

PART – I

Choose the correct answer

20 x 1 = 20

1. A square matrix A of order n has inverse if and only if:
 (a) $\rho(A) > n$ (b) **$\rho(A) = n$** (c) $\rho(A) \neq n$ (d) $\rho(A) < n$
2. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is
 (a) 2 (b) 0 (c) 3 **(d) 1**
3. If $3 \cos^{-1}x \cos^{-1}(4x^3 - 3x)$,
 (a) $x \in \left(\frac{1}{2}, 1\right)$ (b) **$x \in \left[\frac{1}{2}, 1\right]$** (c) $x \in (-\infty, 1]$ (d) $x \in \left[\frac{1}{2}, \infty\right)$
4. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is :
 (a) **$y = kx$** (b) $xy = k$ (c) $\log y = kx$ (d) $y = k \log x$
5. The number of normals that can be drawn from a point to the parabola $y^2 = 4ax$ is
 (a) 3 (b) 2 (c) 0 **(d) 1**
6. If \vec{a} and \vec{b} are parallel vectors then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to:
 (a) 1 (b) 2 **(c) 0** (d) -1
7. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is
 (a) 1 **(b) 2** (c) ∞ (d) 4
8. Suppose that X takes on one of the values 0,1,2. If for some constant k,
 $P(X = i) = kP(X = i - 1)$ for $i = 1,2$ and $P(X = 0) = \frac{1}{7}$, then the value of k is :
 (a) 3 (b) 1 (c) 4 **(d) 2**
9. The maximum value of the function $x^2 e^{-2x}$, $x > 0$ is :
 (a) $\frac{1}{e^2}$ (b) $\frac{1}{e}$ (c) $\frac{4}{e^4}$ (d) $\frac{1}{2e}$
10. The operation *defined by $a * b = \frac{ab}{7}$ is not a binary operation on :
 (a) R (b) Q^+ (c) C **(d) Z**
11. The area between $y^2 = 4x$ and its latus rectum is :
 (a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{5}{3}$ (d) $\frac{4}{3}$
12. Angle between the curves $y^2 = x$ and $x^2 = y$ at the origin is :
 (a) $\frac{\pi}{2}$ (b) $\tan^{-1}\left(\frac{3}{4}\right)$ (c) $\frac{\pi}{4}$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$
13. $|\text{adj}(\text{adj}A)| = |A|^{16}$, then the order of the square matrix A is :
 (a) 2 (b) 3 **(c) 5** (d) 4
14. The value of $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$ is:
 (a) 8 (b) 4 **(c) 2** (d) 6
15. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is:
 (a) $\frac{1}{z}$ (b) z (c) 1 (d) \bar{z}

16. The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is -0.25 ?

- (a) -2 (b) -8 (c) 0 (d) **-4**

17. The value of $\int_0^{\pi/3} \tan x \, dx$ is :

- (a) $-\log 2$ (b) **$\log 2$** (c) $-\log 3$ (d) $\log 3$

18. The number of positive zeros of the polynomial $\sum_{r=0}^n nC_r (-1)^r x^r$ is :

- (a) $< n$ (b) 0 (c) r (d) **n**

19. The Principal value of $\sin^{-1} \left(\frac{-1}{2} \right)$ is :

- (a) $\frac{-\pi}{6}$ (b) 0 (c) $\frac{-\pi}{2}$ (d) $\frac{\pi}{2}$

20. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

- (a) \sqrt{ab} (b) **$2ab$** (c) $\frac{a}{b}$ (d) ab

PART - II

Answer any SEVEN questions

Question number 30 is compulsory

$7 \times 2 = 14$

21. If $|z| = 2$ show that $3 \leq |z + 3 + 4i| \leq 7$.

Solution:

$$\begin{aligned} \text{W. K. T. } & | |z_1| - |z_2| | \leq |z_1 + z_2| \leq |z_1| + |z_2| \\ & | |z| - |3 + 4i| | \leq |z + 3 + 4i| \leq |z| + |3 + 4i| \\ & \text{Since } |z| = 2 \end{aligned}$$

$$|2 - \sqrt{9 + 16}| \leq |z + 3 + 4i| \leq 2 + \sqrt{9 + 16}$$

$$|2 - 5| \leq |z + 3 + 4i| \leq 2 + 5$$

$$|-3| \leq |z + 3 + 4i| \leq 7$$

$$3 \leq |z + 3 + 4i| \leq 7$$

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22. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} +$

$$\sqrt{\frac{n}{l}} = 0.$$

Solution:

G. T. p and q are the roots of the equation $lx^2 + nx + n = 0$

$$\text{Sum of the roots} = p + q = -\frac{n}{l}$$

$$\text{Product of the roots} = pq = \frac{n}{l}$$

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} = \frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} = \frac{p+q}{\sqrt{pq}}$$

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$$= -\frac{\frac{n}{l}}{\sqrt{\frac{n}{l}}} = -\frac{\sqrt{\frac{n}{l}} \sqrt{\frac{n}{l}}}{\sqrt{\frac{n}{l}}} = -\sqrt{\frac{n}{l}}$$

$$\therefore \boxed{\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0}$$

23. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c .

Solution:

$$\text{G. T. } y = 4x + c \text{ and } x^2 + y^2 = 9$$

W. K. T. condition for the line $y = mx + c$ to be a tangent to the circle $x^2 + y^2 = a^2$ is $c^2 = a^2(1 + m^2)$

$$\text{Here } m = 4, \quad a^2 = 9$$

$$c^2 = a^2(1 + m^2)$$

$$c^2 = 9(1 + 4^2)$$

$$c^2 = 9(17)$$

$$c = \pm 3\sqrt{17}$$

24. If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

Solution:

$$\text{Given } r = 10 \text{ cm ; } dr = -0.1 \text{ cm}$$

$$\text{W. K. T. volume of a sphere is } V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = \frac{4}{3}\pi(3r^2)$$

$$dV = 4\pi r^2 dr$$

$$dV = 4\pi(10)^2(-0.1) = -40\pi \text{ cm}^3$$

volume of the sphere decreases about $40\pi \text{ cm}^3$.

25. Evaluate : $\int_b^\infty \frac{1}{a^2 + x^2} dx$, $a > 0, b \in \mathbb{R}$.

Solution:

$$\begin{aligned} \int_b^\infty \frac{1}{a^2 + x^2} dx &= \left[\frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) \right]_b^\infty \\ &= \frac{1}{a} \left[\tan^{-1} \left(\frac{x}{a} \right) \right]_b^\infty \\ &= \frac{1}{a} \left[\tan^{-1} \infty - \tan^{-1} \left(\frac{b}{a} \right) \right] = \frac{1}{a} \left[\frac{\pi}{2} - \tan^{-1} \left(\frac{b}{a} \right) \right] \\ \therefore \int_b^\infty \frac{1}{a^2 + x^2} dx &= \frac{1}{a} \left[\frac{\pi}{2} - \tan^{-1} \left(\frac{b}{a} \right) \right] \end{aligned}$$

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26. Find a parametric form of vector equation of a plane which is at a distance of 7 units from the origin having $3, -4, 5$ as direction ratios of a normal to it.

Solution:

$$\text{Given } p = 7 ; \vec{n} = 3\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{\sqrt{(3)^2 + (-4)^2 + (5)^2}} = \frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{5\sqrt{2}}$$

Vector equation of plane is $\vec{r} \cdot \hat{n} = p$

$$\vec{r} \cdot \left(\frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{5\sqrt{2}} \right) = 7$$

27. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

Solution

$$A \vee B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \vee 1 & 1 \vee 1 \\ 1 \vee 0 & 1 \vee 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A \wedge B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \wedge 1 & 1 \wedge 1 \\ 1 \wedge 0 & 1 \wedge 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

28. Prove that $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ is orthogonal.

Solution:

$$\text{Let } A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$A^T = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$AA^T = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & \sin \theta \cos \theta - \sin \theta \cos \theta \\ \sin \theta \cos \theta - \sin \theta \cos \theta & \cos^2 \theta + \sin^2 \theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$\text{Hence, } A^T A = I_2$$

$$\Rightarrow AA^T = A^T A = I_2$$

Hence, A is orthogonal

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29. Find the equation of tangent to the curve $y = x^2 + 3x - 2$ at the point $(1, 2)$.

Solution:

$$\text{Given } y = x^2 + 3x - 2 \Rightarrow \frac{dy}{dx} = 2x + 3$$

$$m = \left(\frac{dy}{dx} \right)_{(1,2)} = 2(1) + 3 = 5$$

\therefore Equation of tangent is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 2 = 5(x - 1)$$

$$\Rightarrow \boxed{5x - y - 3 = 0}$$

30. Express $e^{\cos \theta + i \sin \theta}$ in a + ib form.

Solution:

$$\begin{aligned} e^{\cos \theta + i \sin \theta} &= e^{\cos \theta} e^{i \sin \theta} \\ &= e^{\cos \theta} (\cos(\sin \theta) + i \sin(\sin \theta)) \\ &= e^{\cos \theta} \cos(\sin \theta) + i e^{\cos \theta} \sin(\sin \theta) \end{aligned}$$

PART – III

Answer any SEVEN questions

Question number 40 is compulsory

7 x 3 = 21

31. Find the equation of the parabola with vertex $(-1, -2)$, axis parallel to y -axis and passing through $(3, 6)$.

Solution:

vertex $(-1, -2)$, axis parallel to y -axis and passing through $(3, 6)$

From diagram, the parabola is open up and axis of symmetry as parallel to y -axis

Then the equation of parabola is

$$(x - h)^2 = 4a(y - k)$$

Here $A(h, k) = A(-1, -2)$

$$(x + 1)^2 = 4a(y + 2) \rightarrow \textcircled{1}$$

It passes through $(3, 6)$

$$(3 + 1)^2 = 4a(6 + 2)$$

$$16 = 4a(8)$$

$$4a = 2 \Rightarrow a = \frac{1}{2}$$

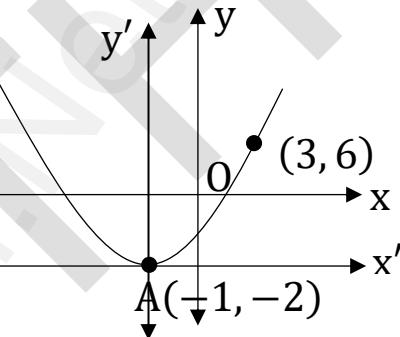
Sub $4a = 2$ in $\textcircled{1}$, we get

$$(x + 1)^2 = 2(y + 2)$$

$$x^2 + 2x + 1 = 2y + 4$$

$$x^2 + 2x + 1 - 2y - 4 = 0$$

$$x^2 + 2x - 2y - 3 = 0$$



32. The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

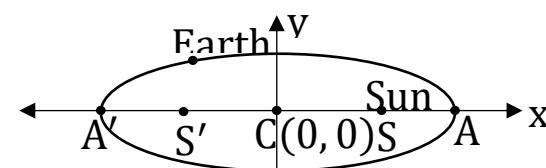
Solution:

$$SA' = a + c = 152 \times 10^6 \rightarrow \textcircled{1}$$

$$SA = a - c = 94.5 \times 10^6 \rightarrow \textcircled{2}$$

$$\textcircled{1} - \textcircled{2} \Rightarrow 2c = 57.5 \times 10^6 = 575 \times 10^5$$

∴ The distance from the Sun to the other focus is 575×10^5 km



33. For what value of x , the inequality $\frac{\pi}{2} < \cos^{-1}(3x - 1) < \pi$ holds?

Solution:

$$\frac{\pi}{2} < \cos^{-1}(3x - 1) < \pi$$

$\because \cos^{-1} x$ is decreasing function

$$\cos \frac{\pi}{2} > 3x - 1 > \cos \pi$$

$$0 > 3x - 1 > -1$$

$$-1 < 3x - 1 < 0$$

$$-1 + 1 < 3x < 0 + 1$$

$$0 < 3x < 1$$

$$0 < x < \frac{1}{3}$$

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34. Find the angle between the straight line $\frac{x+3}{2} = \frac{y-1}{2} = -z$ with coordinate axes.

Solution:

$$\frac{x+3}{2} = \frac{y-1}{2} = -z \Rightarrow \frac{x+3}{2} = \frac{y-1}{2} = \frac{z}{-1}$$

$$\therefore \vec{b} = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{(2)^2 + (2)^2 + (-1)^2}} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$$

Let α, β and γ are the angles made by \hat{b} with + ve x – axis, y – axis and z – axis respectively

$$\therefore \cos \alpha = \frac{2}{3}; \cos \beta = \frac{2}{3}; \cos \gamma = -\frac{1}{3}$$

$$\therefore \alpha = \cos^{-1}\left(\frac{2}{3}\right); \beta = \cos^{-1}\left(\frac{2}{3}\right); \gamma = \cos^{-1}\left(-\frac{1}{3}\right)$$

35. Use the linear approximation to find an approximate values of $(123)^{\frac{2}{3}}$.

Solution:

$$\text{Let } f(x) = x^{\frac{2}{3}} \Rightarrow x = 123; x_0 = 125$$

$$f(x_0) = f(125) = (125)^{\frac{2}{3}} = (5^3)^{\frac{2}{3}} = 25$$

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3x^{\frac{1}{3}}} \Rightarrow f'(x_0) = f'(125) = \frac{2}{3(125)^{\frac{1}{3}}} = \frac{2}{3(5^3)^{\frac{1}{3}}} = \frac{2}{15}$$

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$L(123) = 25 + \frac{2}{15}(123 - 125) = 25 + \frac{2}{15}(-2) = 25 - \frac{4}{15}$$

$$L(123) = 25 - 0.27 = 24.73$$

$$(123)^{\frac{2}{3}} \approx L(123) = 24.73$$

36. Solve the differential equation $x \cos y dy = e^x(x \log x + 1)dx$.

Solution:

$$x \cos y dy = e^x(x \log x + 1)dx$$

$$\cos y dy = e^x \left(\frac{x \log x + 1}{x} \right) dx$$

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$$\begin{aligned}\cos y dy &= e^x \left(\log x + \frac{1}{x} \right) dx \\ \int \cos y dy &= \int e^x \left(\log x + \frac{1}{x} \right) dx \\ \sin y &= e^x (\log x) + c\end{aligned}$$

37. If $F(\alpha) = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

Solution:

$$\begin{aligned}|F(\alpha)| &= \cos \alpha (\cos \alpha - 0) - 0 + \sin \alpha (0 + \sin \alpha) \\ &= \cos^2 \alpha + \sin^2 \alpha = 1 \neq 0 \Rightarrow [F(\alpha)]^{-1} \text{ exists}\end{aligned}$$

$$\begin{aligned}\text{adj } [F(\alpha)] &= \begin{bmatrix} (\cos \alpha - 0) & -(0 - 0) & (0 + \sin \alpha) \\ -(0 - 0) & (\cos^2 \alpha + \sin^2 \alpha) & -(0 + 0) \\ (0 - \sin \alpha) & -(0 - 0) & (\cos \alpha - 0) \end{bmatrix}^T \\ &= \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}^T \\ \text{adj } [F(\alpha)] &= \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix}\end{aligned}$$

$$[F(\alpha)]^{-1} = \frac{1}{|F(\alpha)|} \text{adj } [F(\alpha)]$$

$$[F(\alpha)]^{-1} = \frac{1}{1} \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix} \rightarrow ①$$

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & 0 & \sin(-\alpha) \\ 0 & 1 & 0 \\ -\sin(-\alpha) & 0 & \cos(-\alpha) \end{bmatrix} = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ +\sin \alpha & 0 & \cos \alpha \end{bmatrix} \rightarrow ②$$

From ① and ②, we get

$$[F(\alpha)]^{-1} = F(-\alpha)$$

38. Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent.

Solution:

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T ①	T ②

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From ① and ②, we get

$$\therefore p \rightarrow q \not\equiv q \rightarrow p$$

39. Find z^{-1} , if $z = (2 + 3i)(1 - i)$.

Solution:

$$z = (2 + 3i)(1 - i) = (2 + 3) + i(3 - 2) = 5 + i$$

$$\begin{aligned}\therefore z^{-1} &= \frac{1}{z} = \frac{1}{5+i} = \frac{1}{5+i} \times \frac{5-i}{5-i} \\ &= \frac{5-i}{5^2 + 1^2} = \frac{5-i}{25+1} \\ \therefore z^{-1} &= \frac{5}{26} - i \frac{1}{26}\end{aligned}$$

40. If $a + b + c = 0$ and a, b, c are rational numbers then, prove that the roots of the equation $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are rational numbers.

Solution:

$$\text{G.T. } a + b + c = 0 \Rightarrow b = -a - c$$

$$\text{G.T. } (b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$$

$$A = b + c - a, \quad B = c + a - b, \quad C = a + b - c$$

$$\begin{aligned}\Delta &= B^2 - 4AC = (c + a - (-a - c))^2 - 4(-a - c + c - a)(a - a - c - c) \\ &= (2(c + a))^2 - 4(-2a)(-2c) \\ &= 4(c + a)^2 - 4(4ac) \\ &= 4((c + a)^2 - 4ac) = 4(c - a)^2 \\ &= (2(c - a))^2 \text{ which is perfect square}\end{aligned}$$

Hence, the given equation roots are rational.

PART – IV

Answer ALL questions

7 x 5 = 35

41.(a) Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$.

Solution:

$$z^3 + 8i = 0 \Rightarrow z^3 = -8i \Rightarrow z = (8(-i))^{1/3} \Rightarrow z = 2(-i)^{1/3}$$

$$\therefore z = 2 \left[\cos\left(-\frac{\pi}{2} + 2k\pi\right) + i \sin\left(-\frac{\pi}{2} + 2k\pi\right) \right]^{\frac{1}{3}}, \quad k = 0, 1, 2$$

$$= 2 \left[\cos\left(\frac{-\pi + 4k\pi}{2}\right) + i \sin\left(\frac{-\pi + 4k\pi}{2}\right) \right]^{\frac{1}{3}}, \quad k = 0, 1, 2$$

$$= 2 \left[\cos\left(\frac{-\pi + 4k\pi}{6}\right) + i \sin\left(\frac{-\pi + 4k\pi}{6}\right) \right], \quad k = 0, 1, 2$$

$$\text{when } k = 0 \Rightarrow z = 2 \left[\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right) \right] = 2 \left[\cos\left(\frac{\pi}{6}\right) - i \sin\left(\frac{\pi}{6}\right) \right]$$

$$= 2 \left[\frac{\sqrt{3}}{2} - i \frac{1}{2} \right] = \sqrt{3} - i$$

$$\begin{aligned}\text{when } k = 1 \Rightarrow z &= 2 \left[\cos\left(\frac{3\pi}{6}\right) + i \sin\left(\frac{3\pi}{6}\right) \right] = 2 \left[\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) \right] \\ &= 2[0 + i(1)] = 2i\end{aligned}$$

$$\begin{aligned}\text{when } k = 2 \Rightarrow z &= 2 \left[\cos\left(\frac{7\pi}{6}\right) + i \sin\left(\frac{7\pi}{6}\right) \right] = 2 \left[\cos\left(\pi + \frac{\pi}{6}\right) + i \sin\left(\pi + \frac{\pi}{6}\right) \right] \\ &= 2 \left[-\cos\left(\frac{\pi}{6}\right) - i \sin\left(\frac{\pi}{6}\right) \right] = 2 \left[-\frac{\sqrt{3}}{2} - i \frac{1}{2} \right] = -\sqrt{3} - i\end{aligned}$$

OR

(b) Solve the Linear differential equation $(1 + x + xy^2) \frac{dy}{dx} + (y + y^3) = 0$.

Solution:

$$(1 + x + xy^2) \frac{dy}{dx} + (y + y^3) = 0$$

$$(y + y^3) = -(1 + x + xy^2) \frac{dy}{dx}$$

$$y(1 + y^2) \frac{dx}{dy} = -(1 + x(1 + y^2))$$

$$y(1 + y^2) \frac{dx}{dy} = -1 - x(1 + y^2)$$

$$\div y(1 + y^2)$$

$$\frac{dx}{dy} = -\frac{1}{y(1 + y^2)} - \frac{x(1 + y^2)}{y(1 + y^2)}$$

$$\frac{dx}{dy} + \frac{x}{y} = \frac{-1}{y(1 + y^2)}$$

$$\text{Here, } P = \frac{1}{y}, Q = \frac{-1}{y(1 + y^2)}$$

$$\int P dy = \int \frac{1}{y} dy = \log|y|$$

$$I.F = e^{\int P dy} = e^{\log|y|} = y$$

$$\text{The solution is } x(I.F) = \int Q(I.F) dy + c$$

$$x(y) = \int \frac{-1}{y(1 + y^2)} \cdot y dy + c$$

$$xy = - \int \frac{1}{1 + y^2} dy + c$$

$$xy = -\tan^{-1} x + c$$

$$xy + \tan^{-1} x = c$$

42. (a) Using vector method, prove that $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$.

Solution:

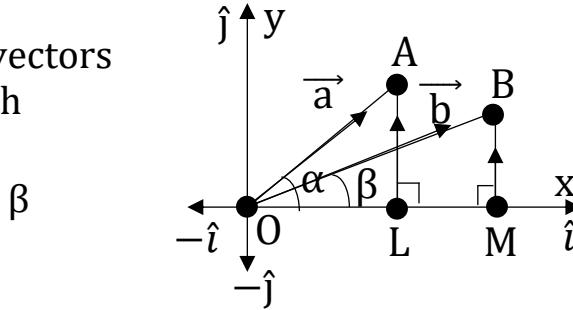
Let $\hat{a} = \overrightarrow{OA}$ and $\hat{b} = \overrightarrow{OB}$ be the unit vectors and which makes angle α and β with positive x – axis respectively.

\therefore The angle between \hat{a} and \hat{b} is $\alpha - \beta$

From the diagram

$$\cos \alpha = \frac{OL}{OA} = \frac{OL}{1} = OL$$

$$\sin \alpha = \frac{LA}{OA} = \frac{LA}{1} = LA$$



$$\cos \beta = \frac{OM}{OB} = \frac{OM}{1} = OM$$

$$\sin \beta = \frac{MB}{OB} = \frac{MB}{1} = MB$$

$$\begin{array}{l|l} \hat{a} = \overrightarrow{OA} = \overrightarrow{OL} + \overrightarrow{LA} & \hat{b} = \overrightarrow{OB} = \overrightarrow{OM} + \overrightarrow{MB} \\ = OL\hat{i} + LA\hat{j} & = OM\hat{i} + MB\hat{j} \\ \hat{a} = \cos \alpha \hat{i} + \sin \alpha \hat{j} & \hat{b} = \cos \beta \hat{i} + \sin \beta \hat{j} \\ \hat{a} \cdot \hat{b} = (\cos \alpha \hat{i} + \sin \alpha \hat{j}) \cdot (\cos \beta \hat{i} + \sin \beta \hat{j}) & \\ \hat{a} \cdot \hat{b} = \cos \alpha \cos \beta + \sin \alpha \sin \beta & \rightarrow \textcircled{1} \end{array}$$

$$\hat{a} \cdot \hat{b} = |\hat{a}| |\hat{b}| \cos(\alpha - \beta) = (1)(1) \cos(\alpha - \beta) = \cos(\alpha - \beta) \rightarrow \textcircled{2}$$

From $\textcircled{1}$ and $\textcircled{2}$, we get

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

OR

- (b) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function $f(x) = \begin{cases} k & 200 \leq x \leq 600 \\ 0 & \text{otherwise} \end{cases}$ Find (i) the value of k
 (ii) the distribution function (iii) the probability that daily sales will fall between 300 litres and 500 litres?

Solution:

(i) Given p. d. f

$$\begin{aligned} \int_{-\infty}^{\infty} f(x) dx &= 1 \\ \int_{200}^{600} k dx &= 1 \\ k[600 - 200] &= 1 \\ k(400) &= 1 \\ k &= \frac{1}{400} \\ \therefore \text{p. d. f is } f(x) &= \begin{cases} \frac{1}{400}, & 200 \leq x \leq 600 \\ 0, & \text{Otherwise} \end{cases} \end{aligned}$$

(ii) The distribution function $F(x)$

$$F(x) = \int_{-\infty}^x f(u) du$$

when $x < 200$

$$F(x) = \int_{-\infty}^x f(u) du = \int_{-\infty}^x 0 du = 0$$

when $200 \leq x \leq 600$

$$\begin{aligned} F(x) &= \int_{-\infty}^{200} f(u) du + \int_{200}^x f(u) du \\ &= 0 + \int_{200}^x \frac{1}{400} du \\ &= \frac{1}{400} [u]_{200}^x \end{aligned}$$

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$$F(x) = \frac{1}{400} [x - 200]$$

When $x > 600$

$$\begin{aligned} F(x) &= \int_{-\infty}^{200} f(u) du + \int_{200}^{600} f(u) du + \int_{600}^x f(u) du \\ &= \int_{-\infty}^{200} 0 du + \int_{200}^{600} \frac{1}{400} du + \int_{600}^x 0 du \\ &= 0 + \frac{1}{400} [u]_{200}^{600} + 0 \\ &= \frac{1}{400} (600 - 200) \end{aligned}$$

$$F(x) = 1$$

$$F(x) = \begin{cases} 0, & -\infty < x < 200 \\ \frac{x - 200}{4}, & 200 \leq x \leq 600 \\ 1, & 600 < x < \infty \end{cases}$$

(iii) probability that daily sales will fall between 300 litres and 500 litres

$$\begin{aligned} P(300 < X < 500) &= P(300 \leq X \leq 500) = F(500) - F(300) \\ &= \frac{500 - 200}{400} - \frac{300 - 200}{400} = \frac{1}{2} \end{aligned}$$

43.(a) Identify the type of conic and find centre, foci, vertices of $18x^2 + 12y^2 - 144x + 48y + 120 = 0$.

Solution:

$18x^2 + 12y^2 - 144x + 48y + 120 = 0$ which is an ellipse

$$18x^2 - 144x + 12y^2 + 48y = -120$$

$$18(x^2 - 8x) + 12(y^2 + 4y) = -120$$

$$18(x^2 - 8x + 16) + 12(y^2 + 4y + 4) = -120 + 18(16) + 12(4)$$

$$18(x - 4)^2 + 12(y + 2)^2 = -120 + 288 + 48$$

$$18(x - 4)^2 + 12(y + 2)^2 = 216$$

÷ by 216

$$\frac{(x - 4)^2}{12} + \frac{(y + 2)^2}{18} = 1$$

$$a^2 = 18 ; b^2 = 12 ; c^2 = a^2 - b^2 = 18 - 12 = 6$$

$$a = 3\sqrt{2} ; b = 2\sqrt{3} ; c = \sqrt{6}$$

$$\text{Here } h = 4 ; k = -2$$

Major axis is parallel to y-axis

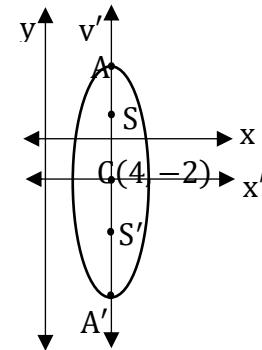
$$\text{Centre : } C(h, k) = C(4, -2)$$

$$\text{Vertices : } A(h, k \pm a) = A(4, -2 \pm 3\sqrt{2})$$

$$A(4, -2 + 3\sqrt{2}) \text{ and } A'(4, -2 - 3\sqrt{2})$$

$$\text{Foci : } S(h, k \pm c) = S(4, -2 \pm \sqrt{6})$$

$$S(4, -2 + \sqrt{6}) \text{ and } S'(4, -2 - \sqrt{6})$$



OR

- (b) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$.

Solution:

$$\text{Let } \alpha = \cos^{-1} x ; \beta = \cos^{-1} y ; \gamma = \cos^{-1} z$$

$$\Rightarrow \cos \alpha = x ; \cos \beta = y ; \cos \gamma = z$$

$$\Rightarrow \sin \alpha = \sqrt{1 - x^2} ; \sin \beta = \sqrt{1 - y^2} ; \sin \gamma = \sqrt{1 - z^2}$$

$$\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi \Rightarrow \alpha + \beta + \gamma = \pi \Rightarrow \alpha + \beta = \pi - \gamma$$

$$\cos(\alpha + \beta) = \cos(\pi - \gamma)$$

$$\cos \alpha \cos \beta - \sin \alpha \sin \beta = -\cos \gamma$$

$$xy - \sqrt{1 - x^2} \sqrt{1 - y^2} = -z$$

$$xy + z = \sqrt{1 - x^2} \sqrt{1 - y^2}$$

$$(xy + z)^2 = (\sqrt{1 - x^2} \sqrt{1 - y^2})^2$$

$$x^2y^2 + 2xyz + z^2 = (1 - x^2)(1 - y^2)$$

$$x^2y^2 + 2xyz + z^2 = 1 - x^2 - y^2 + x^2y^2$$

$$x^2 + y^2 + z^2 + 2xyz = 1$$

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- 44.(a) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8), (-2, -12)$, and $(3, 8)$. He wants to meet his friend at P(7, 60). Will he meet his friend? (Use Gaussian elimination method).

Solution:

$$\text{Given that, } y = ax^2 + bx + c$$

it passes through the points $(-6, 8), (-2, -12)$ and $(3, 8)$

$$36a - 6b + c = 8$$

$$4a - 2b + c = -12$$

$$9a + 3b + c = 8$$

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The matrix form of the system is $AX = B$

$$\text{Where } A = \begin{bmatrix} 36 & -6 & 1 \\ 4 & -2 & 1 \\ 9 & 3 & 1 \end{bmatrix} ; X = \begin{bmatrix} a \\ b \\ c \end{bmatrix} ; B = \begin{bmatrix} 8 \\ -12 \\ 8 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{array} \right]$$

$$\xrightarrow{R_2 \rightarrow R_1 - 9R_2} \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & 12 & -8 & -116 \\ 9 & 3 & 1 & 8 \end{array} \right]$$

$$\xrightarrow{R_3 \rightarrow R_1 - 4R_3} \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & 12 & -8 & -116 \\ 0 & -18 & -3 & -24 \end{array} \right]$$

$$\xrightarrow{R_3 \rightarrow 2R_3 + 3R_2} \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & 12 & -8 & -116 \\ 0 & 0 & -30 & 300 \end{array} \right]$$

it is a row echelon form

$$36a - 6b + c = 8 \rightarrow \textcircled{1}$$

$$12b - 8c = 116 \rightarrow \textcircled{2}$$

$$-30c = 300 \Rightarrow c = -10$$

$$\begin{aligned} \text{Sub } c &= -10 \text{ in } ② \\ 12b - 8(-10) &= 116 \\ 12b &= 116 - 80 \\ 12b &= 36 \end{aligned}$$

$$b = 3$$

$$\begin{aligned} \text{Sub } b &= 3, c = -10 \text{ in } ① \\ 36a - 6(3) - 10 &= 8 \\ 36a &= 8 + 18 + 10 \\ 36a &= 36 \end{aligned}$$

$$a = 1$$

$$\therefore \text{The given path is } y = ax^2 + bx + c \\ \Rightarrow y = x^2 + 3x - 10$$

Since the friend at (7, 60)

$$\begin{aligned} 60 &= 7^2 + 3(7) - 10 \\ 60 &= 49 + 21 - 10 \\ 60 &= 60 \end{aligned}$$

Hence, A boy meet the friend.

OR

(b) Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

Solution:

Let (a, b) be the point of intersection of given curves

$$\begin{aligned} a^2 + 4b^2 &= 8 \rightarrow ① \\ a^2 - 2b^2 &= 4 \rightarrow ② \\ ① - ② &\Rightarrow 6b^2 = 4 \\ &\Rightarrow b^2 = \frac{2}{3} \end{aligned}$$

$$\text{Sub } b^2 = \frac{2}{3} \text{ in } ①$$

$$a^2 + 4\left(\frac{2}{3}\right) = 8 \Rightarrow a^2 = 8 - \frac{8}{3} = \frac{16}{3}$$

$$x^2 + 4y^2 = 8$$

$$2x + 8y \frac{dy}{dx} = 0$$

$$8y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{4y}$$

$$m_1 = \left(\frac{dy}{dx}\right)_{(a,b)} = -\frac{a}{4b}$$

$$m_1 m_2 = \left(-\frac{a}{4b}\right) \left(\frac{a}{2b}\right) = -\frac{a^2}{8b^2} = -\frac{\frac{16}{3}}{8\left(\frac{2}{3}\right)} = -1$$

$$m_1 m_2 = -1$$

\therefore The given two curves cut orthogonally.

- 45.(a)** Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + t(2\hat{i} - \hat{j} + 4\hat{k})$ and perpendicular to plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = 8$.

Solution:

The required equation of plane passing through a point $(1, -1, 3)$ and parallel to vectors $2\hat{i} - \hat{j} + 4\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$.

$$\text{Let } \vec{a} = \hat{i} - \hat{j} + 3\hat{k}; \vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}; \vec{c} = \hat{i} + 2\hat{j} + \hat{k}$$

Vector equation:

$$\vec{r} = \vec{a} + s\vec{b} + t\vec{c} \text{ where } s, t \in \mathbb{R}$$

$$\boxed{\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + s(2\hat{i} - \hat{j} + 4\hat{k}) + t(\hat{i} + 2\hat{j} + \hat{k})} \text{ where } s, t \in \mathbb{R}$$

Cartesian equation:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

$$(x_1, y_1, z_1) = (1, -1, 3)$$

$$(b_1, b_2, b_3) = (2, -1, 4)$$

$$(c_1, c_2, c_3) = (1, 2, 1)$$

$$\begin{vmatrix} x - 1 & y + 1 & z - 3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$(x - 1)(-1 - 8) - (y + 1)(2 - 4) + (z - 3)(4 + 1) = 0$$

$$-9(x - 1) + 2(y + 2) + 5(z - 3) = 0$$

$$-9x + 9 + 2y + 2 + 5z - 15 = 0$$

$$-9x + 2y + 5z - 4 = 0$$

$$\boxed{9x - 2y - 5z + 4 = 0}$$

OR

- (b)** Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

Solution:

$$6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0 \rightarrow \textcircled{1}$$

is a even degree reciprocal equation of type I

Given that $\frac{1}{3}$ is a solution $\Rightarrow 3$ is also solution

$\therefore x = \frac{1}{3}$ and $x = 3$ are the roots

$$\begin{array}{r|ccccc} & 6 & -5 & -38 & -5 & 6 \\ \frac{1}{3} & 0 & 2 & -1 & -13 & -6 \\ \hline & 6 & -3 & -39 & -18 & 0 \\ 3 & 0 & 18 & 45 & 18 & \\ \hline & 6 & 15 & 6 & 0 & \end{array}$$

$$\begin{aligned}\therefore 6x^2 + 15x + 6 &= 0 \\ \div 3 &\Rightarrow 2x^2 + 5y + 2 = 0 \\ \left(x + \frac{4}{2}\right)\left(x + \frac{1}{2}\right) &= 0 \\ \left(x + 2\right)\left(x + \frac{1}{2}\right) &= 0 \\ x = -2, \quad x = -\frac{1}{2} &\end{aligned}$$

Hence, the roots are $-2, -\frac{1}{2}, \frac{1}{3}$ and 3

46.(a) Prove that $p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

Solution:

p	q	r	$\neg q$	$\neg q \vee r$	$p \rightarrow (\neg q \vee r)$	$\neg p$	$\neg p \vee (\neg q \vee r)$
T	T	T	F	T	T	F	T
T	T	F	F	F	F	F	F
T	F	T	T	T	T	F	T
T	F	F	T	T	T	F	T
F	T	T	F	T	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

From ① and ②, we get
 $\therefore p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$

OR

(b) Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

Solution:

Let x be the amount deposited.

$$\begin{aligned}\frac{dx}{dt} &\propto x \\ \frac{dx}{dt} &= kx \\ \frac{dx}{x} &= kdt \\ \int \frac{dx}{x} &= k \int dt \\ \log|x| &= kt + \log|C| \\ \frac{x}{C} &= e^{kt} \\ x = C e^{kt} &\end{aligned} \rightarrow ①$$

$$k = 5\% = \frac{5}{100} = 0.05$$

$$x = C e^{0.05t} \rightarrow \textcircled{2}$$

When $t = 0$, $x = 10,000$

$$10,000 = C e^0 = C(1)$$

$$\boxed{C = 10,000}$$

Sub $C = 10,000$ in $\textcircled{2}$

$$x = 10,000 e^{0.05t} \rightarrow \textcircled{3}$$

When $t = 18$ months, $x = ?$

When $t = 1.5$ years, $x = ?$

$$x = 10,000 e^{0.05(1.5)}$$

$$x = 10,000 e^{0.075}$$

∴ The amount after 18 month is $10,000 e^{0.075}$

47.(a) Find the maximum value of $\frac{\log x}{x}$.

Solution:

$$f(x) = \frac{\log x}{x}, x \in (0, \infty)$$

$$f'(x) = \frac{x\left(\frac{1}{x}\right) - \log x(1)}{x^2} = \frac{1 - \log x}{x^2}$$

$$f''(x) = \frac{x^2\left(0 - \frac{1}{x}\right) - (1 - \log x)(2x)}{x^4} = \frac{-x - 2x + 2x \log x}{x^4} = \frac{-3x + 2x \log x}{x^4} = \frac{x(2 \log x - 3)}{x^4}$$

$$f''(x) = \frac{2 \log x - 3}{x^3}$$

If $f'(x) = 0$, then $\frac{1 - \log x}{x^2} = 0$

$$1 - \log x = 0$$

$$\log x = 1$$

$$e^{\log x} = e^1 \Rightarrow x = e$$

$$\text{At } x = e, f''(e) = \frac{2 \log e - 3}{e^3} = \frac{2 - 3}{e^3} = -\frac{1}{e^3} < 0$$

Hence at $x = e$, $f(x)$ attains a local maximum.

Local Maximum Value : At $x = e$, $f(e) = \frac{\log e}{e} = \frac{1}{e}$

OR

(b) Find the area of the region common to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$.

Solution:

Question not clear about area

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