XII STD MINIMUM PASS STUDY MATERIAL

MALAR TRUST INDIA - EACHAMPALLAM

XIth Std -MATHEMATICS

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6. VECTOR ALGEBRA (19 MARK)

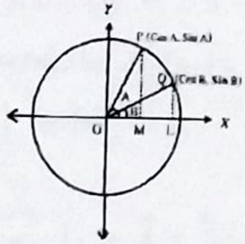
Two questions for full test **Total number of questions : 20**

1) Prove that Cos(A-B) = CosACosB + SinAsinB

Solution:

Let P(CosA, SinA) and Q(CosB, SinB) be any two points on the unit circle with centre at the origin O.

Let $\vec{i}$ and $\vec{j}$ be the unit vectors along the co-ordinate axes.



$\overrightarrow{OP}$ =CosA$\vec{i}$+SinA$\vec{j}$

$\overrightarrow{OQ}$=CosB$\vec{i}$+SinB$\vec{j}$

$\overrightarrow{OQ}.\overrightarrow{OP} = |\overrightarrow{OQ}||\overrightarrow{OP}|\text{Cos}(A-B)$

$= \text{Cos}(A-B)$.........(1)

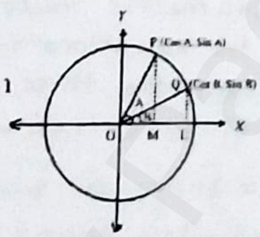
$\overrightarrow{OQ}.\overrightarrow{OP}$ = CosACosB+SinASinB ......(2)

(1)=(2)⇒Cos (A−B)=CosACosB+SinASinB

2) Prove that Sin(A-B) = SinACosB - CosASinB

Solution:

Let P(CosA, SinA) and Q(CosB,SinB) be any two points on the unit circle with centre at the origin O.



Let $\vec{i}$ and $\vec{j}$ be the unit vectors along the co-ordinate axes

$\overrightarrow{OP}$=CosA$\vec{i}$+SinA$\vec{j}$

$\overrightarrow{OQ}$=CosB$\vec{i}$+SinB$\vec{j}$

$\overrightarrow{OQ}\times\overrightarrow{OP}=|\overrightarrow{OQ}||\overrightarrow{OP}|\text{Sin }(A-B)\vec{k}$

$= \text{Sin }(A-B)\vec{k}$....(1)

$$\overrightarrow{OQ}\times\overrightarrow{OP}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ CosB & SinB & 0\\ CosA & SinA & 0\end{vmatrix}$$

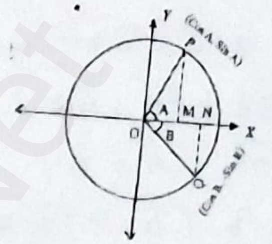
$= \vec{k}$ (SinACosB −CosASinB)...(2)

(1)=(2) ⇒ Sin (A−B)=SinACosB −CosASinB

3) Prove that Cos(A+B) = CosACosB - SinASinB

Solution:

Let P(CosA, SinA) and Q(CosB, -SinB) be any two points on the unit circle with centre at the origin O. Let $\vec{i}$ and $\vec{j}$ be the unit vectors along the co-ordinate axes.



$\overrightarrow{OP}$=CosA$\vec{i}$+SinA$\vec{j}$

$\overrightarrow{OQ}$=CosB$\vec{i}$−SinB$\vec{j}$

$\overrightarrow{OQ}\ .\ \overrightarrow{OP} = |\overrightarrow{OQ}||\overrightarrow{OP}|\text{Cos }(A+B)$

$= \text{Cos}(A+B)$..............(1)

$\overrightarrow{OQ}\ .\ \overrightarrow{OP}$=CosACosB − SinASinB.......(2)

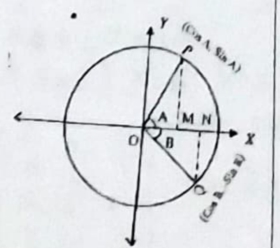
(1)=(2)⇒ Cos (A+B)=CosACosB − SinASinB

4) Prove thatSin (A+B) = SinACosB +CosASinB

Solutons:

Let P(CosA, SinA) and Q (CosB, -SinB) be any two points on the unit circle with centre at the origin O.

Let $\vec{i}$ and $\vec{j}$ be the unit vectors along the co-ordinate axes.



$\overrightarrow{OP}$=CosA$\vec{i}$+SinA$\vec{j}$

$\overrightarrow{OQ}$=CosB$\vec{i}$−SinB$\vec{j}$

$\overrightarrow{OQ}\times\overrightarrow{OP}=|\overrightarrow{OQ}||\overrightarrow{OP}|\text{Sin }(A+B)\vec{k}$

$= \text{Sin }(A+B)\vec{k}$..............(1)

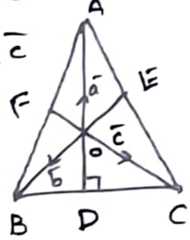
$$\overrightarrow{OQ}\times\overrightarrow{OP}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ CosB & -SinB & 0\\ CosA & SinA & 0\end{vmatrix}$$

=(SinACosB+CosASinB)$\vec{k}$...........(2)

(1)=(2) ⇒ Sin (A+B)=SinACosB+CosASinB

1

5) Prove by vector method that the perpendiculars (altitudes) from the vertices to the opposite sides of a triangle are concurrent.



Let $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$, $\overline{OC} = \bar{c}$

$AD \perp BC$, $OA \perp BC$

$\overline{OA} \cdot \overline{BC} = 0$

$\bar{a} \cdot (\bar{c} - \bar{b}) = 0$

$\bar{a} \cdot \bar{c} - \bar{a}\,\bar{b} = 0$ — ①

$\overline{BE} \perp \overline{CA}$, $\overline{OB} \perp \overline{CA}$

$\overline{OB} \cdot \overline{CA} = 0$

$\bar{b} \cdot (\bar{a} - \bar{c}) = 0$

$\bar{a} \cdot \bar{b} - \bar{b}\,\bar{c} = 0$ — ②

Adding ① + ②

$\bar{a} \cdot \bar{c} - \cancel{\bar{a}\,\bar{b}} + \cancel{\bar{a} \cdot \bar{b}} - \bar{b} \cdot \bar{c} = 0$

$\bar{a}\,\bar{c} - \bar{b} \cdot \bar{c} = 0$

$\overline{OC} \cdot \overline{BA} = 0$ ∴ The altitudes are concurrent

6) If $\bar{a} = -2\hat{i} + 3\hat{j} - 2\hat{k}$, $\bar{b} = 3\hat{i} - \hat{j} + 3\hat{k}$, $\bar{c} = 2\hat{i} - 5\hat{j} + 6\hat{k}$, find $(\bar{a} \times \bar{b}) \times \bar{c}$ and $\bar{a} \times (\bar{b} \times \bar{c})$. State whether they are equal.

$$\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -2 & 3 & -2 \\ 3 & -1 & 3 \end{vmatrix} = \bar{i}(9-2) - \bar{j}(-6+6) + \bar{k}(2-9) = 7\bar{i} - 7\bar{k}$$

$$(\bar{a} \times \bar{b}) \times \bar{c} = \begin{vmatrix} i & j & k \\ 7 & 0 & -7 \\ 2 & -5 & 1 \end{vmatrix} = \bar{i}(-35) - \bar{j}(7+14) + \bar{k}(-35) = -35\hat{i} - 21\hat{j} - 35\hat{k}$$

$$\bar{b} \times \bar{c} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & -1 & 3 \\ 2 & -5 & 1 \end{vmatrix} = \bar{i}(-1+15) - \bar{j}(3-6) + \bar{k}(-15+2) = 14\bar{i} + 3\bar{j} - 13\bar{k}$$

$$\bar{a} \times (\bar{b} \times \bar{c}) = \begin{vmatrix} i & j & k \\ -2 & 3 & -2 \\ 14 & 3 & -13 \end{vmatrix} = \bar{i}(-39+6) - \bar{j}(26+28) + \bar{k}(-6-42) = -33\bar{i} - 54\bar{j} - 48\bar{k}$$

∴ $(\bar{a} \times \bar{b}) \times \bar{c} \neq \bar{a} \times (\bar{b} \times \bar{c})$

7) Find the shortest distance between the two given straight lines $\bar{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + t(-2\hat{i} + \hat{j} - 2\hat{k})$ and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+2}{2}$

$\bar{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + t(-2\hat{i} + \hat{j} - 2\hat{k})$

$\bar{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\bar{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ }

$\bar{c} = 3\bar{i} - 2\hat{k}$, $\bar{d} = 2\bar{i} - \hat{j} + 2\bar{k}$ }

Shortest distance between the points

$$d = \frac{|(\bar{c} - \bar{a}) \times \bar{b}|}{|\bar{b}|}$$

$\bar{c} - \bar{a} = \bar{i} - 3\bar{j} - 6\bar{k}$

$$(\bar{c} - \bar{a}) \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -3 & -6 \\ -2 & 1 & -2 \end{vmatrix} = \bar{i}(6+6) - \bar{j}(-2-12) + \bar{k}(1-6) = 12\bar{i} + 14\bar{j} - 5\bar{k}$$

$$d = \frac{|\bar{c} - \bar{a} \times \bar{b}|}{|\bar{b}|} = \frac{|12\bar{i} + 14\bar{j} - 5\bar{k}|}{|-2\bar{i} + \bar{j} - 2\bar{k}|}$$

$$= \frac{\sqrt{144 + 196 + 25}}{\sqrt{4+1+4}} = \frac{\sqrt{365}}{3}$$

8) Find the Parametric vector, non parametric vector and cartesian form of the equations of the plane passing through the three non-collinear points $(3, 6, -2)$, $(-1, -2, 6)$ and $(6, 4, -2)$

$\bar{a} = 3\bar{i} + 6\bar{j} - 2\bar{k}$, $\bar{b} = -\bar{i} - 2\bar{j} + 6\bar{k}$, $\bar{c} = 6\bar{i} + 4\bar{j} - 2\bar{k}$

Parametric form:

$\bar{r} = \bar{a} + s(\bar{b} - \bar{a}) + t(\bar{c} - \bar{a})$

$\bar{r} = 3\bar{i} + 6\bar{j} - 2\bar{k} + s(-4\bar{i} - 8\bar{j} + 8\bar{k}) + t(3\bar{i} - 10\bar{j} + 0\bar{k})$

Non parametric form:

$(\bar{r} - \bar{a}) \cdot ((\bar{b} - \bar{a}) \times (\bar{c} - \bar{a})) = 0$

$$(\bar{b} - \bar{a}) \times (\bar{c} - \bar{a}) = \begin{vmatrix} i & j & k \\ -4 & -8 & 8 \\ 3 & -10 & 0 \end{vmatrix} = i(80) - j(-24) + k(40+24) = 80\bar{i} + 24\bar{j} + 64\bar{k}$$

$(\bar{r} - 3\bar{i} + 6\bar{j} - 2\bar{k}) \cdot (80\bar{i} + 24\bar{j} + 64\bar{k}) = 0$ $\div 8$

$(\bar{r} - (3\bar{i} + 6\bar{j} - 2\bar{k})) (10\bar{i} + 3\bar{j} + 8\bar{k}) = 0$

$\bar{r} \cdot 10\bar{i} + 3\bar{j} + 8\bar{k} - (30 + 18 - 16) = 0 \Rightarrow \bar{r} \cdot 10\bar{i} + 3\bar{j} + 8\bar{k} - 32 = 0$

Cartesian form:

$(x\bar{i} + y\hat{j} + z\hat{k}) \cdot (10\hat{i} + 3\hat{j} + 8\hat{k}) - 32 = 0$

$\boxed{10x + 3y + 8z - 32 = 0}$

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S. ... KUMAR

Unit-3 Theory of Equations

1. Solve the cubic equation:

$8x^3 - 2x^2 - 7x + 3$

$$\begin{array}{r|rrrr} -1 & 8 & -2 & -7 & 3 \\ & 0 & -8 & 10 & -3 \\ \hline & 8 & -10 & 3 & \boxed{0} \end{array}$$

$8x^2 - 10x + 3$ (24: $-\frac{6}{8}, -\frac{4}{8}$)

$\left(x - \frac{3}{4}\right)\left(x - \frac{1}{2}\right) = 0$

$x = \frac{3}{4}, \frac{1}{2}$

$\therefore$ Roots are $-1, \frac{3}{4}, \frac{1}{2}$

2. Solve the Equations:

$6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

$$\begin{array}{r|rrrrr} 3 & 6 & -35 & 62 & -35 & 6 \\ & 0 & 18 & -51 & 33 & -6 \\ \hline 2 & 6 & -17 & 11 & -2 & \boxed{0} \\ & 0 & 12 & -10 & 2 & \\ \hline \frac{1}{2} & 6 & -5 & 1 & \boxed{0} & \\ & 0 & 3 & -1 & & \\ \hline \frac{1}{3} & 6 & -2 & \boxed{0} & & \\ & 0 & 2 & & & \\ \hline & 6 & \boxed{0} & & & \end{array}$$

$\therefore$ Roots are $3, 2, \frac{1}{2}, \frac{1}{3}$

3. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

$$\begin{array}{r|rrrrr} \frac{1}{3} & 6 & -5 & -38 & -5 & 6 \\ & 0 & 2 & -1 & -13 & -6 \\ \hline 3 & 6 & -3 & -39 & -18 & \boxed{0} \\ & 0 & 18 & 45 & 18 & \\ \hline -\frac{1}{2} & 6 & 15 & 6 & \boxed{0} & \\ & 0 & -3 & -6 & & \\ \hline -2 & 6 & 12 & \boxed{0} & & \\ & 0 & -12 & & & \\ \hline & 6 & \boxed{0} & & & \end{array}$$

$\therefore$ Roots are $\frac{1}{3}, 3, -\frac{1}{2}, -2$

4. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic Progression.

A.P $(a-d)(a)(a+d) = \frac{-b}{a} = +\frac{36}{9} = 4$

$3a = 4, \; a = \frac{4}{3}$

$$\begin{array}{r|rrrr} \frac{4}{3} & 9 & -36 & 44 & -16 \\ & 0 & 12 & -32 & 16 \\ \hline & 9 & -24 & 12 & \boxed{0} \end{array}$$

(12: $-\frac{6}{3}, -\frac{2}{3}$)

$9x^2 - 24x + 12 = 3(3x^2 - 8x + 4)$

$(x-2)\left(x - \frac{2}{3}\right) = 0$

$x = 2, \frac{2}{3}$

$\therefore$ Roots are $\frac{4}{3}, 2, \frac{2}{3}$

5. Solve the Equation: $3x^3 - 26x^2 + 52x - 24 = 0$ if the roots form a geometric progression.

$\left(\frac{a}{r}\right), a, (ar) = \frac{-d}{a} = \frac{+24}{3} = 8 \Rightarrow a^3 = 8, a = 2$

$$\begin{array}{r|rrrr} 2 & 3 & -26 & 52 & -24 \\ & 0 & 6 & -40 & 24 \\ \hline & 3 & -20 & 12 & \boxed{0} \end{array}$$

$3x^2 - 20x + 12$ (36: $-\frac{18}{3}, -\frac{2}{3}$)

$(x-6)\left(x - \frac{2}{3}\right) = 0$

$x = 6, \frac{2}{3}$

$\therefore$ Roots are $2, 6, \frac{2}{3}$

6. Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots.

$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{+6}{2} = 3 \Rightarrow 3\alpha = 6, \alpha = 2.$

$$\begin{array}{r|rrrr} 2 & 2 & -6 & 3 & k \\ & 0 & 4 & -4 & -2 \\ \hline & 2 & -2 & -1 & \boxed{0} \end{array}$$

$\therefore \boxed{k = 2}$

$2x^2 - 2x - 1 \quad a = 2, b = -2, c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+2 \pm \sqrt{4 - 4(2)(-1)}}{2(2)}$

$= \frac{+2 \pm \sqrt{4+8}}{4} = \frac{+2 \pm \sqrt{12}}{4} = \frac{+2 \pm \sqrt{3 \times 4}}{4}$

$= \frac{2 \pm 2\sqrt{3}}{4} \Rightarrow \frac{2(1 \pm \sqrt{3})}{4}$

Roots are $2, \frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}$

7. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1+2i$ and $\sqrt{3}$ are two of its zeros.

$x = 1+2i$, $\quad x-1 = 2i \Rightarrow (x-1)^2 = (2i)^2$

$x^2 - 2x + 1 = -4 \Rightarrow x^2 - 2x + 5 = 0$

$x = \sqrt{3} \quad x^2 = 3, \quad x^2 - 3 = 0$

$x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135 \equiv$

$(x^2 - 2x + 5)(x^2 - 3)(x^2 + px - 9)$

Equating x on both sides.

$-39 = -54 - 15p \Rightarrow \boxed{p = -1}$

$x^2 - x - 9 = 0$

$a = 1, b = -1, c = -9$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{1 \pm \sqrt{1+36}}{2}$

$= \dfrac{1 \pm \sqrt{37}}{2}$

$\therefore$ Roots are $1+2i, 1-2i, \sqrt{3}, -\sqrt{3}$, $\dfrac{1+\sqrt{37}}{2}, \dfrac{1-\sqrt{37}}{2}$

8. Solve: $8x^{3/2n} - 8x^{-3/2n} = 63$

Let take $y = x^{3/2n}$, $\quad \dfrac{1}{y} = x^{-3/2n}$

$8y - \dfrac{8}{y} = 63 \quad$ (cross $\times$ ly)

$\dfrac{8y^2 - 8}{y} = 63$

$8y^2 - 8 = 63y$

$8y^2 - 63y - 8 = 0$

$(y + \frac{1}{8})(y - 8) = 0$

-64: $-64 = \frac{1}{8}, -\frac{64}{8}$

$y = 8, -\frac{1}{8}$

$\boxed{\therefore \text{Solution is } y = 8}$

$y = x^{3/2n} \Rightarrow 8 = x^{3/2n}$

$2^{3} \Rightarrow x^{3/2n}$

$x = 2^{2n}$

$\boxed{x = 4^n}$

9. Solve: $2\sqrt{\dfrac{x}{a}} + 3\sqrt{\dfrac{a}{x}} = \dfrac{b}{a} + \dfrac{6a}{b}$

Take Squares on both sides.

$\left(2\sqrt{\dfrac{x}{a}} + 3\sqrt{\dfrac{a}{x}}\right)^2 = \left(\dfrac{b}{a} + \dfrac{6a}{b}\right)^2$

$4\dfrac{x}{a} + 9\dfrac{a}{x} + 12\sqrt{\dfrac{x}{a}}\sqrt{\dfrac{a}{x}} = \dfrac{b^2}{a^2} + \dfrac{36a^2}{b^2} + 2\cdot\dfrac{b}{a}\cdot\dfrac{6a}{b}$

$4\dfrac{x}{a} + 9\dfrac{a}{x} + \cancel{12} = \dfrac{b^2}{a^2} + \dfrac{36a^2}{b^2} + \cancel{12}$

$\dfrac{4x^2 + 9a^2}{ax} = \dfrac{b^4 + 36a^4}{a^2b^2}$

$(4x^2 + 9a^2)a^2b^2 = (b^4 + 36a^4)ax$

$4x^2a^2b^2 + 9a^4b^2 = b^4ax + 36a^5x$

$4ab^2x^2 - b^4x - 36a^4x + 9a^3b^2 = 0$

$b^2x(4ax - b^2) - 9a^3(4ax - b^2) = 0$

$(4ax - b^2)(b^2x - 9a^3) = 0$

$x = \dfrac{b^2}{4a}, \dfrac{9a^3}{b^2}$

(or)

Aliter:

Let $y = \sqrt{\dfrac{x}{a}} \quad \dfrac{1}{y} = \sqrt{\dfrac{a}{x}}$

$2y + \dfrac{3}{y} = \dfrac{b}{a} + \dfrac{6a}{b} \quad$ (co $\times$ ly)

$\dfrac{2y^2 + 3}{y} = \dfrac{b^2 + 6a^2}{ab}$

$2y^2ab + 3ab = b^2y + 6a^2y$

$2y^2ab + 3ab - b^2y - 6a^2y = 0$

$2ay(by - 3a) - b(by - 3a) = 0$

$(by - 3a)(2ay - b) = 0$

$by - 3a = 0 \quad | \quad 2ay - b = 0$

$by = 3a \quad | \quad 2ay = b$

$y = \dfrac{3a}{b} \quad | \quad y = \dfrac{b}{2a}$ (square the Ans)

$\boxed{x = \dfrac{9a^3}{b^2} \qquad x = \dfrac{b^2}{4a}}$

10. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9-4x^8+4x^7-3x^6+2x^5+x^3+7x^2+7x+2=0$

$p(x) = 9x^9-4x^8+4x^7-3x^6+2x^5+x^3+7x^2+7x+2$

+ − + − + + + + +

1 2 3 4

∴ Total No. of Positive sign changes - 4

∴ Positive real roots = 4

$p(-x) = -9x^9-4x^8-4x^7-3x^6-2x^5-x^3+7x^2-7x+2$ (put odd power -ve)

− − − − − − + − +

1 2 3

∴ No. of sign changes - 3

∴ Total Negative real roots - 3

11. Discuss the maximum possible number of positive & negative roots of the polynomial equation x^2-5x+6 and $x^2-5x+16$. Also draw rough sketch of the graph.

$p(x) = x^2-5x+6$ + − +   1 2

∴ No. of sign changes - 2

Positive real roots - 2

$p(-x) = x^2+5x+6$ + + +

∴ No. of sign changes - 0

Negative real roots - 0

$Q(x) = x^2-5x+16$ + − +   1 2

∴ No. of sign changes - 2

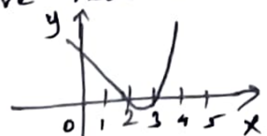
positive real roots - 2

$Q(-x) = x^2+5x+16$ + + +

∴ No of sign changes - 0

Negative real roots - 0

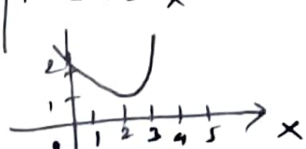
x^2-5x+6



6

−2 −3

x^2+5x+6



12. Show the equation $x^9-5x^5+4x^4+2x^2+1=0$ has at least 6 imaginary solutions.

$p(x) = x^9-5x^5+4x^4+2x^2+1$

+ − + + +

1 2

No. of sign changes = 2 ∴ positive roots - 2

$p(-x) = -x^9+5x^5+4x^4+2x^2+1$ − + + + +

No. of sign changes = 1 ∴ negative roots - 1

Real roots = positive roots + negative roots

= 2 + 1 = 3

Total no. of roots = 9

Imaginary roots = Total roots − Real roots

= 9 − 3 = 6

∴ It has atleast 6 Imaginary solutions.

13. Determine number of positive & negative roots of the equation $x^9-5x^8-14x^7=0$

$p(x) = x^9-5x^8-14x^7$ + − −   1

No. of sign changes - 1 ∴ positive roots = 1

$p(-x) = -x^9-5x^8+14x^7$ − − +

No. of sign changes - 1 ∴ Negative roots = 1.

14. Find the exact number of real & imaginary roots of the equation $x^9+9x^7+7x^5+5x^3+3x=0$

$p(x) = x^9+9x^7+7x^5+5x^3+3x$

+ + + + +

∴ No. of sign changes = 0 ∴ Positive roots = 0

$p(-x) = -x^9-9x^7-7x^5-5x^3-3x$

− − − − −

No. of sign changes = 0 ∴ Negative roots = 0

It has No positive and negative real roots.

∴ 0 is one root. So the given equation has $\boxed{8}$ Imaginary roots.

AJITH KUMAR

UNIT - IV INVERSE TRIGONOMETRIC EQUATIONS

1. Find the value of

$\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{5\pi}{9}\right)$

$\sin(A+B)=\sin A\cos B+\cos A\sin B$

$=\sin^{-1}\left(\sin\left(\frac{5\pi}{9}+\frac{\pi}{9}\right)\right)$

$=\sin^{-1}\left(\sin\frac{\cancel{6}^{2}\pi}{\cancel{9}3}\right)$

$=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right)$

$=\cancel{\sin^{-1}}\left(\sin\cancel{\pi}-\cancel{\sin}\frac{\pi}{3}\right)$

$=\frac{\pi}{3}\in\left[-\pi/2,\pi/2\right]$.

2. Find the value of

$\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17}-\sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$

$=\cos^{-1}\left(\cos\left(\frac{\pi}{7}+\frac{\pi}{17}\right)\right)$

$=\cos^{-1}\left(\cos\left(\frac{17\pi+7\pi}{119}\right)\right)$

$=\cancel{\cos^{-1}}\left(\cancel{\cos}\left(\frac{24\pi}{119}\right)\right)$

$=\frac{24\pi}{119}$ //

3. Find the value of

$\cos\left(\sin^{-1}\left(4/5\right)-\tan^{-1}\left(\frac{3}{4}\right)\right)$

$\cos(x-y)=\cos x\cos y+\sin x\sin y$

Let $x=\sin^{-1}\left(\frac{4}{5}\right)$; $\tan^{-1}\left(\frac{3}{4}\right)=y$

$\sin x=\frac{4}{5}$

$\cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{9}{25}}=3/5$

$\tan y=\frac{3}{4}$

$\sec^2 y=1+\tan^2 y=1+\frac{9}{16}=\frac{25}{16}$.

$\sec y=\frac{5}{4}\Rightarrow\cos y=4/5$

$\sin y=\sqrt{1-\cos^2 y}=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$

$\cos\left(\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right)=\cos(x-y)$

$=\cos x\cos y+\sin x\sin y$

$=\frac{3}{5}\cdot\frac{4}{5}+\frac{4}{5}\cdot\frac{3}{5}$

$=\frac{12}{25}+\frac{12}{25}=\frac{24}{25}$ //

4. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$ show that $x+y+z=xyz$.

$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$.

$\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left(\frac{x+y}{1-xy}\right)+\tan^{-1}z$

$=\tan^{-1}\left(\frac{\frac{x+y}{1-xy}+z}{1-\left(\frac{x+y}{1-xy}\right)z}\right)$

$=\tan^{-1}\left(\frac{\frac{x+y+z-xy}{1-xy}}{1-xy-xz-yz/1-xy}\right)$

$=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]=\pi$

$=\tan\pi=0$

$$\boxed{\therefore x+y+z-xyz=0}$$

5. Prove that $\tan(\sin^{-1}x)=\frac{x}{\sqrt{1-x^2}}$ for $|x|<1$

$\sin^{-1}x=\theta$, $x=\sin\theta$, $-1\le x\le 1$

$\tan(\sin^{-1}x)=\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

$=\frac{x}{\sqrt{1-x^2}}$, $|x|<1$

S. AJITH KUMAR

Sketch the curve $y = f(x) = x^2 - x - 6$

$y = f(x) = x^2 - x - 6$

1. Domain $(-\infty, \infty)$
2. Intercepts $y = f(x) = (x-3)(x+2)$

$y = 0,\ x = -2, 3\ (-2,0)(3,0)$

$x = 0,\ y = -6\ (0,-6)$

3. Critical point

$f'(x) = 2x - 1$

$2x - 1 = 0,\ x = \frac{1}{2}$

4. Local extrema

$f''(x) = 2 > 0$

$\therefore$ Local minimum.

5. Range:

$f(x) = x^2 - x - 6$

$f(1/2) = (1/2)^2 - 1/2 - 6$

$= \frac{1}{4} - \frac{1}{2} - 6$

$= \frac{1 - 2 - 24}{4} = \frac{-25}{4}$

6. Concavity: Upward in the entire real line
7. Inflexion: No points
8. Asymptotes: No Asymptotes

Sketch the curve $y = f(x) = x^3 - 6x - 9$

$y = f(x) = x^3 - 6x - 9$

1. Domain $(-\infty, \infty)$
2. Intercepts:

$y = f(x) = (x-3)(x-3)$

$y = 0,\ x = 3\ (3,0)$

$x = 0,\ y = -9\ (0,-9)$

3. Critical point

$f'(x) = 3x^2 - 6$

$= 3(x^2 - 2) = 0$

$x^2 = 2,\ x = \pm\sqrt{2}$

4. Local extrema

$f''(x) = 6x$

put $x = \sqrt{2}$

$f''(\sqrt{2}) = 6\sqrt{2} > 0$ Local minimum.

5. Range: $f(-\sqrt{2}) = 4\sqrt{2} - 9$
6. Concavity: ~~Con~~ upward in the positive real line. – Downward in the negative real line.
7. Inflexion: $(0,-9)$
8. Asymptotes: No Asymptotes

Sketch the curve $y = \frac{x^2 - 3x}{x - 1}$

$y = f(x) = \frac{x^2 - 3x}{(x-1)}$

$= \frac{x(x-3)}{(x-1)}$

1. Domain: $R \setminus \{1\}$
2. Intercepts:

$y = 0,\ x = 0, 3 \Rightarrow (3,0)$

$x = 0,\ y = 0$

3. Critical point:

$f'(x) = \frac{x^2 - 2x + 3}{(x-1)^2}$

$x = 1$.

4. Local extrema:

No Local maximum or minimum.

5. Concavity: Downwards
6. Inflexion: No
7. Asymptotes:

$\lim_{x \to 1} \frac{x^2 - 3x}{x - 1} = \frac{1-3}{0} = \frac{-2}{0}$

$= -\infty$

$x = 1$ is a vertical Asymptotes.

Sketch the graph of the function $y = 3x / x^2 - 1$

$y = f(x) = \frac{3x}{x^2 - 1}$

1. Domain: $R \setminus \{-1, 1\}$
2. Intercepts:

$y = 0,\ x = 0\ (0,0)$

$x = 0,\ y = 0\ (0,0)$

3. Critical point:

$f'(x) = \frac{-3(x^2+1)}{(x^2-1)^2}$

$x = -1, 1$

4. Local extrema: No
5. ~~Domain~~ Concavity: upward and downward exists.
6. Inflexion: $(0,0)$
7. Asymptotes:

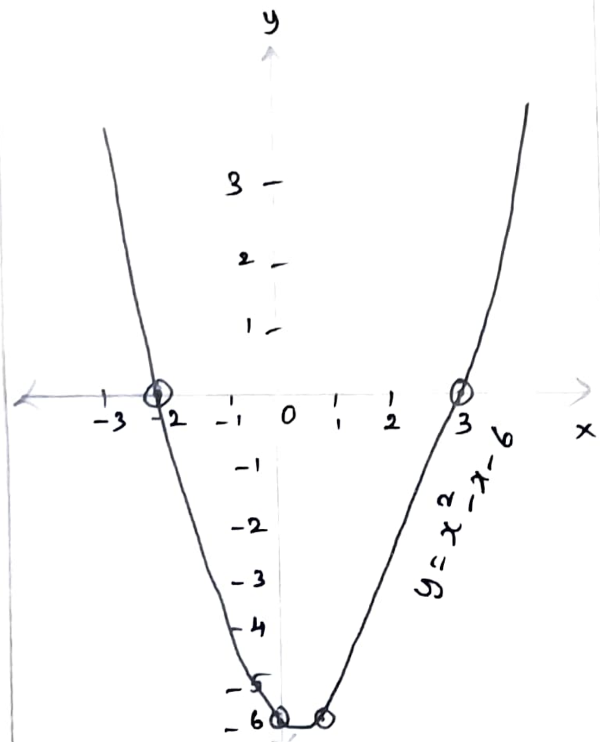
$\lim_{x \to 1} \frac{3x}{x^2 - 1} = \frac{3}{0} = \infty$

$x = -1, x = 1$ are vertical Asymptotes.

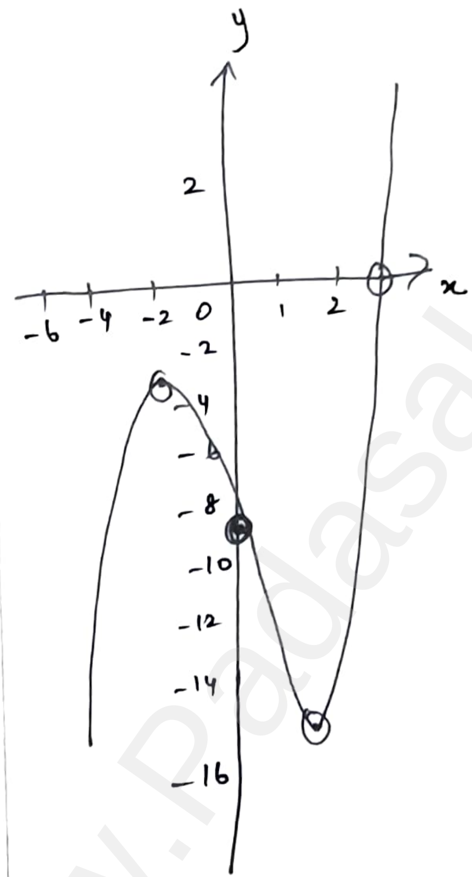
$\lim_{x \to \infty} \frac{3x}{x^2 - 1} = 0,\ y = 0$

$\therefore y = 0$ is horizontal Asymptotes

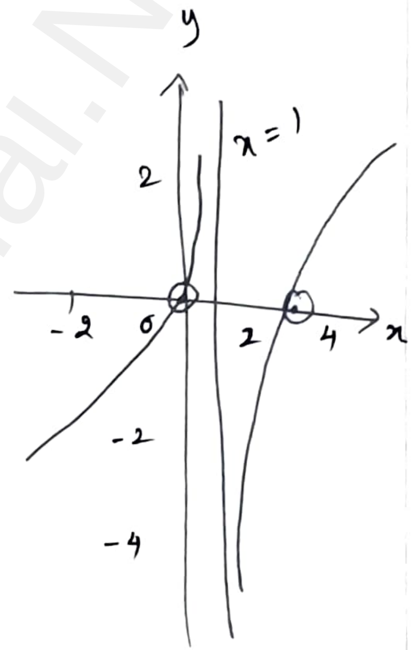
Sketch:



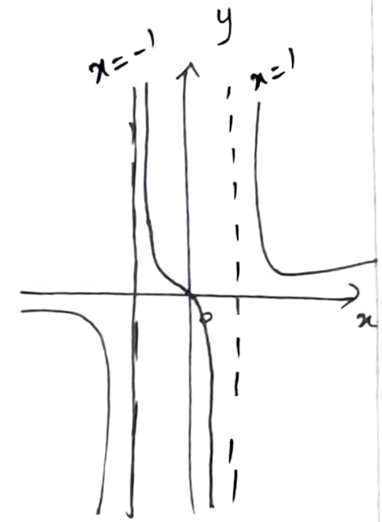
Sketch:



Sketch:



Sketch:



UNIT - 12 DISCRETE MATHEMATICS

1. Verify i) closure property ii) Associative Property iii) existence of identity iv) existence of inverse v) commutative Property for the operation $+_5$ on Z_5 using table corresponding to addition modulo 5.

$+_5$	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[1]	[2]	[3]	[4]
[1]	[1]	[2]	[3]	[4]	[0]
[2]	[2]	[3]	[4]	[0]	[1]
[3]	[3]	[4]	[0]	[1]	[2]
[4]	[4]	[0]	[1]	[2]	[3]

i) All the elements belongs to $+5$. closure property true.

ii) From the table, $+5$ is associative.

iii) From the table Commutative property satisfied

iv) The identity element is [0]

v) The Inverse of 0 is [1], 1 is [4]
[2] is [3], [3] is [2] & 4 is [1]

2. Verify i) closure property ii) associative property iii) existence of inverse v) identity iv) existence of commutative property for the operation $\times_{11}$ on a subset A = [1, 3, 4, 5, 9] of the set of remainders [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

A = [1][3][4][5][9]

$\times_{11}$	[1]	[3]	[4]	[5]	[9]
[1]	[1]	[3]	[4]	[5]	[9]
[3]	[3]	[9]	[1]	[4]	[5]
[4]	(4)	[1]	[5]	[9]	[3]
[5]	[5]	[4]	[9]	[3]	[1]
[9]	[9]	[5]	[3]	[1]	[4]

i) Elements of the table are in the elements of A ∴ Closure property true.

ii) from the table $\times_{11}$ is associative.

ii) from the table $\times_{11}$ is Commutative

iii) from the table the identity element is [1]

iv) the identity element is [1], [3] is [4]

v) The inverse of [1] is [1], [3] is [4] and [9] is [5]
[4] is [3], [5] is [4]

3) Let A be $Q/\{1\}$ Define $*$ on A by $x * y = x+y-xy$ is binary on A? If so, examine all the five properties.

i) $x*y = x+y-xy$ $\quad x*y \neq 1$
$x, y \in A$
∴ Closure property verified.

ii) $x*y = x+y-xy$
$y*x = x+y-xy$ Commutative property satisfied.
$\therefore x*y = y*x$

iii) $x*(y*z) = (x*y)*z$
∴ Associative property satisfied.

iv) Identity element $0 \in A$

v) Inverse of existence : $x = \frac{x}{x-1}$
∴ the Inverse property satisfied.

4) Verify all the five properties on the given set. $m*n = m+n-mn \in N$

i) $m*n = m+n-mn \quad m*n \in N$
∴ closure property satisfied

ii) $m*n = m+n-mn$
$n*m = m+n-mn$
$m*n = n*m$ ∴ Commutative property Satisfied.

iii) $m*(n*p) = (m*n)*p$
∴ Associative property verified.

iv) Existence of Identity
$a*e = a$
$a+e-ae = a$
$e(1-a) = 0$ ∴ $e = 0$

v) $m*n = m+n-me$
$m*n = e \Rightarrow m*n - ne = 0 \Rightarrow n(1-m) = -m$
$n = \frac{-m}{1-m}$
$\boxed{n = \frac{m}{m-1}}$

5. Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} . x \in R - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so examine all the properties.

1) Closure property:

$a = \begin{pmatrix} x & x \\ x & x \end{pmatrix}, b = \begin{pmatrix} y & y \\ y & y \end{pmatrix}$

$a * b = \begin{pmatrix} x & x \\ x & x \end{pmatrix} * \begin{pmatrix} y & y \\ y & y \end{pmatrix} = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in M$

$\therefore$ closure property verified.

2) Commutative property

$a * b = \begin{pmatrix} x & x \\ x & x \end{pmatrix} * \begin{pmatrix} y & y \\ y & y \end{pmatrix} = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}$

$b * a = \begin{pmatrix} y & y \\ y & y \end{pmatrix} * \begin{pmatrix} x & x \\ x & x \end{pmatrix} = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}$

$a * b = b * a$ $\therefore$ Commutative property verified.

3) Associative property:

$a * (b * c) = (a * b) * c$

$\therefore$ associative property verified.

4) Identity property:

$a * e = a$

$\begin{pmatrix} x & x \\ x & x \end{pmatrix} * \begin{pmatrix} e & e \\ e & e \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$

$\begin{pmatrix} 2xe & 2xe \\ 2xe & 2xe \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$

$2xe = x$

$e = 1/2 \in R - \{0\}$

$\therefore \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \in M$

$\therefore$ Identity property satisfied.

5) Existence of Inverse property:

$a * a^{-1} = e$

$\begin{pmatrix} x & x \\ x & x \end{pmatrix} \begin{pmatrix} x^{-1} & x^{-1} \\ x^{-1} & x^{-1} \end{pmatrix} = \begin{pmatrix} e & e \\ e & e \end{pmatrix}$

$\begin{pmatrix} 2x\,x^{-1} & 2x\,x^{-1} \\ 2x\,x^{-1} & 2x\,x^{-1} \end{pmatrix} = \begin{pmatrix} e & e \\ e & e \end{pmatrix}$

$2x\,x^{-1} = e$ $\quad \therefore e = 1/2$

$x^{-1} = e/2x$

$x^{-1} = 1/4x \in R - \{0\}$

$\therefore \begin{pmatrix} 1/4x & 1/4x \\ 1/4x & 1/4x \end{pmatrix}$

$\therefore$ Inverse property verified.

6. Define an operation $*$ on Q as follows $a * b = \left(\frac{a+b}{2}\right)$, $a, b \in Q$. Verify the properties

1) Closure property:

$a * b = \frac{a+b}{2}$

$a, b \in Q$ $\qquad \frac{a+b}{2} \in Q$

$+$ is binary on Q $\qquad \boxed{a * b \in Q}$

$a + b \in Q$

$\therefore$ closure property verified.

2) Commutative property

$a * b = b * a$

$a * b = \frac{a+b}{2}$, $b * a = \frac{b+a}{2}$

$\therefore$ Commutative property satisfied.

3) Associative property:

$a * (b * c) = (a * b) * c$

$a * \frac{b+c}{2} = \frac{a+b}{2} * c$

$\frac{a * \frac{b+c}{2}}{2} = \frac{\frac{a+b}{2} + c}{2}$

$\frac{a+b+2c}{4} = \frac{a+b+2c}{4}$

$\therefore$ Associative property satisfied.

4) Identity property:

$a * e = a$

$\frac{a+e}{2} = a$

$a + e = 2a$

$e = 2a - 2$

$\boxed{e = a}$ eq Identity should be unique

$\therefore$ Identity does not exist.

5) Existence of Inverse property:

Existence of Inverse does not exit.

S. MOZITH KUMAR