Exercise 8.1

1. Find the range and coefficient of range of the following data.

(i) 63, 89, 98, 125, 79, 108, 117, 68

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

(i) 63,89,98,125,79,108,117,68

Solution:

Let us arrange the given data in the ascending order 63,68,79,89,98,108,117,125
Largest value L=125 Smallest value S=63

Range = L-S

= 125-63=62

$$\text{Coefficient of Range} = \frac{L-S}{L+S}$$

$$= \frac{125\ \ 63}{125\ \ 63} = \frac{62}{188} = 0.33$$

Answer R=62 Coefficient of Range = 0.33

(ii) 43.5,13.6,18.9,38.4,61.4,29.8

Solution:

Let us arrange in ascending order 13.6,18.9,29.8,38.4,43.5,61.4 Largest value L=61.4 Smallest value S= 13.6

Range = L-S

61.4-13.6 = 47.8

$$\text{Coefficient of Range} = \frac{L-S}{L+S}$$

$$= \frac{61.4-13.6}{61.4+13.6}$$

$$= \frac{47.8}{75} = 0.64$$

Answer: R=47.8 Coefficient of Range = 0.64

2. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

Solution:

Range R = 36.8

Smallest value S = 13.4

R = L-S

36.8= L-13.4

L= 36.8+13.4=50.2

Answer: Largest Value = 50.2

3. Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Solution:

Largest value L=650

Smallest value S =400

Range R = L-S

=650-450=250

Answer : R=250.

4. A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages yet to be completed by them.

Solution:

By Assumed Mean Method :

Pages yet to be completed are 28, 25, 23, 30, 27, 24, 25, and 23

Assumed mean A=25 n = 8

x_i	$d_i=x_i-A$ $d_i=x_i-35$	d_i^2
23	-2	4
23	-2	4
24	-1	1
25	0	0
25	0	0
27	2	4
28	3	9
30	5	25
	$\sum d_i=5$	$\sum d_i^2=47$

Standard deviation

$$\sigma=\sqrt{\frac{\sum d_i^2}{n}-\left(\frac{\sum d_i}{n}\right)^2}$$

$$=\sqrt{\frac{47}{8}-\left(\frac{5}{8}\right)^2}$$

$$=\sqrt{\frac{47}{8}-\frac{25}{64}}$$

$$=\sqrt{\frac{351}{64}}$$

$$=\frac{18.735}{8}$$

$$\sigma=2.34$$

Ans: S.D of the pages to be completed = 2.34

5. Find the variance and standard deviation of the wages of 9 workers given below: ₹310, ₹290, ₹320, ₹280, ₹300, ₹290, ₹320, ₹310, ₹280.

Solution Mean Method :

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{280+280+290+290+300+310+310+232+320}{9}$$

$$= \frac{2700}{9} = 300$$

x_i	$d_i = x_i - \bar{x}$ $= x_i - 300$	d_i^2
280	–20	400
280	–20	400
290	–10	100
290	–10	100
300	0	0
310	10	100
310	10	100
320	20	400
320	20	400
	$\Sigma d_i = -5$	$\Sigma d_i^2 = 2000$

Variance

$$\sigma^2 = \frac{\Sigma d_i^2}{n}$$

$$= \frac{2000}{9}$$

$$= 222.22$$

Standard deviation = 14.91

$$\sigma = \sqrt{\text{Variance}}$$

$$= \sqrt{222.22}$$

Answer : Variance = 222.22

S.D = 14.91

6. A wall clock strikes the bell once at 1 o' clock, 2 times at 2 o' clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.

Solution :

The number of strikes the bell make a day.

$= 2\,(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12)$

Number of times strikes in a day.

$$2\left[\frac{n(n+1)}{2}\right]$$

$$2\left(\frac{12\times13}{2}\right)$$

$= 2 \times 78 = 156$

Standard deviation :

S.D of the fist n natural numbers

$$\sigma = \sqrt{\frac{n^2-1}{12}}$$

S.D of number of strikes in a day

$$= 2\sqrt{\frac{n^2-1}{12}}$$

$$= 2\sqrt{\frac{12^2-1}{12}}$$

$$= 2\sqrt{\frac{144-1}{12}}$$

$$= 2\sqrt{\frac{143}{12}} \qquad = 2\sqrt{11.92}$$

$= 2 \times 3.45$

$= 6.90$

Answer : Standard deviation = 6.9

7. Find the standard deviation of first 21 natural numbers.

Solution :

Standard deviation $\sigma = \sqrt{\frac{n^2-1}{12}}$

$$= \sqrt{\frac{21^2-1}{12}} = \sqrt{\frac{441-1}{12}}$$

$$= \sqrt{\frac{440}{12}} = \sqrt{36.6}$$

$$= 6.05$$

Answer : S.D of 21 natural numbers = 6.05

8. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Solution :

$\sigma = 4.5$

each value decreased by 5.

The standard deviation will not change when we subtract some fixed constant to all the values.

$\therefore$ new standard deviation is 4.5

Answer : New S.D $\sigma = 4.5$

9. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation

Solution :

$\sigma = 3.6$

each value is divided by 3.

When we divide each data by 3 the standard deviation is also get divided by 3.

$\therefore$ new standard deviation $\sigma = \frac{3.6}{3} = 1.2$

and new variance $\sigma^2 = (1.2)^2$

$= 1.44$

Answer : New variance = 1.44

New S.D = 1.2

10. The rainfall recorded in various places of five districts in a week are given below.

Rainfall (in mm)	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

Find its standard deviation.

Solution : Assumed mean method A = 60

x_i	f_i	$d_i = x_i - A = x_i - 60$	$f_i d_i$	$f_i d_i^2$
45	5	–15	–75	1125
50	13	–10	–130	1300
55	4	–5	–20	100
60	9	0	0	0
65	5	5	25	125
70	4	10	40	400
	N = 40		$\Sigma f_i d_i = -160$	$\Sigma f_i d_i^2 = 3050$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma fidi^2}{N} - \left(\frac{\Sigma fidi}{N}\right)^2}$$

$$= \sqrt{\frac{305\cancel{0}}{4\cancel{0}} - \left(\frac{-16\cancel{0}}{4\cancel{0}}\right)^2}$$

$$= \sqrt{76.25 - 16}$$

$$= \sqrt{60.25}$$

$$= 7.76$$

Answer : Standard deviation $\sigma \cong = 7.76$

11. In a study about viral fever, the number of people affected in a town were noted as

Age in years	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of people affected	3	5	16	18	12	7	4

Find its standard deviation.

Solution : Assumed mean method A = 35

Class Interval	Mid value x_i	f_i	$d_i = x_i - A = x_i - 35$	$f_i d_i$	$f_i d_i^2$
0 – 10	5	3	–30	–90	2700
10 – 20	15	5	–20	–100	2000
20 – 30	25	16	–10	–160	1600
30 – 40	35	18	0	0	0
40 – 50	45	12	10	120	1200
50 – 60	55	7	20	140	2800
60 – 70	65	4	30	120	3600
		N = 65		$\Sigma f_i d_i = 30$	$\Sigma f_i d_i^2 = 13900$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma fidi^2}{N} - \left(\frac{\Sigma fidi}{N}\right)^2}$$

$$= \sqrt{\frac{13900}{65} - \left(\frac{30}{65}\right)^2}$$

$$= \sqrt{\frac{13900}{65} - \frac{900}{4225}} \quad = \sqrt{213.85 - 0.21}$$

$$= \sqrt{213.64}$$

$$= 14.62$$

Answer : S.D $\sigma \cong = 14.6$

12. The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.

Diameter(cm)	21-24	25-28	29-32	33-36	37-40	41-44
Number of plates	15	18	20	16	8	7

Solution :

Step Deviation method

$A = 30.5$ $C = 4$

$d_i = \frac{x_i - 30.5}{4}$

Diameter	Mid value x_i	f_i	$di = \frac{x_i - A}{c}$	$f_i d_i$	$f_i d_i^2$
21 – 24	22.5	15	-2	-30	60
25 – 28	26.5	18	-1	-18	18
29 – 32	30.5	20	0	0	0
33 – 36	34.5	16	1	16	16
37 – 40	38.5	8	2	16	32
41 – 44	42.5	7	3	21	63
		N = 84		$\Sigma f_i d_i = 5$	$\Sigma f_i d_i^2 = 189$

Standard deviation

$$\sigma = c \times \sqrt{\frac{\Sigma fidi^2}{N} - \left(\frac{\Sigma fidi}{N}\right)^2}$$

$$= 4 \times \sqrt{\frac{189}{84} - \left(\frac{5}{84}\right)^2}$$

$$= 4 \times \sqrt{\frac{189}{84} - \frac{.25}{7059}} \quad = 4 \times \sqrt{2.25 - 0.0035}$$

$$= 4 \times \sqrt{2.2465}$$

$= 4 \times 1.5 = 6.0'$

Answer : S.D $\sigma \cong = 6$

13. The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation.

Time taken(sec)	8.5-9.5	9.5-10.5	10.5-11.5	11.5-12.5	12.5-13.5
Number of students	6	8	17	10	9

Solution :

Step Deviation method

A = 11 C = 1 $d_i = \frac{x_i - A}{c} = \frac{x_i - 11}{1}$

Time taken	Mid value x_i	f_i	$d_i = \frac{x_i - A}{c}$	$f_i d_i$	$f_i d_i^2$
8.5 – 9.5	9	6	–2	–12	24
9.5 – 10.5	10	8	–1	–8	8
10.5 – 11.5	11	17	0	0	0
11.5 – 12.5	12	10	1	10	10
12.5 – 13.5	13	9	2	18	36
		N = 50		$\Sigma f_i d_i = 8$	$\Sigma f_i d_i^2 = 78$

Standard deviation

$$\sigma = c \times \sqrt{\frac{\Sigma fidi^2}{N} - \left(\frac{\Sigma fidi}{N}\right)^2}$$

$$= 1 \times \sqrt{\frac{78}{50} - \left(\frac{8}{50}\right)^2}$$

$$= \sqrt{\frac{78}{50} - \frac{64}{2500}} \qquad = \sqrt{1.56 - 0.026}$$

$$= \sqrt{1.534}$$

Answer : S.D $\sigma \cong$ = 1.24

14. For a group of 100 candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as 40 and 27. Find the correct mean and standard deviation.

Solution :

$n = 100 \qquad \bar{x} = 60 \qquad \sigma = 15$

Correct values 45 and 72 wrong values 40 and 27

$\bar{x} = \frac{\Sigma x}{n} \Rightarrow \Sigma x = n \times \bar{x}$

$\therefore$ Wrong total = $100 \times 60 = 600$

Correct Total = Wrong total – Wrong values + Correct values

$= 6000 - 40 - 27 + 45 + 72$

$= 6050$

Correct mean $\bar{x} = \frac{\text{Correct total}}{n} = \frac{6050}{100} = 60.5$

Correct mean $\bar{x} = 60.5$

Standard deviation $\sigma = \sqrt{\frac{\Sigma xi^2}{n} - \left(\frac{\Sigma xi}{n}\right)^2}$

$15 = \sqrt{\frac{\Sigma xi^2}{100} - (60)^2} \qquad \left[\therefore \frac{\Sigma xi}{n} = \bar{x}\right]$

$15^2 = \frac{\Sigma x_i^2}{100} - (60)^2$

$\frac{\Sigma xi^2}{100} = 15^2 + 60^2 = 225 + 3600 = 3825$

Incorrect $\Sigma x_i^2 = 3825 \times 100 = 382500$

Correct $\Sigma x_i^2 = 382500 - (40)^2 - (27)^2 + 45^2 + 72^2$

$= 382500 - 1600 - 729 + 2025 + 5184$

$= 382500 - 2329 + 7209$

$= 387380$

Correct $\sigma = \sqrt{\frac{\text{correct}\Sigma xi^2}{n} - \left(\frac{\text{correct}\Sigma xi}{n}\right)^2}$

$= \sqrt{\frac{387380}{100} - (60.5)^2}$

$= \sqrt{3873.8 - 3660.25}$

$= \sqrt{213.55} = 14.61$

Answer : Correct mean = 60.5

Correct S.D $\sigma \cong 14.6$

15. The mean and variance of seven observations are 8 and 16 respectively. If five of these are 2, 4, 10, 12 and 14, then find the remaining two observations.

Solution :

Let the two observations be p and q.

$\overline{x} = 8$ and $\sigma^2 = 16$, $n = 7$

five observations are 2, 4, 10, 12 and 14.

x_i	x_i^2
2	4
4	16
10	100
12	144
14	196
p	p^2
q	q^2

$\overline{x} = 8$

$\frac{\Sigma x}{n} = 8$

$\frac{42 + p + q}{7} = 8$

$42 + p + q = 56$

$p + q = 56 - 42 = 14$

$\boxed{p + q = 14}$ -----(1)

$\Sigma x = 4^2 + p + q$

$\Sigma x^2 = 460 + p^2 + q^2$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma xi}{n}\right)^2$$

$$16 = \frac{460 + p^2 + q^2}{7} - \bar{x} \qquad \left[\therefore \frac{\Sigma x_i}{n} = \bar{x}\right]$$

$$16 = \frac{460 + p^2 + q^2}{7} - 8^2$$

$$\frac{460 + p^2 + q^2}{7} = 64 + 16 = 80$$

$460 + p^2 + q^2 = 560$

$p^2 + q^2 = 560 - 460 = 100$

$(p + q)^2 - 2pq = 100$

$14^2 - 2pq = 100$

$196 - 2pq = 100$

$2pq = 96$

$\boxed{pq = 48}$ ---(2)

Solving (1) and (2)

From (2) $\Rightarrow$ $q = \frac{48}{p}$ substitute in (1)

(1) $\Rightarrow p + \frac{48}{p} = 14$

$\frac{p^2 + 48}{p} = 14 \Rightarrow p^2 + 48 = 14p$

$p^2 - 14p + 48 = 0$

$(p - 8)(p - 6) = 0$

$p - 8 = 0$ (or) $p - 6 = 0$

$\therefore p = 8$ and $p = 6$.

If $p = 8$, $q = \frac{48}{8} = 6$

If $p = 6$, $q = \frac{48}{6} = 8$

Ans :

The two observations are 6 and 8.

Exercise 8.2

1. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

Solution :

S.D $\sigma = 6.5$ mean $\bar{x} = 12.5$

Coefficient of variation $C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{6.5}{12.5} \times 100$

Ans : Coefficient of variation = 52 %

2. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Solution :

S.D $\sigma = 12.5$ C.V = 25.6

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$25.6 = \frac{1.2}{\bar{x}} \times 100$$

$$\bar{x} = \frac{1.2}{25.6} \times 100$$

$$= 4.96$$

Answer : mean = 4.69

3. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Solution :

$\bar{x} = 15$ C.V = 48

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$48 = \frac{\sigma}{15} \times 100$$

$$\sigma = \frac{48 \times 15}{100} = 7.2$$

Answer : Standard deviation = 7.2

4. If $n = 5$, $\bar{x} = 6$, $\Sigma x^2 = 765$, then calculate the coefficient of variation.

Solution :

$n = 5$ $\bar{x} = 6$

$\Sigma x^2 = 765$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$$

$$= \sqrt{\frac{765}{5} - 6^2} = \sqrt{153 - 36} = \sqrt{117}$$

$\sigma = 10.817$

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{10.817}{6} \times 100$$

$$= 1.8028 \times 100 = 180.28\%$$

Answer :

Coefficient of variation = 180.28 %

5. Find the coefficient of variation of 24, 26, 33, 37, 29, 31.

Solution :

Arrange in ascending order 24, 26, 29, 31, 33, 37.

$$\bar{x} = \frac{24+26+29+31+33+37}{6} = \frac{180}{6} = 30$$

$\overline{x} = 30$

x_i	$d_i = x_i - \overline{x}$ $= x_i - 30$	d_i^2
24	–6	36
26	–4	16
29	–1	1
31	1	1
33	3	9
37	7	49
	$\Sigma d_i = 0$	$\Sigma d_i^2 = 112$

$$\sigma = \frac{\Sigma d_i^2}{n}$$

$$= \sqrt{\frac{112}{6}} \qquad = \sqrt{18.67}$$

$$= 4.32$$

$$C.V = \frac{\sigma}{x} \times 100$$

$$= \frac{4.32}{30} \times 100 = 14.4$$

Answer :

Coefficient of variation = 14.4 %

6. The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.

Solution :

Let as arrange in ascending order 38, 40, 43, 44, 46, 47, 49, 53

$$\bar{x} = \frac{38+40+43+44+46+47+49+53}{8} = \frac{360}{8} = 45$$

$\overline{x} = 45$

x_i	$d_i = x_i - \overline{x}$ $= x_i - 45$	d_i^2
38	–7	49
40	–5	25
43	–2	4
44	–1	1
46	1	1
47	2	4
49	4	16
53	8	64
	$\Sigma d_i = 0$	$\Sigma d_i^2 = 164$

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$$

$$= \sqrt{\frac{164}{8}} = \sqrt{20.5}$$

$\sigma = 4.53$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.53}{45} \times 100$$

$= 0.1007 \times 100 = 10.07\ \%$

Answer : Coefficient of variation = 10.07 %

7. The total marks scored by two students Sathya and Vidhya in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?

Solution :

$n = 5$

Vidhya	Sathya
total marks $\sum x = 480$	total marks $\sum x = 460$
S.D $\sigma = 2.4$	S.D $\sigma = 4.6$
$\bar{x} = \frac{\Sigma x}{n} = \frac{480}{5}$	$\bar{x} = \frac{\Sigma x}{n} = \frac{460}{5}$
$\bar{x} = 96$	$\bar{x} = 92$
$C.V = \frac{\sigma}{\bar{x}} \times 100$	$C.V = \frac{\sigma}{\bar{x}} \times 100$
$= \frac{2.4}{96} \times 100 = 2.5$	$= \frac{4.6}{92} \times 100 = 5$

C.V of Vidhya < C.V of Sathya.

Answer : Vidhya is more consistent.

8. The mean and standard deviation of marks obtained by 40 students of a class in three subjects Mathematics, Science and Social Science are given below.

Subject	Mean	SD
Mathematics	56	12
Science	65	14
Social Science	60	10

Which of the three subjects shows highest variation and which shows lowest variation in marks?

Solution :

Mathematics	Science	Social Science
$\overline{x} = 56\ \sigma = 12$	$\overline{x} = 65\ \sigma = 14$	$\overline{x} = 60\ \sigma = 10$
$C.V = \frac{\sigma}{x} \times 100$	$C.V = \frac{\sigma}{x} \times 100$	$C.V = \frac{\sigma}{x} \times 100$
$= \frac{12}{56} \times 100$	$= \frac{14}{65} \times 100$	$= \frac{10}{60} \times 100$
$= 0.2143 \times 100$	$= 0.2154 \times 100$	$= 0.1667 \times 100$
$= 21.43\ \%$	$= 21.54\ \%$	$= 16.67\ \%$

C.V Maths = 2143 % C.V of Science = 21.54 % C.V of Social = 16.67 %

Answer : Highest variation is Science.

Lowest variation is Social Science.

9. The temperature of two cities *A* and *B* in a winter season are given below.

Temperature of city A (in degree Celsius)	18	20	22	24	26
Temperature of city B (in degree Celsius)	11	14	15	17	18

Find which city is more consistent in temperature changes?

Solution :

Find the coefficient of variation for city A and city B.

City A

$$\bar{x} = \frac{18+20+22+24+26}{5}$$

$$= \frac{110}{5} = 22$$

$\overline{x} = 22$

x_i	$d_i = x_i - \overline{x} = x_i - 22$	d_i^2
18	-4	16
20	-2	4
22	0	0
24	2	4
26	4	16
	$\Sigma d_i = 0$	$\Sigma d_i^2 = 40$

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$$

$$= \sqrt{\frac{40}{5}} = \sqrt{8}$$

$\sigma = 2.828$

$$C.V = \frac{\sigma}{x} \times 100$$

$$= \frac{2.828}{22} \times 100$$

$= 12.85\ \%$

City B

$$\bar{x} = \frac{11+14+15+17+18}{5}$$

$$= \frac{75}{5} = 15$$

$\overline{x} = 15$

x_i	$d_i = x_i - \overline{x} = x_i - 22$	d_i^2
18	-4	16
20	-2	4
22	0	0
24	2	4
26	4	16
	$\Sigma d_i = 0$	$\Sigma d_i^2 = 40$

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$$

$$= \sqrt{\frac{30}{5}} = \sqrt{6}$$

$\sigma = 2.449$

$$C.V = \frac{\sigma}{x} \times 100$$

$$= \frac{2.449}{15} \times 100$$

$= 16.33\ \%$

C.V of city A < C.V of city B.

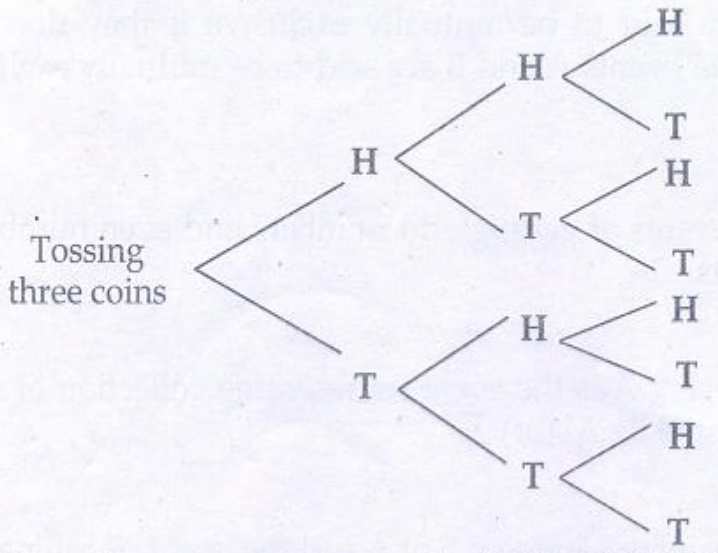
Answer : City A is more consistent.

Exercise 8.3

1. Write the sample space for tossing three coins using tree diagram.

Solution :

When we tossing a coin the outcomes are Head (H) and Tail (T).

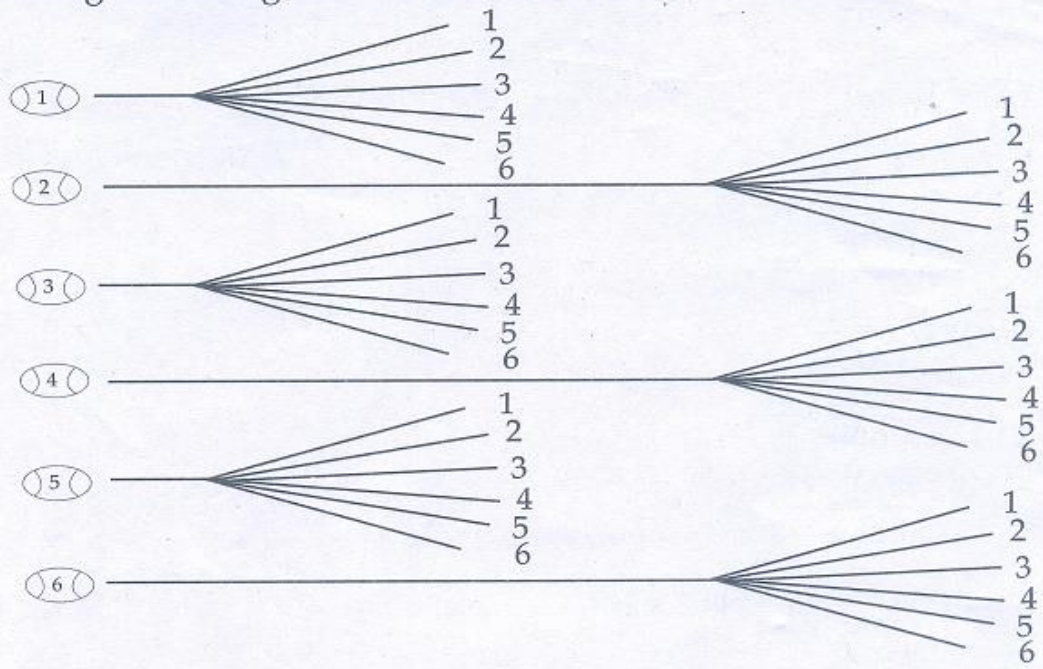


Free tree diagram sample sapce S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

2. Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram).

Solution :

A bag containing 6 balls numbered 1 to 6.



sample sapce S = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

3. If A is an event of a random experiment such that $P(A) : P(\bar{A}) = 17:15$ and $n(S)=640$ then find (i) $P(\bar{A})$ (ii) $n(A)$.

Solution :

$P(A) : P(\overline{A}) = 17 : 15$ and $n(S) = 640$

$\therefore$ Let $P(A) = 17x$ and $P(\overline{A}) = 15x$

we have $P(A) + P(\overline{A}) = 1$

$$17x + 15x = 1$$

$$32x = 1$$

$$x = \frac{1}{32}$$

$$\therefore P(\overline{A}) = 15x$$

$$= 15\left(\frac{1}{32}\right)$$

$$p(\overline{A}) = \frac{15}{32}$$

and $p(A) = 17x = \frac{17}{32}$

$$\therefore \frac{n(A)}{n(S)} = \frac{17}{32}$$

$$\frac{n(A)}{640} = \frac{17}{32}$$

$$n(A) = \frac{17}{\cancel{32}} \times \cancel{640}^{20} = 340$$

Answer : $p(A) = \frac{15}{32}$ $n(A) = 340$

3.B

Solution :

Ayan throws two dice once

sample sapce S = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

n (S) = 36

Let A be the event of getting product of the numbers on the dice is 36.

$\therefore$ A = {(6, 6)} n (A) = 1

$$p(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$$

Krishna throws one die

$\therefore$ S = {1, 2, 3, 4, 5, 6} n (S) = 6

Let B be the event of getting squares the number on the die is 36.

$\therefore$ B = $\{(6^2)\}$ = {36} n (B) = 1

$$p(B) = \frac{n(B)}{n(S)} = \frac{1}{6}$$

Comparing the probability of an event A and B.

Answer : Krishna getting the better change.

4. A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution :

A coin is tossed thrice.

S = {HHH, HTH, HHT, THH, TTH, THT, HTT, TTT}

$\therefore$ n (S) = 8

Let A be the event of getting two consecutive tails

A = {TTH, HTT, TTT}

n (A) = 3

$$p(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

Answer : P (getting two consecutive tails) $\frac{3}{8}$

5. At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won?

Solution :

$n(S) = 1000$

Let A be the event of getting a perfect square number greater than 500.

(i) $A = \{23^3, 24^2, 25^2, 26^2, 27^2, 28^2, 29^2, 30^2, 31^2\}$

$A = \{529, 576, 625, 676, 729, 784, 841, 900, 961\}$

$\therefore n(A) = 9$

$\therefore p(A) = \frac{n(A)}{n(S)} = \frac{9}{1000}$

(ii) The card is not replaced & Let B be the event of winning second player if the first has won.

$\therefore n(S) = 999$ and $n(B) = 8$

$\therefore p(A) = \frac{n(A)}{n(S)} = \frac{8}{999}$

6. A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find x.

Solution :

A bag has 12 blue balls and x red balls.

$\therefore$ n (S) = 12 + x

(i) Let R be the event of getting red ball.

$\therefore$ n (R) = x

$$p(R) = \frac{n(R)}{n(S)} = \frac{x}{12+x}$$

(ii) 8 more red balls are put in the bag.

$\therefore$ n (S) = 20 + x

$$\therefore p(R_1) = \frac{x+8}{20+x}$$

Given P (R_1) = 2 P (R)

$$\frac{x+8}{20+x} = 2\left(\frac{x}{12+x}\right)$$

$$(12 + x)(x + 8) = 2x(20 + x)$$

$$12x + x^2 + 8x + 96 = 40x + 2x^2$$

$$2x^2 - x^2 + 40x - 20x - 96 = 0$$

$$x^2 + 20x - 96 = 0$$

$$(x + 24)(x - 4) = 0$$

$x + 24 = 0$ (or) $x - 4 = 0$

$x \neq -24$ $x = 4$

Answer : Value of x = 4

$$\text{(i) } p(R) = \frac{4}{16} = \frac{1}{4}$$

7. Two unbiased dice are rolled once. Find the probability of getting

(i) a doublet (equal numbers on both dice)

(ii) the product as a prime number

(iii) the sum as a prime number

(iv) the sum as 1

Solution :

Two dice are rolled

sample sapce S = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

n (S) = 36

(i) Let A be the event of getting a doublet

A = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)}

n (A) = 6

$$p(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) Let B be the event of getting a product is a prime number

B = {(1, 2) (1, 3) (1, 5) (2, 1) (3, 1) (5, 1)}

n (B) = 6

$$p(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(iii) Let C be the event of getting a sum is prime number.

C = {(1, 1) (1, 2) (1, 4) (1, 6) (2, 1) (2, 3) (2, 5) (3, 2) (3, 4) (4, 1) (4, 3) (5, 2) (5, 6) (6, 1) (6, 5)}

n (C) = 15

(iv) Let D be the event of getting a sum is 1.

∴ D is an impossible event D = { }

∴ P (D) = 0

8. Three fair coins are tossed together. Find the probability of getting

(i) all heads

(ii) atleast one tail

(iii) atmost one head

(iv) atmost two tails

Solution :

Three fair coins are tossed

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

n (S) = 8

(i) Let A be the event of getting all heads

A = {HHH} n (H) = 1

$$p(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

(ii) Let B be the event of getting atleast one tail

B = {HHT, HTH, THH, HTT, THT, TTH, TTT}

n (B) = 7

$$p(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$$

(iii) Let C be the event of getting atmost one head.

C = {HTT, THT,TTH, TTT}

n (C) = 4

$$p(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(iv) Let D be the event of getting atmost two tails

D = {HHH, HHT, HTH, THH, TTH, THT, HTT}

∴ n (D) = 7

$$p(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$$

9. Two dice are numbered 1,2,3,4,5,6 and 1,1,2,2,3,3 respectively. They are rolled and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.

Solution :

$S = \{(1,1)\ (1,1)\ (1,2)\ (1,2)\ (1,3)\ (1,3)$
$(2,1)\ (2,1)\ (2,2)\ (2,2)\ (2,3)\ (2,3)$
$(3,1)\ (3,1)\ (3,2)\ (3,2)\ (3,3)\ (3,3)$
$(4,1)\ (4,1)\ (4,2)\ (4,2)\ (4,3)\ (4,3)$
$(5,1)\ (5,1)\ (5,2)\ (5,2)\ (5,3)\ (5,3)$
$(6,1)\ (6,1)\ (6,2)\ (6,2)\ (6,3)\ (6,3)\}$

$n(S) = 36$

Let A_1 be the event of getting sum 2.

$A_1 = \{(1,1)\ (1,1)\}$ $\quad n(A_1) = 2$

$$p(A_1) = \frac{n(A_1)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

Let A_2 be the event of getting sum 3.

$A_2 = \{(1,2)\ (1,2)\ (2,1)\ (2,1)\}$ $\quad n(A_2) = 4$

$$p(A_2) = \frac{n(A_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Let A_3 be the event of getting a sum 4.

$A_3 = \{(2,2)\ (2,2)\ (3,1)\ (3,1)\ (1,3)\ (1,3)\}$ $\quad n(A_3) = 6$

$$p(A_3) = \frac{n(A_3)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Let A_4 be the event of getting a sum 5.

$A_4 = \{(2,3)\ (2,3)\ (3,2)\ (3,2)\ (4,1)\ (4,1)\}$ $\quad n(A_4) = 6$

$$p(A_4) = \frac{n(A_4)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Let A_5 be the event of getting a sum 6.

$A_5 = \{(3,3)\ (3,3)\ (4,2)\ (4,2)\ (5,1)\ (5,1)\}$ $\quad n(A_5) = 6$

$$p(A_5) = \frac{n(A_5)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Let A_6 be the event of getting a sum 7.

$A_6 = \{(4,3)\ (4,3)\ (5,2)\ (5,2)\ (6,1)\ (6,1)\}$ $\quad n(A_6) = 6$

$$p(A_6) = \frac{n(A_6)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Let A_7 be the event of getting a sum 8.

$A_7 = \{(5,3)\ (5,3)\ (6,2)\ (6,2)\}$ $\quad n(A_7) = 4$

$$p(A_7) = \frac{n(A_7)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Let A_8 be the event of getting a sum 9.

$A_8 = \{(6,3)\ (6,3)\}$ $\quad n(A_8) = 2$

$$p(A_8) = \frac{n(A_8)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

10. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is

(i) White

(ii) black or red

(iii) not white

(iv) neither white nor black

Solution :

A bag contains 5 red balls, 6 white balls, 7 green balls, and 8 black balls.

$\therefore n(S) = 5 + 6 + 7 + 8 = 26$

(i) Let A be the event of getting 'white balls'

$n(A) = 6$

$$p(A) = \frac{n(A)}{n(S)} = \frac{6}{26} = \frac{3}{13}$$

(ii) Let B be the event of getting 'black or red'

$n(B) = 8 + 5 = 13$

$$p(B) = \frac{n(B)}{n(S)} = \frac{13}{26} = \frac{1}{2}$$

(iii) Let C be the event of getting 'nota white ball'

$n(C) = 26 - 6 = 20$

$$p(C) = \frac{n(C)}{n(S)} = \frac{20}{26} = \frac{10}{13}$$

(iv) Let D be the event of getting 'neither white nor black'

$n(D) = 5 + 7 = 12$

$$p(D) = \frac{n(D)}{n(S)} = \frac{12}{36} = \frac{6}{13}$$

11. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is 3/8 then, find the number of defective bulbs.

Solution :

Let the number of defective bulbs are x

No. on non-defective bulbs = 20

$\therefore$ n (S) = 20 + x

Let A be the event of getting defective bulbs

$$p(A) = \frac{3}{8}$$

$$\frac{n(A)}{n(S)} = \frac{3}{8}$$

$$\frac{x}{20+x} = \frac{3}{8}$$

$$8x = 3\ (20 + x)$$

$$8x = 60 + 3x$$

$$8x - 3x = 60$$

$$5x = 60 \Rightarrow x = \frac{60}{5} = 12$$

$\therefore$ x = 12

Answer : Number of defective bulbs = 12.

12. The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is

(i) a clavor

(ii) a queen of red card

(iii) a king of black card

Solution :

In a deck of 52 cards,

the king and queen of diamonds

queen and jack of hearts

jack and king of spades are removed

$\therefore$ n (S) = 52 – 6 = 46

(i) Let A be the event of getting 'a clavor'

$\therefore$ n (A) = 13

$$p(A) = \frac{n(A)}{n(S)} = \frac{13}{46}$$

(ii) Let B be the event of getting 'a queen of red card'

$\therefore$ n (B) = 0

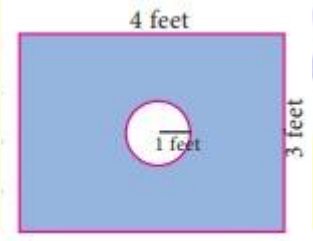
$$p(B) = \frac{n(B)}{n(S)} = \frac{0}{46} = 0$$

(iii) Let C be the event of getting 'a king of black card'

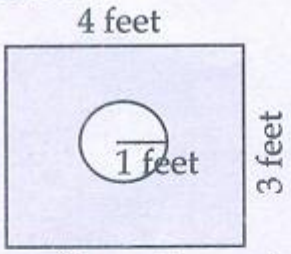
$\therefore$ n (C) = 1 [1 clavor black card]

$$p(C) = \frac{n(C)}{n(S)} = \frac{1}{46}$$

13. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game?



Solution :



Area of a rectangular region = $3 \times 4 = 12$ feet.

$\therefore n(S) = 12$

Let A be event of getting 'win the game'.

$\therefore$ A denote the circular region = πr^2

$= 3.14 \times 1^2 = 3.14$

$$p(A) = \frac{n(A)}{n(S)} = \frac{3.14}{12} = \frac{314}{1200} = \frac{157}{600}$$

Answer :

Probability of win the game = $\frac{157}{600}$

14. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on

(i) the same day

(ii) different days

(iii) consecutive days?

Solution :

Priya and Vidhya visiting a particular shop in the same week.

S = {(Mon, Mon) (Mon, Tue) Mon, Wed) (Mon, Thurs) (Mon, Fri) (Mon, Sat)

(Tue, Mon) (Tue, Tue) (Tue, Wed) (Tue, Thurs) (Tue, Fri) (Tue, Sat)

(Wed, Mon) (Wed, Tue) (Wed, Wed) (Wed, Thurs) (Wed, Fri) (Wed, Sat)

(Thrus, Mon) (Thurs, Tue) (Thurs, Wed,) (Thurs, Thurs) (Thurs, Fri) (Thurs, Sat)

(Fri, Mon) (Fri, Tue) (Fri, Wed) (Fri, Thurs) (Fri, Fri) (Fri, Sat)

(Sat, Mon) (Sat, Tue) (Sat, Wed) (Sat, Thurs) (Sat, Fri) (Sat, Sat)}

n (S) = 36

(i) Let A be the event of getting 'the same day'

A = {(Mon, Mon) (Tue, Tue) (Wed, Wed) (Thurs, Thurs) (Fri, Fri) (Sat, Sat)}

$\therefore$ n (A) = 6

$$p(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

15. In a game, the entry fee is 150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry. If one or two heads shows she gets her entry fee back. If she throwns 3 heads she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.

(ii) Let B the event of getting 'different day'

B is Complementary event of A

$\therefore$ n (B) = 36 – 6 = 30

$$p(B) = \frac{n(B)}{n(S)} = \frac{30}{36} = \frac{5}{6}$$

(iii) Let C the event of getting 'consecutive days'

C = {(Mon, Tue) (Tue, Wed) (Wed, Thurs) (Thurs, Fri) (Fri, Sat)}

$\therefore$ n (C) = 5

$$p(C) = \frac{n(C)}{n(S)} = \frac{5}{6}$$

Solution :

The game consists of tossing a coin 3 times

S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

n (S) = 8

(i) Let A be the event of getting 'double entry fee' if she throws 3 heads

$\therefore$ A = {HHH}

$\therefore$ n (A) = 1

$$p(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

(ii) Let B the event of getting 'just gets her entry fee' if she throws one or two heads

B = {HTT, THT, HTT, HHT, THH, HTH}

$\therefore$ n (B) = 6

$$p(B) = \frac{n(B)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

(iii) Let C the event of getting 'loses the entry fee' if she gets three tails

C = {TTT}

$\therefore$ n (C) = 1

$$p(C) = \frac{n(C)}{n(S)} = \frac{1}{8}$$

Exercise 8.4

1. If $P(A) = 2/3$, $P(B) = 2/5$, $P(A \cup B) = 1/3$ then find $P(A \cap B)$.

Solution :

$$p(A)=\frac{2}{3} \quad p(B)=\frac{2}{5} \quad p(A\cup B)=\frac{1}{3}$$

Using Addition theorem of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{1}{3}=\frac{2}{3}+\frac{2}{5}-(p(A\cap B)$$

$$\frac{1}{3}=\frac{10+6}{15}-p(A\cap B)$$

$$\frac{1}{3}=\frac{16}{15}-p(A\cap B)$$

$$p(A\cap B)=\frac{16}{15}-\frac{1}{3}=\frac{16-5}{15}=\frac{11}{15}$$

Answer : $p(A\cap B)=\frac{11}{15}$

2. A and B are two events such that, $P(A) = 0.42$, $P(B) = 0.48$, and $P(A \cap B) = 0.16$.

Find

(i) P,(not A)

(ii) P,(not B)

(iii) P,(A or B)

Solution :

$P(A) = 0.42 \quad P(B) = 0.48 \quad P(A \cap B) = 0.16$

(a) $P(\text{not } A) = P(\overline{A}) = 1 - P(A)$

$= 1 - 0.42 = 0.58$

(b) $P(\text{not } B) = P(\overline{B}) = 1 - P(B)$

$= 1 - 0.48 = 0.52$

(c) $P(A \text{ or } B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16$

$= 0.90 - 0.16 = 0.74$

$P(A \cup B) = 0.74$

3. If *A* and *B* are two mutually exclusive events of a random experiment and *P*(not *A*) = 0.45, *P* (*A* U *B*) = 0.65, then find *P* (*B*).

Solution :

$P(\text{not } A) = P(\overline{A}) = 0.45 \quad P(A \cup B) = 0.65$

$\therefore P(A) = 1 - P(\overline{A})$

$= 1 - 0.45 = 0.55$

A and B are two mutually exclusive events then

$P(A \cup B) = P(A) + P(B)$

$0.65 = 0.55 + P(B)$

$P(B) = 0.65 - 0.55 = 0.10$

Answer : $P(B) = 0.1$

4. The probability that atleast one of *A* and *B* occur is 0.6. If *A* and *B* occur simultaneously with probability 0.2, then find $P(\overline{A}) + P(\overline{B})$.

Solution :

$P(A \cup B) = 0.6 \qquad P(A \cap B) = 0.2$

Using addition theorem of probability,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = P(A) + P(B) - 0.2$

$0.6 + 0.2 = P(A) + P(B)$

$0.8 = 1 - P(\overline{A}) + 1 - P(\overline{B})$

$= 2 - (P(\overline{A}) + P(\overline{B}))$

$P(\overline{A}) + P(\overline{B}) = 2 - 0.8 = 1.2$

Answer : $P(\overline{A}) + P(\overline{B}) = 1.2.$

5. The probability of happening of an event *A* is 0.5 and that of *B* is 0.3. If *A* and *B* are mutually exclusive events, then find the probability that neither *A* nor *B* happen.

Solution:

$P(A) = 0.5, \qquad P(B) = 0.3$

If A and B are mutually exclusive then

$P(A \cup B) = P(A) + P(B)$

$= 0.5+0.3 = 0.8$

$P(\text{neither A nor B}) = P(\overline{A} \cap \overline{B})$

$= P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - 0.8 = 0.2$

Answer: P(neither A nor B happen) = 0.2

6. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution:

Two dice are rolled once.

$S = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$n(S) = 36$

Let A be the event of getting even number on first die

$A = \{(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$n(A) = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$$

Let B be the event of getting total of face sum as 8

$B = \{(2,6)\ (3,5)\ (4,4)\ (5,3)\ (6,2)\}$

$n(B) = 5$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

$A \cap B = \{ (2,6)\ (4,4)\ (6,2)\}$

$$n(A \cap B) = 3\ P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$$

Using addition theorem of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$$

$$= \frac{20}{36} = \frac{5}{9}$$

Answer: P(even number on first die 0. a total of face sum 8) = $\frac{5}{9}$

7. From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of it being either a red king or a black queen.

Solution:

From a pack of 52 cards,

$n(S) = 52$

Let A be the event of getting a red king

$n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{52}$$

Let B be the event of getting a black queen

$n(B) = 2$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{52}$$

Since A and B are mutually exclusive events

$P(A \cap B) = 0$

Using addition theorem of probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{52} + \frac{2}{52}$$

$$= \frac{4}{52} = \frac{1}{13}$$

Answer:

P(red king or black queen) = $\frac{1}{13}$

8. A box contains cards numbered 3, 5, 7, 9, … 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Solution:

A box contains cards numbered 3, 5, 7, . . . 37.

$S = \{3, 5, 7, 9, 11, 13, \ldots . 35, 37\} \Rightarrow n(S) = 18$

Let A be the event of getting multiples of 7

$A = \{7, 21, 35\} \Rightarrow n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{18}$$

Let B be the event of getting a prime number

$B = \{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37\}$

$n(B) = 11$

$$P(B) = \frac{n(B)}{n(S)} = \frac{11}{18}$$

$A \cap B = \{7\} \quad n(A \cap B) = 1$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{18}$$

Using addition theorem of probability,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{3}{18} + \frac{11}{18} - \frac{1}{18}$$

$$= \frac{13}{18}$$

Answer:

P(multiples of 7 or a prime number) = $\frac{13}{18}$

9. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

Solution:

Three coins are tossed

S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

$n(S) = 8$

Let A be the event of getting atmost 2 tails

A = {HHH, HHT, HTH, THH, TTH, THT, HTT}

$n(A) = 7$

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

Let B be the event of getting at least 2 heads

B = {HHH, HHT, HTH, THH}

$n(B) = 4$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{8}$$

$A \cap B$ = {HHH, HHT, HTH, THH}

$n(A \cap B) = 4$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{4}{8}$$

Using addition theorem of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{7}{8} + \frac{4}{8} - \frac{4}{8}$$

$$= \frac{7}{8}$$

Answer:

P(getting at most 2 tails or at least 2 heads) $= \frac{7}{8}$

10. The probability that a person will get an electrification contract is 3/5 and the probability that he will not get plumbing contract is 5/8. The probability of getting atleast one contract is 5/7. What is the probability that he will get both?

Solution:

Let A be the event of a person will get an electrification contract and B be a person will get plumbing contract.

$$P(A) = \frac{3}{5}\ P(B') = \frac{5}{8} \text{ and } P(A \cup B) = \frac{5}{7}$$

$$P(B) = 1 - P(B')$$

$$= 1 - \frac{5}{8} = \frac{3}{8}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{5}{7} = \frac{3}{5} + \frac{3}{8} - P(A \cap B)$$

$$\frac{5}{7} = \frac{24+15}{40} - P(A \cap B)$$

$$\therefore P(A \cap B) = \frac{39}{40} - \frac{5}{7}$$

$$= \frac{273-200}{280} = \frac{73}{280}$$

Answer: $P(\text{getting both}) = \frac{73}{280}$

11. In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years?

Solution:

Total People = 8000

$n(S) = 8000$

Let A be the event of choosing a females

$n(A) = 3000$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3000}{8000}$$

Let B be the event of choosing over 50 years

$n(B) = 1300$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1300}{8000}$$

$A \cap B$ = 30% females over 50 years

$$n(A \cap B) = \frac{30}{1\cancel{00}} \times 30\cancel{00} = 900$$

$$P(A \cap B) = \frac{900}{8000}$$

Using addition theorem of probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{3000}{8000} + \frac{1300}{8000} - \frac{900}{8000}$$

$$= \frac{3400}{8000} = \frac{34}{80} = \frac{17}{40}$$

Answer:

P(either a female or over 50 years) = $\frac{17}{40}$

12. A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.

Solution:

Three coins are tossed

$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

$n(S) = 8$

Let A be the event of getting exactly two heads

$A = \{HHT, HTH, THH\} \Rightarrow n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

Let B be the event of getting atleast one tail

$B = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

$n(B) = 7$

$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$

Let C be the event of getting consecutively two heads

$C = \{HHT, THH, HHH\} \Rightarrow n(C) = 3$

$P(C) = \frac{n(C)}{n(S)} = \frac{3}{8}$

$A \cap B = \{HHT, HTH, THH\}$

$n(A \cap B) = 3$

$P(A \cap B) = \frac{3}{8}$

$B \cap C = \{HHT, THH\}$

$n(B \cap C) = 2$

$P(B \cap C) = \frac{2}{8}$

$C \cap A = \{HHT, THH\}$

$n(C \cap A) = 2$

$P(C \cap A) = \frac{2}{8}$

$A \cap B \cap C = \{HHT, THH\}$

$n(A \cap B \cap C) = 2$

$P(A \cap B \cap C) = \frac{2}{8}$

$P(A \cup B \cup C) = P(A)+P(B)+P(C)-P(A \cap B) -P(B \cap C)-P(C \cap A)+P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8}$

$= \frac{8}{8} = 1$

Answer: P(atleast one tail or exactly two heads or consecutively two heads) = 1

13. If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A \cap B) = 1/6$. $P(B \cap C) = 1/4$, $P(A \cap C) = 1/8$, $P(A \cup B \cup C) = 9/10$, $P(A \cap B \cap C) = 1/15$, then find $P(A)$, $P(B)$ and $P(C)$?

Solution:

$$P(A\cap B) = \frac{1}{6}, P(B\cap C) = \frac{1}{4}, \quad P(A\cap C) = \frac{1}{8},$$

$$P(A\cup B\cup C) = \frac{9}{10}, P(A\cap B\cap C) = \frac{1}{15}$$

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$$

$$\frac{9}{10} = P(A)+P(B)+P(C)-\frac{1}{6}-\frac{1}{4}-\frac{1}{8}+\frac{1}{15}$$

$$\frac{9}{10} = P(A)+P(B)+P(C)-\left(\frac{4+6+3}{24}\right)+\frac{1}{15}$$

$$\frac{}{10} = P(A)+P(B)+P(C)-\frac{13}{24}+\frac{1}{15}$$

$$P(A)+P(B)+P(C) = \frac{9}{10}+\frac{13}{24}-\frac{1}{15}$$

$$= \frac{108+65-8}{120} = \frac{165}{120} = \frac{11}{8}$$

$$P(A)+P(B)+P(C) = \frac{11}{8} \text{ ------(1)}$$

Given $P(B) = 2P(A)$ and $P(C) = 3P(A)$

Substitute in (1)

$$P(A)+2P(A)+3P(A) = \frac{11}{8}$$

$$6P(A) = \frac{11}{8}$$

$$P(A) = \frac{11}{48}$$

$$P(B) = 2\times\frac{11}{48} = \frac{11}{24}$$

$$P(C) = 3\times\frac{11}{48} = \frac{11}{16}$$

Answer: $P(A) = \frac{11}{48}, P(B) = \frac{11}{24}, P(C) = \frac{11}{16}$

14. In a class of 35, students are numbered from 1 to 35. The ratio of boys to girls is 4:3. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with prime roll number or a girl with composite roll number or an even roll number.

Solution:

Total number of students = 35

$n(S) = 35$

ratio of boys and girls is 4:3

Let Number of boys be $4x$ and Number of girls be $3x$

$4x+3x = 35$

$7x = 35$

$x = 5$

No. of boys $= 4\times5 = 20$

No. of girls $= 3\times5 = 15$

Roll numbers begin with boys and end with girls

Boys roll numbers are 1, 2, 3 20

Girls roll numbers from 21, 22, 23, ...35

Let A be the event of selecting a boy with prime roll number

$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$n(A) = 8$

$P(A) = \frac{n(A)}{n(S)} = \frac{8}{35}$

Let B be the event of selecting a girl with composite roll number

$B = \{21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35\}$

$n(B) = 12$

$P(B) = \frac{n(B)}{n(S)} = \frac{12}{35}$

Let C be the event of selecting an even roll number

$C = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34\}$

$n(B) = 17$

$P(B) = \frac{n(C)}{n(S)} = \frac{17}{35}$

$A\cap B = \{\}$ $n(A\cap B) = 0,$ $P(A\cap B) = 0$

$B\cap C = \{22, 24, 26, 28, 30, 32, 34\}$

$n(B\cap C) = 7$ $P(B\cap C) = \frac{7}{35}$

$C\cap A = \{2\}$ $n(C\cap A) = 1,$ $P(C\cap A) = \frac{1}{35}$

$A\cap B\cap C = \{\}$ $n(A\cap B\cap C) = 0,$ $P(A\cap B\cap C) = 0$

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$$

$$= \frac{8}{35}+\frac{12}{35}+\frac{17}{35}-0-\frac{7}{35}-\frac{1}{35}+0$$

$$= \frac{37}{35}-\frac{8}{35} = \frac{29}{35}$$

Answer: P(a boy with prime roll number or a girl with composite number or an even roll number) $= \frac{29}{35}$

Multiple choice questions

1. Which of the following is not a measure of dispersion?

(1) Range

(2) Standard deviation

(3) Arithmetic mean

(4) Variance

2. The range of the data 8, 8, 8, 8, 8 . . . 8 is

(1) 0

(2) 1

(3) 8

(4) 3

Solution

Range = L - S = 8 - 8 = 0

3. The sum of all deviations of the data from its mean is

(1) Always positive

(2) always negative

(3) zero

(4) non-zero integer

Solution: $\Sigma(x-\bar{x})=0$

4. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is

(1) 40000

(2) 160900

(3) 160000

(4) 30000

Solution:

$n = 100 \quad \bar{x} = 40 \quad \sigma = 3$

$$\sigma^2 = \frac{\sum x^2}{n} - (\bar{x})^2$$

$$9 = \frac{\sum x^2}{100} - (40)^2$$

$$\frac{\sum x^2}{100} = 9 + 1600 = 1609$$

$$\Sigma x^2 = 160900$$

5. Variance of first 20 natural numbers is

(1) 32.25

(2) 44.25

(3) 33.25

(4) 30

Solution:

$$\sigma = \frac{n^2 - 1}{12} = \frac{20^2 - 1}{12} = \frac{400 - 1}{12} = \frac{399}{12} = 33.25$$

6. The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is

(1) 3

(2) 15

(3) 5

(4) 225

Solution:

$\sigma = 3$

new $\sigma = 3 \times 5 = 15$

new $\sigma^2 = 15^2 = 225$

7. If the standard deviation of x, y, z is p then the standard deviation of $3x + 5$, $3y + 5$, $3z + 5$ is

(1) $3p + 5$

(2) $3p$

(3) $p + 5$

(4) $9p + 15$

Solution:

S.D unchaged when 5 added to x, y, z and it is multiplied by 3 new S.D is also multiplied by 3.

new σ is 3p

8. If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is

(1) 3.5

(2) 3

(3) 4.5

(4) 2.5

Solution:

$$\sigma = \frac{C.V \times \bar{x}}{100} = \frac{87.5 \times 4}{100} = 3.5$$

9. Which of the following is incorrect?

(1) $P(A) > 1$

(2) $0 \leq P(A) \leq 1$

(3) $P(\phi) = 0$

(4) $P(A) + P(\bar{A}) = 1$

10. The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is

(1) $\frac{q}{p+q+r}$ (2) $\frac{p}{p+q+r}$

(3) $\frac{p+q}{p+q+r}$ (4) $\frac{p+r}{p+q+r}$

Ans: (2)

Solution:

$n(S) = p+q+r$

$P(\text{Selecting red marble}) = \frac{p}{p+q+r}$

11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is

(1) 3/10

(2) 7/10

(3) 3/9

(4) 7/9

Solution:

S = Unit digits = { 0, 1, ...9} n(S) = 10

P(page number chosen is less than 7) = $\frac{7}{10}$

12. The probability of getting a job for a person is *x*/3 . If the probability of not getting the job is 2/3 then the value of *x* is

(1) 2

(2) 1

(3) 3

(4) 1.5

Solution:

P(getting a job)+P(not getting job) = 1

$\frac{x}{3}+\frac{2}{3} = 1 \Rightarrow x+2 = 3 \Rightarrow x = 3-2 = 1$

13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is 1/9 , then the number of tickets bought by Kamalam is

(1) 5

(2) 10

(3) 15

(4) 20

Solution:

$n(S) = 135 \quad P(A) = \frac{1}{9}$

$\frac{n(A)}{n(S)} = \frac{1}{9} \Rightarrow \frac{n(A)}{135} = \frac{1}{9}$

$n(A) = \frac{135}{9} = 15$

14. If a letter is chosen at random from the English alphabets $\{a,b,...,z\}$, then the probability that the letter chosen precedes x

(1) 12/13

(2) 1/13

(3) 23/26

(4) 3/26

Solution:

n(S) = 26

P(letter chosen precedes x) = $\frac{23}{26}$

15. A purse contains 10 notes of ₹2000, 15 notes of ₹500, and 25 notes of ₹200. One note is drawn at random. What is the probability that the note is either a ₹500 note or ₹200 note?

(1) 1/5

(2) 3/10

(3) 2/3

(4) 4/5

Solution:

n(S) = 10+15+25 = 50

P(either Rs. 500 or Rs. 200 note) = $\frac{15}{50}+\frac{25}{50}=\frac{40}{50}=\frac{4}{5}$

Unit Exercise

1. The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Compute the missing frequencies f_1 and f_2.

Class Interval	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

Solution: $\Sigma f = 50$ $\bar{x} = 62.8$

C.I	Mid value x	f	$d = x-A$ $d = x-70$	fd
0–20	10	5	–60	–300
20–40	30	f_1	–40	$-40f_1$
40–60	50	10	–20	–200
60–80	$\boxed{70} = A$	f_2	0	0
80–100	90	7	20	140
100–120	110	8	40	320
		$\Sigma f = 30+f_1+f_2$		$\Sigma fd = -40-40f_1$

$$\bar{x} = A + \frac{\sum fd}{\sum f}$$

$$62.8 = 70 + \frac{-40-40f_1}{50}$$

$$62.8-70 = \frac{-40-40f_1}{50}$$

$$-7.2 \times 50 = -40-40f_1$$

$$-360 = -40-40f_1$$

$$-360+40 = -40f_1$$

$$-40f_1 = -320$$

$$f_1 = \frac{\cancel{-320}f_1}{\cancel{-40}}$$

$$\Sigma f = 30+f_1+f_2$$

$$50 = 30+f_1+f_2$$

$$f_1+f_2 = 50-30$$

$$f_1+f_2 = 20$$

$$8+f_2 = 20$$

$$f_2 = 20-8 = 12$$

Answer: $f_1 = 8, f_2 = 12$

2. The diameter of circles (in mm) drawn in a design are given below.

Diameters	33-36	37-40	41-44	45-48	49-52
Number of circles	15	17	21	22	25

Calculate the standard deviation.

Solution:

C.I	x_i	f_i	$d_i = x_i - A$	$f_i d_i$	$f_i d_i^2$
33–36	34.5	15	–8	–120	960
37–40	38.5	17	–4	–68	272
41–44	42.5	21	0	0	0
45–48	46.5	22	4	88	352
49–52	50.5	25	8	200	1600
		$N = 100$		$\Sigma f_i d_i = 100$	$\Sigma f_i d_i^2 = 3184$

$$\sigma = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

$$= \sqrt{\frac{3184}{100} - \left(\frac{100}{100}\right)^2}$$

$$= \sqrt{31.84 - 1}$$

$$= \sqrt{30.84} = 5.55$$

$$\sigma = 5.55$$

Answer: Standard deviation $\sigma = 5.55$

3. The frequency distribution is given below.

x	k	$2k$	$3k$	$4k$	$5k$	$6k$
f	2	1	1	1	1	1

Solution:

Variance = 160

x_i	f_i	$d_i = x_i - A$ $d_i = x_i - 4k$	$f_i d_i$	$f_i d_i^2$
k	2	-3k	-6k	$18k^2$
2k	1	-2k	-2k	$4k^2$
3k	1	-k	-k	k^2
$\boxed{4k} = A$	1	0	0	0
5k	1	k	k	k^2
6k	1	2k	2k	$3k^2$
	N = 7		$\Sigma f_i d_i = -6k$	$\Sigma f_i d_i^2 = 28k^2$

$$\sigma^2 = \frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2$$

$$= \frac{28k^2}{7} - \left(\frac{-6k}{7}\right)^2$$

$$= \frac{28k^2}{7} - \frac{36k^2}{49}$$

$$= \frac{196k^2 - 36k^2}{49} = \frac{160k^2}{49}$$

$$k^2 = \frac{49 \times 160}{160} = 49$$

$$k = \sqrt{49} = 7 \qquad [\because K \text{ is positive}]$$

Answer: Value of k = 7

4. The standard deviation of some temperature data in degree celsius (^oC) is 5. If the data were converted into degree Farenheit (^oF) then what is the variance?

Solution:

S.D $\sigma_C = 5^oC$

Converted to Fahreheit

$$F = \left(\frac{9}{5}C\right) + 32$$

$\sigma_F \frac{9}{5}\sigma_C$ [∴IF we add (or) subtract the new S.D won't change)

$\therefore \sigma_F = \frac{9}{5} \times 5 = 9_2$ Variance $= 9^2 = 81$

Answer: Variance = 81

5. If for a distribution, $\sum (x - 5) = 3$, $\sum (x - 5)^2 = 43$, and total number of observations is 18, find the mean and standard deviation.

Solution: $\Sigma(x-5) = 3$

$\Sigma(x-5)^2 = 43$ $\quad n = 18$

$\Sigma(x-5) = 3$

$\Sigma x - 5\Sigma 1 = 3$

$\Sigma x - 5 \times 18 = 3$

$\Sigma x - 90 = 3$

$\Sigma x = 3 + 90 = 93$

$$\text{Mean } \bar{x} = \frac{\sum x}{n} = \frac{93}{18}$$

$\Sigma(x-5)^2 = 43$

$\Sigma(x^2 - 10x + 25) = 43$

$\Sigma x^2 - 10\Sigma x + 25\Sigma 1 = 43$

$\Sigma x^2 - 10 \times 93 + 25 \times 18 = 43$

$\Sigma x^2 - 930 + 450 = 43$

$\Sigma x^2 = 43 + 930 - 450$

$\Sigma x^2 = 523$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$= \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$$

$$= \sqrt{\frac{523}{18} - (5.17)^2}$$

$$= \sqrt{29.06 - 26.73}$$

$$= \sqrt{2.33} = 1.53$$

Answer: mean = 5.17,

standard deviation = 1.53

6. Prices of peanut packets in various places of two cities are given below. In which city, prices were more stable?

Prices in city A	20	22	19	23	16
Prices in city B	10	20	18	12	15

Solution: City A: $\bar{x} = \frac{20+22+19+23+16}{5} = \frac{100}{5} = 20$

x_i	$d_i = x_i - \bar{x}$ $d_i = x_i - 20$	d_i^2
16	-4	16
19	-1	1
20	0	0
22	2	4
23	3	9
		$\Sigma d^2 = 30$

$$\sigma^2 = \sqrt{\frac{\sum d_i^2}{n}}$$

$$= \sqrt{\frac{30}{5}} = \sqrt{6} = 2.45$$

$$C.V_1 = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{2.45}{20} \times 10\cancel{0}$$

$$= 2.45 \times 5 = 12.25$$

$$C.V_1 = 12.25$$

City B: $\bar{x} = \frac{10+20+18+12+15}{5} = \frac{75}{5} = 15$

x_i	$d_i = x_i - \bar{x}$ $d_i = x_i - 20$	d_i^2
16	-4	16
19	-1	1
20	0	0
22	2	4
23	3	9
		$\Sigma d^2 = 30$

$$\sigma^2 = \sqrt{\frac{\sum d_i^2}{n}}$$

$$= \sqrt{\frac{68}{5}} = \sqrt{13.6} = 3.69$$

$$C.V_2 = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{3.69}{15} \times 100$$

$$= \frac{369}{15} = 24.6$$

$$C.V_2 = 24.6$$

$C.V_1 = 12.25$ $C.V_2 = 24.6$

$C.V_1 < C.V_2$

Answer: Prices in City A were more stable

7. If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.

Solution:

range = 20

$L - S = 20$ -----(1)

Coefficient of range = 0.2

$$\frac{L-S}{L+S} = 0.2$$

$$\frac{20}{L+S} = 0.2$$

$$L+S = \frac{20}{0.2}$$

$$= \frac{200}{2} = 100$$

$L+S = 100$ ----(2)

Solving (1) & (2)

$L - S = 20$

$L + S = 100$

$2L = 120$

$$L = \frac{120}{2} = 60$$

$S = 100 - 60 = 40$

Answer: Largest value = 60

Smallest value = 40

8. If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.

Solution:

Two dice are rolled.

$S = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$n(S) = 36$

Let A be the event of getting the product of face value 6.

$A = \{(1,6)\ (2,3)\ (3,2)\ (6,1)\}$

$n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36}$$

Let B be the event of getting the difference of face values 5.

$B = \{(1,6)\ (6,1)\} \Rightarrow n(B) = 2$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36}$$

$A \cap B = \{(1,6)\ (6,1)\}$

$n(A \cap B) = 2$

$$P(A \cap B) = \frac{2}{36}$$

Using addition theorem of probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{4}{36} + \frac{2}{36} - \frac{2}{36}$$

$$= \frac{4}{36} = \frac{1}{9}$$

$$P(A \cup B) = \frac{1}{9}$$

Answer: P(product of face value 6 (or) difference of face values 5) $= \frac{1}{9}$

9. In a two children family, find the probability that there is at least one girl in a family.

Solution:

In a family, two children are there

S = { father, mother and two children}

n(S) = 4

Let A be the event of selecting at least one girl in a family.

n(A) = 3[Since two children may be girl and mother]

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

Answer: P(at least one girl) = $\frac{3}{4}$

10. A bag contains 5 white and some black balls. If the probability of drawing a black ball from the bag is twice the probability of drawing a white ball then find the number of black balls.

Solution:

A bag contains 5 white and some black balls.

Let the number of black balls be x

n(S) = 5+x

Let B & W denote the event of drawing a black and white balls.

$$\text{Given } P(B) = 2P(W)$$

$$\frac{n(B)}{n(S)} = 2\frac{n(W)}{n(S)}$$

$$\frac{x}{5+x} = 2\left(\frac{5}{5+x}\right)$$

$$\frac{x}{5+x} = \frac{10}{5+x} \Rightarrow x = 10$$

Answer: number of black balls = 10

11. The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Tamil examination?

Solution:

Let E and T denote the event of passing English and Tamil examination.

$P(E \cap T) = 0.5 \quad P(\overline{E} \cap \overline{T}) = 0.1$

$P(E) = 0.75$

We have $P(\overline{E} \cap \overline{T}) = P(\overline{E \cup T}) = 1 - P(E \cup T)$

$P(E \cup T) = P(E) + P(T) - P(E \cap T)$

$= 0.75 + P(T) - 0.5$

$P(E \cup T) = 0.25 + P(T)$

$P(\overline{E} \cap \overline{T}) = 1 - P(E \cup T)$

$0.1 = 1 - [0.25 + P(T)]$

$0.1 = 1 - 0.25 - P(T)$

$0.1 = 0.75 - P(T)$

$P(T) = 0.75 - 1$

$P(T) = 0.65$

$P(T) = \frac{65}{100} = \frac{13}{20}$

Answer:

Probability of passing Tamil = $\frac{13}{20}$

12. The King, Queen and Jack of the suit spade are removed from a deck of 52 cards. One card is selected from the remaining cards. Find the probability of getting (i) a diamond (ii) a queen (iii) a spade (iv) a heart card bearing the number 5.

Solution:

King, Queen and Jack of spade are removed

n(S) = 52–3 = 49

i) Let A be the event of getting a diamond card

n(A) = 13

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{49}$$

ii) Let B be the event of getting a queen card

n(B) = 3

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{49}$$

iii) Let C be the event of getting a spade card

n(C) = 10

$$P(C) = \frac{n(C)}{n(S)} = \frac{10}{49}$$

iv) Let D be the event of getting 5 of heart card.

n(D) = 1

$$P(D) = \frac{n(D)}{n(S)} = \frac{1}{49}$$

Points to Remember

- Range = L–S (L - Largest value, S - Smallest value)
- Coefficient of range = $\frac{L-S}{L+S}$; Variance $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$
- Standard deviation $\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$
- Standard deviation (ungrouped data)

(i) Direct method $\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$

(ii) Mean method $\sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$

(iii) Assumed mean method $\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$

(iv) Step deviation method $\sigma = c \times \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$

- Standard deviation of first *n* natural numbers $\sigma = \sqrt{\frac{n^2 - 1}{12}}$
- Standard deviation (grouped data)

(i) Mean method $\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}}$

(ii)Assumed mean method $\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

(iii) Step deviation method $\sigma = C \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

- Coefficient of variation C.V = $\frac{\sigma}{\bar{x}} \times 100\%$
- If the C.V. value is less, then the observations of corresponding data are consistent.

If the C.V. value is more then the observations of corresponding are inconsistent.

- In a random experiment, the set of all outcomes are known but exact outcome is not known.
- The set of all possible outcomes is called sample space.
- A, B are said to be mutually exclusive events if $A \cap B = \phi$
- Probability of event E is $P(E) = n(E)/n(S)$

(i) The probability of sure event is 1 and the probability of impossible event is 0.

(ii) $0 \leq P(E) \leq 1$;(iii) $P(\bar{E}) = 1 - P(E)$

- If A and B are mutually exclusive events then $P(A \cup B) = P(A) + P(B)$.

(i) $P(A \cap \bar{B}) = P(\text{only}A) = P(A) - P(A \cap B)$

(ii) $P(A \cap \bar{B}) = P(\text{only}B) = P(B) - P(A \cap B)$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, for any two events A, B.
- For any three events A, B, C

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$