CENTUM ACHIEVERS' ACADEMY

56,KASTHURI BAI 4TH STREET,GANAPATHY, CBE-06.PH.NO.7667761819

XII STD(MATHS)	FULL PORTION –9	TIME : 2 ½ Hrs MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : (20× 1 = 20)

1. If adj $A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and adj $B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj (AB) is

   (1) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (2) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (3) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (4) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

2. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

   (1) 2 (2) 4 (3) 3 (4) 1

3. The principal argument of $\frac{3}{-1+i}$ is

   (1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

4. The solution of the equation $|z| - z = 1 + 2i$ is

   (1) $\frac{3}{2} - 2i$ (2) $-\frac{3}{2} + 2i$ (3) $2 - \frac{3}{2}i$ (4) $2 + \frac{3}{2}i$

5. The number of real numbers in $[0,2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

   (1) 2 (2) 4 (3) 1 (4) ∞

6. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

   (1) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (2) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (3) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

7. $\sin(\tan^{-1} x), |x| < 1$ is equal to

   (1) $\frac{x}{\sqrt{1-x^2}}$ (2) $\frac{1}{\sqrt{1-x^2}}$ (3) $\frac{1}{\sqrt{1+x^2}}$ (4) $\frac{x}{\sqrt{1+x^2}}$

8. The radius of the circle passing through the point (6,2) two of whose diameter are $x + y = 6$ and $x + 2y = 4$ is (1) 10 (2) $2\sqrt{5}$ (3) 6 (4) 4

9. The circle passing through $(1,-2)$ and touching the axis of x at (3,0) passing through the point

   (1) $(-5,2)$ (2) $(2,-5)$ (3) $(5,-2)$ (4) $(-2,5)$

10. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \frac{1}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

    (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

11. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1,9]$ is

    (1) 2 (2) 2.5 (3) 3 (4) 3.5

12. The maximum value of the product of two positive numbers, when their sum of the squares is 200 , is
    (1) 100 (2) $25\sqrt{7}$ (3) 28 (4) $24\sqrt{14}$

13. If $w(x,y,z) = x^2(y-z) + y^2(z-x) + z^2(x-y)$, then $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$ is
    (1) $xy + yz + zx$ (2) $x(y+z)$ (3) $y(z+x)$ (4) 0

14. If $f(x) = \int_0^x t\cos t\, dt$, then $\frac{df}{dx} =$
    (1) $\cos x - x\sin x$ (2) $\sin x + x\cos x$ (3) $x\cos x$ (4) $x\sin x$

15. The value of $\int_0^{\pi} \frac{dx}{1+5^{\cos x}}$ is
    (1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{2}$ (4) 2π

16. If $\sin\ x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is
    (1) $\log\ \sin\ x$ (2) $\cos\ x$ (3) $\tan\ x$ (4) $\cot\ x$

17. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of a is
    (1) 2 (2) -2 (3) 1 (4) -1

18. A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is
    (1) 6 (2) 4 (3) 3 (4) 2

19. If X is a binomial random variable with expected value 6 and variance 2.4, then $P(X = 5)$ is
    (1) $\binom{10}{5}\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$ (2) $\binom{10}{5}\left(\frac{3}{5}\right)^{10}$ (3) $\binom{10}{5}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^6$ (4) $\binom{10}{5}\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$

20. Which one is the contrapositive of the statement $(p \vee q) \rightarrow r$?
    (1) $\neg r \rightarrow (\neg p \wedge \neg q)$ (2) $\neg r \rightarrow (p \vee q)$
    (3) $r \rightarrow (p \wedge q)$ (4) $p \rightarrow (q \vee r)$

PART-II

(i) Answer any SEVEN questions. **$(7\times 2 = 14)$**

(ii) Qn.No.30 is compulsory

21. If $|z| = 2$ show that $3 \leq |z + 3 + 4i| \leq 7$
22. Solve : $\sin^2\ x - 5\sin\ x + 4 = 0$
23. Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.
24. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c.
25. Prove that $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$.
26. If the mass $m(x)$ (in kilograms) of a thin rod of length x (in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is $x = 3$ and $x = 27$ metres.
27. Find Δf and df for the function f for the indicated values of $x, \Delta x$ and compare
    $f(x) = x^2 + 2x + 3\ ;\ \ x = -0.5, \Delta x = dx = 0.1$

28. Evaluate $\int_0^{\frac{\pi}{2}} \begin{vmatrix} \cos^4 x & 7 \\ \sin^5 x & 3 \end{vmatrix} dx$.

29. Using binomial distribution find the mean and variance of X for the following experiments

    (i) A fair coin is tossed 100 times, and X denote the number of heads.

30. If F is the constant force generated by the motor of an automobile of mass M, its velocity V is given by $M\frac{dV}{dt} = F - kV$, where k is a constant. Express V in terms of t given that $V = 0$ when $t = 0$.

PART-III

(i) Answer any SEVEN questions. **$(7\times 3 = 21)$**

(ii) Qn.No.40 is compulsory

31. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O_2$. Hence, find A^{-1}.

32. If $\omega \neq 1$ is a cube root of unity, show that

    (i) $(1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6 = 128$.

    (ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\cdots\left(1+\omega^{2^{11}}\right) = 1$.

33. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$.

34. Solve : $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

35. Find the magnitude and the direction cosines of the torque about the point $(2,0,-1)$ of a force $2\hat{\imath} + \hat{\jmath} - \hat{k}$, whose line of action passes through the origin.

36. Find the smallest possible value of $x^2 + y^2$ given that $x + y = 10$.

37. Solve : $\frac{dy}{dx} = (3x + y + 4)^2$.

38. Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum.

39. Establish the equivalence property connecting the bi-conditional with conditional:

    $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

40. If $w(x,y,z) = x^2 + y^2 + z^2, x = e^t, y = e^t \sin t$ and $z = e^t \cos t$, find $\frac{dw}{dt}$.

PART-IV

Answer the following questions. **$(7\times 5 = 35)$**

41. a) The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c, 0 \leq t \leq 100$ where $a, b,$ and c are constants. It has been found that the speed at times $t - 3, t - 6,$ and $t - 9$ seconds are respectively, 64, 133 , and 208 miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian elimination method.). **(OR)**

    b) Solve $x^4 + 3x^3 - 3x - 1 = 0$

42. a) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x - y + z = 4,\ x - 2y - 2z = 9, 2x + y + 3z = 1$. **(OR)**

b) Suppose z_1, z_2, and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

43. a) Two coast guard stations are located 600 km apart at points $A(0,0)$ and $B(0,600)$. A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship. **(OR)**

b) Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$. if $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$.

44. a) Solve : $(1 + x + xy^2)\frac{dy}{dx} + (y + y^3) = 0$ **(OR)**

b) If $w(x, y) = xy + \sin(xy)$, then prove that $\frac{\partial^2 w}{\partial y\,\partial x} = \frac{\partial^2 w}{\partial x\,\partial y}$.

45. a) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0), (2,2-1)$ and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$. **(OR)**

b) Find the dimensions of the rectangle with maximum area that can be inscribed in circle of radius 10 cm.

46. a) Solve: $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$. **(OR)**

b) Find the area of the region bounded by the curve $2 + x - x^2 + y = 0$, x-axis, $x = -3$ and $x = 3$.

47. a) Define an operation $*$ on $\mathbb{Q}$ as follows: $a * b = \left(\frac{a+b}{2}\right)$; $a, b \in \mathbb{Q}$. Examine the closure, commutative, associative , the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$. **(OR)**

b) If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights
(i) exactly 10 will have a useful life of at least 600 hours;
(ii) at least 11 will have a useful life of at least 600 hours;
(iii) at least 2 will not have a useful life of at least 600 hours.

SARATH KUMAR.S,
P.G.ASSISTANT,
COIMBATORE-6.