XII MATHS- NCERT_CBSE
CHAPTER-6 APPLICATION OF DERIVATIVES

Chapter 6

6.4 Tangents and Normals

Equations of tangent and normal at a point to a Curve

Tangent of a curse is a line which touches the curve at only one point and normal is a line perpendicular to the tangent and passing through the point of contact.

NOTE:

If given curve is $y=f(x)$ and $P(x_1, y_1)$ is a point on it, then

(1) The Slope of the tangent(curve) at P is $m=\frac{dy}{dx}$ at $P(x_1, y_1)$

(2) The Slope of the normal at P is $\frac{-1}{m}$

(3) The equation of the tangent at $P(x_1, y_1)$ is $y - y_1 = m(x - x_1)$

(4) The equation of the normal at $P(x_1, y_1)$ is $y - y_1 = \frac{-1}{m}(x - x_1)$

(5) If the tangent(curve) at P makes an θ, then the slope of the tangent at P is $m = \tan\theta$

θ	0°	30°	45°	60°	90°	120°	135°	150°
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$\frac{-1}{\sqrt{3}}$

(6) We know that the Slope of a line joining the two given points $(x_1. y_1)$ and (x_2, y_2) is $m = \frac{y_2-y_1}{x_2-x_1}$.

(7) If slope m and the y-intercept c, then the equation of the line is $y=mx+c$

(8) If the equation of the line is of the form $ax + by + c = 0$

$\Rightarrow y=\frac{-a}{b}x - \frac{-c}{b}$ then its slope is $m=\frac{-a}{b} = \frac{-\text{coefficient of } x}{\text{coefficient of } y}$

Remark

(a) If the tangent to a curve is **horizontal** at a point $P(x_1, y_1)$, then

(i) the derivative at that point that is **Slope is 0**.

(ii) at that point $P(x_1, y_1)$, the equation of the **tangent** is $y = y_1$ and

(iii) the equation of the **normal** is $x = x_1$.

(b) If the tangent to a curve is **vertical** at a point $P(x_1, y_1)$, then

(i) the derivative exists and infinite that is **Slope is ∞** at the point.

(ii) at that point $P(x_1, y_1)$, the equation of the **tangent** is $x = x_1$ and

(iii) the equation of the **normal** is $y = y_1$.

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

Example 14

Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$.

SOLUTION:

Given curve is $y = x^3 - x$

$$\frac{dy}{dx} = 3x^2 - 1$$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\ x=2}$

$= 3(4) - 1$

$= 11$

m= 11

EXERCISE 6.3

1. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.

SOLUTION:

Given curve is $y = 3x^4 - 4x$

$$\frac{dy}{dx} = 12x^3 - 4$$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\ x=4}$

$= 12(4)^3 - 4$

$= 12(64) - 4$

$= 768 - 4$

$= 764$

m= 764

2. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}$, $x \neq 2$ at $x = 10$.

SOLUTION:

Given curve is $y = \frac{x-1}{x-2}$, $x \neq 2$

$$\frac{dy}{dx} = \frac{(x-2)(1)-(x-1)(1)}{(x-2)^2}$$

$$= \frac{x-2-x+1}{(x-2)^2} = \frac{-1}{(x-2)^2}$$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\ x=10}$

$= \frac{-1}{(10-2)^2}$

$= \frac{-1}{64}$

m= $\frac{-1}{64}$

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

XII MATHS- NCERT_CBSE

CHAPTER-6 APPLICATION OF DERIVATIVES

3. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose x-coordinate is 2.

SOLUTION:

Given curve is $y = x^3 - x + 1$

$$\frac{dy}{dx} = 3x^2 - 1$$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\ x=2}$

$= 3(4) - 1$

$= 11$

m= 11

4. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

SOLUTION:

Given curve is $y = x^3 - 3x + 2$

$$\frac{dy}{dx} = 3x^2 - 3$$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\ x=3}$

$= 3(9) - 3$

$= 24$

m= 24

16. Show that the tangents to the curve $y = 7x^3 + 11$ at the points where $x = 2$ and $x = -2$ are parallel.

SOLUTION:

for(a)

Given curve is $y = 7x^3 + 11$ .... (1)

$$\frac{dy}{dx} = 21x^2$$

Let the point of contact of the tangent be $P(x_1,y_1)$

∴Slope of the tangent is **m1** $= \left(\frac{dy}{dx}\right)_{at\ x=2} = 21(4) = 84$

∴Slope of the tangent is **m2** $= \left(\frac{dy}{dx}\right)_{at\ x=-2} = 21(4) = 84$

∴ $m1 = m2$

∴ At **x = 2 and x = – 2, the tangents are parallel.**

XII MATHS- NCERT_CBSE

CHAPTER-6 APPLICATION OF DERIVATIVES

5. Find the slope of the normal to the curve $x = a\cos^3\theta,\ y = b\sin^3\theta$ at $\theta = \frac{\pi}{2}$

SOLUTION:

Given curve is $x = a\cos^3\theta$, $y = b\sin^3\theta$

Take $x = a\cos^3\theta$

$$\frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta) = -3a\cos^2\theta.\sin\theta$$

Take $y = b\sin^3\theta$

$$\frac{dy}{d\theta} = 3b\sin^2\theta(\cos\theta) = 3b\sin^2\theta.\cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3b\sin^2\theta.\cos\theta}{-3a\cos^2\theta.\sin\theta} = \frac{-b.\sin\theta}{a.\cos\theta}$$

$\therefore$ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\ \theta=\frac{\pi}{2}}$

$$= \left(\frac{-b.\sin\theta}{a.\cos\theta}\right)_{at\ \theta=\frac{\pi}{2}}$$

$$= \frac{-b.\sin(\frac{\pi}{2})}{a.\cos(\frac{\pi}{2})}$$

$$m = \frac{-b(1)}{a(0)}$$

Slope of the normal $= \frac{-1}{m} = \frac{0}{b} = 0$

6. Find the slope of the normal to the curve $x = 1 - a\sin\theta,\ y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}$

SOLUTION:

Given curve is $x = 1 - a\sin\theta$, $y = b\cos^2\theta$

Take $x = 1 - a\sin\theta$

$$\frac{dx}{d\theta} = 0 - a\cos\theta = -a\cos\theta$$

Take $y = b\cos^2\theta$

$$\frac{dy}{d\theta} = 2b\cos\theta(-\sin\theta) = -2b\cos\theta\sin\theta$$

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-2b\cos\theta\sin\theta}{-a\cos\theta} = \frac{2b.\sin\theta}{a}$$

$\therefore$ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\ \theta=\frac{\pi}{2}}$

$$= \left(\frac{2b.\sin\theta}{a}\right)_{at\ \theta=\frac{\pi}{2}}$$

$$= \frac{2b.\sin(\frac{\pi}{2})}{a}$$

$$m = \frac{2b}{a}$$

Slope of the normal $= \frac{-1}{m} = \frac{-a}{2b}$

7. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the *x*-axis.

SOLUTION:

Given curve is $y = x^3 - 3x^2 - 9x + 7$ **.........(1)**

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

Slope of the line X-axis is $m_1 = 0$

Slope of the parallel X-axis is $m_2 = m_1 = 0 = m$ **(say)**

Let the point of contact of the tangent be $P(x_1,y_1)$

$\therefore$ Slope of the tangent is $m = \left(\frac{dy}{dx}\right)_{(x1,y1)} = 3x_1^2 - 6x_1 - 9$

$0 = 3x_1^2 - 6x_1 - 9$ **[$\because$ slope m = 0]**

$\div$ by 3, $x_1^2 - 2x_1 - 3 = 0$

$(x_1 + 1)(x_1 - 3) = 0$

$x_1 = -1$ and $x_1 = 3$

At $x_1 = -1$, the given equation (1) $\Rightarrow y_1 = x_1^3 - 3x_1^2 - 9x_1 + 7$

$\Rightarrow y_1 = (-1)^3 - 3(-1)^2 - 9(-1) + 7$

$= -1 - 3 + 9 + 7$

$= 14$

At $x_1 = 3$, the given equation (1) $\Rightarrow y_1 = x_1^3 - 3x_1^2 - 9x_1 + 7$

$\Rightarrow y_1 = (3)^3 - 3(3)^2 - 9(3) + 7$

$= 27 - 27 - 27 + 7$

$= -20$

∴ The required points are (−1, 14) and (3, −20)

8. Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

SOLUTION:

Given curve is $y = (x - 2)^2$ ....(1)

$$\frac{dy}{dx} = 2(x - 2)$$

Slope of the chord joining the points (2, 0) and (4, 4) is

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{4-2} = \frac{4}{2} = 2$$

Slope of the line parallel the chord is $m_2 = m_1 = 2 = m$ **(say)**

Let the point of contact of the tangent be $P(x_1, y_1)$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{(x1,y1)}$

$2 = 2(x_1 - 2)$ [∵ slope m = 2]

$1 = x_1 - 2$

$x_1 = 3$

At $x_1 = 3$, the given equation (1) $\Rightarrow y_1 = (x_1 - 2)^2$

$\Rightarrow y_1 = (3 - 2)^2$

$= 1$

∴ The required point is (3, 1)

9. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.

SOLUTION:

Given curve is $y = x^3 - 11x + 5$ ....(1)

$$\frac{dy}{dx} = 3x^2 - 11$$

Slope of the tangent **y = x – 11** is **m = 1**

Let the point of contact of the tangent be $P(x_1, y_1)$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{(x1,y1)}$

$1 = 3x_1^2 - 11$ **[∵ slope m = 1]**

$12 = 3x_1^2$

$x_1^2 = 4$

$x_1 = \pm 2$

At $x_1 = 2$, the given equation (1) $\Rightarrow y_1 = x_1^3 - 11x_1 + 5$

$\Rightarrow y_1 = 8 - 22 + 5$

$= -9$

At $x_1 = -2$, the given equation (1) $\Rightarrow y_1 = x_1^3 - 11x_1 + 5$

$\Rightarrow y_1 = -8 + 22 + 5$

$= 19$

∴ The required points are (2, – 9) and (–2, 19)

Example 17

Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

SOLUTION:

Given curve is $\frac{x^2}{4} + \frac{y^2}{25} = 1$ **………(1)**

Differentiate w.r.to x

$$\frac{2x}{4} + \frac{2y}{25} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{2x}{4} \times \frac{25}{2y} = \frac{-25x}{4y}$$

for (i) parallel to x-axis

Slope of the line X-axis is $\mathbf{m_1 = 0}$

Slope of the parallel X-axis is $\mathbf{m_2 = m_1 = 0 = m}$ **(say)**

Let the point of contact of the tangent be P(x_1,y_1)

∴ Slope of the tangent is **m**= $\left(\frac{dy}{dx}\right)_{(x1,y1)} = \frac{-25x_1}{4y_1}$

$0 = \frac{-25x_1}{4y_1}$ **[∵ slope m = 0]**

$x_1 = 0$

At $x_1 = 0$, the given equation (1) $\Rightarrow \frac{x_1^2}{4} + \frac{y_1^2}{25} = 1$

$\Rightarrow y_1^2 = 25$

$\Rightarrow y_1 = \pm 5$

∴ The required points are (0, −5) and (0, 5)

for (ii) parallel to x-axis

Slope of the line Y-axis is $\mathbf{m_1} = \frac{1}{0}$

Slope of the parallel Y-axis is $\mathbf{m_2} = \mathbf{m_1} = \frac{1}{0} = \mathbf{m}$ **(say)**

Let the point of contact of the tangent be P(x_1,y_1)

∴ Slope of the tangent is $\mathbf{m} = \left(\frac{dy}{dx}\right)_{(x1,y1)} = \frac{-25x_1}{4y_1}$

$$\frac{1}{0} = \frac{-25x_1}{4y_1} \qquad [\because \text{slope m} = 0]$$

$$4y_1 = 0$$

$$y_1 = 0$$

At $y_1 = 0$, the given equation (1) $\Rightarrow \frac{x_1^2}{9} + \frac{y_1^2}{16} = 1$

$$\Rightarrow \frac{x_1^2}{4} = 1$$

$$\Rightarrow x_1^2 = 4$$

$$\Rightarrow x_1 = \pm 2$$

∴ The required points are (−2, 0) and (2, 0)

EXERCISE 6.3

13. Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

SOLUTION:

Given curve is $\frac{x^2}{9} + \frac{y^2}{16} = 1$ **.........(1)**

Differentiate w.r.to x

$$\frac{2x}{9} + \frac{2y}{16} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{2x}{9} \times \frac{16}{2y} = \frac{-16x}{9y}$$

for (i) parallel to x-axis

Slope of the line X-axis is $\mathbf{m_1} = \mathbf{0}$

Slope of the parallel X-axis is $\mathbf{m_2} = \mathbf{m_1} = \mathbf{0} = \mathbf{m}$ **(say)**

Let the point of contact of the tangent be P(x_1,y_1)

∴ Slope of the tangent is $\mathbf{m} = \left(\frac{dy}{dx}\right)_{(x1,y1)} = \frac{-16x_1}{9y_1}$

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

$0 = \frac{-16x_1}{9y_1}$ **[∵ slope m = 0]**

$-16x_1 = 0$

$x_1 = 0$

At $x_1 = 0$, the given equation (1) $\Rightarrow \frac{x_1^2}{9} + \frac{y_1^2}{16} = 1$

$\Rightarrow \frac{y_1^2}{16} = 1$

$\Rightarrow y_1^2 = 16$

$\Rightarrow y_1 = \pm 4$

∴ The required points are (0, −4) and (0, 4)

for (ii) parallel to *x*-axis

Slope of the line Y-axis is $m_1 = \frac{1}{0}$

Slope of the parallel Y-axis is $m_2 = m_1 = \frac{1}{0} = m$ **(say)**

Let the point of contact of the tangent be $P(x_1, y_1)$

∴ Slope of the tangent is $m = \left(\frac{dy}{dx}\right)_{(x1,y1)} = \frac{-16x_1}{9y_1}$

$\frac{1}{0} = \frac{-16x_1}{9y_1}$ **[∵ slope m = 0]**

$9y_1 = 0$

$y_1 = 0$

At $y_1 = 0$, the given equation (1) $\Rightarrow \frac{x_1^2}{9} + \frac{y_1^2}{16} = 1$

$\Rightarrow \frac{x_1^2}{9} = 1$

$\Rightarrow x_1^2 = 9$

$\Rightarrow x_1 = \pm 3$

∴ The required points are (−3, 0) and (3, 0)

17. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

SOLUTION:

Given curve is $y = x^3$ ....(1)

$$\frac{dy}{dx} = 3x^2$$

Let the point of contact of the tangent be $P(x_1,y_1)$

Slope of the tangent = y_1 is $m = y_1$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{(x1,y1)}$

$y_1 = 3{x_1}^2$ .... (2) [∵ slope m = y_1]

At $P(x_1,y_1)$, the given equation (1) $\Rightarrow y_1 = {x_1}^3$

$\Rightarrow 3{x_1}^2 = {x_1}^3$

$\Rightarrow {x_1}^3 - 3{x_1}^2 = 0$

$\Rightarrow {x_1}^2 (x_1 - 3) = 0$

$\Rightarrow x_1 = 0\ ,\ x_1 = 3$

At $x_1 = 0$, (2)$\Rightarrow$ $y_1 = 0$

At $x_1 = 3$, (2)$\Rightarrow$ $y_1 = 27$

∴ The required points are (0, 0) and (3, 27)

Example 15

Find the point at which the tangent to the curve $y = \sqrt{4x - 3} - 1$ has its slope $\frac{2}{3}$.

SOLUTION:

Given curve is $y = \sqrt{4x - 3} - 1$ ....(1)

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4x-3}}(4) - 0$$

$$= \frac{2}{\sqrt{4x-3}}$$

Given Slope of the tangent **m** $= \frac{2}{3}$

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

Let the point of contact of the tangent be P(x_1,y_1)

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{(x1,y1)}$

$\frac{2}{3} = \frac{2}{\sqrt{4x_1 - 3}}$ [∵ slope m = $\frac{2}{3}$]

$3 = \sqrt{4x_1 - 3}$

$4x_1 - 3 = 9$

$4x_1 = 12$

$x_1 = 3$

At x_1= 3, the given equation (1) $\Rightarrow y_1 = \sqrt{4x_1 - 3} - 1$

$\Rightarrow y_1 = \sqrt{12 - 3} - 1$

$y_1 = 2$

Therefore, the required point is (3, 2).

Example 16

Find the equation of all lines having slope 2 and being tangent to the curve $y + \frac{2}{x-3} = 0$.

SOLUTION:

Given curve is $y + \frac{2}{x-3} = 0$

$y = \frac{-2}{x-3}$ .... (1)

$\frac{dy}{dx} = \frac{-2}{(x-3)^2}(-1) = \frac{2}{(x-3)^2}$

Given Slope of the tangent **m = 2**

Let the point of contact of the tangent be P(x_1,y_1)

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\ (x_1,y_1)}$

$2 = \frac{2}{(x_1-3)^2}$

$(x_1 - 3)^2 = 1$

$x_1 - 3 = \pm 1$

$x_1 = \pm 1 + 3$

$x_1 = 4 , 2$

At $x_1 = 2$, the given equation (1) $\Rightarrow y_1 = \frac{-2}{2-3} = 2$

At $x_1 = 4$, the given equation (1) $\Rightarrow y_1 = \frac{-2}{4-3} = -2$

Therefore, the required point is (2, 2) and (4, −2) .

The equation of the tangent at **$P(x_1, y_1)=(2, 2)$ with slope $m = 2$** is

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 2(x - 2)$$
$$y - 2 = 2x - 4$$
$$2x - y - 2 = 0$$

The equation of the tangent at **$P(x_1, y_1)=(4, -2)$ with slope $m = 2$** is

$$y - y_1 = m(x - x_1)$$
$$y + 2 = 2(x - 4)$$
$$y + 2 = 2x - 8$$
$$2x - y - 10 = 0$$

Example 18

Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

SOLUTION:

Given curve is $y = \frac{x-7}{(x-2)(x-3)}$ …. (1)

Note that on x-axis, y = 0.

(1)$\Longrightarrow$ x = 7.

∴ the curve cuts the x-axis at (7, 0).

(1)$\Longrightarrow$ $y = \frac{x-7}{x^2-5x+6}$

$$\frac{dy}{dx} = \frac{(x^2-5x+6)(1)-(x-7)(2x-5)}{(x^2-5x+6)^2}$$

$$\frac{dy}{dx} = \frac{(x^2-5x+6)-(x-7)(2x-5)}{(x^2-5x+6)^2}$$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\,(7,0)} = \frac{20}{20^2}$

$$m = \frac{1}{20}$$

The equation of the tangent at **$P(x_1, y_1)=(7, 0)$ with slope $m = \frac{1}{20}$** is

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{1}{20}(x - 7)$$
$$20y = x - 7$$
$$x - 20y - 7 = 0$$

Example 19

Find the equations of the tangent and normal to the curve $x^{2/3} + y^{2/3} = 2$ at (1, 1).

SOLUTION:

Given curve is $x^{2/3} + y^{2/3} = 2$ ....(1)

Differentiate with respect to x

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

÷ by $\frac{2}{3}$,

$$\frac{dy}{dx} = \frac{-x^{-1/3}}{y^{-1/3}} = \frac{-y^{1/3}}{x^{1/3}}$$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\,(1,1)} = -1$

m= – 1

The equation of the tangent at **$P(x_1, y_1)=(1, 1)$ with slope m = – 1** is

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -1\,(x - 1)$$
$$y - 1 = -1x + 1$$
$$x + y - 2 = 0$$

The equation of the Normal at **$P(x_1, y_1)=(1, 1)$ with slope m = – 1** is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$
$$y - 1 = \frac{-1}{-1}(x - 1)$$
$$y - 1 = x - 1$$
$$x - y = 0$$

 PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

XII MATHS- NCERT_CBSE
CHAPTER-6 APPLICATION OF DERIVATIVES

EXERCISE 6.3

14. Find the equations of the tangent and normal to the given curves at the indicated points:

(i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ **at (0, 5)**

(ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ **at (1, 3)**

(iii) $y = x^3$ **at (1, 1)**

(iv) $y = x^2$ **at (0, 0)**

(v) $x = \cos t,\ y = \sin t$ **at** $t = \frac{\pi}{4}$

SOLUTION:

for(i)

Given curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\ (0,5)}$

$= -10$

$$m = -10$$

The equation of the tangent at $P(x_1, y_1)=(0, 5)$ with slope $m = -10$ is

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -10(x - 0)$$

$$y - 5 = -10x$$

$$10x + y - 5 = 0$$

The equation of the Normal at $P(x_1, y_1)=(0, 5)$ with slope $m = -10$ is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 5 = \frac{-1}{-10}(x - 0)$$

$$10(y - 5) = x$$

$$10y - 50 = x$$

$$x - 10y + 50 = 0$$

for(ii)

Given curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\,(1,3)} = 4 - 18 + 26 - 10$

m= 2

The equation of the tangent at **P(x_1, y_1)=(1, 3) with slope m = 2** is

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 2(x - 1)$$

$$y - 3 = 2x - 2$$

$$2x - y + 1 = 0$$

The equation of the Normal at **P(x_1, y_1)=(1, 3) with slope m = 2** is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 3 = \frac{-1}{2}(x - 1)$$

$$2(y - 3) = -1(x - 1)$$

$$2y - 6 = -x + 1$$

$$x + 2y - 7 = 0$$

for(iii)

Given curve is $y = x^3$

$$\frac{dy}{dx} = 3x^2$$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\,(1,1)} = 3$

m= 3

The equation of the tangent at **P(x_1, y_1)=(1, 1) with slope m = 3** is

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 3(x - 1)$$

$$y - 1 = 3x - 3$$

$$3x - y - 2 = 0$$

The equation of the Normal at **P(x_1, y_1)=(1, 1) with slope m = 3** is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 1 = \frac{-1}{3}(x - 1)$$

$3(y - 1) = -1(x - 1)$

$3y - 3 = -x + 1$

$\mathbf{x + 3y - 4 = 0}$

for(iv)

Given curve is $y = x^2$

$\frac{dy}{dx} = 2x$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\,(0,\ 0)} = 0$

m= 0

The equation of the tangent at $P(x_1, y_1)=(0, 0)$ **with slope m = 0** is

$y - y_1 = m(x - x_1)$

$y - 0 = 0(x - 0)$

$y = 0$

(i.e) The tangent to the curve is X-axis at (0, 0)

The equation of the Normal at $P(x_1, y_1)=(0, 0)$ **with slope m = 0** is

$y - y_1 = \frac{-1}{m}(x - x_1)$

$y - 0 = \frac{-1}{0}(x - 0)$

$0 = -x$

$x = 0$

(i.e) The Normal to the curve is Y-axis at (0, 0)

for(v)

Given curve is $x = \cos t, y = \sin t$ at $t = \frac{\pi}{4}$

Take $x = \cos t$

$\frac{dx}{dt} = -\sin t$

Take $y = \sin t$

$\frac{dy}{dt} = \cos t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t}$$

$\therefore$ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{at\ t=\frac{\pi}{4}} = \left(\frac{\cos t}{-\sin t}\right)_{at\ t=\frac{\pi}{4}} = -1$

$$\mathbf{m = -1}$$

At $t = \frac{\pi}{4}$, $\quad x = \cos t = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$,

$\quad y = \sin t = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

The equation of the tangent at $P(x_1, y_1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ **with slope** $m = -1$ is

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{\sqrt{2}} = -1\left(x - \frac{1}{\sqrt{2}}\right)$$

$$y - \frac{1}{\sqrt{2}} = -x + \frac{1}{\sqrt{2}}$$

$$x + y - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$$

$$x + y - \frac{2}{\sqrt{2}} = 0$$

$$x + y - \sqrt{2} = 0$$

The equation of the Normal at $P(x_1, y_1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ **with slope** $m = -1$ is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - \frac{1}{\sqrt{2}} = \frac{-1}{-1}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$y - \frac{1}{\sqrt{2}} = x - \frac{1}{\sqrt{2}}$$

$$x - y - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = 0$$

$$x - y = 0$$

XII MATHS- NCERT_CBSE

CHAPTER-6 APPLICATION OF DERIVATIVES

15. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(a) parallel to the line $2x - y + 9 = 0$

(b) perpendicular to the line $5y - 15x = 13$.

SOLUTION:

for(a)

Given curve is $y = x^2 - 2x + 7$ …. (1)

$\frac{dy}{dx} = 2x - 2$

Slope of the given line $2x - y + 9 = 0$ is $m_1 = 2$

∴Slope of the line parallel to it is $m_2 = m_1 = 2 = m$ (say)

Let the point of contact of the tangent be $P(x_1,y_1)$

∴ Slope of the tangent is $m = \left(\frac{dy}{dx}\right)_{at\ (x1,y1)}$

$2 = 2x_1 - 2$

$4 = 2x_1$

$x1 = 2$

∴ At $P(x_1,y_1)$, (1)⟹ $y1 = 4 - 4 + 7$

$y1 = 7$

∴ the point of contact of the tangent be $P(x_1,y_1) = (2, 7)$

The equation of the tangent at $P(x_1, y_1) = (2, 7)$ **with slope $m = 2$** is

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 2(x - 2)$$

$$y - 7 = 2x - 4$$

$$2x - y + 3 = 0$$

for(b)

Given curve is $y = x^2 - 2x + 7$ …. (1)

$\frac{dy}{dx} = 2x - 2$

Slope of the given line $5y - 15x = 13$⟹ is $m_1 = 3$

∴Slope of the line perpendicular to it is $m_2 = \frac{-1}{m_1} = \frac{-1}{3} = m$ (say)

Let the point of contact of the tangent be $P(x_1,y_1)$

∴ Slope of the tangent is $m = \left(\frac{dy}{dx}\right)_{at\ (x1,y1)}$

$\frac{-1}{3} = 2x_1 - 2$

$-1 = 6x_1 - 6$

$x_1 = \frac{5}{6}$

∴ At $P(x_1,y_1)$, (1)⟹ $y_1 = \frac{25}{36} - \frac{5}{3} + 7 = \frac{25-60+252}{36}$

$y_1 = \frac{217}{36}$

∴ the point of contact of the tangent be $P(x_1,y_1) = (\frac{5}{6}, \frac{217}{36})$

The equation of the tangent at $P(x_1, y_1) = (\frac{5}{6}, \frac{217}{36})$ **with slope** $m = \frac{-1}{3}$ is

$$y - y_1 = m(x - x_1)$$

$$y - \frac{217}{36} = \frac{-1}{3}(x - \frac{5}{6})$$

$$36y - 217 = -12x + 10$$

$$12x + 36y + 227 = 0$$

21. Find the equation of the normal to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

SOLUTION:

Given curve is $y = x^3 + 2x + 6$ .... (1)

$\frac{dy}{dx} = 3x^2 + 2$

Slope of the given line $x + 14y + 4 = 0$ is $m_1 = \frac{-1}{14}$

∴Slope of the normal which is parallel to it is $m_2 = m_1 = \frac{-1}{14}$ (say)

∴Slope of the tangent is $m = 14$

Let the point of contact of the tangent be $P(x_1,y_1)$

∴ Slope of the tangent is $m = \left(\frac{dy}{dx}\right)_{at\ (x1,y1)}$

$14 = 3x_1^2 + 2$

$3x_1^2 = 14 - 2 = 12$

$x_1^2 = 4$

$x_1 = \pm 2$

∴ At $P(x_1,y_1)$, (1)⟹ $y_1 = x_1^3 + 2x_1 + 6$

If $x_1 = 2$ then $y_1 = 8 + 4 + 6 = 18$

$y_1 = 18$

∴ the point of contact of the tangent be $P(x_1,y_1) = (2, 18)$

The equation of the normal at $P(x_1, y_1) = (2, 18)$ **with slope** $m = \frac{-1}{14}$ is

$$y - y_1 = m(x - x_1)$$

$$y - 18 = \frac{-1}{14}(x - 2)$$

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

$14y - 252 = -x + 2$

$$x + 14y - 254 = 0$$

If $x_1 = -2$ **then** $y_1 = -8 - 4 + 6 = -6$

$$y_1 = -6$$

∴ **the point of contact of the tangent be** $P(x_1,y_1) = (-2, -6)$

The equation of the normal at $P(x_1, y_1) = (-2, -6)$ **with slope** $m = \frac{-1}{14}$ is

$$y - y_1 = m(x - x_1)$$

$$y + 6 = \frac{-1}{14}(x + 2)$$

$$14y + 84 = -x - 2$$

$$x + 14y + 86 = 0$$

25. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ **which is parallel to the line** $4x - 2y + 5 = 0$ **.**

SOLUTION:

Given curve is $y = \sqrt{3x - 2}$ ....(1)

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{3x-2}}(3) = \frac{-3}{2\sqrt{3x-2}}$$

Slope of the line $4x - 2y + 5 = 0$ is $m_1 = 2$

Slope of the parallel line(tangent) is $m_2 = m_1 = 2 = m$ (say)

Let the point of contact of the tangent be $P(x_1,y_1)$

∴ Slope of the tangent is $m = \left(\frac{dy}{dx}\right)_{(x1,y1)} = \frac{-3}{2\sqrt{3x_1 - 2}}$

$$2 = \frac{-3}{2\sqrt{3x_1 - 2}} \quad [\because \text{slope } m = 2]$$

$$4\sqrt{3x_1 - 2} = -3$$

Taking squaring on both side

$$16(3x_1 - 2) = 9$$

$$48x_1 - 32 = 9$$

$$48x_1 = 41$$

$$x_1 = \frac{41}{48}$$

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

At $x_1 = \frac{41}{48}$, the given equation (1)⇒ $y_1 = \sqrt{3x_1 - 2}$

$= \sqrt{3\left(\frac{41}{48}\right) - 2}$

$= \sqrt{\frac{41}{16} - 2}$

$= \sqrt{\frac{9}{16}}$

$y_1 = \frac{3}{4}$

∴ the point of contact of the tangent be $P(x_1,y_1) = \left(\frac{41}{48}, \frac{3}{4}\right)$.

The equation of the tangent at $P(x_1, y_1)= \left(\frac{41}{48}, \frac{3}{4}\right)$ with slope m= 2 is

$y - y_1 = m(x - x_1)$

$y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$

$y - \frac{3}{4} = 2x - \frac{41}{24}$

Multiply by 24, $24y - 18 = 48x - 41$

$48x - 24y - 23=0$

20. Find the equation of the normal at the point (am^2,am^3) for the curve $ay^2 = x^3$.

SOLUTION:

Given curve is $ay^2 = x^3$ .... (1)

$2ay\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{2ay}$

∴ Slope of the tangent is m1 $= \left(\frac{dy}{dx}\right)_{(am^2,am^3)}$

$= \left(\frac{3x^2}{2ay}\right)_{(am^2,am^3)}$

$= \frac{3(am^2)^2}{2a(am^3)}$

$= \frac{3a^2m^4}{2a^2m^3}$

$= \frac{3m}{2}$

∴ **Slope of the normal is** $m_2 = \frac{-1}{\frac{3m}{2}} = \frac{-2}{3m} = M(say)$

The equation of the normal at **P(x_1, y_1)=** (am^2, am^3) **with slope** $M = \frac{-2}{3m}$ is

$$y - y_1 = M(x - x_1)$$

$$y - am^3 = \frac{-2}{3m}(x - am^2)$$

$$3my - 3am^4 = -2x + 2am^2$$

$$2x + 3my - 3am^4 - 2am^2 = 0$$

$$2x + 3my - am^2(3m^2 + 2) = 0$$

22. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

SOLUTION:

Given curve is $y^2 = 4ax$ .... (1)

$$2y\frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

∴ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{(at^2,2at)}$

$$= \left(\frac{2a}{y}\right)_{(at^2,2at)}$$

$$= \frac{2a}{2at}$$

$$= \frac{1}{t}$$

The equation of the tangent at **P(x_1, y_1)=** $(at^2, 2at)$ **with slope** $m = \frac{1}{t}$ is

$$y - y_1 = m(x - x_1)$$

$$y - 2at = \frac{1}{t}(x - at^2)$$

$$yt - 2at^2 = x - at^2$$

$$yt = x - at^2 + 2at^2$$

$$yt = x + at^2$$

The equation of the normal at **P(x_1, y_1)=** $(at^2, 2at)$ **with slope** $m = \frac{1}{t}$ is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 2at = -t(x - at^2)$$

$$y - 2at = -tx + at^3$$

$$y + tx = 2at + at^3$$

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

XII MATHS- NCERT_CBSE

CHAPTER-6 APPLICATION OF DERIVATIVES

23. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

SOLUTION:

Given curve is $x = y^2$ …. (1)

$xy = k$ …. (2)

To find the point of intersection of the curves (1) & (2)

Put $x = y^2$ in (2)

(2) $\Rightarrow y^3 = k$

$\Rightarrow y = k^{1/3}$

(1) $\Rightarrow x = k^{2/3}$

$\therefore$ **the point of intersection is $P(x_1, y_1) = (k^{2/3}, k^{1/3})$**

Differentiate (1) with respect to x, we have

$1 = 2y\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2y}$

$\therefore$ **Slope of the curve (1) is m1** $= \left(\frac{dy}{dx}\right)_{(k^{2/3}, k^{1/3})}$

$= \left(\frac{1}{2y}\right)_{(k^{2/3}, k^{1/3})}$

$m1 = \frac{1}{2k^{1/3}}$

Differentiate (2) with respect to x, we have

$x\frac{dy}{dx} + y = 0$

$\frac{dy}{dx} = \frac{-y}{x}$

$\therefore$ **Slope of the curve (2) is m2** $= \left(\frac{dy}{dx}\right)_{(k^{2/3}, k^{1/3})}$

$= \left(\frac{-y}{x}\right)_{(k^{2/3}, k^{1/3})}$

$m2 = \frac{-k^{1/3}}{k^{2/3}}$

$m2 = \frac{-1}{k^{1/3}}$

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

$\therefore$ **m1 × m2** $= \frac{1}{2k^{1/3}} \times \frac{-1}{k^{1/3}} = \frac{-1}{2k^{2/3}} = \frac{-1}{2(k^2)^{1/3}}$

$= \frac{-1}{2\left(\frac{1}{8}\right)^{1/3}}$ **if $8k^2 = 1$**

$= \frac{-1}{2\left(\frac{1}{2}\right)}$

$$m1 \times m2 = -1$$

$\therefore$ **Given to curves cut at right angle (orthogonally).**

24. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

SOLUTION:

Given curve is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ **………(1)**

Differentiate with respect to x

$$\frac{2x}{a^2} - \frac{2y}{b^2} \times \frac{dy}{dx} = 0$$

$$\frac{2x}{a^2} = \frac{2y}{b^2} \times \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{xb^2}{ya^2}$$

$\therefore$ Slope of the tangent is **m** $= \left(\frac{dy}{dx}\right)_{(x_0,y_0)} = \left(\frac{xb^2}{ya^2}\right)_{(x_0,y_0)} = \frac{x_0 b^2}{y_0 a^2}$

The equation of the tangent at **$P(x_1, y_1) = (x_0, y_0)$ with slope $m = \frac{x_0 b^2}{y_0 a^2}$** is

$$y - y_1 = m(x - x_1)$$

$$y - y_0 = \frac{x_0 b^2}{y_0 a^2}(x - x_0)$$

$$yy_0 a^2 - {y_0}^2 a^2 = xx_0 b^2 - {x_0}^2 b^2$$

$$xx_0 b^2 - yy_0 a^2 = {x_0}^2 b^2 - {y_0}^2 a^2$$

$\div$ **by a^2b^2, we have**

$$\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = \frac{{x_0}^2}{a^2} - \frac{{y_0}^2}{b^2}$$

$$\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1 \quad [\because \text{by } (1)$$

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010

XII MATHS- NCERT_CBSE
CHAPTER-6 APPLICATION OF DERIVATIVES

The equation of the normal at **P(x_1, y_1)=** (x_0, y_0) **with slope m=** $\frac{x_0 b^2}{y_0 a^2}$ is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - y_0 = \frac{-y_0 a^2}{x_0 b^2}(x - x_0)$$

$$y x_0 b^2 - x_0 y_0 b^2 = -x y_0 a^2 + x_0 y_0 a^2$$

$$x y_0 a^2 + y x_0 b^2 = x_0 y_0 a^2 + x_0 y_0 b^2$$

÷ by $x_0 y_0$, we have

$$\frac{x a^2}{x_0} + \frac{y b^2}{y_0} = a^2 + b^2$$

^^^^^^^^^^^^^^^^^

PREPARED BY S. SARAVANAN, M.Sc., B.Ed., PGDCA,
DISTRICT COORDINATOR (SS) DPO, RAMANATHAPURAM, CELL:9787619010