Chapter 6

Application of Derivatives

Extrema using Second Derivative Test To Find the Maximum and Minimum Values of the Given Function

Theorem 4 (The Second Derivative Test)

Suppose that c is **a critical point** at which $f'(c) = 0$, that $f'(x)$ exists in a neighborhood of c, and that $f''(c)$ exists. Then **f(x)** has

(i) a relative **maximum value** at c **if** $f''(c) < 0$ and

(ii) a relative **minimum value** at c **if** $f''(c) > 0$.

(iii) **If $f''(c) = 0$, the test is not informative.**

Example 32

Find the local maximum and local minimum of the function

$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

SOLUTION:

Given function is $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$ ....(1)

$f'(x) = 12x^3 + 12x^2 - 24x$

$= 12x(x^2 + x - 2)$

$= 12x(x + 2)(x - 1)$

$f''(x) = 36x^2 + 24x - 24$

$= 12(3x^2 + 2x - 2)$ ....(2)

To Find the Critical points at which $f'(x)=0$

(i.e) $12x(x + 2)(x - 1) = 0$

$\Rightarrow x = 0, x = -2$ and $x = 1$ **are critical points**

If $x = -2$ then $f''(-2) = 12(12 - 4 - 2) = 72 > 0$ [∵by (2)]

∴By second Derivative Test,

the function f has minimum value at $x = -2$

∴ The minimum value of the function is

(1)⇒ $f(-2) = 3(-2)^4 + 4(-2)^3 - 12(-2)^2 + 12$

$= 3(16) + 4(-8) - 12(4) + 12$

$= 48 - 32 - 48 + 12$

∴ $f(-2) = -20$

If $x = 0$ then $f''(0) = 12(-2) = -24 < 0$ [∵by (2)]

∴By second Derivative Test,

the function f has maximum value at $x = 0$

∴ The maximum value of the function at $x = 0$ is

(1)⇒ $f(0) = 12$

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If x = 1 then $f''(1) = 12(3 + 2 – 2) = 36 > 0$ [∵by (2)]

∴By second Derivative Test,

the function f has minimum value at x = 1

∴ The minimum value of the function is

(1)⇒ $f(1) = 3(1)^4 + 4(1)^3 – 12(1)^2 + 12$

$= 3 + 4 – 12 + 12$

∴ $f(1) = 7$

Example 33

Find all the points of the local maxima and local minima of the function $f(x) = 2x^3 − 6x^2 + 6x + 5$

SOLUTION:

Given function is $f(x) = 2x^3 − 6x^2 + 6x + 5$ ……….(1)

$f'(x) = 6x^2 – 12x + 6$

$= 6(x^2 – 2x + 1)$

$= 6(x – 1)(x –1)$

$f''(x) = 12x – 12$

$=12(x – 1)$ ………….(2)

To Find the Critical points at which $f'(x)=0$

(i.e) $6(x – 1)(x –1) = 0$

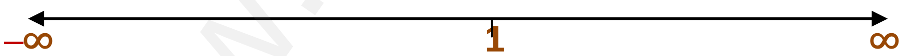
⇒ x = 1 is a critical point

If x = 1 then $f''(1) = 0$ [∵by (2)]

∴By second Derivative Test,

If $f''(c) = 0$, the test is not informative.

Using First Derivative Test,



We have the intervals (–∞, 1) and (1,∞)

(i)If x∈ (–∞, 1), suppose x = –1 then $f'(–1) = 6(–1 – 1)(–1 –1)$

$= 6(–2)(–2)$

$= 24>0$

∴ If x∈ (–∞, 1) then $f'(x)≥0$

∴ f is strictly increasing function on the intervals (–∞, 1).

(ii)If x∈(1,∞), suppose x =2 then $f'(2) = 6(2 – 1)(2 –1)$

$= 6(1)(1)$

$= 6 >0$

∴ If x∈ (1,∞) then $f'(x)≥0$

∴ f is strictly increasing function on the intervals (1,∞).

From (i) and (ii),
If $f'(x)$ is positive on both sides (no change) at $x = 1$, then
By First Derivative Test
f(x) neither a local minimum nor a local maximum

EXERCISE 6.5

3. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:

(i) $f(x) = x^2$

SOLUTION:

Given function is $f(x) = x^2$ ....(1)

$f'(x) = 2x$

$f''(x) = 2$ ....(2)

To Find the Critical points at which $f'(x)=0$

(i.e) $2x = 0$

$\Rightarrow$ $x = 0$ is a critical point

If $x = 0$ then $f''(0) = 2>0$ [$\because$by (2)

$\therefore$By second Derivative Test,

the function f has minimum value at $x = 0$

$\therefore$ The minimum value of the function at $x=0$ is

(1) $\Rightarrow f(0) = 0$

(ii) $g(x) = x^3 - 3x$

SOLUTION:

Given function is $g(x) = x^3 - 3x$ ....(1)

$g'(x) = 3x^2 - 3$

$= 3(x^2 - 1)$

$= 3(x + 1)(x - 1)$

$g''(x) = 6x$ ....(2)

To Find the Critical points at which $g'(x)=0$

(i.e) $3(x + 1)(x - 1) = 0$

$\Rightarrow$ $x = -1$ and $x = 1$ are critical points

If $x = -1$ then $g''(-2) = 6(-1) = -6 < 0$ [$\because$by (2)

$\therefore$By second Derivative Test,

the function g(x) has maximum value at $x = -1$

$\therefore$ The maximum value of the function is

(1)$\Rightarrow g(-1) = (-1)^3 - 3(-1)$

$\therefore g(-1) = 1 + 3 = 4$

If x = 1 then $g''(1) = 6(1) = 6 > 0$ [∵by (2)

∴By second Derivative Test,

the function g(x) has minimum value at x = 1

∴ The minimum value of the function at x = 1 is

(1)⇒ $g(1)= (1)^3 - 3(1)$

∴ $g(1) = 1 - 3 = -2$

(iii) $h(x) = \sin x + \cos x$, $0 < x < \frac{\pi}{2}$

SOLUTION:

Given function is $h(x) = \sin x + \cos x$ ……….(1)

$h'(x) = \cos x - \sin x$

$h''(x) = -\sin x - \cos x$

$= -(\sin x + \cos x)$ ………….(2)

To Find the Critical points at which $h'(x)=0$

(i.e) $\cos x - \sin x = 0$

$\cos x = \sin x$, $0 < x < \frac{\pi}{2}$

⇒ $x = \frac{\pi}{4}$ is a critical point

If $x = \frac{\pi}{4}$ then $h''(\frac{\pi}{4}) = -(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}) = -(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}})$

$= -\frac{2}{\sqrt{2}} = -\sqrt{2} < 0$ [∵by (2)

∴By second Derivative Test,

the function h(x) has maximum value at $x = \frac{\pi}{4}$

∴ The maximum value of the function is

(1)⇒ $h(\frac{\pi}{4}) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

(iv) $f(x) = \sin x - \cos x$, $0 < x < 2\pi$

SOLUTION:

Given function is $f(x) = \sin x - \cos x$ ……….(1)

$f'(x) = \cos x + \sin x$

$f''(x) = -\sin x + \cos x$

$= \cos x - \sin x$ ………….(2)

To Find the Critical points at which $f'(x)=0$

(i.e) $\cos x + \sin x = 0$

$\cos x = -\sin x$

$\cos x = \cos(\frac{\pi}{2} + x)$, $0 < x < 2\pi$

[∵If $\cos x = \cos\alpha$ then $x = 2n\pi \pm \alpha$ where $n \in Z$ with $\alpha \in [0, \pi]$

& if n=0,1,2

⇒ $x = -(\frac{\pi}{2} + x)$, $x = 2\pi - (\frac{\pi}{2} + x)$ & $x = 4\pi - (\frac{\pi}{2} + x)$

$\Rightarrow x = -\frac{\pi}{2} - x, \quad x = 2\pi - \frac{\pi}{2} - x \;\&\; x = 4\pi - \frac{\pi}{2} - x$

$\Rightarrow 2x = -\frac{\pi}{2}, \quad 2x = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2} \;\&\; 2x = 4\pi - \frac{\pi}{2} = \frac{7\pi}{2}$

$\Rightarrow x = -\frac{\pi}{4}, \quad x = \frac{3\pi}{4} \;\&\; x = \frac{7\pi}{4}$

$\Rightarrow x = \frac{3\pi}{4} \;\&\; x = \frac{7\pi}{4}$, $0 < x < 2\pi$ are critical points

If $x = \frac{3\pi}{4}$ then $f''(\frac{3\pi}{4}) = \cos\frac{3\pi}{4} - \sin\frac{3\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$= -\frac{2}{\sqrt{2}} = -\sqrt{2} < 0$ [∵by (2)

∴By second Derivative Test,

the function f(x) has maximum value at $x = \frac{3\pi}{4}$

∴ The maximum value of the function is

$(1) \Rightarrow f(\frac{3\pi}{4}) = \sin\frac{3\pi}{4} - \cos\frac{3\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

If $x = \frac{7\pi}{4}$ then $f''(\frac{7\pi}{4}) = \cos\frac{7\pi}{4} - \sin\frac{7\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} = \sqrt{2} > 0$ [∵by (2)

∴By second Derivative Test,

the function f(x) has minimum value at $x = \frac{7\pi}{4}$

∴ The minimum value of the function is

$(1) \Rightarrow f(\frac{7\pi}{4}) = \sin\frac{7\pi}{4} - \cos\frac{7\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

(v) $f(x) = x^3 - 6x^2 + 9x + 15$

SOLUTION:

Given function is $f(x) = x^3 - 6x^2 + 9x + 15$ ……….(1)

$f'(x) = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x - 1)(x - 3)$

$f''(x) = 6x - 12$

$= 6(x - 2)$ ………….(2)

To Find the Critical points at which $f'(x) = 0$

(i.e) $3(x - 1)(x - 3) = 0$

$\Rightarrow$ x = 1 and x = 3 are critical points

If x = 1 then $f''(1) = 6(1 - 2) = -6 < 0$ [∵by (2)

∴By second Derivative Test,

the function f has maximum value at x = 1

∴ The maximum value of the function at x = 1 is

$(1) \Rightarrow f(1) = (1)^3 - 6(1)^2 + 9(1) + 15 = 1 - 6 + 9 + 15 = 19$

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If x = 3 then $f''(3) = 6(3-2) = 6 > 0$ [∵by (2)]

∴By second Derivative Test,

the function f has minimum value at x = 3

∴ The minimum value of the function at x = 3 is

(1)⇒ $f(3) = (3)^3 - 6(3)^2 + 9(3) + 15$

$= (27) - 6(9) + 9(3) + 15$

$= 27 - 54 + 27 + 15$

∴ f(3)= 15

(vi) $g(x) = \frac{x}{2} + \frac{2}{x}$, x>0

SOLUTION:

$g(x) = \frac{x}{2} + \frac{2}{x}$ ……(1)

$g'(x) = \frac{1}{2} - \frac{2}{x^2}$

$g''(x) = \frac{4}{x^3}$ ……(2)

To Find the Critical points at which $g'(x)=0$

(i.e) $\frac{1}{2} - \frac{2}{x^2} = 0$

$\frac{2}{x^2} = \frac{1}{2}$

$x^2 = 4$

⇒ x = – 2 and x = 2

⇒ x = 2 is a critical point [∵x>0

If x = 2 then $g''(x) = \frac{4}{(2)^3} = \frac{4}{8} = \frac{1}{2} > 0$ [∵by (2)

∴By second Derivative Test,

the function g(x) has minimum value at x = 2

∴ The maximum value of the function at x = 2 is

(1)⇒ $g(2) = \frac{2}{2} + \frac{2}{2} = 1 + 1 = 2$

(vii) $g(x) = \frac{1}{x^2+2}$

SOLUTION:

$g(x) = \frac{1}{x^2+2}$ ……(1)

$g'(x) = \frac{-2x}{(x^2+2)^2}$

$g''(x) = \frac{(x^2+2)(-2)-(-2x)(2x)}{(x^2+2)^4}$

$= \frac{-2x^2-4+4x^2}{(x^2+2)^4}$

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$$= \frac{2x^2-4}{(x^2+2)^4}$$

$$g''(x) = \frac{2(x^2-2)}{(x^2+2)^4} \quad ......(2)$$

To Find the Critical points at which $g'(x)=0$

(i.e) $\frac{-2x}{(x^2+2)^2} = 0$

$-2x = 0$

$\Rightarrow$ $x = 0$ is a critical point

If $x = 0$ then $g''(0) = \frac{2(-2)}{(2)^4} = \frac{-4}{16} = \frac{-1}{4} < 0$ [∵by (2)

∴By second Derivative Test,

the function g(x) has maximum value at $x = 0$

∴ The maximum value of the function at $x = 0$ is

(1)⇒ $g(0)= \frac{1}{0+2} = \frac{1}{2}$

(viii) $f(x) = x\sqrt{1-x}$, $0 < x < 1$

SOLUTION:

$$f(x) = x\sqrt{1-x}$$

$$f'(x) = x.\frac{1}{2\sqrt{1-x}}(-1) + \sqrt{1-x}\,(1)$$

$$= \frac{-x}{2\sqrt{1-x}} + \sqrt{1-x}$$

$$= \frac{-x+2(1-x)}{2\sqrt{1-x}}$$

$$= \frac{2-3x}{2\sqrt{1-x}}$$

$$f''(x) = \frac{(2\sqrt{1-x})(-3)-(2-3x).\left[\frac{-1}{2\sqrt{1-x}}\right]}{(2\sqrt{1-x})^2}$$

$$= \frac{(-6\sqrt{1-x})+\frac{2-3x}{2\sqrt{1-x}}}{4(1-x)}$$

$$= \frac{\frac{-12(1-x)+(2-3x)}{2\sqrt{1-x}}}{4(1-x)}$$

$$= \frac{-12+12x+2-3x}{8(1-x)\sqrt{1-x}}$$

$$= \frac{9x-10}{8(1-x)^{3/2}}$$

To Find the Critical points at which $f'(x)=0$

(i.e) $\frac{2-3x}{2\sqrt{1-x}} = 0$

$2-3x = 0$

$\Rightarrow$ $x = \frac{2}{3}$ is a critical point

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If $x = \frac{2}{3}$ then $f''(0) = \frac{9\left(\frac{2}{3}\right)-10}{8\left(1-\frac{2}{3}\right)^{3/2}} = \frac{6-10}{8\left(\frac{1}{3}\right)^{3/2}} = \frac{-4}{8\left(\frac{1}{3}\right)^{3/2}} < 0$ [∵by (2)

∴By second Derivative Test,

the function f(x) has maximum value at $x = \frac{2}{3}$

∴ The maximum value of the function at $x = \frac{2}{3}$ is

$(1) \Rightarrow f\left(\frac{2}{3}\right) = \frac{2}{3} \cdot \sqrt{1 - \frac{2}{3}} = \frac{2}{3} \cdot \sqrt{\frac{1}{3}} = \frac{2}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{9}$

Chapter 6

Application of Derivatives

6.6.Extrema using First Derivative Test

Theorem-3 (First Derivative Test)

Let $(c, f(c))$ be a critical point of function $f(x)$ that is continuous on an open interval I containing c . If $f(x)$ is differentiable on the interval, then

(i) If $f'(x)$ changes from negative to positive at c , then $f(x)$ has a local minimum $f(c)$.

(ii) If $f'(x)$ changes from positive to negative at c , then $f(x)$ has a local maximum $f(c)$.

(iii) If $f'(x)$ is positive on both sides of c or negative on both sides of c then $f(x)$ is neither a local minimum nor a local maximum.

NOTE-1:

(i) (c, f (c)) be a critical point of function f (x) at x=c, for $f'(x)=0$ or $f'(x)=\infty$

(ii) (c, f (c)) be a stationary point of function f (x) at x=c, only for $f'(x)=0$

NOTE-2:

(i) If sinx=0 then $x = n\pi$, $n\epsilon Z$ (i.e) $x = 0, \pm\pi, \pm2\pi, \pm3, \ldots$

(ii) If cosx=0 then $x = (2n+1)\frac{\pi}{2}$, $n\epsilon Z$ (i.e) $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} \ldots$

(iii) If tanx=0 then $x = n\pi$, $n\epsilon Z$ (i.e) $x = 0, \pm\pi, \pm2\pi, \pm3, \ldots$

(iv) If sinx= sinα then $x= n\pi + (-1)^n.\alpha$, where $n\epsilon Z$ with $\alpha\epsilon \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(v) If cosx = cosα then $x= 2n\pi \pm \alpha$ where $n\epsilon Z$ with $\alpha\epsilon [0, \pi]$

(vi) If tanx = tanα then $x = n\pi + \alpha$ where $n\epsilon Z$ with $\alpha\epsilon\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Example 29

Find all points of local maxima and local minima of the function *f* given by $f(x) = x^3 - 3x + 3$.

SOLUTION:

Given function is $f(x) = x^3 - 3x + 3$ ....(1)

$f'(x) = 3x^2 - 3$

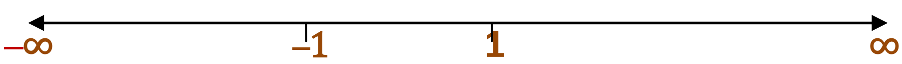
$= 3(x^2 - 1)$

$= 3(x + 1)(x - 1)$

To Find the Critical points at which $f'(x)=0$

$\Rightarrow 3(x + 1)(x - 1)=0$

$\Rightarrow x = -1$ and $x = 1$ are critical points



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We have the intervals $(-\infty,-1)$, $(-1,1)$ and $(1,\infty)$

(i)If $x\epsilon(-\infty,-1)$, suppose $x = -3$ then $f'(-3) = 6(-3 + 2)(-3 - 1)$

$= 6(-1)(-4)$

$= 24>0$

$\therefore$ If $x\epsilon(-\infty,-1)$ then $f'(x)\geq 0$

$\therefore$ f is strictly increasing function on the intervals $(-\infty,-1)$.

(ii)If $x\epsilon(-1,1)$, suppose $x = 0$ then $f'(0) = 6(0 + 2)(0 - 1)$

$= 6(2)(-1)$

$= -12<0$

$\therefore$ If $x\epsilon(-1,1)$ then $f'(x)\leq 0$

$\therefore$ f is strictly decreasing function on the intervals $(-1,1)$.

(iii)If $x\in(1,\infty)$, suppose $x = 2$ then $f'(2) = 6(2 + 2)(2 - 1)$

$= 6(4)(1)$

$= 24>0$

$\therefore$ If $x\in(1,\infty)$ then $f'(x)\geq 0$

$\therefore$ f is strictly increasing function on the intervals $(1,\infty)$.

From (i) and (ii),

If $f'(x)$ changes from positive to negative at $x = -1$, then

By First Derivative Test

At $x= -1$, $f(x)$ has a local maximum $f(-1)$

$f(-1) = (-1)^3 - 3(-1) + 3$ [∵By (1)]

$= (-1) + 3 + 3$

$= 5$

From (ii) and (iii),

If $f'(x)$ changes from negative to positive at $x = 1$, then

By First Derivative Test

At $x= 1$, $f(x)$ has a local minimum $f(1)$

$f(1) = (1)^3 - 3(1) + 3$ [∵By (1)]

$= 1 - 3 + 3$

$= 1$

Example 30

Find all the points of local maxima and local minima of the function *f* given by $f(x) = 2x^3 - 6x^2 + 6x + 5$

SOLUTION:

Given function is $f(x) = 2x^3 - 6x^2 + 6x + 5$ ……….(1)

$f'(x) = 6x^2 - 12x + 6$

$= 6(x^2 - 2x + 1)$

$= 6(x - 1)(x - 1)$

To Find the Critical points at which $f'(x)=0$

$\Rightarrow 6(x - 1)(x - 1) = 0$

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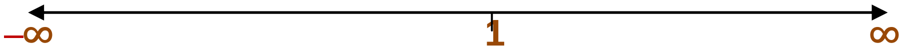
$\Rightarrow x = 1$ twice are critical points



We have the intervals $(-\infty, 1)$ and $(1,\infty)$

(i)If $x\epsilon\ (-\infty, 1)$, suppose $x = -1$ then $f'(-1) = 6(-1-1)(-1-1)$
$= 6(-2)(-2)$
$= 24>0$

$\therefore$ If $x\epsilon\ (-\infty, 1)$ then $f'(x)\geq 0$

$\therefore$ f is strictly increasing function on the intervals $(-\infty, 1)$.

(ii)If $x\in(1,\infty)$, suppose $x = 2$ then $f'(2) = 6(2-1)(2-1)$
$= 6(1)(1)$
$= 6 > 0$

$\therefore$ If $x\in\ (1,\infty)$ then $f'(x)\geq 0$

$\therefore$ f is strictly increasing function on the intervals $(1,\infty)$.

From (i) and (ii),

If $f'(x)$ is positive on both sides (no change) at $x = 1$, then

By First Derivative Test

f(x) neither a local minimum nor a local maximum

EXERCISE 6.5

1. Find the maximum and minimum values, if any, of the following functions given by

(i) $f(x) = (2x-1)^2 + 3$ (ii) $f(x) = 9x^2 + 12x + 2$

(iii) $f(x) = -(x-1)^2 + 10$ (iv) $g(x) = x^3 + 1$

SOLUTION:

for(i)

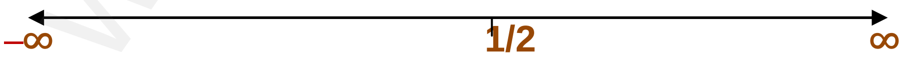
Given function is $f(x) = (2x-1)^2 + 3$ ……….(1)

$f'(x) = 2(2x-1)(2)$
$= 4(2x-1)$

To Find the Critical points at which $f'(x)=0$

$\Rightarrow 4(2x-1)=0$

$\Rightarrow x = \frac{1}{2}$ is a critical point



We have the intervals $(-\infty, \frac{1}{2})$ and $(\frac{1}{2},\infty)$

(i)If $x\in(-\infty, \frac{1}{2})$, suppose $x = 0$ then $f'(0) = 4(0-1) = 4(-1) = -4<0$

$\therefore$ If $x\epsilon\ (-\infty, \frac{1}{2})$ then $f'(x)<0$

$\therefore$ f is strictly decreasing function on the intervals $(-\infty, \frac{1}{2})$.

(ii)If $x \in (\frac{1}{2}, \infty)$, suppose $x = 3$ then $f'(3) = 4(6-1) = 4(5) = 20 > 0$

$\therefore$ If $x \in (\frac{1}{2}, \infty)$ then $f'(x) > 0$

$\therefore$ f is strictly increasing function on the intervals $(\frac{1}{2}, \infty)$.

From (i) and (ii),

If $f'(x)$ changes from negative to positive at $x = \frac{1}{2}$, then

By First Derivative Test

At $x = \frac{1}{2}$, f (x) has a local minimum $f(\frac{1}{2})$

$$f(\tfrac{1}{2}) = [2(\tfrac{1}{2}) - 1]^2 + 3 \quad [\because \text{By } (1)]$$
$$= 0 + 3$$
$$= 3$$

for(ii)

Given function is $f(x) = 9x^2 + 12x + 2$ ……….(1)

$f'(x) = 18x + 12$

$= 6(3x + 2)$

To Find the Critical points at which $f'(x) = 0$

$\Rightarrow 6(3x + 2) = 0$

$\Rightarrow x = \frac{-2}{3}$ is a critical point

We have the intervals $(-\infty, \frac{-2}{3})$ and $(\frac{-2}{3}, \infty)$

(i)If $x \in (-\infty, \frac{-2}{3})$, suppose $x = -1$ then $f'(-1) = 6(-3 + 2)$

$= 6(-1) = -6 < 0$

$\therefore$ If $x \in (-\infty, \frac{-2}{3})$ then $f'(x) < 0$

$\therefore$ f is strictly decreasing function on the intervals $(-\infty, \frac{-2}{3})$.

(ii)If $x \in (\frac{-2}{3}, \infty)$, suppose $x = 0$ then $f'(0) = 6(0 + 2) = 6(2) = 18 > 0$

$\therefore$ If $x \in (\frac{-2}{3}, \infty)$ then $f'(x) > 0$

$\therefore$ f is strictly increasing function on the intervals $(\frac{-2}{3}, \infty)$.

From (i) and (ii),

If $f'(x)$ changes from negative to positive at $x = \frac{-2}{3}$, then

By First Derivative Test

At $x = \frac{-2}{3}$, f (x) has a local minimum $f(\frac{-2}{3})$

$$f(\tfrac{-2}{3}) = 9(\tfrac{-2}{3})^2 + 12(\tfrac{-2}{3}) + 2 \quad [\because \text{By } (1)]$$
$$= 9(\tfrac{4}{9}) + 4(-2) + 2$$
$$= 4 - 8 + 2 = -2$$

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for(iii)

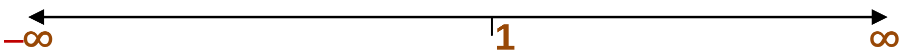
Given function is $f(x) = -(x-1)^2 + 10$ ……….(1)

$f'(x) = -2(x-1)$

To Find the Critical points at which $f'(x)=0$

$\Rightarrow -2(x-1)=0$

$\Rightarrow x = 1$ is a critical point



We have the intervals $(-\infty, 1)$ and $(1,\infty)$

(i)If $x\in(-\infty, 1)$, suppose $x = 0$ then $f'(0) = -2(0-1) = -2(-1) = 2 > 0$

∴ If $x\in(-\infty, 1)$ then $f'(x)>0$

∴ f is strictly increasing function on the intervals $(-\infty, 1)$.

(ii)If $x\in(1,\infty)$, suppose $x = 3$ then $f'(3) = -2(3-1) = -2(2) = -4 < 0$

∴ If $x\in(1,\infty)$ then $f'(x)<0$

∴ f is strictly decreasing function on the intervals $(1,\infty)$.

From (i) and (ii),

If $f'(x)$ changes from positive to negative at $x = 1$, then

By First Derivative Test

At x=1, f (x) has a local minimum f (1)

$f(1) = -(1-1)^2 + 10$ [∵By (1)]

$= 10$

for(iv)

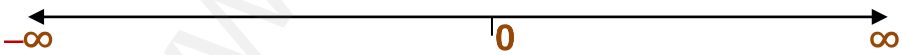
Given function is $g(x) = x^3 + 1$ ……….(1)

$g'(x) = 3x^2$

To Find the Critical points at which $g'(x)=0$

$\Rightarrow 3x^2=0$

$\Rightarrow x = 0$ is a critical point



We have the intervals $(-\infty, 0)$ and $(0,\infty)$

(i)If $x\in(-\infty, 0)$, suppose $x = -1$ then $f'(-1) = 3(-1)^2 = 3(1) = 3 > 0$

∴ If $x\in(-\infty, 0)$ then $f'(x)>0$

∴ f is strictly increasing function on the intervals $(-\infty, 0)$.

(ii)If $x\in(0,\infty)$, suppose $x = 1$ then $f'(1) = 3(1)^2 = 3(1) = 3 > 0$

∴ If $x\in(0,\infty)$ then $f'(x)>0$

∴ f is strictly decreasing function on the intervals $(0,\infty)$.

From (i) and (ii),

If $f'(x)$ is positive on both sides (no change) at $x = 0$, then

By First Derivative Test

f(x) neither a local minimum nor a local maximum