Exercise 1.1

1. Find $A \times B$, $A \times A$ and $B \times A$

(i) $A = \{2, -2, 3\}$ and $B = \{1, -4\}$ (ii) $A = B = \{p, q\}$ (iii) $A = \{m, n\}$; $B = \phi$

Solution:

i) $A = \{2, -2, 3\}$ and $B = \{1, -4\}$

$A \times B = \{2, -2, 3\} \times \{1, -4\}$

$= \{(2, 1) (2, -4) (-2, 1) (-2, -4) (3, 1),)(3, -4)\}$

$A \times A = \{2, -2, 3\} \times \{2, -2, 3\}$

$= \{(2, 2) (2, -2) (2, 3) (-2, 2) (-2, -2) (-2, 3) (3, 2) (3, -2) (3, 3)\}$

$B \times A = \{1, -4\} \times \{2, -2, 3\}$

$= \{(1, 2) (1, -2) (1, 3) (-4, 2) (-4, -2) (-4, 3)\}$

Note:
HERE
$A \times A = A \times B = B \times A$
Since the element
of the set A and B are equal

ii) $A = B = \{p, q\}$

$A \times B = \{p, q\} \times \{p, q\}$

$= \{(p, p) (p, q) (q, p) (q, q)\}$

$A \times A = \{p, q\} \times \{p, q\}$

$= \{(p, p) (p, q) (q, p) (q, q)\}$

$B \times A = \{p, q\} \times \{p, q\}$

$= \{(p, p) (p, q) (q, p) (q, q)\}$

Note:
HERE
$A \times A = A \times B = B \times A$
Since the element
of the set A and B are equal

iii) If $A = \{m, n\}$; $B = \emptyset$

$A \times B = \{\}$

$A \times A = \{m, n\} \times \{m, n\}$

$= \{(m, m) (m, n) (n, m) (n, n)\}$

$B \times A = \{\}$

Note:
HERE
$A \times A = A \times B = B \times A$
Since the element
of the set A and B are equal

2. Let $A = \{1,2,3\}$ and $B = \{x \mid x$ is a prime number less than 10$\}$.
Find $A \times B$ and $B \times A$.

Solution:

Let $A = \{1, 2, 3\}$; $B = \{2, 3, 5, 7\}$

$A \times B = \{(1, 2) (1, 3) (1, 5) (1, 7) (2, 2) (2, 3) (2, 5) (2, 7) (3, 2) (3, 3) (3, 5) (3, 7)\}$

$B \times A = \{2, 3, 5, 7\} \times \{1, 2, 3\}$

$= \{(2, 1) (2, 2) (2, 3) (3, 1) (3, 2) (3, 2) (3, 3) (3, 3) (5, 1) (5, 2) (5, 3) (5, 3) (7, 1) (7, 2) (7, 3)\}$

3. If $B \times A = \{(-2, 3),(-2, 4),(0, 3),(0, 4),(3, 3),(3, 4)\}$ find A and B.

Solution:

B = {set of all first coordinates of elements of B × A}

$\therefore B = \{(-2, 0, 3)\}$

A = {set of all second co ordinates of element of B × A}

$\therefore A = \{3, 4\}$

4. If $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$, Show that $A \times A = (B \times B) \cap (C \times C)$.

Solution:

L.H.S $A \times A$

If $A = \{5, 6\}$ and $A = \{5, 6\}$

$A \times A = \{(5, 5) (5, 6) (6, 5) (6, 6)\}$ ———— ❶

R.H.S $(B \times B) \cap (C \times C)$

$B \times B = \{4, 5, 6\} \times \{4, 5, 6\}$

$= \{(4, 4) (4, 5) (4, 6) (5, 4) (5, 5) (5, 6) (6, 4) (6, 5) (6, 6)\}$

$C \times C = \{5, 6, 7\} \times \{5, 6, 7\}$

$= \{(5, 5) (5, 6) (5, 7) (6, 5) (6, 6) (6, 6) (6, 7) (7, 5) (7, 6) (7, 7)\}$

$(B \times B) \cap (C \times C) = \{(5, 5) (5, 6) (6, 5) (6, 6)\}$ ———— ❷

$\therefore A \times A = (B \times B) \cap (C \times C)$

L.H.S = R.H.S

5. Given A={1,2,3}, *B* = {2,3,5}, *C* = {3,4} and *D* = {1,3,5}, check if

$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

Solution:

L.H.S $(A \cap C) \times (B \cap D)$

$A \cap C = \{1, 2, 3\} \cap \{3, 4\}$

$= \{3\}$

$B \cap D = \{2, 3, 5\} \cap \{1, 3, 5\}$

$= \{3, 5\}$

$(A \cap C) \times (B \cap D) = \{3\} \times \{3, 5\}$

$= \{(3,3)\ (3, 5)$ ——— ❶

R.H.S $(A \times B) \cap (C \times D)$

$A \times B = \{1, 2, 3\} \times \{2, 3, 5\}$

$= \{ (1, 2)\ (1, 3)\ (1, 5)\ (2, 2)\ (2, 3)\ (2, 5)\ (3, 2)\ (3, 3)\ (3, 5)\}$

$C \times D = \{ (3, 4) \times (1, 3, 5)\}$

$= \{(3, 1)\ (3, 3)\ (3, 5)\ (4, 1)\ (4, 3)\ (4, 5)\}$

$(A \times B) \cap (C \times D) = \{ (3, 3)\ (3, 5) \}$ ——— ❷

$\therefore (A \cap C) \times (B \cap D) = (A \times B) \cap (C \cap D)$

L.H.S = R.H.S $\Rightarrow$ True

6. Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Solution:

i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

L.H.S $A \times (B \cup C)$

$B \cup C = \{2, 3, 4\} \cup \{3, 5\}$

$= \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$

$= \{(0, 2) (0, 3) (0, 4) (0, 5) (1, 2) (1, 3) (1, 4) (1, 5)\}$ —— ❶

R.H.S $(A \times B) \cup (A \times C)$

$A \times B = \{0, 1\} \times \{2, 3, 4\}$

$= \{(0, 2) (0, 3) (0, 4) (1, 2) (1, 3) (1, 4)\}$

$A \times C = \{0, 1\} \times \{3, 5\}$

$= \{(0, 3) \quad (0, 5) (1, 3) (1, 5)\}$

$(A \times B) \cup (A \times C) = \{(0, 2) (0, 3) (0, 4) (0, 5) (1, 2) (1, 3) (1, 4) (1, 5)\}$ —— ❷

$\therefore$ L.H.S = R.H.S

ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

L.H.S $A \times (B \cap C)$

$B \cap C = \{2, 3, 4\} \cap \{3, 5\}$

$= \{3\}$

$A \times (B \cap C) = \{0, 1\} \times \{3\}$

$= \{(0, 3) (1, 3)\}$ —— ❶

R.H.S $(A \times B) \cap (A \times C)$

$A \times B = \{0, 1\} \times \{2, 3, 4\}$

$= \{(0, 2) (0, 3) (0, 4) (1, 2) (1, 3) (1, 4)\}$

$A \times C = \{0, 1\} \times \{3, 5\}$

$= \{(0, 3) (0, 5) (1, 3) (1, 5)\}$

$(A \times B) \cap (A \times C) = \{(0, 3) (1, 3)\}$ —— ❷

L.H.S = R.H.S

iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

L.H.S $(A \cup B) \times C$

$A \cup B = \{0, 1\} \cup \{2, 3, 4\}$

$= \{0, 1, 2, 3, 4\}$

$(A \cup B) \times C = \{0, 1, 2, 3, 4\} \times \{3, 5\}$

$= \{(0, 3) (0, 5) (1, 3) (1, 5) (2, 3) (2, 5) (3, 3) (3, 5) (4, 3) (4, 5)\}$ —— ❶

R.H.S $(A \times C) \cup (B \times C)$

$(A \times C) = \{(0, 1) \times (3, 5)\}$

$= \{(0, 3) (0, 5) (1, 3) (1, 5)\}$

$B \times C = \{2, 3, 4\} \times \{3, 5\}$

$= \{(2, 3) (2, 5) (3, 3) (3, 5) (4, 3) (4, 5)\}$

$(A \times C) \cup (B \times C) = \{(0, 3) (0, 5) (1, 3) (1, 5) (2, 3) (2, 5) (3, 3) (4, 3) (4, 5)\}$ —— ❷

L.H.S = R.H.S (from 1 and 2)

7. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

Solution:

Let A = {1, 2, 3, 4, 5, 6, 7}   B = {2, 3, 5, 7}   C = {2}

i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

L.H.S $(A \cap B) \times C$

$A \cap B$ = {1, 2, 3, 4, 5, 6, 7} $\cap$ {2, 3, 5, 7}

= {2, 3, 5, 7}

$A \cap B \times C$ = {2, 3, 5, 7} × {2}

= {(2, 3) (3, 2) (5, 2) (7, 2)} —— ❶

R.H.S $(A \times C) \cap (B \times C)$

$A \times C$ = {1, 2, 3, 4, 5, 6, 7} × {2}

= {(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2) (7, 2)}

$B \times C$ = {2, 3, 5, 7} × {2}

= {(2, 2) (3, 2) (5, 2) (7, 2)}

$(A \times C) \cap B \times C$ = {(2, 2) (3, 2) (5, 2) (7, 2)} —— ❷

L.H.S = R.H.S

ii) $A \times (B - C) = (A \times B) - (A \times C)$

L.H.S $A \times (B - C)$

B – C = {2, 3, 5, 7} – {2}

= {3, 5, 7}

A × (B – C) = {1, 2, 3, 4, 5, 6, 7} × {3, 5, 7}

= {(1, 3) (1, 5) (1, 7) (2, 3) (2, 5) (2, 7) (3, 3) (3, 5) (3, 7) (4, 3) (4, 5) (4, 7) (5, 3) (5, 5) (5, 7) (6, 3) (6, 5) (6, 7) (7, 3) (7, 5) (7, 7) }—❶

R.H.S $(A \times B) - (A \times C)$

(A × B) = {1, 2, 3, 4, 5, 6, 7} × {2, 3, 5, 7}

= {(1, 2) (1, 3) (1, 5) (1, 7) (2, 2) (2, 3) (2, 5) (2, 7) (3, 2) (3, 3) (3, 5) (3, 7) (4, 2) (4, 3) (4, 5) (4, 7) (5, 2) (5, 3) (5, 5) (5, 7) (6, 2) (6, 3) (6, 5) (6, 7) (7, 2) (7, 3) (7, 5) (7, 7)}

A × C = {1, 2, 3, 4, 5, 6, 7} × {2}

= {(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2) (7, 2)}

(A × B) – (A × C)

={(1, 3) (1, 5) (1, 7) (2, 3) (2, 5) (2, 7) (3, 3) (3, 5) (3, 7) (4, 3) (4, 5) (4, 7) (5, 3) (5, 5) (5, 7) (6, 3) (6, 5) (6, 7) (7, 3) (7, 5) (7, 7)}—— ❷

L.H.S = R.H.S (From 1 and 2)}

Exercise 1.2

1. Let $A = \{1,2,3,7\}$ and $B = \{3,0,-1,7\}$, which of the following are relation from A to B ?

(i) $R_1 = \{(2,1), (7,1)\}$

(ii) $R_2 = \{(-1,1)\}$

(iii) $R_3 = \{(2,-1), (7,7), (1,3)\}$

(iv) $R_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$

Solution:-

i) Here $(2, 1)$ and $(7, 1) \notin A \times B$
Thus R_1 is not a relation from A to B

ii) Here $(-1, 1) \notin A \times B$
Thus R_2 is not a relation from A to B

iii) $R_3 \subseteq A \times B$
Thus R_3 is not a relation from A to B

iv) Here $(0, 3)$; $(0, 7) \notin A \times B$
Thus R_4 is not a relation from A to B

2. Let $A=\{1,2,3,4,...,45\}$ and R be the relation defined as "is square of" on A. Write R as a subset of $A \times A$. Also, find the domain and range of R.

Solution:

$1^2 = 1$; $2^2 = 4$; $3^2 = 9$; $4^2 = 16$;
$5^2 = 25$; $6^2 = 36$
$R = \{(1, 1) (2, 4) (3, 9) (4, 16) (5, 25) (6, 36)\}$
$R \in A \times A$
Domain of $R = \{1, 2, 3, 4, 5, 6\}$
Range of $R = \{1, 4, 9, 16, 25, 36\}$

3. A Relation R is given by the set $\{(x , y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$.

Determine its domain and range.

Solution:

Given y = $x + 3$

Put $x = 0$;	$y = 0 + 3 = 3$
Put $x = 1$;	$y = 1 + 3 = 4$
Put $x = 2$;	$y = 2 + 3 = 5$
Put $x = 3$;	$y = 3 + 3 = 6$
Put $x = 4$;	$y = 4 + 3 = 7$
Put $x = 5$;	$y = 5 + 3 = 8$

Doman of R = { 0, 1, 2, 3, 4, 5}

Range of R = { 3, 4, 5, 6, 7, 8}

4. Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x,y) | x = 2y, x \in \{2,3,4,5\}, y \in \{1,2,3,4\}$

(ii) $\{(x,y) | y = x+3, x, y$ are natural numbers $< 10\}$

Solution:

i) $x = 2y \Rightarrow y = \frac{x}{2}$

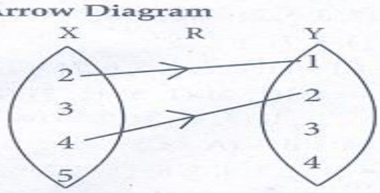
Put $x = 2$; $y = \frac{2}{2} = 1$

$x = 3$; $y = \frac{3}{2}$

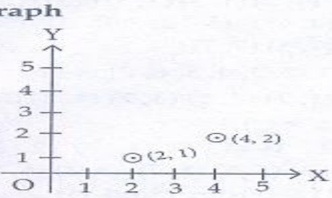
$x = 4$; $y = \frac{4}{2} = 2$

$x = 5$; $y = \frac{5}{2}$

a. Arrow Diagram



b. graph



C, A set in roster form

{(2, 1) (4, 2)}

ii) {(x, y)/ y = x + 3 are natural numbers < 10

$x = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$y = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Given $y = x + 3$

Put $x = 1;$ $y = 1 + 3 = 4$

$x = 2;$ $y = 2 + 3 = 5$

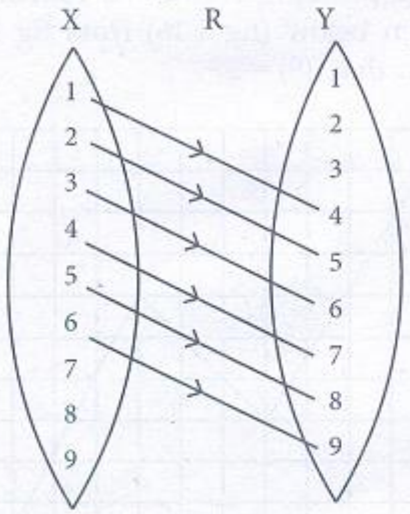
$x = 3;$ $y = 3 + 3 = 6$

$x = 4;$ $y = 4 + 3 = 7$

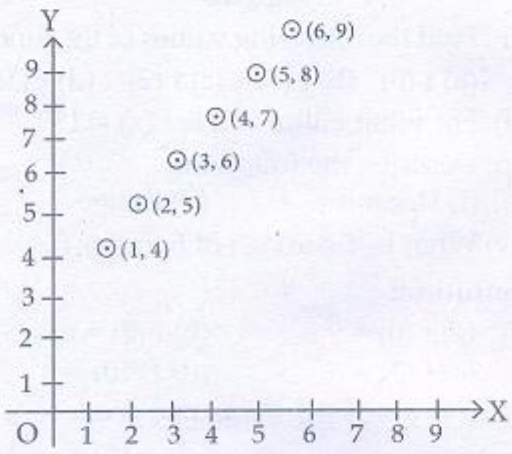
$x = 5;$ $y = 5 + 3 = 8$

$x = 6;$ $y = 6 + 3 = 9$

a, an arrow diagram



b, a graph



c, a set in roster

{(1, 4) (2, 5) (3, 6) (4, 7) (5, 8) (6, 9)}

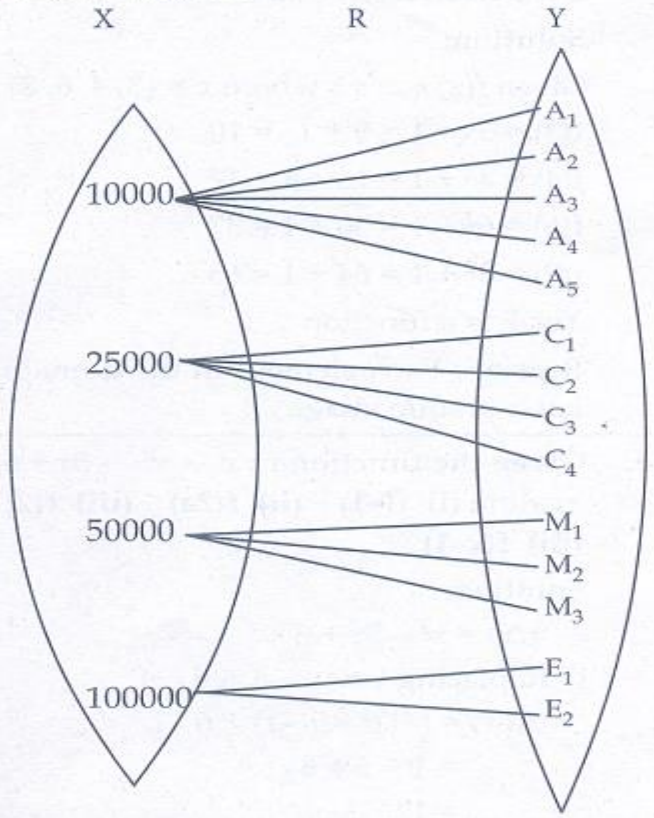
5. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹10,000, ₹25,000, ₹50,000 and ₹1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation R is defined by xRy, where x is the salary given to person y, express the relation R through an ordered pair and an arrow diagram.

Solution:

Ordered pair

$\{(10000, A_1)\ (10000, A_2)\ (10000, A_3)\ (10000, A_4)\ (10000, A_5)\ (25000, C_1)\ (25000, C_2)\ (25000, C_3)\ (25000, C_4)\ (50000, M_1)\ (50000, M_2)\ (50000, M_3)\ (100000, E_1)\ (100000, E_2)$

Arrow diagram



Exercise 1.3

1. Let $f = \{(x,y) \mid x,y \in \mathbf{N}$ and $y = 2x\}$ be a relation on N. Find the domain, co-domain and range. Is this relation a function?

Solution:

Given $y = 2x$ where $x, y \in N$

Put $x = 1$; $y = 2(1) = 2$

$x = 2$; $y = 2(2) = 4$

$x = 3$; $y = 2(3) = 6$

$x = 4$; $y = 2(4) = 8$ Ect.......

Domain = {1, 2, 3, 4.................}

Codomain = {1, 2, 3, 4.................}

Range = {2, 4, 6, 8.................}

Yes it is a function

Reason : – Each element of x has an unique image in y.

2. Let $X = \{3, 4, 6, 8\}$. Determine whether the relation $\mathbf{R} = \{(x, f(x)) \mid x \in X, f(x) = x^2 + 1\}$ is a function from X to $\mathbf{N}$?

Solution:

Given $f(x) = x^2+1$ where $x = \{3, 4, 6, 8\}$

$f(3) = 3^2 + 1 = 9 + 1 = 10$

$f(4) = 4^2 + 1 = 16 + 1 = 17$

$f(6) = 6^2 + 1 = 36 + 1 = 37$

$f(8) = 8^2 + 1 = 64 + 1 = 65$

Yes R is a function

Reason : Each element in the domain of f has a unique image

3. Given the function $f : x \to x^2 - 5x + 6$, evaluate

(i) $f(-1)$

(ii) $f(2a)$

(iii) $f(2)$

(iv) $f(x - 1)$

Solution:

$f(x) = x^2 - 5x + 6$

i) Replacing x with -1 we get

$f(-1) = (-1)^2 - 5(-1) + 6$

$= 1 + 5 + 6$

$= 12$

ii) Replacing x with 2a we get

$f(2a) = (2a)^2 - 5(2a) + 6$

$= 4a^2 - 10a + 6$

iii) Replacing x with 2 we get

$f(2) = (2)^2 - 5(2) + 6$

$= -6 + 6$

$= 0$

iv) Replacing x with x–1 we get

$f(x-1) = (x-1)^2 - 5(x-1) + 6$

$= x^2 - 2x + 1 - 5x + 5 + 6$

$= x^2 - 7x + 12$

4. A graph representing the function $f(x)$ is given in Fig.1.16 it is clear that $f(9) = 2$.

Fig. 1.16

(i) Find the following values of the function

(a) $f(0)$

(b) $f(7)$

(c) $f(2)$

(d) $f(10)$

(ii) For what value of x is $f(x) = 1$?

(iii) Describe the following (i) Domain (ii) Range

(iv) What is the image of 6 under f?

Solution:

i) (a) $f(0) = 9$ (b) $f(7) = 6$

(c) $f(2) = 6$ (d) $f(10) = 0$

ii) if $f(x) = 1$ the value of x is 9.5

iii) Domain = $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Range = $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

iv) The image of 6 under f is 5

5. Let $f(x) = 2x+5$. If $x \neq 0$ then find $\frac{f(x+2)-f(2)}{x}$

Solution:

Given $f(x) = 2x + 5$

$f(x+2) = 2(x+2) + 5$

$= 2x + 4 + 5$

$= 2x + 9$

$f(2) = 2(2) + 5$

$= 4 + 5 = 9$

$$\frac{f(x+2)-f(2)}{x} = \frac{2x+9-9}{x} = \frac{2x}{x} = 2$$

6. A function f is defined by $f(x) = 2x - 3$

(i) find $[f(0) + f(1)] / 2$.

(ii) find x such that $f(x) = 0$

(iii) find x such that $f(x) = x$

(iv) find x such that $f(x) = f(1 - x)$

Solution:

Given $f(x) = 2x - 3$

$$f(0) = 2(0) - 3 = 0 - 3 = -3$$

$$f(1) = 2(1) - 3 = 2 - 3 = -1$$

i) $\dfrac{f(0) + f(1)}{2} = \dfrac{-3-1}{2} = \dfrac{-4}{2} = -2$

ii) Given $f(x) = 0$

$$2x - 3 = 0$$

$$2x = 3 \Rightarrow x = \frac{3}{2}$$

iii) Given $f(x) = x$

$$2x - 3 = x$$

$$2x - x = 3$$

$$x = 3$$

iv) Given $f(x) = f(1-x)$

$$2x - 3 = 2(1-x) - 3$$

$$2x - 3 = 2 - 2x - 3$$

$$2x - 3 = -1 - 2x$$

$$2x + 2x = -1 + 3$$

$$4x = 2 \Rightarrow x = \frac{2}{4} = \frac{1}{2}$$

7. An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown (Fig.1.17). Express the volume V of the box as a function of x.

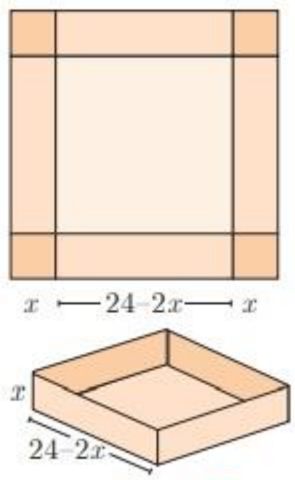


Fig. 1.17

Solution:

Given : length $= 24 - 2x$

breadth $= 24 - 2x$

height $= x$

Volume of the box $= l \times b \times h$

$= (24 - 2x) \times (24 - 2x) \times x$

$= (24 - 2x)^2 \times x$

$= (576 - 96x + 4x^2) \times x$

$= 4x^2 - 96x^2 + 576\,x$

8. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x\ 2) = (f(x))2$

Solution:

Given $f(x) = 3 - 2x$

$f(x^2) = 3 - 2x^2$

$[f(x^2)] = 3 - 2x^2$

$[f(x^2)] = (3 - 2x^2)$

Given $f(x^2) = [f(x^2)]^2$

$3 - 2x^2 = (3 - 2x)^2$

$3 - 2x^2 = 9 - 12x + 4x^2$

$\Rightarrow 4x^2 + 2x^2 - 12x = 3 - 9$

$6x^2 - 12x = -6$

$6x^2 - 12x + 6 = 0$

36	
-6	-6
6x	6x

Factorise the quadratic equation

$6x^2 - 12x + 6 = (6x - 6)(6x - 6)$

$(6x - 6) = 0$

$6x = 6 \Rightarrow x = 1$

9. A plane is flying at a speed of 500 km per hour. Express the distance *d* travelled by the plane as function of time *t* in hours.

Solution:

Given speed = 500 km / hr

time = t hours

Distance = speed × time

= 500 × t = 500 t

10. The data in the adjacent table depicts the length of a woman's forehand and her corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length(x) as $y = ax + b$, where a, b are constants.

Length 'x' of forehand (in cm)	Height 'y' (in inches)
45.5	65.5
35	56
45	65
50	69.5
55	74

(i) Check if this relation is a function.

(i) Find a and b.

(iii) Find the height of a woman whose forehand length is 40 cm.

(iv) Find the length of forehand of a woman if her height is 53.3 inches.

Solution:

i) Yes this relation is a function

ii) Given y = ax + b

put $x = 55$ y = 75;

75 = 55 a + b ——— ❶

Put $x = 42$; y = 62

62 = 42a + b ——— ❷

Solving ❶ and ❷

$$\begin{array}{l} 55a + b = 75 \\ 42a + b = 62 \\ (-) \quad (-) \quad (-) \\ \hline 13a = 13 \end{array}$$

$$a = \frac{13}{13}$$

a = 1

Subtitute a = 1 in equ ❶

75 = 55(1) + b

75 = 55 + b

b = 75 – 55 = 20

So a = 1 and b = 20

iii) forehand length (x) = 48cm

height (y) = ? [∴ a = 1b = 20]

Given y = ax + b

y = 1(48) + 20

= 48 + 20

= 68 inches

iv) height (y) = 60.54 inches

Forehand length (x) = ?

Given y = ax + b

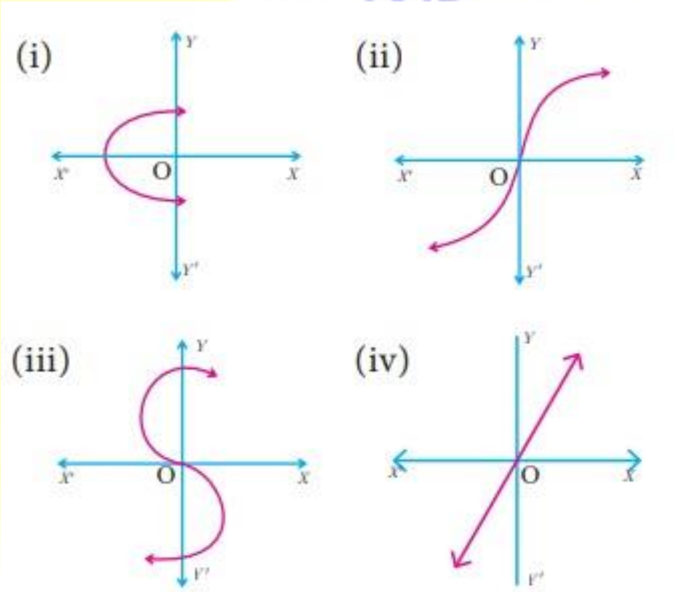
60.54 = (1)x + 20

x = 60.54 – 20

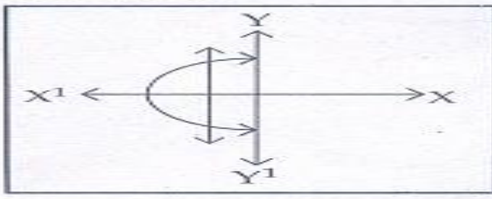
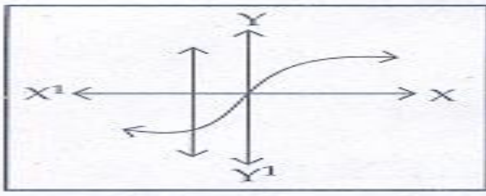
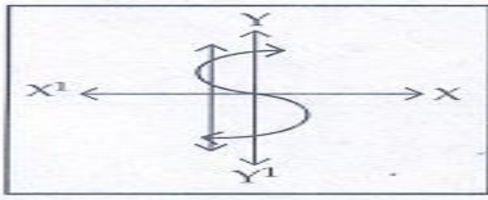
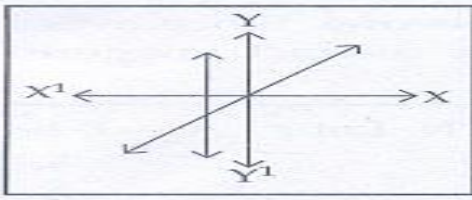
= 40.54 cm

Exercise 1.4

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.



Solution:

	Do not represent a function as the vertical line meet the curves in two points.
	Represent a function as the vertical line meet the curve in one point.
	Do not represent a function as the vertical line meet the curves in two points.
	Represent a function as the vertical line meet the curve in one point.

2. Let $f : A \rightarrow B$ be a function defined by $f(x) = (x/2) - 1$, where $A = \{2, 4, 6, 10, 12\}$, $B = \{0, 1, 2, 4, 5, 9\}$. Represent f by

(i) set of ordered pairs;

(ii) a table;

(iii) an arrow diagram;

(iv) a graph

Solution:

Given $f(x) = \frac{x}{2} - 1$

$f(2) = \frac{2}{2} - 1 = 1 - 1 = 0$

$f(4) = \frac{4}{2} - 1 = 2 - 1 = 1$

$f(6) = \frac{6}{2} - 1 = 3 - 1 = 2$

$f(10) = \frac{10}{2} - 1 = 5 - 1 = 4$

$f(12) = \frac{12}{2} - 1 = 6 - 1 = 5$

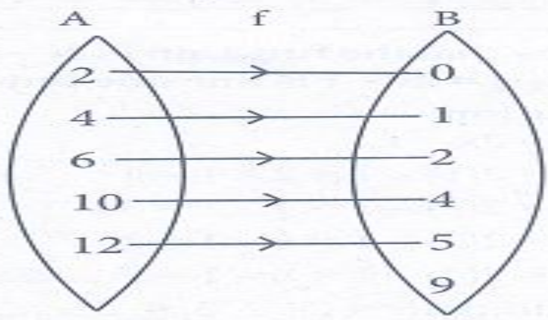
i) Set of ordered pairs

$f = \{(2, 0) (4, 1) (6, 2) (10, 4) (12, 5)\}$

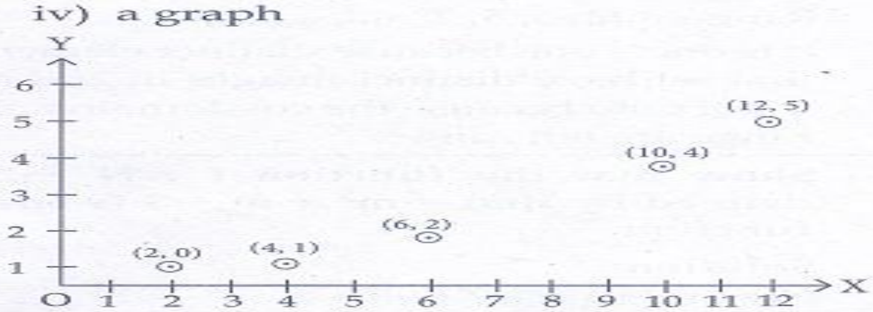
ii) a table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

iii) an arrow diagram



iv) a graph



3. Represent the function $f = \{(1, 2),(2, 2),(3, 2),(4, 3),(5, 4)\}$ through

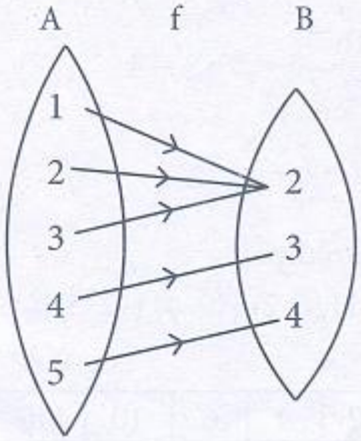
(i) an arrow diagram

(ii) a table form

(iii) a graph

Solution:

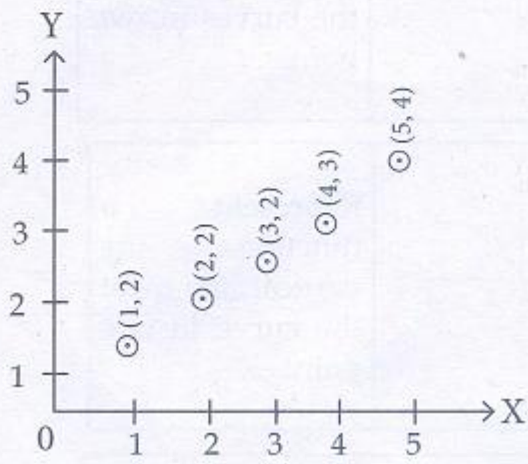
i) an arrow diagram



ii) a table

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

iii) a graph



4. Show that the function $f : \mathbf{N} \to \mathbf{N}$ defined by $f(x) = 2x - 1$ is one-one but not onto.

Solution:

$f(x) = 2x - 1$

$f(1) = 2(1) - 1 = 2 - 1 = 1$

$f(2) = 2(2) - 1 = 4 - 1 = 3$

$f(3) = 2(3) - 1 = 6 - 1 = 5$

$f(4) = 2(4) - 1 = 8 - 1 = 7$ .... etc

co-domain = {1, 2, 3, 4, 5.................}

Range = {1, 3, 5, 7.................}

It is one – one because distinct elements of first set have distinct images in 2nd set. It is not onto because the co-domain and the range are not same.

5. Show that the function $f : \mathbf{N} \to \mathbf{N}$ defined by $f(m) = m_2 + m + 3$ is one-one function.

Solution:

Given $f(m) = m^2 + m + 3$

$f(1) = (1)^2 + 1 + 3 = 5$

$f(2) = (2)^2 + 2 + 3 = 9$

$f(3) = (3)^2 + 3 + 3 = 15$...................etc

So the function f is one-one. Since every element in 1st set have distinct image in 2nd set.

6. Let $A = \{1, 2, 3, 4\}$ and $B = \mathbf{N}$ Let $f: A \to B$ be defined by $f(x) = x_3$ then,

(i) find the range of f

(ii) identify the type of function

Solution:

Given $f(x) = x^3$

$f(1) = 1^3 = 1$

$f(2) = 2^3 = 8$

$f(3) = 3^3 = 27$

$f(4) = 4^3 = 64$

i) Range = {1, 8, 27, 64}

ii) Type of function is one-one and into function

7. In each of the following cases state whether the function is bijective or not. Justify your answer .

(i) $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 2x + 1$)

(ii) $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 3 - 4x^2$

Solution:

i) Given f(x) = 2(x) + 1

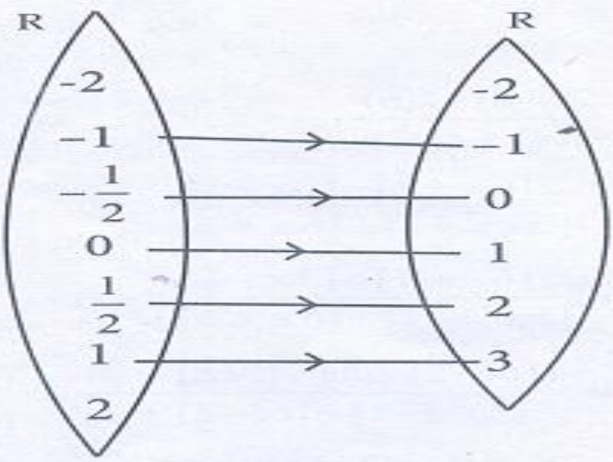
$f(0) = 2(0) + 1 = 1$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 2$

$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) + 1 = 0$

$f(1) = 2(1) + 1 = 3$

$f(-1) = 2(-1) + 1 = -1$



So it is a bijective function.

ii) $f : R \to R$ defined by $f(x) = 3 - 4x^2$

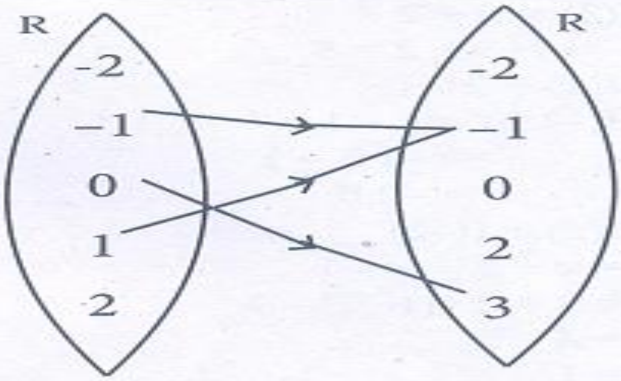
Solution:

Given f(x) = 3 – 4x2

$f(0) = 3 - 4(0) = 3$

$f(1) = 3 - 4(1)^2 = -1$

$f(-1) = 3 - 4(-1)^2 = -1$



So it is one–one but onto ∵ $0 \in R$ has no pre image in first set

So it is not bijective.

8. Let $A = \{-1,1\}$ and $B = \{0, 2\}$. If the function $f: A \to B$ defined by $f(x) = ax + b$ is

an onto function? Find a and b.

Solution:

Given $f(x) = ax + b$

$f(-1) = a(-1) + b = 0$

$-a + b = 0$ ——— (1)

$f(1) = a(1) + b = 2$

$a + b = 2$ ——— (2)

Solving (1) and (2) we get

$a = 1 ; b = 1$

co-domain and range are not equal So it not on-to function.

9. If the function f is defined by $f(x) = \begin{cases} x+2 & \text{if } x > 1 \\ 2 & \text{if } -1 \le x \le 1 \\ x-1 & \text{if } -3 < x < -1 \end{cases}$; find the values of

(i) $f(3)$

(ii) $f(0)$

(iii) $f(-1 \cdot 5)$

(iv) $f(2) + f(-2)$

$$f(x) = \begin{cases} x+2 & \text{if } x > 1 \\ 2 & \text{if } -1 \le x \le 1 \\ x-1 & \text{if } -3 < x < -1 \end{cases}$$

find the values of

i) f(3) ii) f(0)

iii) f(−1.5) iv) f(2) + f(−2)

Solution:

$$f(x) = \begin{cases} x+2 & \text{if } x = \{2, 3, 4, 5......\} \\ 2 & \text{if } x = \{-1, 0, 1\} \\ x-1 & \text{if } x \quad x = \{-2\} \end{cases}$$

i) $f(3) = x + 2$

$= 3 + 2 = 5$

ii) $f(0) = 2$

iii) $f(-1.5) = x - 1$

$= -1.5 - 1$

$= -2.5$

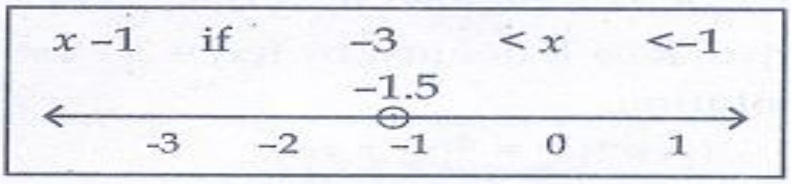
iv) $f(2) + f(-2)$

$= x + 2 + x - 1$

$= 2 + 2 + (-2) - 1$

$= 4 - 3$

$= 1$



10. A function $f : [-5, 9] \to \mathbf{R}$ is defined as follows: $f(x) =$

$$\begin{cases} 6x+1 & \text{if } -5 \le x < 2 \\ 5x^2 - 1 & \text{if } 2 \le x < 6 \\ 3x - 4 & \text{if } 6 \le x \le 9 \end{cases}$$

Find

(i) $f(-3) + f(2)$

(ii) $f(7) - f(1)$

(iii) $2f(4) + f(8)$

(iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

Solution:

$$f(x) = \begin{cases} 6x+1 & \text{if } x = \{-5,-4,-3,-2,-1,0,1\} \\ 5x^2-1 & \text{if } x = \{2,3,4,5\} \\ 3x-4 & \text{if } x = \{6,7,8,9\} \end{cases}$$

i) $f(-3) + f(2)$

$= (6x+1) + (5x^2-1)$

$= [6(-3)+1] + [5(2)^2 - 1]$

$= (-18+1) + [20-1]$

$= -17 + 19$

$= 2$

ii) $f(7) - f(1)$

$= (3x-4) - (6x+1)$

$= [3(7)-4] - [6(1)+1]$

$= [21-4] - [6+1]$

$= 17 - 7$

$= 10$

iii) $2f(4) + f(8)$

$= 2[5x^2-1] + [3x-4]$

$= 2[5(4)^2 - 1] + [3(8) - 4]$

$= (2 \times 79) + (24-4)$

$= 158 + 20$

$= 178$

iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

$= \frac{2[6x+1]-[3x-4]}{[5x^2-1]+[6x+1]}$

$= \frac{2[6x+1]-[3x-4]}{[5x^2-1]+[6x+1]}$

$= \frac{2[6(-2)+1]-[3(6)-4]}{[5(4)^2-1]+[6(-2)+1]}$

$= \frac{2[-12+1]-[18-4]}{[80-1]+[-12+1]}$

$= \frac{[2x-11]-14}{79-11}$

$= \frac{-22-14}{68}$

$= \frac{-36}{68}$

$= \frac{-9}{17}$

11. The distance *S* an object travels under the influence of gravity in time *t* seconds is given by $S(t) = 1/2\, gt^2 + at + b$ where, (*g* is the acceleration due to gravity), *a*, *b* are constants. Check if the function *S*(*t*) is one-one.

Solution:

Given $s(t) = \frac{1}{2} gt^2 + at + b$

Let time = secs

constants a, b = m, n

Let $S(t_1) = S(t_2)$

$$\frac{1}{2} gt_1^2 + mt_1 + n = \frac{1}{2} gt_2^2 + mt_2 + n$$

So $t_1 = t_2$

$\therefore S(t)$ is one – one

12. The function '*t*' which maps temperature in Celsius (*C*) into temperature in Fahrenheit (*F*) is defined by *t*(*C*) = *F* where *F* = 9/5 (*C*) + 32 . Find,

(i) *t*(0)

(ii) *t*(28)

(iii) *t*(-10)

(iv) the value of *C* when *t*(*C*) = 212

(v) the temperature when the Celsius value is equal to the Farenheit value.

Solution:

Given t(c) = F

$F = \frac{9c}{5} + 32$

i) $t(0) = \frac{0}{5} + 32$

ii) $t(28) = \frac{9 \times 28}{5} + 32$

$= \frac{252}{5} + 32$

$= 50.4 + 32 = 82.4$

iii) $t(-10) = \frac{9 \times -\cancel{10}^{2}}{\cancel{5}} + 32$

$= -18 + 32$

$= 24$

iv) The value of c when t(c) = 212

t(c) = 212

$\frac{9C}{5} + 32 = 212$

$9C + 160 = 212 \times 5$

$9C = 1060 - 160$

$9C = 900 \Rightarrow C = 100°C$

v) the temperature when the celsius value is equal to the Farenheit value

$C = \frac{9C}{5} + 32$

$5C = 9C + 160$

$4C = -160$

$C = -40$

Exercise 1.5

1. Using the functions f and g given below, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

(i) $f(x) = x - 6$, $g(x) = x^2$

(ii) $f(x) = 2/x$, $g(x) = 2x^2 - 1$

(iii) $f(x) = \frac{x+6}{3}$ $g(x) = 3 - x$

(iv) $f(x) = 3 + x$, $g(x) = x - 4$

(v) $f(x) = 4x^2 - 1$, $g(x) = 1 + x$

Solution:

i) $f(x) = x-6,\ g(x) = x^2$ $[\because f(x) = x - 6]$

$fog(x) = f(g(x)) = f(x^2)$

$= x^2 - 6$

$gof(x) = g(f(x)) = g(x-6)$

$= (x-6$

$\therefore fog \neq gof$

ii) $f(x) = \frac{2}{x};\ g(x) = 2x^2 - 1$ $[\because g(x) = x^2]$

$fog(x) = f(g(x)) = f(2x^2-1)$ $= \frac{2}{2x^2-1}$

$\left[\because f(x) = \frac{2}{x}\right]$

$gof(x) = g(f(x)) = g\left(\frac{2}{x}\right)$

$= 2\left(\frac{2}{x}\right) - 1$ $[\because g(x) = 2x^2 - 1]$

$= 2 \times \frac{4}{x^2} - 1$

$= \frac{8}{x^2} - 1$

$\therefore fog \neq gof$

iii) $f(x) = \frac{x+6}{3}$, $g(x) = 3 - x$

$fog(x) = f(g(x) = f(3-x)$

$= \frac{3-x+6}{3}$

$\because f(x) = \frac{x+6}{3}$

$gof(x) = g(f(x)) = g\left(\frac{x+6}{3}\right)$

$= 3-\left(\frac{x+6}{3}\right)$

$\because g(x) = 3-x$

$= \frac{9-x-6}{3}$

$= \frac{3-x}{3}$ $\therefore fog \neq gof$

iv) $f(x) = 3 + x;\ g(x) = x - 4$

$fog(x) = f(g(x)) = f(x-4)$

$= 3 + x - 4$ $\because g(x) = x - 4$

$= x - 1$

$gof(x) = g(f(x) = g(3 + x)$

$= 3 + x - 4$

$= x - 1$

So $fog = gof$

v) $f(x) = 4x^2 - 1,\ g(x) = 1 + x$

$fog(x) = f(g)(x)) = f(1 + x)$

$= 4(1 + x)^2 - 1$

$= 4(1 + 2x + x^2) - 1$ $\therefore f(x) = 4x^2 - 1$

$= 4 + 8x + 4x^2 - 1$

$= 4x^2 + 8x + 3$ $\therefore g(x) = 1 + x$

$gof(x) = g(f(x) = g(4x^2 - 1)$

$= 1 + 4x^2 - 1$

$= 4x^2$

So $fog \neq gof$

2. Find the value of k, such that $f \circ g = g \circ f$

(i) $f(x) = 3x + 2$, $g(x) = 6x - k$

(ii) $f(x) = 2x - k$, $g(x) = 4x + 5$

Solution :

i) $f(x) = 3x + 2 \qquad g(x) = 6x - K$

$fog = gof$ (Given)

$fog(x) = gof(x)$

$f(g(x)) = g(f(x))$

$f(6x - k) = g(3x + 2)$

$3(6x - k) + 2 = 6(3x + 2) - k$

$18x - 3k + 2 = 18x + 12 - k$

$18x - 3k + 2 = 18x + 12 - k$

$2k = -10$

$k = -5$

ii) $f(x) = 2x - k$, $g(x) = 4x + 5$

Given $fog = gof$

$fog(x) = gof(x)$

$f(g)(x) = g(f(x)$

$f(4x + 5) = g(2x - k)$

$2(4x + 5) - k = 4(2x - k) + 5$

$8x + 10 - k = 8x - 4k + 5$

$3k = -5$

$k = -\frac{5}{3}$

3. If $f(x) = 2x - 1$, $g(x) = [x+1]/2$ show that $f \circ g = g \circ f = x$

Solution:

$$fog\,(x) = f(g)(x))$$

$$= f\left(\frac{x+1}{2}\right)$$

$$= \not{2}\left(\frac{x+1}{\not{2}}\right) - 1$$

$$= x + 1 - 1 = x$$

$$gof = gof(x)$$

$$= g(f(x))$$

$$= g(2x-1)$$

$$= \frac{2x \not{-1} \not{+1}}{2}$$

$$= \frac{2x}{2}$$

$$= x$$

So $fog = gof = x$

Hence proved

4. (i) If $f(x) = x^2 - 1$, $g(x) = x - 2$ find a, if $g \circ f(a) = 1$.

(ii) Find k, if $f(k) = 2k - 1$ and $f \circ f(k) = 5$.

Solution:

$f(x) = x^2 - 1;\quad g(x) = x - 2$

Given $gof(a) = 1$

$g(f(a)) = 1$

$g(a^2 - 1) = 1$

$a^2 - 1 - 2 = 1$

$a^2 - 3 = 1$

$a^2 = 4$

$a = \pm 2$

ii) $f(k) = 2k - 1$ given

$fof(k) = 5$

$f(f(k)) = 5$

$f(2k-1) = 5$

$2(2k-1) - 1 = 5$

$4k - 2 - 1 = 5$

$4k - 3 = 5 \Rightarrow 4k = 8 \Rightarrow k = 2$

5. Let $A, B, C \subseteq \mathbf{N}$ and a function $f : A \to B$ be defined by $f(x) = 2x + 1$ and $g : B \to C$ be defined by $g(x) = x^2$. Find the range of $f \circ g$ and $g \circ f$.

Solution:

Given $f(x) = 2x + 1$ $g(x) = x^2$

$$fog = fog(x) = f(g(x)) = f(x^2) = 2x^2 + 1$$

$$gof = gof(x) = g(f(x)) = g(2x + 1) = (2x+1)^2$$

Range of fog and gof is

$\{y/y = 2x^2 + 1, x \in N\}; \{y/y = (2x+1)^2, x \in N\}$

6. Let $f(x) = x^2 - 1$. Find (i) $f \circ f$ (ii) $f \circ f \circ f$

Solution:

Given $f(x) = x^2 - 1$

a) $fof(x) = f(f(x))$

$= f(x^2 - 1)$

$= (x^2 - 1)^2 - 1$

$= x^4 - 2x^2 + 1 - 1$

$= x^4 - 2x^2$

b) $fofof = fofof(x)$

$= fof(f(x))$

$= fof(x^2 - 1)$

$= f(f(x^2 - 1))$

$= f[(x^2 - 1)^2 - 1]$

$= f[x^4 - 2x^2 + 1 - 1]$

$= f[x^4 - 2x^2] \Rightarrow (x^4 - 2x^2)^2 - 1$

7. If $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one?

Solution:

If $f(x) = f(y)$

$x^5 = y^5$

and hence $x = y$

Thus f is one - one

If $g(x) = g(y)$

$x^4 = y^4$

and hence $x = y$

Thus g is one - one

$$fog = fog(x)$$
$$= f(g(x))$$
$$= f(x^4)$$
$$= (x^4)5$$
$$= x^{20}$$

If $fog(x) = fog(y)$

$x^{20} = y^{20}$

hence $x = y$ $\Rightarrow$ Thus fog is one - one

8. Consider the functions $f(x)$, $g(x)$, $h(x)$ as given below. Show that $(f \circ g) \circ h = f \circ (g \circ h)$ in each case.

(i) $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$

(ii) $f(x) = x^2$, $g(x) = 2x$ and $h(x) = x + 4$

(iii) $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 3x - 5$

Solution:

i) $f(x) = x-1 \quad g(x) = 3x + 1 \quad h(x) = x^2$

$fog(x) = f(g(x)) = f(3x+1)$
$= (3x + 1 - 1)$
$= 3x$

$(fog)oh = (fog)\, oh\, (x)$
$= fog(h(x))$
$= fog(x^2)$
$= 3x^2$ ——— ❶

$goh(x) = g(h(x)) = g(x^2)$
$= 3x^2 + 1$

$fo(goh) = fo(goh(x))$
$= f(3x^2+1)$

$fo(goh) = fo(goh(x))$
$= f(3x^2 + 1)$
$= 3x^2 + 1 - 1$
$= 3x^2$ ——— ❷

from ❶ and ❷

$(fog)oh = fo(goh)$

ii) $f(x) = x^2 \quad g(x) = 2x \quad h(x) = x+4$

$fog(x) = f(g(x) = f(2x)$
$= (2x)^2$
$= 4x^2$

$(fog)oh = (fog)\, oh(x)$
$= fog(h(x)$
$= fog(x+4)$
$= 4(x+4)^2$
$= 4(x^2 + 8x + 16)$
$= 4x^2 + 32x + 64$ ——— ❶

$goh = goh = (x) = g(h(x))$
$= g\,(x + 4)$
$= 2\,(x + 4) = 2x + 8$

$fo\,(goh) = fo\,(goh)(x)$
$= fo\,(2x+8)$
$= (2x + 8)^2$
$= 4x^2 + 32x + 64$ ——— ❷

from ❶ and ❷

$(fog)oh = fo(goh).$

$f(x) = x - 4 \quad g(x) = x^2$

$h(x)=3x-5$

$fog(x) = fo(x^2)$
$= x^2 - 4$

$(fog)oh = (fog)oh(x)$
$= fog(3x - 5)$
$= (3x - 5)^2 - 4$
$= 9x^2 - 30x + 25 - 4$
$= 9x^2 - 30x + 21$ ——— ❶

$goh(x) = go(3x-5)$
$= (3x-5)^2$
$= 9x^2 - 30x + 25$

$fo(goh)x = fo(9x^2 - 30x + 25)$
$= 9x^2 - 30x + 25 - 4$
$= 9x^2 - 30x + 21$ ——— ❷

from ❶ and ❷

$(fog)oh = fo(goh)$

9. Let $f = \{(-1, 3),(0,-1),(2,-9)\}$ be a linear function from $\mathbb{Z}$ into $\mathbb{Z}$. Find $f(x)$.

Solution:

$f(x) = -4x - 1$

Check

$y = -4x - 1$

put $x = -1$

$= -4(-1) - 1$

put $x = 0$

$= -4(0) - 1$

$= 0 - 1$

$= -1$

10. In electrical circuit theory, a circuit $C(t)$ is called a linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$, where a,b are constants. Show that the circuit $C(t) = 3t$ is linear.

Solution:

Let $c(t_1) = 3t_1$ and $c(t_2) = 3t_2$

Given $c(at_1 + bt_2) = ac(t_1) + bc(t_2)$

$ac(t_1) + bc(t_2) = ac(t_1) + bc(t_2)$

$a \times 3t_1 + b \times 3t_2 = a \times 3t_1 + b \times 3t_2$

Satisfies the superposition principle

So $c(t) = 3t$ is linear.

Multiple choice questions

1. If $n(A\times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is

(1) 1

(2) 2

(3) 3

(4) 6

Solution

$n(A \times B) = 6$

$n(A) = 2$

$$n(B) = \frac{n(A\times B)}{n(A)} = \frac{6}{2} = 3$$

2. $A = \{a,b, p\}$, $B = \{2, 3\}$, $C = \{p,q,r,s\}$ then $n[(A \cup C)\times B]$ is

(1) 8

(2) 20

(3) 12

(4) 16

Solution

$A \cup C = \{a, b, p, q, r, s\} \Rightarrow n(AUC) = 6$

$B = \{2, 3\} \Rightarrow n(B) = 2$

$n[(A \cup C \times B)] = 6 \times 2 = 12$

3. If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$ then state which of the following statement is true.

(1) $(A\times C) \subset (B \times D)$

(2) $(B \times D) \subset (A\times C)$

(3) $(A\times B) \subset (A\times D)$

(4) $(D \times A) \subset (B \times A)$

Solution

$A \times C = \{1, 2\} \times \{5, 6\}$

$= \{(1, 5) (1, 6) (2, 5) (2, 6)\}$

$B \times D = \{1, 2, 3, 4\} \times \{5, 6, 7, 8\}$

$= \{(1, 5) (1, 6) (1, 7) (1, 8) (2, 5) (2, 6) (2, 7)$
$(3, 5) (3, 6) (3, 7) (3, 8) (4, 5) (4, 6) (4, 7) (4, 8)$

$(A \times C) \subset (B \times D)$

4. If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a set B, then the number of elements in B is

(1) 3

(2) 2

(3) 4

(4) 8

Solution

$2^{pq} = 1024$ $n(A) = 5 = p$

$2^{5q} = 2^{10}$ $n(B) = ? = q$

$5q = 10$

$q = 2$

5. The range of the relation $R = \{(x, x^2) \mid x$ is a prime number less than 13$\}$ is

(1) {2,3,5,7}

(2) {2,3,5,7,11}

(3) {4,9,25,49,121}

(4) {1,4,9,25,49,121}

Solution

Prime number less than 13 are

{2, 3, 5, 7, 11}

Given $f(x) = x^2$

$f(2) = 2^2 = 4$

$f(3) = 3^2 = 9$

$f(5) = 5^2 = 25$

$f(7) = 7^2 = 49$

$f(11) = 11^2 = 121$

Range = {4, 9, 25, 49, 121}

6. If the ordered pairs ($a + 2$, 4) and (5, $2a + b$) are equal then (a,b) is

(1) (2, –2)

(2) (5,1)

(3) (2,3)

(4) (3, –2)

Solution

$$a + 2 = 5$$
$$a = 5 - 2$$
$$a = 3$$
$$2a + b = 4$$
$$2(3) + b = 4$$
$$6 + b = 4$$
$$b = 4 - 6$$
$$b = -2$$

7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is

(1) M^n

(2) N^m

(3) $2^{mn} - 1$

(4) 2^{mn}

Solution

Total number of relations = 2^{pq} = 2^{mn}

8. If {(a, 8),(6,b)} represents an identity function, then the value of a and b are respectively

(1) (8,6)

(2) (8,8)

(3) (6,8)

(3) (6,6)

9. Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 8, 9, 10\}$. A function $f: A \to B$ given by $f = \{(1, 4),(2, 8),(3, 9),(4,10)\}$ is a

(1) Many-one function

(2) Identity function

(3) One-to-one function

(4) Into function

10. If $f(x) = 2x^2$ and $g(x) = 1/3x$ then $f \circ g$ is

(1) $\frac{3}{2x^2}$ (2) $\frac{2}{3x^2}$

(3) $\frac{2}{9x^2}$ (4) $\frac{1}{6x^2}$

Ans: (3)

Solution

$$fog(x) = f(g(x)) = f\left(\frac{1}{3x}\right)$$

$$= 2\left(\frac{1}{3x}\right)^2$$

$$2 \times \left(\frac{1}{9x^2}\right) \Rightarrow \frac{2}{9x^2}$$

11. If $f: A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to

(1) 7

(2) 49

(3) 1

(4) 4

12. Let f and g be two functions given by

$f = \{(0,1),(2, 0),(3,-4),(4, 2),(5, 7)\}$

$g = \{(0, 2),(1, 0),(2, 4),(-4, 2),(7, 0)\}$

then the range of $f \circ g$ is

(1) {0,2,3,4,5}

(2) {–4,1,0,2,7}

(3) {1,2,3,4,5}

(4) {0,1,2}

Solution

Every image of g has an image in f

So fog = {0, 1, 2}

13. Let $f(x) = \sqrt{1+x^2}$ then

(1) $f(xy) = f(x).f(y)$

(2) $f(xy) \geq f(x).f(y)$

(3) $f(xy) \leq f(x).f(y)$

(4) None of these

Solution

Let $f(x) = \sqrt{1+x^2}$

$f(y) = \sqrt{(1+y^2)}$

$f(xy) = \sqrt{(1+x^2y^2)}$

$f(xy) = f(x).\ f(y)$

$\sqrt{(1+x^2y^2)} = \sqrt{1+x^2}\ .\ \sqrt{(1+y^2)}$

$\sqrt{1+x^2y^2} = \sqrt{(1+x^2)(1+y^2)}$

square on both sides

$1 + x^2y^2 = (1+x^2)\ (1+y^2)$

$1 + x^2y^2 = 1 + x^2 + y^2 + x^2y^2$

so $1 + x^2y^2 \leq 1 + x^2 + y^2 + x^2y^2$

14. If $g = \{(1,1),(2, 3),(3, 5),(4, 7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of a and b are

(1) (–1,2))

(2) (2, –1)

(3) (–1, –2)

(4) (1,2)

Solution

$g(x) = \alpha x + \beta$

x y
(1, 1)

$g(1) = \alpha + \beta = 1$ ——— ❶

$g(2) = 2\alpha + \beta = 3$ ——— ❷

x y
(2, 3)

Saving ❶ and ❷

$\alpha = 2$ $\beta = -1$

15. $f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is

(1) linear

(2) cubic

(3) reciprocal

(4) quadratic

Solution

$f(x) = (x+1)^3 - (x-1)^3$

$= x^3 + 3x^2 +$

$3x + 1 - x^3 + 3x^2 - 3x + 1$

$= 6x^2 + 2$ is a quadratic function

UNIT EXERCISE

1. If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and (-2, 5) are equal, then find x and y.

Solution:

Given $x^2 - 3x = -2$

$x^2 - 3x + 2 = 0$

$(x - 1)(x + 2) = 0$

$x = 1$ and $x = 2$

+2

$\frac{-1}{x} \Big| \frac{-2}{x}$

Given $y^2 + 4y = 5$

$y^2 + 4y - 5 = 0$

$(y-1)(y+5) = 0$

$y - 1$ and $y = -5$

-5

$\frac{-1}{y} \Big| \frac{+5}{y}$

The value of x is 1 and 2

The value of y is 1 and -5

2. The cartesian product $A \times A$ has 9 elements among which (–1, 0) and (0,1) are found. Find the set A and the remaining elements of $A \times A$

Solution:

The set A = {5, 6, 7, 8}

The remaining elements of A × A is

{(-1, -1) (-1, 1) (0, -1) (0, 0) (1, -1) (1, 0) (1, 1)}

3. Given that $f(x) = \begin{cases} \sqrt{x-1} & x \geq 1 \\ 4 & x < 1 \end{cases}$. Find

(i) $f(0)$

(ii) $f(3)$

(iii) $f(a + 1)$ in terms of a.(Given that $a \geq 0$)

Solution:

$$f(x)=\begin{cases}\sqrt{x-1} & \text{if} \quad x=\{1,2,3,4.........\}\\ 4 & \text{if} \quad x=\{0,-1,-2.........\}\end{cases}$$

i) f(0) = 4

ii) $f(3) = \sqrt{x-1} = \sqrt{3-1} = \sqrt{2}$

iii) $f(a+1) = \sqrt{x-1} = \sqrt{a+1-1} = \sqrt{a}$

4. Let $A = \{9,10,11,12,13,14,15,16,17\}$ and let $f: A \rightarrow N$ be defined by $f(n)$ = the highest prime factor of $n \in A$. Write f as a set of ordered pairs and find the range of f.

Solution:

f(n) = the highest prime factor

f(9) = 3 (factors 1, 3, 9)

f(10) = 5 (factors 1, 2, 5)

f(11) = 11 (factors 1, 11)

f(12) = 3 (factors 1, 2, 3, 4, 6, 12)

f(13) = 13 (factors 1, 13)

f(14) = 7 (factors 1, 2, 7, 14)

f(15) = 5 (factors 1, 3, 5, 15)

f(16) = 2 (factors 1, 2, 4, 8, 16)

f(17) = 17 (factors 1, 17)

Set of ordered pair {(9, 3) (10, 5) (11, 11) (12, 3) (13, 13) (14, 7) (15, 5) (16, 2) (17, 7)}

Range of f = {(2, 3, 5, 11, 13, 17}

5. Find the domain of the function $f(x) = \sqrt{1+\sqrt{1-\sqrt{1-x^2}}}$

Solution:

$f(x)=\sqrt{1+\sqrt{1+\sqrt{1-\sqrt{1-x^2}}}}$

Domain = R

Reason : $f(0)=\sqrt{1+\sqrt{1-\sqrt{1-0^2}}}=\sqrt{1}$

$f(1)=\sqrt{1+\sqrt{1-\sqrt{1-1^2}}}=\sqrt{2}$

$f(2)=\sqrt{1+\sqrt{1-\sqrt{1-2^2}}}=\sqrt{3}$

$f(-1)=\sqrt{1+\sqrt{1-\sqrt{1-(-1^2)}}}=\sqrt{2}$

...........................so on.

6. If $f(x) = x^2$, $g(x) = 3x$ and $h(x) = x - 2$, Prove that $(f \circ g) \circ h = f \circ (g \circ h)$

Solution:

fog(x) = f(g(x) = f(3x)
= $(3x)^2$
= $9x^2$

(fog)oh(x) = fog(h(x)
= fog(x–2)
= $9\,(x-2)^2$
= $9[x^2 - 4x + 4]$
= $9x^2 - 36x + 36$ ——— ❶

goh(x) = g(h(x) = g(x–2)
= 3(x–2)
= 3x –6

fo(goh)(x) = fo(3x – 6)
= (3x – 6)2
= 9x2 – 36x + 36 ——— ❷

from ❶ and ❷ we get
(fog)oh = fo(goh)

7. Let $A = \{1, 2\}$ and $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify whether A×C is a subset of B×D?

Solution:

$A \times C = \{1, 2\} \times \{5, 6\}$

$= \{(1, 5) (1,6) (2, 5) (2, 6)\}$ —— ❶

$B \times D = \{1, 2, 3, 4\} \times \{5, 6, 7, 8\}$

$= \left\{ \begin{matrix} (1,5)\ (1,6)\ (1,7)\ (1,8)\ (2,5)\ (2,6) \\ (2,7)\ (2,8)\ (3,5)\ (3,6) \\ (3,7)\ (3,8)\ (4,5)\ (4,6)\ (4,7)\ (4,8) \end{matrix} \right\}$

—— ❷

from and it is clear

$A \times C \subset B \times D$

8. If $f(x) = [x-1]/[x+1]$, $x \neq 1$ show that $f(f(x)) = -1/x$ provided $x \neq 0$

Solution:

Given $f(x) = \dfrac{x-1}{x+1}$

$f(x) = \left(\dfrac{x-1}{x+1}\right)$

$= \dfrac{\dfrac{x-1}{x+1} - 1}{\dfrac{x-1}{x+1} + 1}$

$= \dfrac{\dfrac{x-1-(x+1)}{x+1}}{\dfrac{x-1+x+1}{x+1}}$

$= \dfrac{-2}{-2x} \Rightarrow = \dfrac{-1}{x}$ proved

9. The functions *f* and *g* are defined by $f(x) = 6x + 8$; $g(x) = [x-2]/3$

(i). Calculate the value of $gg(1/2)$

(ii) Write an expression for $gf(x)$ in its simplest form.

Solution:

Given $f(x) = 6x + 8$

$g(x) = \frac{x-2}{3}$

$g\,g\left(\frac{1}{2}\right) = g\left(\frac{x-2}{3}\right)$ where $x = \frac{1}{2}$

$= g\left(\frac{\frac{1}{2}-2}{3}\right)$

$= g\left(\frac{-1}{2}\right)$

$= \frac{x-2}{3}$ where $x = -\frac{1}{2}$

$= \frac{-\frac{1}{2}-2}{3}$

$= \frac{-\frac{5}{2}}{3} \Rightarrow \frac{-5}{2}\times\frac{1}{3} = \frac{-5}{6}$

ii) Write an expression for g f(x) in its simplest form

Given : $f(x) = 6x + 8$

$g(x) = \frac{x-2}{3}$

$f(x) = g(6x + 8)$

$= \frac{x-2}{3}$ where $x = 6x + 8$

$= \frac{6x+8-2}{3}$

$= \frac{6x+6}{3} \Rightarrow \frac{6(x+1)}{3}$

$= 2(x + 1)$

10. Write the domain of the following real functions

(i) $f(x) = \frac{2x+1}{x-9}$ (ii) $p(x) = \frac{-5}{4x^2+1}$

(iii) $g(x) = \sqrt{x-2}$

(iv) $h(x) = x + 6$

Solution:

i) $f(x) = \frac{2x+1}{x-9}$

Domain = R – {9}

ii) $p(x) = \frac{-5}{4x^2+1}$

Domain = R

iii) $g(x) = \sqrt{x-2}$

Domain = {2, 3, 4, 5..........}

iv) $h(x) = x + 6$

Domain = R

HINT

If $x = 9$

$f(x) = \frac{2(9)+1}{9-9}$

$= \frac{18+1}{0}$

= Not defined

HINT

If x = 0 and less than 0

$g(0) = \sqrt{0-2} = \sqrt{2} \notin R$

Points to Remember

- The Cartesian Product of A with B is defined as $A \times B = \{(a, b) \mid \text{for all } a \in A, b \in B\}$
- A relation R from A to B is always a subset of $A \times B$. That is $R \subseteq A \times B$
- A relation R from X to Y is a function if for every $x \in X$ there exists only one $y \in Y$.
- A function can be represented by

(i) an arrow diagram

(ii) a tabular form

(ii) a set of ordered pairs

(iv) a graphical form

- Some types of functions

(i) One-one function

(ii) Onto function

(iii) Many-one function

(iv) Into function

- Identity function $f(x) = x$
- Reciprocal function $f(x) = 1/x$
- Constant function $f(x) = c$
- Linear function $f(x) = ax + b$, $a \neq 0$
- Quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$
- Cubic function $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$
- For three non-empty sets A, B and C, if $f : A \to B$ and $g : B \to C$ are two functions, then the composition of f and g is a function $g\ f : A \to C$ will be defined as $g \circ f(x) = g(f(x))$ for all $x \in A$.
- If f and g are any two functions, then in general, $f \circ g \neq g \circ f$
- If f, g and h are any three functions, then $f \circ (g \circ h) = (f \circ g) \circ h$