CENTUM ACHIEVERS' ACADEMY

56,KASTHURI BAI 4TH STREET,GANAPATHY, CBE-06.PH.NO.7667761819

XII STD(MATHS)	FULL PORTION –8	TIME : 2 ½ Hrs
		MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : **(20× 1 = 20)**

1. If $\rho(A) \neq \rho([A \mid B])$, then the system $AX = B$ of linear equations is

   (1) consistent and has a unique solution (2) consistent

   (3) consistent and has infinitely many solution (4) inconsistent

2. If $0 \leq \theta \leq \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0, (\cos\theta)x - y + z = 0,$ $(\sin\theta)x + y - z = 0$ has a non-trivial solution then θ is

   (1) $\frac{2\pi}{3}$ (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{4}$

3. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

   (1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{2}$

4. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

   (1) cis $\frac{2\pi}{3}$ (2) cis $\frac{4\pi}{3}$ (3) $-$cis $\frac{2\pi}{3}$ (4) $-$cis $\frac{4\pi}{3}$

5. If α, β, and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then the value of $\sum\frac{1}{\beta\gamma}$

   (1) $-\frac{q}{r}$ (2) $-\frac{p}{q}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

6. The polynomial $x^3 - 2x + 3$ has

   (1) one negative and two imaginary zeros (2) two positive and one negative

   (3) three real zeros (4) no zeros

7. If $x = \frac{1}{5}$, the value of $\cos(\cos^{-1}x + 2\sin^{-1}x)$ is

   (1) $-\sqrt{\frac{24}{25}}$ (2) $\sqrt{\frac{24}{25}}$ (3) $\frac{1}{5}$ (4) $-\frac{1}{5}$

8. If $\cot^{-1}x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1}x$ is

   (1) $-\frac{\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $-\frac{\pi}{5}$

9. The locus of a point whose distance from $(-2,0)$ is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is

   (1) a parabola (2) a hyperbola (3) an ellipse (4) a circle

10. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

    (1) $2ab$ (2) ab (3) $\sqrt{ab}$ (4) $\frac{a}{b}$

11. If the planes $\vec{r}\cdot(2\hat{\imath}-\lambda\hat{\jmath}+\hat{k})=3$ and $\vec{r}\cdot(4\hat{\imath}+\hat{\jmath}-\mu\hat{k})=5$ are parallel, then the value of λ and μ are

(1) $\frac{1}{2},-2$ (2) $-\frac{1}{2},2$ (3) $-\frac{1}{2},-2$ (4) $\frac{1}{2},2$

12. One of the closest points on the curve $x^2-y^2=4$ to the point $(6,0)$ is

(1) $(2,0)$ (2) $(\sqrt{5},1)$ (3) $(3,\sqrt{5})$ (4) $(\sqrt{13},-\sqrt{3})$

13. The slope of the line normal to the curve $f(x)=2\cos 4x$ at $x=\frac{\pi}{12}$ is

(1) $-4\sqrt{3}$ (2) -4 (3) $\frac{\sqrt{3}}{12}$ (4) $4\sqrt{3}$

14. If $w(x,y)=x^y, x>0$, then $\frac{\partial w}{\partial x}$ is equal to

(1) $x^y\log x$ (2) $y\log x$ (3) yx^{y-1} (4) $x\log y$

15. The value of $\int_0^1 x(1-x)^{99}dx$ is

(1) $\frac{1}{11000}$ (2) $\frac{1}{10100}$ (3) $\frac{1}{10010}$ (4) $\frac{1}{10001}$

16. If $\int_0^x f(t)dt=x+\int_x^1 tf(t)dt$, then the value of $f(1)$ is

(1) $\frac{1}{2}$ (2) 2 (3) 1 (4) $\frac{3}{4}$

17. The degree of the differential equation $y(x)=1+\frac{dy}{dx}+\frac{1}{1\cdot2}\left(\frac{dy}{dx}\right)^2+\frac{1}{1\cdot2\cdot3}\left(\frac{dy}{dx}\right)^3+\cdots$ is

(1) 2 (2) 3 (3) 1 (4) 4

18. The solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is

(1) $x\phi\left(\frac{y}{x}\right)=k$ (2) $\phi\left(\frac{y}{x}\right)=kx$ (3) $y\phi\left(\frac{y}{x}\right)=k$ (4) $\phi\left(\frac{y}{x}\right)=ky$

19. $var(5X+4)$ is

(1) $25var(X)$ (2) $5var(X)$ (3) $var(X)$ (4) $4var(X)$

20. In the last column of the truth table for $\neg(p\vee\neg q)$ the number of final outcomes of the truth value ' F ' are

(1) 1 (2) 2 (3) 3 (4) 4

PART-II

(i) Answer any SEVEN questions. **(7× 2 = 14)**

(ii) Qn.No.30 is compulsory

21. Prove that $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal.

22. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9}+i\sin\frac{2k\pi}{9}\right)$.

23. If α and β are the roots of the quadratic equation $17x^2+43x-73=0$, construct a quadratic equation whose roots are $\alpha+2$ and $\beta+2$.

24. Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$ for $x>0$.

25. Find the equations of tangent and normal to the parabola $x^2+6x+4y+5=0$ at $(1,-3)$.

26. Find the local extremum of the function $f(x)=x^4+32x$.

27. Find the area of the region bounded by x-axis, the sine curve $y=\sin x$, the lines $x=0$ and $x=2\pi$.

28. Express physical statements in the form of differential equation - For a certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature.

29. On $\mathbb{Z}$, define $*$ by $(m * n) = m^n + n^m : \forall m, n \in \mathbb{Z}$. Is $*$ binary on $\mathbb{Z}$?

30. Find the acute angle between the planes $\vec{r} \cdot (2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) = 11$ and $4x - 2y + 2z = 15$

PART-III

(i) Answer any SEVEN questions. **$(7\times 3 = 21)$**

(ii) Qn.No.40 is compulsory

31. Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

32. Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$.

33. Obtain the condition that the roots of $x^3 + px^2 + qx + r = 0$ are in A.P.

34. The parabolic communication antenna has a focus at 2 m distance from the vertex of the antenna. Find the width of the antenna 3 m from the vertex.

35. Evaluate the limits, if necessary use l'Hôpital Rule: $\lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$

36. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and the approximate percentage change in the area.

37. Evaluate the integrals using properties of integration : $\int_0^{2\pi} x\log\left(\frac{3+\cos x}{3-\cos x}\right)dx$

38. Solve the differential equations: $\sin\frac{dy}{dx} = a, y(0) = 1$

39. The probability density function of the random variable X is given by $f(x) = \begin{cases} 16xe^{-4x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$ find the mean and variance of X.

40. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, if $6x^2 < 1$.

PART-IV

Answer the following questions. **$(7\times 5 = 35)$**

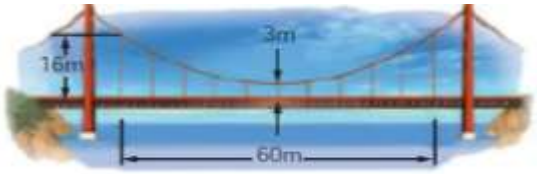
41. a) Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method. (OR)

b) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

42. a) a) Solve:(i) $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$ (ii) $\cot^{-1} x - \cot^{-1}(x+2) = \frac{\pi}{12}, x > 0$. **(OR)**

b) Find the coordinate of the foot of the perpendicular drawn from the point $(-1,2,3)$ to the line $\vec{r} = (\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + t(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$. Also, find the shortest distance from the given point to the line

43. a) Parabolic cable of a 60 m portion of the roadbed of a suspension bridge are positioned as shown below.Vertical Cables are to be spaced every 6 m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



(OR)

b) Let z_1, z_2, and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$.

Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$.

44. a) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.**(OR)**

b) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

45. a) Find the intervals of monotonicities and hence find the local extremum for the function

$f(x) = 2x^3 + 3x^2 - 12x$ **(OR)**

b) Let X be a random variable denoting the life time of an electrical equipment having probability density function $f(x) = \begin{cases} ke^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$ Find (i) the value of k (ii) Distribution function

(iii) $P(X < 2)$ (iv) calculate the probability that X is at least for four unit of time (v) $P(X = 3)$.

46. a) Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.**(OR)**

b) Let $f(x, y) = \sin(xy^2) + e^{x^3+5y}$ for all $(x, y) \in \mathbb{R}^2$. Prove $\frac{\partial^2 f}{\partial x\, \partial y} = \frac{\partial^2 f}{\partial y\, \partial x}$

47. a) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the commutative and associative properties,the existence of identity, existence of inverse properties for the operation $*$ on M. **(OR)**

b) A pot of boiling water at 100°C is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C, and another 5 minutes later it has dropped to 65°C. Determine the temperature of the kitchen.

SARATH KUMAR S, P.G.ASST. CBE-6