $\frac{1}{4}$.Suppose he tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time. **(EX 11.5 – 2)**

43. Prove $p \to (q \to r) \equiv (p \wedge q) \to r$ without using truth table. **(EX 12.2 – 14)**

44. Solve: $\frac{dy}{dx} + 2y = e^{-x}$.**(EG 10.22)**

IV ANSWERS THE FOLLOWING QUESTIONS: **$8 \times 5 = 40$**

45. $(\boldsymbol{a})$Find the value of (ii) $sin\left(tan^{-1}\left(\frac{1}{2}\right) - cos^{-1}\left(\frac{4}{5}\right)\right)$.**(EX 4.3 - 4)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.**(EX 7.8 – 7)**

46. $(\boldsymbol{a})$Solve the following systems of linear equations by Cramer's rule: $(iv)\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$ **(EX 1.4 - 1)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Suppose $z_1, z_2$and$z_3$are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$.If $z_1 = 1 + i\sqrt{3}$, then find $z_2$andz_3.**(EG 2.36)**

47. $(\boldsymbol{a})$At a water fountain, water attains a maximum height of $4m$at horizontal distance of 0.5 $m$from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 m from the point of origin. **(EX 5.5 - 3)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$if it is known that $\frac{1}{3}$is a solution. **(EX 3.5 - 7)**

48. $(\boldsymbol{a})$Find the vector parametric, vector non-parametric and Cartesian formof the equation of the plane passing through the points $(-1,2,0), (2,2,-1)$and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.**(EG 6.44)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Prove that $g(x, y) = x\log\left(\frac{y}{x}\right)$is homogeneous. What is the degree? Verify Euler's Theorem for g.**(EX 8.7 – 3)**

49. $(\boldsymbol{a})$If the curves $ax^2 + by^2 = 1$and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$.**(EG 7.17)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Father of a family wishes to divide his square field bounded by $x = 0, x = 4, y = 4$ and $y = 0$ along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them. **(EX 9.8 – 8)**

50. $(\boldsymbol{a})$Solve:$\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$.**(EG 10.15)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Solve the following system of homogenous equations. $(i) 3x + 2y + 7z = 0, 4x - 3y - 2z = 0, 5x + 9y + 23z = 0$. **(EX 1.7 - 1)**

51. $(\boldsymbol{a})$Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r} = \left(6\vec{i} - \vec{j} + \vec{k}\right) + s\left(-\vec{i} + 2\vec{j} + \vec{k}\right) + t\left(-5\vec{i} - 4\vec{j} - 5\vec{k}\right)$.**(EX 6.7 – 7)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Verify whether the following compound propositions are tautologies or contradictions or contingency $(iv)\big((p \rightarrow q) \wedge (q \rightarrow r)\big) \rightarrow (p \rightarrow r)$**(EX 12.2 – 7)**

52. $(\boldsymbol{a})$A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws. (i) Find the probability mass function. (ii) Find the cumulative distribution function. (iii) Find $P(3 \leq X < 6)$ (iv) Find $P(X \geq 4)$.**(EG 11.8)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Find the equations of tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$which are parallel to$10x - 3y + 9 = 0$.**(EX 5.4 - 2)**

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2023 MODEL FULL PORTION QUESTION PAPERS - 8
MATHEMATICS

MARKS: 100 **TIME: 3 HOURS + 15 MIN.**

I CHOOSE THE CORRECT ANSWERS: $\mathbf{20 \times 1 = 20}$

1. If $A$is a 3×3non-singular matrix such that $AA^T = A^T$and $B = A^{-1}A^T$, then $BB^T =$
$(a)\ A$ $(b)\ B$ $(c)\ I$ $(d)\ B^T$

2. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$.The system has infinitely many solutions if
$(a)\ \lambda = 7, \mu \neq -5$ $(b) \lambda = -7, \mu = 5$ $(c)\ \lambda \neq 7, \mu \neq -5$, $(d) \lambda = 7, \mu = -5$

3. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$is
$(a)\ \frac{1}{2}$ $(b)\ 1$ $(c)\ 2$ $(d)\ 3$

4. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$is
$(a)\ 0$ $(b)\ 1$ $(c) - 1$ $(d)\ i$

5. If $x^3 + 12x^2 + 10ax + 1999$definitely has a positive root, if and only if
$(a)\ a \geq 0$ $(b)\ a > 0$ $(c)\ a < 0$ $(d)\ a \leq 0$

6. $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$. Then $x$is a root of the equation
$(a) x^2 - x - 6 = 0$ $(b)\ x^2 - x - 12 = 0$ $(c) x^2 + x - 12 = 0$ $(d) x^2 + x - 6 = 0$

7. If $\cot^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha})$, then $\cos 2u$is equal to
$(a)\ \tan^2\alpha$ $(b)\ 0$ $(c) - 1$ $(d)\ \tan 2\alpha$

8. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$is
$(a)\ 4(a^2 + b^2)$ $(b)\ 2(a^2 + b^2)$ $(c)\ \ a^2 + b^2$ $(d)\ \frac{1}{2}(a^2 + b^2)$

9. The centre of the circle inscribed in a square formed by the lines $x^2 - 8 - 12 = 0$and $y^2 - 14y + 45 = 0$is
$(a)\ (4,7)$ $(b)\ (7,4\ )$ $(c)\ (9,4)$ $(d)\ (4,9)$

10. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$,then
$(a)\ c = \pm 3$ $(b)\ c = \pm\sqrt{3}$ $(c)\ c > 0$ $(d)\ 0 < c < 1$

11. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$and $\vec{\gamma}$,then
$(a)[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 1$ $(b)[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = -1$ $(c)[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ $(d)[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 2$

12. If the planes $\vec{r}.(2\vec{i} - \lambda\vec{j} + \vec{k}) = 3$and $\vec{r}.(4\vec{i} + \vec{j} - \mu\vec{k}) = 5$are parallel, then the value of λ and μ are
$(a) \frac{1}{2}, -2$ $(b)\ \frac{-1}{2}, 2$ $(c) \frac{-1}{2}, -2$ $(d)\ \frac{1}{2}, 2$

13. A stone is thrown up vertically. The height it reaches at time $t$seconds is given by the tangent to the curve $y^2 + xy\ + 9 = 0$is vertical when
$(a) y = 0$ $(b) y = \pm\ 3$ $(c) y = 1$ $(d) y = \pm 3$

14. The point of inflection of the curve $y = (x - 1)^3$is
$(a)(0,0)$ $(b)(0,1)$ $(c)(1,0)$ $(d)(1,1)$

15. If $w(x,y,z) = x^2(y-z) + y^2(z-x) + z^2(x-y)$, then is
    $(a)\ xy + yz + zx$ $(b)\ x(y+z)$ $(c)\ y(z+x)$ $(d)\ 0$
16. The value of $\int_0^{\frac{\pi}{6}} \cos^3 3x\, dx$ is
    $(a)\frac{2}{3}$ $(b)\frac{2}{9}$ $(c)\frac{1}{9}$ $(d)\frac{1}{3}$
17. The solution of $\frac{dy}{dx} + p(x)y = 0$ is
    $(a)\ y = ce^{\int pdx}$ $(b)\ y = ce^{-\int pdx}$ $(c)\ x = ce^{-\int pdy}$ $(d)\ x = ce^{\int pdy}$
18. The population P in any year t is such that the rate of increase in the population is proportional to the population. Then
    $(a)\ P = Ce^{kt}$ $(b)\ P = Ce^{-kt}$ $(c)\ P = Ckt$ $(d)\ P = C$
19. A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is
    $(a)\ 6$ $(b)\ 4$ $(c)\ 3$ $(d)\ 2$
20. Subtraction is not a binary operation in
    $(a)\mathbb{R}$ $(b)\mathbb{Z}$ $(c)\mathbb{N}$ $(d)\mathbb{Q}$

II ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER 32 IS COMPULSARY. $\mathbf{8 \times 2 = 16}$

21. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$is non-singular, find A^{-1}.**(EG 1.2)**
22. If $\omega \neq 1$ is a cube root of unity, show that$(1-\omega+\omega^2)^6(1+\omega-\omega^2)^6 = 128$. **(EX 2.8 - 8)**
23. Show that, if $p, q, r$are rational, the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$are rational. **(EG 3.13)**
24. Is$cos^{-1}(-x) = \pi - cos^{-1}(x)$true? Justify your answer. **(EX 4.2 - 3)**
25. A circle of radius 3 units touches both the axes. Find the equations of all possible circles formed in the general form. **(EG 5.8)**
26. Prove that $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$.**(EG 6.19)**
27. Expand the polynomial $f(x) = x^2 - 3x + 2$ in powers of $x - 1$.**(EX 7.4 – 4)**
28. In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree. $g(x,y,z) = \frac{\sqrt{3x^2+5y^2+z^2}}{4x+7y}$ **(EX 8.7 – 1)**
29. Evaluate the following definite integrals:$(i)\ \int_3^4 \frac{dx}{x^2-4}$**(EX 9.3 – 1)**
30. Express each of the following physical statements in the form of differential equation.(i) Radium decays at a rate proportional to the amount Q present. **(EX 10.2 – 1(i))**
31. Determine whether $*$ is a binary operation on the sets given below.$(ii) a * b = min\ (a,b)$on $A = \{1,2,3,4,5\}$**(EX 12.1 – 1)**
32. The mean and variance of a binomial variate$X$ are respectively 2 and 1.5. Find $(i)\ P(X=0)$**(EG 11.21)**

III ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER 44 IS COMPULSARY. $\mathbf{8 \times 3 = 24}$

33. Find $adj(adj\ (A))$if $(adj\ (A)) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$. **(EX 1.1 -10)**
34. Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$are the vertices of an equilateral triangle. **(EG 2.14)**
35. Prove that a line cannot intersect a circle at more than two points. **(EG 3.14)**

36. Sketch the graph of $y = \sin\left(\frac{1}{3}x\right)$for $0 \leq x < 6\pi$.**(EX 4.1 - 3)**

37. If $y = 2\sqrt{2}x + c$is a tangent to the circle $x^2 + y^2 = 16$, find the value of c.**(EX 5.1 - 8)**

38. Show that the lines $\vec{r} = (6\vec{i} + \vec{j} + 2\vec{k}) + s(\vec{i} + 2\vec{j} - 3\vec{k})$and $\vec{r} = (3\vec{i} + 2\vec{j} - 2\vec{k}) + t(2\vec{i} + 4\vec{j} - 5\vec{k})$ are skew lines and hence find the shortest distance between them. **(EX 6.5 - 2)**

39. A beacon makes one revolution every 10seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45°with the shore? **(EX 7.1 – 7)**

40. If $U(x, y, z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$.**(EX 8.4 – 4)**

41. Show that $\int_0^1 (\tan^{-1} x + \tan^{-1}(1 - x))dx = \frac{\pi}{2} - \log 2$**(EG 9.28)**

42. Show that $y = e^{-x} + mx + n$ is a solution of differential equation $e^x\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.**(EX 10.4 – 4)**

43. Show that $(ii)\ \neg(p \to q) \equiv p \wedge \neg q$.**(EX 12.2 – 8)**

44. If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X + 3) = 10$ and $E(X + 3)^2 = 116$, find μ and σ^2.**(EX 11.4 – 3)**

IV ANSWERS THE FOLLOWING QUESTIONS: **$8 \times 5 = 40$**

45. $(\boldsymbol{a})$By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \to CO_2 + H_2O$. (The above is the reaction that is taking place in the burning of organic compound called isoprene.) **(EG 1.39)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$If $2\cos\alpha = x + \frac{1}{x}and2\cos\beta = y + \frac{1}{y}$show that $(i)\ \frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$ $(ii) x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$. **(EX 2.8 - 4)**

46. $(\boldsymbol{a})$Solve:$2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$. **(EX 3.5 - 4)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6$m$along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.**(EX 5.5 - 5)**

47. $(\boldsymbol{a})$Find parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r} = (\vec{i} - \vec{j} + 3\vec{k}) + t(2\vec{i} - \vec{j} + 4\vec{k})$ and perpendicular to plane $\vec{r}.(\vec{i} + 2\vec{j} + \vec{k}) = 8$.**(EX 6.7 – 5)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Sketch the curve $(x) = x^2 - x - 6$.. **(EG 7.69)**

48. $(\boldsymbol{a})$If $V(x, y) = e^x(x\cos y - y\sin y)$, then prove that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$.**(EX 8.4 – 7)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$The curve $y = (x - 2)^2 + 1$ has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2. Find the area bounded by the curve and the chord PQ.**(EX 9.8 – 9)**

49. $(\boldsymbol{a})$A tank initially contains 50 litres of pure water. Starting at time $t = 0$a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time $t > 0$.**(EX 10.8 – 10)**

$(\boldsymbol{OR})$

($\boldsymbol{b}$)Find the area of the region bounded by the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$.**(EX 9.8 – 4)**

50. ($\boldsymbol{a}$)A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find (i) the probability mass function (ii) the cumulative distribution function (iii) $P(4 \leq X < 10)$$(iv)$ $P(X \geq 6)$**(EX 11.2 – 2)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Find the vertex, focus, directrix, and length of the latus rectum of the parabola $x^2 - 4x - 5y - 1 = 0$.**(EG 5.17)**

51. ($\boldsymbol{a}$)If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k},\ \vec{b} = 3\vec{i} + 5\vec{j} + 2\vec{k},\ \vec{c} = -\vec{i} - 2\vec{j} + 3\vec{k}$, verify that $(i)(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$**(EX 6.3 - 4)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent **(EX 12.2 – 10)**

52. ($\boldsymbol{a}$)A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$. (i)At what times the particle changes direction? (ii) Find the total distance travelled by the particle in the first 4 seconds. (iii) Find the particle's acceleration each time the velocity is zero. **(EX 7.1 – 3)**

($\boldsymbol{OR}$)

($\boldsymbol{b}$)Determine the values of λ for which the following system of equations $x + y + 3z = 0, 4x + 3y + \lambda z = 0, 2x + y + 2z = 0$,has$(i)$a unique solution (ii) a non-trivial solution. **(EX 1.7 - 2)**

LONDON KRISHNAMOORTHY MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

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2023 MODEL FULL PORTION QUESTION PAPERS - 9

MATHEMATICS

MARKS: 100 TIME: 3 HOURS + 15 MIN.

I CHOOSE THE CORRECT ANSWERS: $20 \times 1 = 20$

1. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = adj\,A$ and $C = 3A$, then $\frac{|adj\,B|}{|C|} =$
   (a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1
2. Which of the following is/are correct? (i) Adjoint of a symmetric matrix is also a symmetric matrix. (ii) Adjoint of a diagonal matrix is also a diagonal matrix. (iii) If A is a square matrix of order n and λ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$. (iv) $A(adj\,A) = A(adj\,A) = |A|I$
   (a) Only (i) (b) (ii) and (iii) (c) (iii) and (iv) (d) (i), (ii) and (iv)
3. If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to
   (a) 1 (b) -1 (c) $\sqrt{3}i$ (d) $-\sqrt{3}i$
4. If $(1+i)(1+2i)(1+3i)\ldots(1+ni) = x + iy$, then $2.5.10\ldots(1+n^2)$ is
   (a) 1 (b) i (c) $x^2 + y^2$ (d) $1 + n^2$
5. The polynomial $x^3 - kx^2 + 9x$ has three real roots if and only if, k satisfies
   (a) $|k| \leq 6$ (b) $k = 0$ (c) $|k| > 6$ (d) $|k| \geq 6$
6. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is
   (a) $[1,2]$ (b) $[-1,1]$ (c) $[0,1]$ (d) $[-1,0]$
7. The length of the diameter of the circle which touches the x -axis at the point (1,0) and passes through the point (2,3).
   (a) $\frac{6}{5}$ (b) $\frac{5}{3}$ (c) $\frac{10}{3}$ (d) $\frac{3}{5}$
8. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is
   (a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$
9. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]\}^2$ is equal to
   (a) 81 (b) 9 (c) 27 (d) 18
10. If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - 5\vec{k}$, $\vec{c} = 3\vec{i} + 5\vec{j} - \vec{k}$, then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is
    (a) $-17\vec{i} + 21\vec{j} - 97\vec{k}$ (b) $17\vec{i} + 21\vec{j} - 123\vec{k}$
    (c) $-17\vec{i} - 21\vec{j} + 97\vec{k}$ (d) $-17\vec{i} - 21\vec{j} - 97\vec{k}$
11. The vector equation $\vec{r} = (\vec{i} - 2\vec{j} - \vec{k}) + t(6\vec{i} - \vec{k})$ represents a straight line passing through the points
    (a) $(0,6,-1)$ and $(1,-2,-1)$ (b) $(0,6,-1)$ and $(-1,-4,-2)$
    (c) $(1,-2,-1)$ and $(1,4,-2)$ (d) $(1,-2,-1)$ and $(0,-6,1)$
12. $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by
    (a) 2 (b) 2.5 (c) 3 (d) 3.5
13. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

(a) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left[0, \frac{\pi}{4}\right]$

14. If $u(x, y) = x^2 + 3xy + y - 2019$, then $\left(\frac{\partial u}{\partial x}\right)_{(4,5)}$ is equal to

(a) -4 (b) -3 (c) -7 (d) 13

15. The value of $\int_0^a \left(\sqrt{a^2 - x^2}\right)^3 dx$ is

(a) $\frac{\pi a^3}{16}$ (b) $\frac{3\pi a^4}{16}$ (c) $\frac{3\pi a^2}{8}$ (d) $\frac{3\pi a^4}{8}$

16. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(a) $xy = k$ (b) $y = k \log x$ (c) $y = kx$ (d) $\log y = kx$

17. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of a is

(a) 2 (b) -2 (c) 1 (d) -1

18. A computer salesperson knows from h is past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?

(a) $\frac{57}{20^3}$ (b) $\frac{57}{20^2}$ (c) $\frac{19^3}{20^3}$ (d) $\frac{57}{20}$

19. A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is rolled and the sum is determined. Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is

(a) 1 (b) 2 (c) 3 (d) 4

20. Which one is the inverse of the statement $(p \vee q) \rightarrow (p \wedge q)$?

(a) $(p \wedge q) \rightarrow (p \vee q)$ (b) $\neg(p \vee q) \rightarrow (p \wedge q)$
(c) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$ (d) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$

II ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER32 IS COMPULSARY. $\mathbf{8 \times 2 = 16}$

21. Find the following $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$ **(EG 2.10)**
22. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$. **(EX 3.6 - 4)**
23. For what value of x , the inequality $\frac{\pi}{2} < \cos^{-1}(3x - 1) < \pi$ holds? **(EX 4.2 - 7)**
24. Find the equations of the tangent and normal to the circle $x^2 + y^2 = 25$ at $P(-3,4)$. **(EG 5.11)**
25. Find the length of the perpendicular from the point $(1, -2, 3)$ to the plane $x - y + z = 5$. **(EX 6.9 – 6)**
26. If $\lim\limits_{\theta \to 0} \left(\frac{1 - \cos m\theta}{1 - \cos n\theta}\right) = 1$, then prove that $m = \pm n$. **(EG 7.37)**
27. A sphere is made of ice having radius 10 cm. Its radius decreases from $10cm$ to 9.8 cm. Find approximations for the following: (i) Change in the volume **(EX 8.1 – 5)**
28. Evaluate $\int_0^1 x dx$, as the limit of a sum. **(EG 9.2)**
29. For each of the following differential equations, determine its order, degree (if exists) $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} + 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4 = 0$ **(EX 10.1 – 1)**
30. Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where (i) $n = 6, p = \frac{1}{3}, k = 3$ **(EX 11.5 – 1)**
31. On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n + n^m : m, n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$? **(EX 12.1 – 2)**

32. If $adjA = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$,find A^{-1}.**(EG 1.6)**

III ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER44 IS COMPULSARY. $\mathbf{8 \times 3 = 24}$

33. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$,show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.**(EX 1.1 -11)**
34. If $z_1, z_2$and $z_3$are complex numbers such that$|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$,find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$.**(EG 2.12)**
35. Find the condition that the roots of $x^3 + ax^2 + bx + c = 0$, are in the ratio $p{:}q{:}r$. **(EG 3.5)**
36. Find $(i)\ tan^{-1}(-\sqrt{3})$ $(ii)\ tan^{-1}\left(\tan\left(\frac{3\pi}{5}\right)\right)$ $(iii)\ \tan(\tan^{-1}(2019))$**(EG 4.9)**
37. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis. **(EX 5.2 - 7)**
38. With usual notations, in any triangle ABC,prove the following by vector method. $(iii) c^2 = a^2 + b^2 - 2ab\cos C$.**(EG 6.1)**
39. Show that the value in the conclusion of the mean value theorem for$(i) f(x) = \frac{1}{x}$on a closed interval of positive numbers $[a, b]$is $\sqrt{ab}$.**(EX 7.3 – 5)**
40. Let $U(x, y, z) = x^2 - xy + \sin z\,, x, y, z \in \mathbb{R}$. Find the linear approximation for U at $(2, -1, 0)$.**(EG 8.17)**
41. Evaluate: $\int_{-4}^{4} |x + 3| dx$**(EG 9.15)**
42. A lottery with 600 tickets gives one prize of `200, four prizes of `100, and six prizes of ` 50. If the ticket costs is ` 2, find the expected winning amount of a ticket. **(EX 11.4 – 8)**
43. Verify whether the following compound propositions are tautologies or contradictions or contingency $(ii)\big((p \vee q) \wedge \neg p\big) \rightarrow q$**(EX 12.2 – 7)**
44. Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later? **(EX 10.8 – 5)**

IV ANSWERS THE FOLLOWING QUESTIONS: $\mathbf{8 \times 5 = 40}$

45. $(\boldsymbol{a})$For each of the following functions find the $f_x, f_y$and show that $f_{xy} = f_{yx}$. $(ii)\ f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$.**(EX 8.4 – 2)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$find the products $AB$and $BA$and hence solve the system of equations $x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2$.**(EX 1.3 - 2)**

46. $(\boldsymbol{a})$Prove that among all the rectangles of the given perimeter, the square has the maximum area. **(EX 7.8 – 8)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150 $m$tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower. **(EX 5.5 - 6)**

47. $(\boldsymbol{a})$Find the foot of the perpendicular drawn from the point $(5,4,2)$ to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$.Also, find the equation of the perpendicular. **(EX 6.5 - 7)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$If $z=x+iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right)=0$.Show that the locus of $z$is $2x^2+2y^2+x-2y=0$.**(EX 2.6 - 2)**

48. $(\boldsymbol{a})$Using integration find the area of the region bounded by triangle ABC, whosevertices A, B, and C are $(-1,1), (3,2)$, and $(0,5)$ respectively. **(EG 9.60**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Test for consistency of the following system of linear equations and if possible solve: $x+2y-z=3, 3x-y+2z=1, x-2y+3z=3, x-y+z+1=0$. **(EG 1.29)**

49. $(\boldsymbol{a})$In a murder investigation, a corpse was found by a detective at exactly$8p.m$. Being alert, the detective also measured the body temperature and found it to be $70°F$. Two hours later, the detective measured the body temperature again and found it to be $60°F$. If the room temperature is $50°F$, and assuming that the body temperature of the person before death was $98.6°F$, at what time did the murder occur? **(EG 10.29)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Identify the type of conic and find centre, foci, vertices, and directrices of each of the following: $(i)\frac{(x-3)^2}{225}+\frac{(y-4)^2}{289}=1$.**(EX 5.2 - 8)**

50. $(\boldsymbol{a})$Find the equation of a straight line passing through the point of intersection of the straight lines $\vec{r}=(\vec{i}+3\vec{j}-\vec{k})+t(2\vec{i}+3\vec{j}+2\vec{k})$ and $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z+3}{4}$, and perpendicular to both straight lines. **(EG 6.34)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s.When the base of the ladder is 8 metres from the wall. (i) How fast is the top of the ladder moving down the wall? (ii) At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing? **(EX 7.1 – 9)**

51. $(\boldsymbol{a})$Expand $\tan x$in ascending powers of $x$upto5^{th}power for $-\frac{\pi}{2}<x<\frac{\pi}{2}$.**(EG 7.31)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$The probability density function of X is given by $f(x)=\begin{cases} ke^{\frac{-x}{3}}, & for\ x>0 \\ 0, & for\ x\leq 0\end{cases}$ Find (i) the value of k (ii) the distribution function (iii) $P(X<3)(iv)$ $P(5\leq X)(v)$ $P(X\leq 4)$.**(EX 11.3 – 4)**

52. $(\boldsymbol{a})$Form the equation whose roots are the squares of the roots of the cubic equation $x^3+ax^2+bx+c=0$.**(EG 3.6)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$A pot of boiling water at $100°C$ is removed from a stove at time $t=0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to $80°C$, and another 5 minuteslater it has dropped to $65°C$ .Determine the temperature of the kitchen. **(EX 10.8 – 9)**

LONDON KRISHNAMOORTHY MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

M-10 **+2**

2023 MODEL FULL PORTION QUESTION PAPERS - 10

MATHEMATICS

MARKS: 100 TIME: 3 HOURS + 15 MIN.

I CHOOSE THE CORRECT ANSWERS: $20 \times 1 = 20$

1. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then the values of x and y are respectively,
   (a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (b) $log(\Delta_1/\Delta_3), log(\Delta_2/\Delta_3)$ 3
   (c) $log(\Delta_2/\Delta_1), log(\Delta_3/\Delta_1)$ (d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$
2. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x - y + z = 0$ has a non-trivial solution then θ is
   (a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$
3. The solution of the equation $|z| - z = 1 + 2i$ is
   (a) $\frac{3}{2} - 2i$ (b) $\frac{-3}{2} + 2i$ (c) $2 - \frac{3}{2}i$ (d) $2 + \frac{3}{2}i$
4. Let z_1, z_2 and z_3 be complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$, then ${z_1}^2 + {z_2}^2 + {z_3}^2$ is
   (a) 3 (b) 2 (c) 1 (d) 0
5. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is
   (a) mn (b) $m + n$ (c) m^n (d) n^m
6. If $\sin^{-1} x = 2\sin^{-1} \alpha$ has a solution, then
   (a) $|\alpha| \le \frac{1}{\sqrt{2}}$ (b) $|\alpha| \ge \frac{1}{\sqrt{2}}$ (c) $|\alpha| < \frac{1}{\sqrt{2}}$ (d) $|\alpha| > \frac{1}{\sqrt{2}}$
7. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is
   (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$
8. An ellipse has OB as semi minor axes, F and F' its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is
   (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$
9. The volume of the parallelepiped with its edges represented by the vectors $\vec{i} + \vec{j}$, $\vec{i} + 2\vec{j}$, $\vec{i} + \vec{j} + \pi\vec{k}$ is
   (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$
10. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to
    (a) 2 (b) -1 (c) 1 (d) 0
11. One of the closest points on the curve $x^2 - y^2 = 4$ to the point $(6,0)$ is
    (a) $(2,0)$ (b) $(\sqrt{5}, 1)$ (c) $(3, \sqrt{5})$ (d) $(\sqrt{13}, -\sqrt{3})$
12. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is
    (a) $0.3xdx\ m^3$ (b) $0.3x\ m^3$ (c) $0.3\ x^2 m^3$ (d) $0.03\ x^3 m^3$

13. The value of $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \left[\frac{2x^7-3x^5+7x^3-x+1}{\cos^2 x}\right] dx$ is
(a) 4 (b) 3 (c) 2 (d) 0

14. The value of $\int_0^{\pi} \sin^4 x\, dx$ is
(a) $\frac{3\pi}{10}$ (b) $\frac{3\pi}{8}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

15. The order of the differential equation of all circles with centre at (h, k) and radius 'a' is
(a) 2 (b) 3 (c) 4 (d) 1

16. The solution of the differential equation $\frac{dy}{dx} = 2xy$ is
(a) $y = Ce^{x^2}$ (b) $y = 2x^2 + C$ (c) $y = Ce^{-x^2} + C$ (d) $y = x^2 + C$

17. On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is
(a) $\frac{11}{243}$ (b) $\frac{3}{8}$ (c) $\frac{1}{243}$ (d) $\frac{5}{243}$

18. If X is a binomial random variable with expected value 6 and variance 2.4, Then $P(X = 5)$is
(a) $\binom{10}{5}\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$ (b) $\binom{10}{5}\left(\frac{3}{5}\right)^{10}$ (c) $\binom{10}{5}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^6$ (d) $\binom{10}{5}\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$

19. A binary operation on a set S is a function from
(a) $S \to S$ (b) $(S \times S) \to S$ (c) $S \to (S \times S)$ (d) $(S \times S) \to (S \times S)$

20. The dual of $\neg(p \vee q) \vee [p \vee (p \wedge \neg r)]$is
(a) $\neg(p \wedge q) \wedge [p \vee (p \wedge \neg r)]$ (b) $(p \vee q) \wedge [p \wedge (p \vee \neg r)]$
(c) $\neg(p \wedge q) \wedge [p \wedge (p \wedge r)]$ (d) $\neg(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

II ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER 32 IS COMPULSARY. **$8 \times 2 = 16$**

21. Verify $(AB)^{-1} = B^{-1}A^{-1}$with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.**(EG 1.9)**

22. Find the rank of the following matrices by minor method: $(iii) \begin{bmatrix} 1 & -2 & -10 \\ 3 & -6 & -31 \end{bmatrix}$**(EX 1.2 - 1)**

23. A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away. **(EX 3.1 - 12)**

24. Find all the values of $x$such that $(ii)\ -8\pi \le x \le 8\pi$and$\sin x = -1$**(EX 4.1 - 1)**

25. Find centre and radius of the following circles. $(ii) x^2 + y^2 + 6x - 4y + 4 = 0$.**(EX 5.1 - 11)**

26. A person learnt 100 words for an English test. The number of words the person remembers in $t$days after learning is given by $W(t) = 100 \times (1 - 0.1t^2), 0 \le t \le 10$.What is the rate at which the person forgets the words 2 days after learning? **(EG 7.3)**

27. Let $V(x, y, z) = xy + yz + zx, x, y, z \in \mathbb{R}$. Find the differential dV.**(EX 8.5 – 5)**

28. Evaluate the following integrals using properties of integration: $(i) \int_{-5}^{5} x \cos\left(\frac{e^x-1}{e^x+1}\right) dx$ **(EX 9.3 – 2)**

29. Express each of the following physical statements in the form of differential equation.(iii) For a certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature. **(EX 10.2 – 1(iii))**

30. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$.Find the probability that exactly 3 of the 5 components tested survive. **(EX 11.5 – 4)**

31. Check whether the statement $p \to (q \to p)$ is a tautology or a contradiction without using the truth table. **(EX 12.2 – 12)**

32. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a}, \vec{b}, \vec{c}]^2 = \frac{1}{4}|\vec{a}|^2|\vec{b}|^2$. **(EX 6.2 - 10)**

III ANSWER ANY EIGHT OF THE FOLLOWING QUESTIONS.
QUESTION NUMBER 44 IS COMPULSARY. $\mathbf{8 \times 3 = 24}$

33. Find the inverse of the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$ by Gauss-Jordan method. **(EG 1.20)**
34. Given the complex number $z = 2 + 3i$, represent the complex numbers in Argand diagram. $(ii) z, -iz$ and $z - iz$. **(EX 2.2 - 2)**
35. If the equations $x^2 + px + q = 0$, and $x^2 + p'x + q' = 0$, have a common root, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$. **(EX 3.1 - 10)**
36. Find the value (iii) $cos^{-1}\left(cos\frac{\pi}{7}cos\frac{\pi}{17} - sin\frac{\pi}{7}sin\frac{\pi}{17}\right)$. **(EX 4.2 - 5)**
37. A line $3x + 4y + 10 = 0$ cuts a chord of length 6 units on a circle with centre of the circle (2,1). Find the equation of the circle in general form. **(EG 5.7)**
38. Find the vector and Cartesian equations of the plane passing through the point with position vector $2\vec{i} + 6\vec{j} + 3\vec{k}$ and normal to the vector $\vec{i} + 3\vec{j} + 5\vec{k}$. **(EX 6.6 - 3)**
39. Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally. **(EX 7.2 – 10)**
40. Let $g(x, y) = x^2 - yx + \sin(x + y)$, $x(t) = e^{3t}$, $y(t) = t^2$, $t \in \mathbb{R}$. Find $\frac{dg}{dt}$. **(EG 8.19)**
41. Solve $y' = \sin^2(x - y + 1)$. **(EG 10.13)**
42. If X is the random variable with distribution function $F(x)$ given by, $f(x) = \begin{cases} 0, & -\infty \le x < 0 \\ \frac{1}{2}(x^2 + x), & 0 \le x < 1 \\ 1, & 1 \le x < \infty \end{cases}$ then find (i) the probability density function $f(x)$ (ii) $P(0.3 \le X \le 0.6)$ **(EX 11.3 – 6)**
43. Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$ **(EX 12.2 – 11)**
44. Evaluate $\int_0^1 x^3 dx$, as the limit of a sum. **(EG 9.3)**

IV ANSWERS THE FOLLOWING QUESTIONS: $\mathbf{8 \times 5 = 40}$

45. $(\boldsymbol{a})$ Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + \lambda z = \mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(EX 1.6 - 3)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$ Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$. **(EG 3.28)**

46. $(\boldsymbol{a})$ Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$, P.T. $\left|\frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3}\right| = r$. **(EG 2.15)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$ A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3 m from the end in contact with x –axis is an ellipse. Find the eccentricity. **(EX 5.5 - 7)**

47. $(\boldsymbol{a})$ Find the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$. **(EG 6.33)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$A manufacturer wants to design an open box having a square base and a surface area of 108 $sq.cm$. Determine the dimensions of the box for the maximum volume. **(EX 7.8 – 10)**

48. $(\boldsymbol{a})$The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. find the following in calculating the area of the circular plate: (i) Absolute error (ii) Relative error (iii) Percentage error **(EX 8.1 – 4)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Find the area of the region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$.**(EX 9.8 – 5)**

49. $(\boldsymbol{a})$Let X be a random variable denoting the life time of an electrical equipment having probability density function $f(x) = \begin{cases} ke^{-2x}, & for\ x > 0 \\ 0, & for\ x \leq 0 \end{cases}$.Find (i) the value of k (ii) Distribution function (iii) $P(X < 2)$$(iv)$ calculate the probability that X is at least for four unit of time (v) $P(X = 3)$.**(EG 11.15)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.**(EX 4.5 - 5)**

50. $(\boldsymbol{a})$Find the foci, vertices and length of major and minor axis of the conic $4x^2 + 36y^2 + 40x - 288y + 532 = 0$.**(EG 5.20)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car? **(EX 7.1 – 10)**

51. $(\boldsymbol{a})$Find the area of the region in the first quadrant bounded by the parabola$y^2 = 4x$, the line $x + y = 3$ and y −axis. **(EG 9.58)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent. **(EG 6.7)**

52. $(\boldsymbol{a})$Solve: $(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$**(EX 10.7 – 4)**

$(\boldsymbol{OR})$

$(\boldsymbol{b})$If F is the constant force generated by the motor of an automobile of mass M, its velocity V is given by $M\frac{dV}{dt} = F - kV$,where k is a constant. Express V in terms of t given that $V = 0$when $t = 0$.**(EX 10.5 – 1)**