Exercise 6.1

1. Prove the following identities.

(i) $\cot\theta + \tan\theta = \sec\theta \operatorname{cosec}\theta$

(ii) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Solution:

(i) L.H.S $= \cot\theta + \tan\theta$

$= \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$

$= \sec\theta \operatorname{cosec}\theta$ = R.H.S

(ii) L.H.S $= \tan^4\theta + \tan^2\theta$

$= \tan^2\theta(\tan^2\theta + 1)$

$= \tan^2\theta \times \sec^2\theta$ --------(1)

R.H.S $= \sec^4\theta - \sec^2\theta$

$= \sec^2\theta(\sec^2\theta - 1)$

$= \sec^2\theta \times \tan^2\theta$ --------(2)

From (1) & (2)

L.H.S = R.H.S

2. Prove the following identities.

(i) $\frac{1-\tan^2\theta}{\cot^2\theta-1}=\tan^2\theta$

(ii) $\frac{\cos\theta}{1+\sin\theta}=\sec\theta-\tan\theta$

Solution:

(i)

$$\text{L.H.S}=\frac{1-\tan^2\theta}{\cot^2-1}$$

$$=\frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}-1}$$

$$=\frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}}$$

$$=\frac{\cancel{\cos^2\theta-\sin^2\theta}}{\cos^2\theta}\times\frac{\sin^2\theta}{\cancel{\cos^2\theta-\sin^2\theta}}$$

$$=\frac{\sin^2\theta}{\cos^2\theta}=\tan^2\theta=\text{R.H.S}$$

(ii) Solution

$$\text{L.H.S}=\frac{\cos\theta}{1+\sin\theta}\text{X}\frac{1-\sin\theta}{1-\sin\theta}$$

$$=\frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$=\frac{\cancel{\cos\theta}\,(1-\sin\theta)}{\cos^{\cancel{2}}\theta}$$

$$=\frac{1-\sin\theta}{\cos\theta}=\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}$$

$$=\text{Sec}\,\theta-\tan\theta=\text{RHS}$$

3. Prove the following identities.

(i) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

(ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

(i) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

(ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

$$LHS = \sqrt{\frac{1+\sin\theta}{1-\sin\theta} X \frac{1+\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} = \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \text{Sec}\,\theta + \tan\theta = \text{RHS}.$$

(ii) Solution

$$LHS = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{1+Sin\theta}{1-Sin\theta} X \frac{1+Sin\theta}{1-Sin\theta}}$$

$$+ \sqrt{\frac{1-Sin\theta}{1+Sin\theta} X \frac{1-Sin\theta}{1-Sin\theta}}$$

$$= \sqrt{\frac{(1+Sin\theta)^2}{1-Sin^2\theta}} + \sqrt{\frac{(1-Sin\theta)^2}{1-Sin^2\theta}}$$

$$= \sqrt{\frac{(1+Sin\theta)^2}{Cos^2\theta}} + \sqrt{\frac{(1-Sin\theta)^2}{Cos^2\theta}}$$

$$= \frac{1+Sin\theta}{Cos\theta} + \frac{1-Sin\theta}{Cos\theta} = \frac{1+Sin\theta+1-Sin\theta}{Cos\theta}$$

$$= \frac{2}{Cos\theta} = 2\,\text{Sec}\,\theta = \text{RHS}$$

4. Prove the following identities.

(i) $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

(ii) $(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = 1 + (\sec\theta + \operatorname{cosec}\theta)^2$

(i) Solution:

$$\begin{aligned}
\text{LHS} &= \sec^6\theta \\
&= (\sec^2\theta)^3 \\
&= (1+\tan^2\theta)^3 \\
&= (1)^3 + 3(1)^2(\tan\theta)^2 \\
&\quad + 3(1)(\tan^2\theta)^2 + (\tan^2\theta)^3 \\
&= 1 + 3\tan^2\theta + 3\tan^4\theta + \tan^6\theta \\
&= \tan^6\theta + 3\tan^2\theta(1+\tan^2\theta) + 1 \\
&= \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1 \\
&= \text{RHS.}
\end{aligned}$$

(ii) Solution:

$$\begin{aligned}
\text{L.H.S} &= (\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 \\
&= \sin^2\theta + \sec^2\theta + 2\sin\theta\sec\theta \\
&\quad + \cos^2\theta + \operatorname{cosec}^2\theta + 2\cos\theta\operatorname{cosec}\theta \\
&= 1 + 2\sin\theta \times \frac{1}{\cos\theta} + 2\cos\theta \times \frac{1}{\sin\theta} \\
&\quad + \sec^2\theta + \operatorname{cosec}^2\theta \\
&= 1 + 2\left[\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right] + \sec^2\theta \\
&\quad + \operatorname{cosec}^2\theta \\
&= 1 + 2\left[\frac{1}{\cos\theta\sin\theta}\right] + \sec^2\theta + \operatorname{cosec}^2\theta \\
&= 1 + 2\sec\theta\operatorname{cosec}\theta + \sec^2\theta + \operatorname{cosec}^2\theta \\
&= 1 + (\sec\theta + \operatorname{cosec}\theta)^2 = \text{R.H.S}
\end{aligned}$$

5. Prove the following identities.

(i) $\sec^4\theta\,(1-\sin^4\theta) - 2\tan^2\theta = 1$

(ii) $\dfrac{\cot\theta - \cos\theta}{\cot\theta + \cos\theta} = \dfrac{\operatorname{cosec}\theta - 1}{\operatorname{cosec} + 1}$

(i) Solution:

$$\text{LHS} = \sec^4\theta\,(1-\sin^4\theta) - 2\tan^2\theta$$

$$= \frac{1}{\cos^4\theta}(1+\sin^2\theta)\,(1-\sin^2\theta) - 2\frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1}{(\cos^2\theta)^2}(1+\sin^2\theta)\cos^2\theta - 2\frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1+\sin^2\theta}{\cos^2\theta} - 2\frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1+\sin^2\theta-2\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} = 1 = \text{R.H.S}$$

(ii) Solution:

$$\text{L.H.S} = \frac{\cot\theta-\cos\theta}{\cot\theta+\cos\theta}$$

$$= \frac{\frac{\cos\theta}{\sin\theta}-\cos\theta}{\frac{\cos\theta}{\sin\theta}+\cos\theta} = \frac{\frac{\cos\theta-\cos\theta\sin\theta}{\sin\theta}}{\frac{\cos\theta+\cos\theta\sin\theta}{\sin\theta}}$$

$$= \frac{\cos\theta-\cos\theta\sin\theta}{\cos\theta+\cos\theta\sin\theta} = \frac{\cos\theta\,(1-\sin\theta)}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{1-\sin\theta}{1+\sin\theta} = \frac{\frac{1}{\sin\theta}-\frac{\sin\theta}{\sin\theta}}{\frac{1}{\sin\theta}+\frac{\sin\theta}{\sin\theta}}$$

$$= \frac{\operatorname{cosec}\theta-1}{\operatorname{cosec}\theta+1} = \text{R.H.S}$$

6. Prove the following identities.

(i) $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$

(ii) $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

(i) $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$ (ii) $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

(i) Solution:

$$\text{LHS} = \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$$

$$= \frac{(\sin A - \sin B)(\sin A + \sin B) + (\cos A - \cos B)(\cos A + \cos B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{(1 - \cos^2 A) - (1 - \cos^2 B) + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{\cancel{1} - \cancel{\cos^2 A} - \cancel{1} + \cancel{\cos^2 B} + \cancel{\cos^2 A} - \cancel{\cos^2 B}}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= 0 = \text{R.H.S}$$

(ii) Solution:

$$\text{LHS} = \frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A}$$

$$= \frac{\cancel{(\sin A + \cos A)}(\sin^2 A - \sin A \cos A + \cos^2 A)}{\cancel{(\sin A + \cos A)}} + \frac{\cancel{(\sin A - \cos A)}(\sin^2 A + \sin A \cos A + \cos^2 A)}{\cancel{(\sin A - \cos A)}}$$

$$= (\sin^2 A - \sin A \cos A + \cos^2 A) + (\sin^2 A + \sin A \cos A + \cos^2 A)$$

$$= 1 - \sin A \cos A + 1 + \sin A \cos A$$

$$= 2 = \text{R.H.S}$$

7. (i) If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

(ii) If $\sqrt{3}\sin\theta - \cos\theta = 0$, then show that $\tan 3\theta = (3\tan\theta - \tan^3\theta) / (1 - 3\tan^2\theta)$

(i) Solution:

Given $\sin\theta + \cos\theta = \sqrt{3}$

$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$1 + 2\sin\theta\cos\theta = 3$

$2\sin\theta\cos\theta = 3 - 1 = 2$

$\sin\theta\cos\theta = \frac{2}{2}$

$\sin\theta\cos\theta = 1$ ---------(1)

L.H.S $= \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} \quad \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{1} = 1 = \text{R.H.S}$

(ii) Solution

given $\sqrt{3}\sin\theta - \cos\theta = 0$

$\sqrt{3}\sin\theta = \cos\theta$

$\frac{Sin\theta}{Cos\theta} = \frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^0$

$\boxed{\theta = 30^0}$

LHS $= \tan 3\theta$

$= \tan 3(30^0)$

$= \tan 90^0 = \text{Un defined}$ ------ (1)

RHS $= \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$= \frac{3\tan 30^0 - \tan^3 30^0}{1 - 3\tan^2 30}$

$= \frac{3\left(\frac{1}{\sqrt{3}}\right) - \left(\frac{1}{\sqrt{3}}\right)^3}{1 - 3\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{\sqrt{3} - \frac{1}{3\sqrt{3}}}{1 - 3\frac{1}{3}} = \frac{\sqrt{3} - \frac{1}{3\sqrt{3}}}{1 - 1} = \frac{\sqrt{3} - \frac{1}{3\sqrt{3}}}{0}$

$=$ Undefined ---------- (2)

From (1) & (2)

LHS = RHS

8. (i) If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, then prove that $(m^2 + n^2)\cos^2\beta = n^2$

(ii) If $\cot\theta + \tan\theta = x$ and $\sec\theta - \cos\theta = y$, then prove that $(x^2y)^{2/3} - (xy^2)^{2/3} = 1$

(i) Solution :

$\frac{\cos\alpha}{\cos\beta} = m$ | $\frac{\cos\alpha}{\sin\beta} = n$

$\cos\alpha = \cos\beta$ | $\frac{m\cos\beta}{\sin\beta} = n$

$m\cos\beta = n\sin\beta$

$m^2\cos^2\beta = n^2\sin^2\beta$

$m^2\cos^2\beta = n^2(1-\cos^2\beta)$

$m^2\cos^2\beta = n^2 - n^2\cos^2\beta$

$m^2\cos^2\beta + n^2\cos^2\beta = n^2$

$(m^2+n^2)\cos^2\beta = n^2$

Hence proved.

(ii) Solution:

given $\cot\theta + \tan\theta = x$ | $\sec\theta - \cos\theta = y$

$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = x$ | $\frac{1}{\cos\theta} - \cos\theta = y$

$\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta} = x$ | $\frac{1-\cos^2\theta}{\cos\theta} = y$

$\frac{1}{\sin\theta\cos\theta} = x$ | $\frac{\sin^2\theta}{\cos\theta} = y$

$x = \frac{1}{\sin\theta\cos\theta}$

$$(x^2y)^{\frac{2}{3}} = \left[\frac{1}{\cancel{\sin^2\theta}\cos\theta} \times \frac{\cancel{\sin^2\theta}}{\cos\theta}\right]^{\frac{2}{3}}$$

$$= \left(\frac{1}{\cos^3\theta}\right)^{\frac{2}{3}} = \frac{1}{\cos^2\theta}$$

$$(xy^2)^{\frac{2}{3}} = \left[\frac{1}{\cancel{\sin\theta}\cos\theta} \times \frac{\cancel{\sin^3\theta}}{\cos^2\theta}\right]^{\frac{2}{3}}$$

$$= \left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{2}{3}} = \frac{\sin^2\theta}{\cos^2\theta}$$

$$\text{L.H.S} = (x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}}$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta}$$

$= 1 = \text{R.H.S}$

9. (i) If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then prove that $q(p^2 - 1) = 2p$

(ii) If $\sin\theta(1 + \sin^2\theta) = \cos^2\theta$, then prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

(i) Solution:

Given $\sin\theta + \cos\theta = p$

$(\sin\theta + \cos\theta)^2 = p^2$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = p^2$

$1 + 2\sin\theta\cos\theta = p^2$ -------- (1)

$\sec\theta + \operatorname{cosec}\theta = q$

$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q$

$\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = q$

$\frac{p}{\sin\theta\cos\theta} = q$

$\sin\theta\cos\theta = \frac{p}{q}$ ---------(2)

Use (2) in (1)

$1 + 2\frac{p}{q} = p^2$

$2p = q(p^2 - 1)$

$q(p^2 - 1) = 2p$

Hence Proved

(ii) Solution:

Given $\sin\theta(1 + \sin^2\theta) = \cos^2\theta$

$\sin\theta[1 + 1 - \cos^2\theta] = \cos^2\theta$

$\sin\theta[2 - \cos^2\theta] = \cos^2\theta$

Take square on both sides

$\sin^2\theta[2 - \cos^2\theta]^2 = \cos^4\theta$

$(1 - \cos^2\theta)[(2)^2 - 2(2)(\cos^2\theta) + (\cos^2\theta)^2] = \cos^4\theta$

$(1 - \cos^2\theta)[4 - 4\cos^2\theta + \cos^4\theta] = \cos^4\theta$

$4 - 4\cos^2\theta + \cos^4\theta - 4\cos^2\theta + 4\cos^4\theta - \cos^6\theta = \cos^4\theta$

$4 - 8\cos^2\theta - 4\cos^4\theta - \cos^6\theta - \cos^4\theta = 0$

$\cos^6\theta - 4\cos^4\theta - 8\cos^2\theta = 4$

Hence Proved

10. If $\frac{\cos\theta}{1+\sin\theta} = \frac{1}{a}$, then prove that $\frac{a^2-1}{a^2+1} = \sin\theta$

Solution:

Given $\frac{\cos\theta}{1+\sin\theta} = -$

$$\frac{\cos^2\theta}{(1+\sin\theta)^2} = \frac{1}{a^2}$$

$$\frac{1-\sin^2\theta}{(1+\sin\theta)^2} = \frac{1}{a^2}$$

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\sin\theta)^2} = \frac{1}{a^2}$$

$$\frac{1-\sin\theta}{1+\sin\theta} = \frac{1}{a^2}$$

$$a^2 - a^2\sin\theta = 1+\sin\theta$$

$$a^2 - 1 = a^2\sin\theta + \sin\theta$$

$$a^2 - 1 = (a^2+1)\sin\theta$$

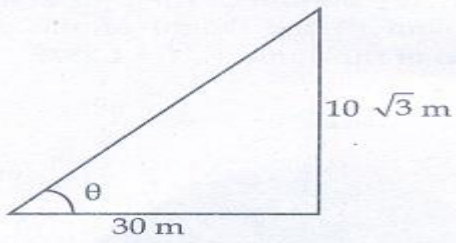
$$\sin\theta = \frac{a^2-1}{a^2+1}$$

Hence Proved

Exercise 6.2

1. Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m.

Solution:



$\tan\theta = \frac{opp}{adj} = \frac{10\sqrt{3}}{30}$

$\tan\theta = \tan 30^\circ$

$\theta = 30^\circ$

2. A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.

Solution:

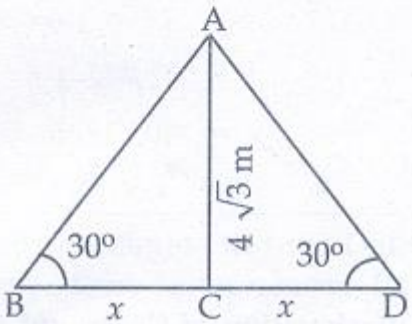
$\tan\theta = \frac{opp}{adj}$

$\tan 30^\circ = \frac{4\sqrt{3}}{x}$

$\frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{x}$

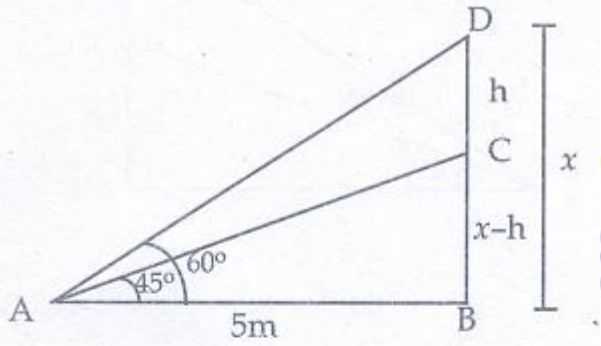
$x = 4\sqrt{3} \times \sqrt{3} = 12\text{m}$

Width of the road = $2x = 2 \times 12 = 24\text{m}$



3. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5 m away from the wall, what is the height of the window? ($\sqrt{3}$ = 1.732)

Solution:



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 45^0 = \frac{x-h}{5}$$

$$1 = \frac{x-h}{5}$$

$$5 = x-h$$

$$x = h+5 \text{-------(1)}$$

$$\tan 60^0 = \frac{x}{5}$$

$$\sqrt{3} = \frac{x}{5}$$

$$x = 5\sqrt{3} \text{ -----(2)}$$

From (1) & (2)

$$h+5 = 5\sqrt{3}$$

$$h = 5\sqrt{3} - 5$$

$$= (\sqrt{3} - 1)5$$

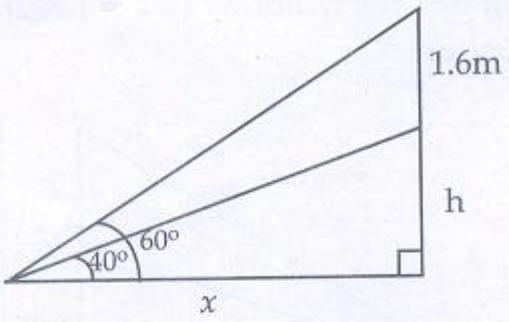
$$= (1.732 - 1)5$$

$$= (0.732)\ 5$$

$$h = 3.66 \text{ m}$$

4. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 40° . Find the height of the pedestal. ($\sqrt{3} = 1.732$)

Solution:



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 40^0 = \frac{h}{x}$$

$$0.8391 = \frac{h}{x}$$

$$x = \frac{h}{0.8391} \text{ ------(1)}$$

$$\tan 60^o = \frac{1.6+h}{x}$$

$$\sqrt{3} = \frac{1.6+h}{x}$$

$$x = \frac{1.6+h}{\sqrt{3}} \text{ -------(2)}$$

From (1) & (2)

$$\frac{h}{0.8391} = \frac{1.6+h}{\sqrt{3}}$$

$$\sqrt{3}\,h = 0.8391 \times 1.6 + 0.8391\,h$$

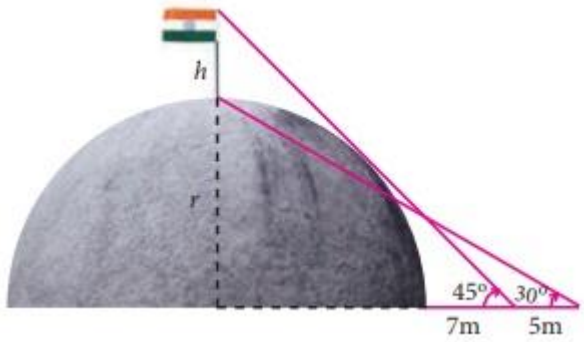
$$1.732\,h - 0.8391\,h = 1.3426$$

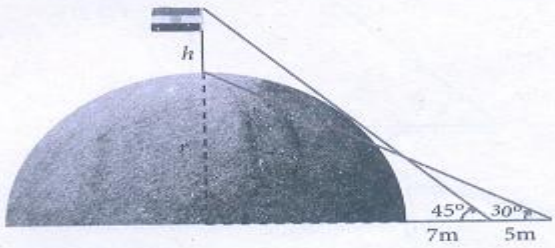
$$0.8929\,h = 1.3426$$

$$h = \frac{1.3426}{0.8929}$$

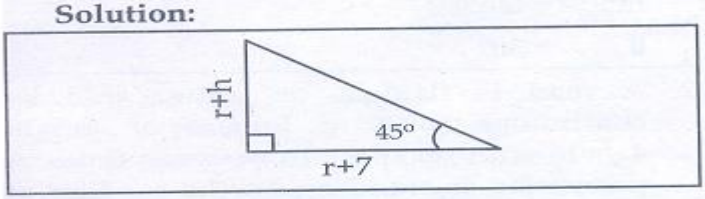
$$h = 1.50 \text{ m}$$

5. A flag pole '*h*' metres is on the top of the hemispherical dome of radius '*r*' metres. A man is standing 7 m away from the dome. Seeing the top of the pole at an angle 45° and moving 5 m away from the dome and seeing the bottom of the pole at an angle 30° . Find (i) the height of the pole (ii) radius of the dome. ($\sqrt{3} = 1.732$)





Solution:



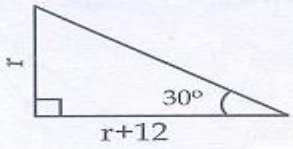
$\tan\theta = \frac{opp}{adj}$

$\tan 45^\circ = \frac{r+h}{r+7}$

$1 = \frac{r+h}{r+7}$

$r+7 = r+h$

$\Rightarrow h = 7m$



$\tan 30^\circ = \frac{\quad}{12}$

$\frac{1}{\sqrt{3}} = \frac{r}{r+12}$

$r+12 = \sqrt{3}\,r$

$\sqrt{3}\,r - r = 12$

$(\sqrt{3}-1)r = 12$

$(1.732 - 1) = 12$

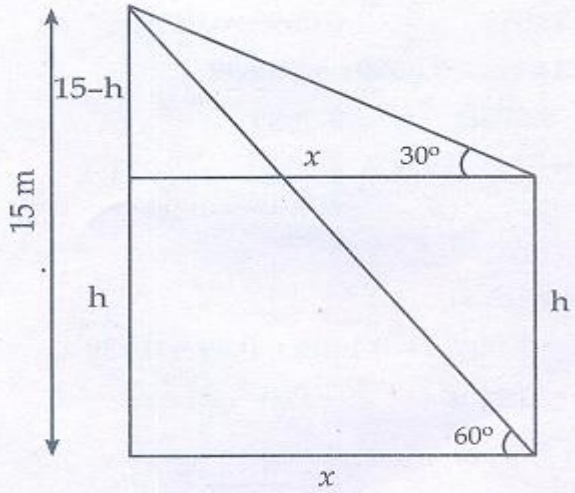
$0.732 \times r = 12$

$r = \frac{12}{0.732}$

$r = 16.39\ m$

6. The top of a 15 m high tower makes an angle of elevation of 60° with the bottom of an electronic pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole?

Solution:



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30^\circ = \frac{15-h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{15-h}{x}$$

$$x = (15-h)\sqrt{3} \quad \text{------(1)}$$

$$\tan 60^\circ = \frac{15}{x}$$

$$\sqrt{3} = \frac{15}{x}$$

$$x = \frac{15}{\sqrt{3}} \quad \text{------(2)}$$

(1) = (2)

$$\frac{15}{\sqrt{3}} = (15-h)\sqrt{3}$$

$$15 = (15-h)\,3$$

$$15-h = \frac{15}{3} = 5$$

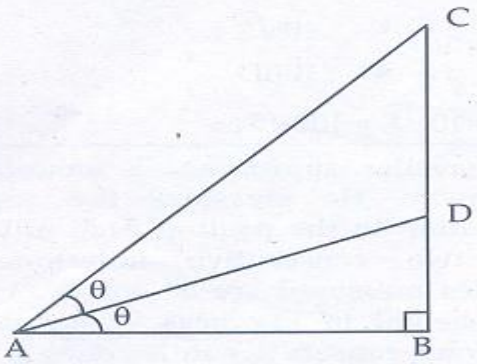
$$15-h = 5$$

$$15-5 = h$$

$$h = 10\text{m}$$

7. A vertical pole fixed to the ground is divided in the ratio 1:9 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a place on the ground, 25 m away from the base of the pole, what is the height of the pole?

Solution:



Let BC be the pole and A be the point 25m away from the pole.

$BD : CD = 1:9$

$BD = \frac{1}{10} BC \Rightarrow BC = 10BD$

$\tan\theta = \frac{opp}{adj}$

$\tan 2\theta = \frac{BC}{AB}$

$\tan 2\theta = \frac{BC}{25}$ -----(1)

$\Delta ABD, \tan\theta = \frac{BD}{AB}, \tan\theta = \frac{BD}{25}$

$\tan 2\theta = \frac{BC}{25}$

$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{BC}{25}$

$2\tan\theta \times 25 = BC(1-\tan^2\theta)$

$\cancel{50}^{2} \times \frac{BD}{\cancel{25}} = 10BD\left[1-\left(\frac{BD}{25}\right)^2\right]$

$\frac{\cancel{2}BD}{1} = \cancel{10}^{5}BD\left(\frac{625-BD^2}{\cancel{625}_{125}}\right)$

$125 = 625-BD^2$

$BD^2 = 625-125$

$BD^2 = 500$

$BD = \pm\sqrt{500} = \sqrt{5\times5\times2\times2\times5}$

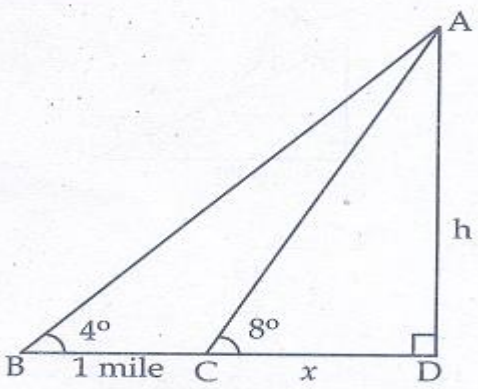
$BD = 10\sqrt{5}$

$BC = 10BD$

$= 10\times10\sqrt{5} = 100\sqrt{5}$ m

8. A traveler approaches a mountain on highway. He measures the angle of elevation to the peak at each milestone. At two consecutive milestones the angles measured are 4° and 8° . What is the height of the peak if the distance between consecutive milestones is 1 mile. (tan 4° = 0.0699, tan 8° = 0. 1405)

Solution:



$\tan\theta = \frac{opp}{adj}$

$\tan 40^\circ = \frac{h}{1+x}$

$0.0699 = \frac{h}{1+x}$

$h = (0.0699)(1+x)$ --------(1)

$\tan 8^\circ = \frac{h}{x}$

$0.1405 = \frac{h}{x}$

$h = 0.1405x$ ----------(2)

From (1) & (2)

$0.1405x = 0.0699+0.0699x$

$0.1405x - 0.0699x = 0.0699$

$0.0706x = 0.0699$

$x = \frac{0.0699}{0.0706}$

$x = 0.99$

Use in (2)

$h = 0.1405x = 0.1405 \times 0.99 = 0.139$

$h \simeq 0.14$ m

Exercise 6.3

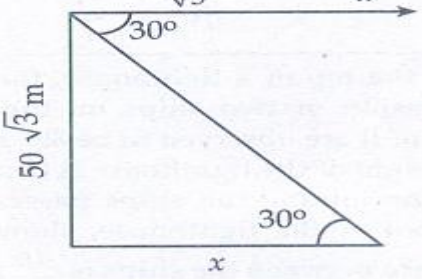
1. From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.

Solution:

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30^\circ = \frac{50\sqrt{3}}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{x}$$



$$x = 50\sqrt{3} \times \sqrt{3} = 50 \times 3$$

$$x = 150 \text{ m}$$

2. The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120 m, find the height of the first building.

Solution:

$$\tan\theta = \frac{opp}{adj}$$

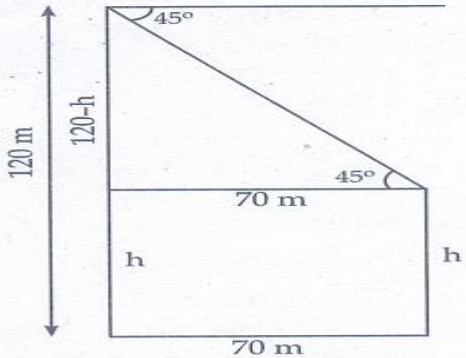
$$\tan 45^\circ = \frac{120 - h}{70}$$

$$1 = \frac{120 - h}{70}$$

$$70 = 120 - h$$

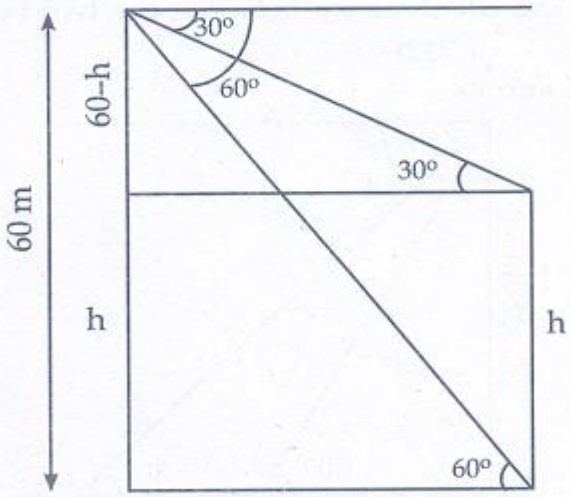
$$h = 120 - 70 = 50$$

$$h = 50 \text{ m}$$



3. From the top of the tower 60 m high the angles of depression of the top and bottom of a vertical lamp post are observed to be 38° and 60° respectively. Find the height of the lamp post. (tan 38° = 0.7813, $\sqrt{3}$ = 1.732)

Solution:



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 38^\circ = \frac{60-h}{x}$$

$$0.7813 = \frac{60-h}{x}$$

$$x = \frac{60-h}{0.7813} \text{ --------(1)}$$

$$\tan 60^\circ = \frac{60}{x}$$

$$\sqrt{3} = \frac{60}{x}$$

$$x = \frac{60}{\sqrt{3}} = 20\sqrt{3} \text{ --------(2)}$$

From (1) & (2)

$$\frac{60-h}{0.7813} = 20\sqrt{3}$$

$$60 - h = 20(1.732) \times 0.7813$$

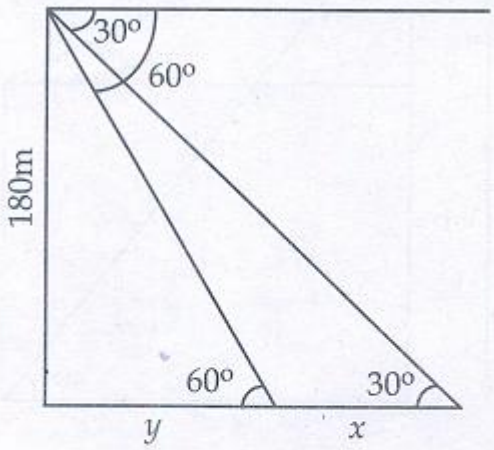
$$60 - h = 27.06$$

$$h = 60 - 27.06 = 32.94$$

$$h = 32.94 \text{ m}$$

4. An aeroplane at an altitude of 1800 m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two boats. ($\sqrt{3} = 1.732$)

Solution:



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30^\circ = \frac{1800}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{1800}{x+y}$$

$$x+y = 1800\sqrt{3}$$

$$y = 1800\sqrt{3} - x \text{ ------(1)}$$

$$\tan 60^\circ = \frac{1800}{y}$$

$$\sqrt{3} = \frac{1800}{y}$$

$$y = \frac{1800}{\sqrt{3}} = 600\sqrt{3} \text{ ------(2)}$$

(1) = (2) $600\sqrt{3} = 1800\sqrt{3} - x$

$$x = 1800\sqrt{3} - 600\sqrt{3}$$

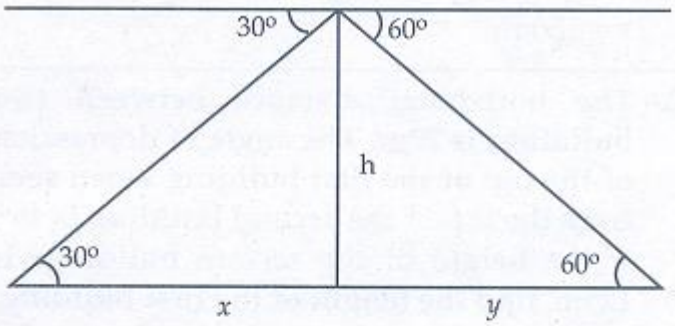
$$= 1200\sqrt{3}$$

$$= 1200(1.732)$$

$$x = 2078.4 \text{ m}$$

5. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is *h* meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $4h/\sqrt{3}$ m.

Solution:



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30^\circ = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = \sqrt{3}\,h \quad \text{------(1)}$$

$$\tan 60^\circ = \frac{h}{y}$$

$$\sqrt{3} = \frac{h}{y}$$

$$y = \frac{h}{\sqrt{3}}$$

$$\text{Distance} = x+y$$

$$= \frac{h}{\sqrt{3}} + h\sqrt{3}$$

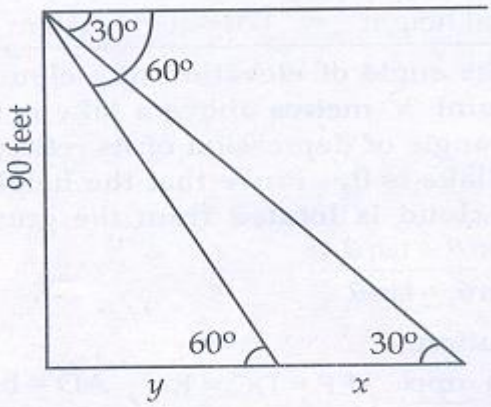
$$= \frac{h+3h}{\sqrt{3}}$$

$$x+y = \frac{4h}{\sqrt{3}} \text{ m}$$

6. A lift in a building of height 90 feet with transparent glass walls is descending from the top of the building. At the top of the building, the angle of depression to a fountain in the garden is 60°. Two minutes later, the angle of

depression reduces to 30°. If the fountain is $30\sqrt{3}$ feet from the entrance of the lift, find the speed of the lift which is descending.

Solution:



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 60^\circ = \frac{90}{y}$$

$$\sqrt{3} = \frac{90}{y}$$

$$y = \frac{90}{\sqrt{3}} = 30\sqrt{3} \quad \text{-------(1)}$$

$$y = 30\sqrt{3}$$

$$\tan 30^\circ = \frac{90}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{90}{x+y}$$

$$x+y = 90\sqrt{3} \quad \text{-------(2)}$$

$$x+30\sqrt{3} = 90\sqrt{3} \text{ by (1)}$$

$$x = 90\sqrt{3} - 30\sqrt{3}$$

$$x = 60\sqrt{3}$$

$$\text{distance} = x = 60\sqrt{3}$$

$$\text{speed} \times \text{time} = 60\sqrt{3}$$

$$\text{speed} \times 2 = 60\sqrt{3}$$

$$\text{speed} = \frac{60\sqrt{3}}{2}$$

$$\text{Speed} = 30\sqrt{3}$$

Exercise 6.4

1. From the top of a tree of height 13 m the angle of elevation and depression of the top and bottom of another tree are 45° and 30° respectively. Find the height of the second tree. ($\sqrt{3}$=1.732)

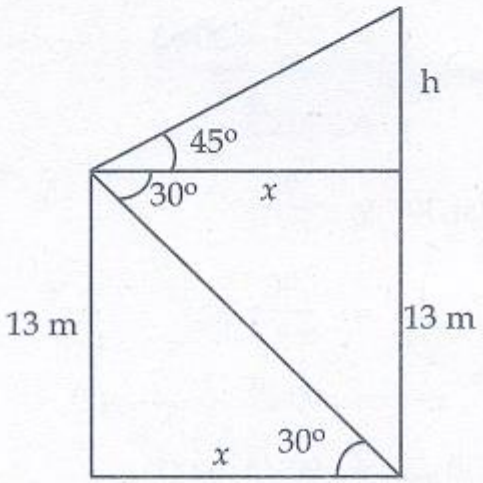
Solution:

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 45^\circ = \frac{h}{x}$$

$$1 = \frac{h}{x}$$

$$x = h \text{ -------(1)}$$



$$\tan 30^\circ = \frac{13}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{13}{x}$$

$$x = 13\sqrt{3} \text{ --------(2)}$$

From (1) & (2)

$$h = 13\sqrt{3} = 13 \times 1.732$$

$$h = 22.516 \text{ m}$$

Height of the second tree = 13+h

$$= 13 + 22.52$$

$$= 35.52 \text{ m}$$

2. A man is standing on the deck of a ship, which is 40 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30° . Calculate the distance of the hill from the ship and the height of the hill. ($\sqrt{3}$=1.732)

Solution:

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 60^\circ = \frac{h}{x}$$

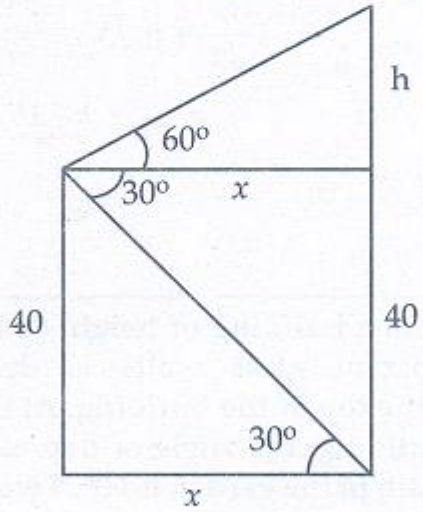
$$\sqrt{3} = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \text{ --------(1)}$$

$$\tan 30^\circ = \frac{40}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{40}{x}$$

$$x = 40\sqrt{3} \text{ -------(2)}$$



from (1) & (2)

$$40\sqrt{3} = \frac{h}{\sqrt{3}}$$

$$h = 40\sqrt{3} \times \sqrt{3} = 40 \times 3 = 120$$

$$h = 120 \text{ m}$$

$$\text{Total height} = 120+40 \text{ m} = 160 \text{ m}$$

3. If the angle of elevation of a cloud from a point 'h' metres above a lake is θ_1 and the angle of depression of its reflection in the lake is θ_2. Prove that the height that the cloud is located from the ground is $\frac{h(\tan\theta_1 + \tan\theta_2)}{\tan\theta_2 - \tan\theta_1}$.

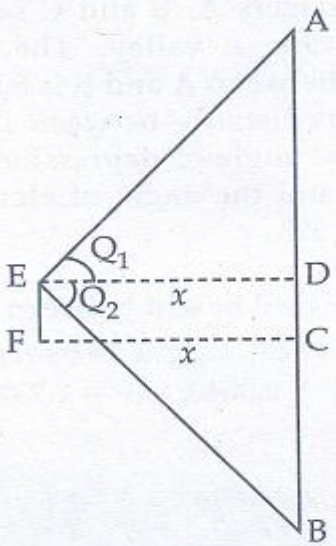
Solution:

$\tan\theta \frac{opp}{adj}$, $EF = DC = h$, $AC = BC = b$, $AD = b - h$, $BD = b + h$

$$\tan\theta_1 = \frac{AD}{ED} = \frac{b-h}{x}, \text{---------} (1)$$

$$\tan\theta_2 = \frac{BD}{EP} = \frac{b+h}{x}, \text{---------} (2)$$

(1) & (2)



(1) ÷ (2)

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{b-h}{b+h}$$

$$\tan\theta_1 (b+h) = \tan\theta_2 (b-h)$$

$$b\tan\theta_1 + h\tan\theta_1 = b\tan\theta_2 - h\tan\theta_2$$

$$h\tan\theta_1 + h\tan\theta_2 = b\tan\theta_2 - b\tan\theta_1$$

$$h(\tan\theta_1 + \tan\theta_2) = b(\tan\theta_2 - b\tan\theta_1)$$

$$\Rightarrow b(\tan\theta_2 - \tan\theta_1) = h(\tan\theta_1 + \tan\theta_2)$$

$$b = h\left(\frac{\tan\theta_1 + \tan\theta_2}{\tan\theta_2 + \tan\theta_1}\right)$$

4. The angle of elevation of the top of a cell phone tower from the foot of a high apartment is 60° and the angle of depression of the foot of the tower from the top of the apartment is 30° . If the height of the apartment is 50 m, find the height of the cell phone tower. According to radiations control norms, the minimum height of a cell phone tower should be 120 m. State if the height of the above mentioned cell phone tower meets the radiation norms.

Solution:

$\tan\theta \frac{opp}{adj}$

$\tan 30° - \frac{50}{x}$

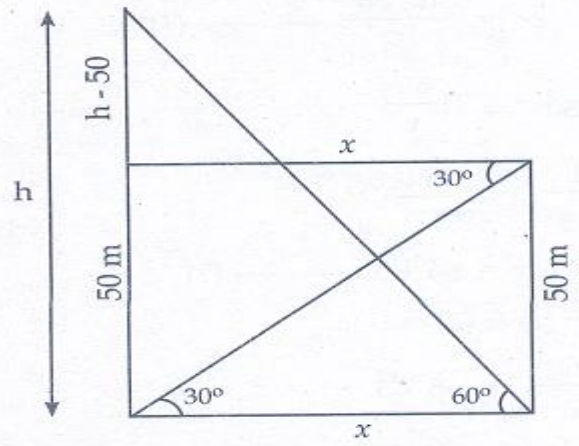
$\frac{1}{\sqrt{3}} = \frac{50}{x}$

$x = 50\sqrt{3}$ ---------- (1)

$\tan 60° = \frac{h}{x}$

$\sqrt{3} = \frac{h}{x}$

$x = \frac{h}{\sqrt{3}}$



from (1) & (2)

$\frac{h}{\sqrt{3}} = 50\sqrt{3}$

$h = 50\sqrt{3} \times \sqrt{3} = 50 \times 3 = 150$

h = 150 m

Yes, meets the radiation norm.

5. The angles of elevation and depression of the top and bottom of a lamp post from the top of a 66 m high apartment are 60° and 30° respectively. Find

(i) The height of the lamp post.

(ii) The difference between height of the lamp post and the apartment.

(iii) The distance between the lamp post and the apartment. ($\sqrt{3}$=1.732)

Solution:

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 60° = \frac{h - 66}{x}$$

$$\sqrt{3} = \frac{h - 66}{x}$$

$$x = \frac{h - 66}{\sqrt{3}} \quad \text{---------- (1)}$$

$$\tan 30° = \frac{66}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{66}{x}$$

$$x = 66\sqrt{3} \quad \text{---------- (2)}$$

From (1) & (2)

$$\frac{h - 66}{\sqrt{3}} = 66\sqrt{3}$$

$$h - 66 = 66\sqrt{3} \times 66 \times 3 = 198$$

$$h = 198 + 66$$

i) $\boxed{h = 264 \text{ m}}$

ii) The difference of lamp post and the apartment

$= 264 - 66 = 198$ m.

iii) The distance between L ampost and apartment

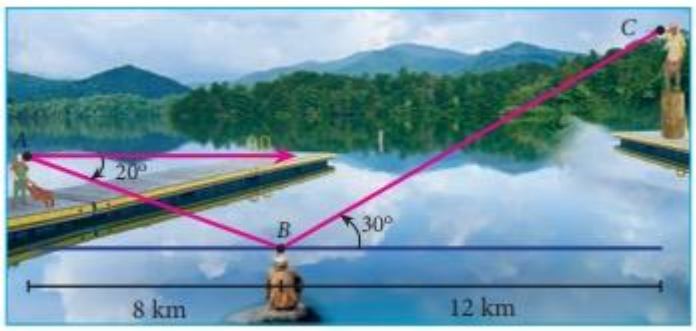
$$x = 66\sqrt{3} = 66 \times 1.732$$

$$x = 114.31\text{m}$$

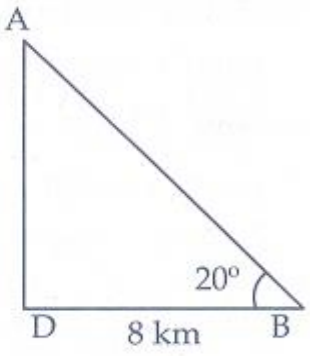
6. Three villagers *A*, *B* and *C* can see each other across a valley. The horizontal distance between *A* and *B* is 8 km and the horizontal distance between *B* and *C* is 12 km. The angle of depression of B from *A* is 20° and the angle of elevation of *C* from *B* is 30° . Calculate

(i) the vertical height between *A* and *B*.

(ii) the vertical height between *B* and *C*. (tan20° = 0.3640, $\sqrt{3}$=1.732)



Solution:



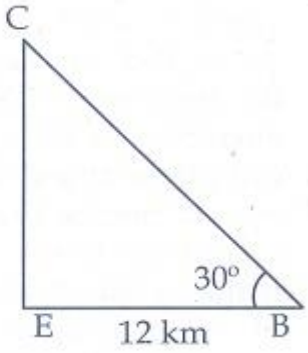
$$\tan\theta = \frac{opp}{adj}$$

$$\tan 20^\circ = \frac{AD}{BD}$$

$$0.3640 = \frac{AD}{8}$$

$$AD = 0.3640 \times 8 = 2.91 \text{ km}$$

$$\boxed{AD = 2.91 \text{ km}}$$



$$\tan 30^\circ = \frac{CE}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{CE}{12}$$

$$\frac{12}{\sqrt{3}} = CE$$

$$CE = 4\sqrt{3} = 4 \times 1.732$$

$$\boxed{CE = 6.93 \text{ km}}$$

Multiple choice questions

1. The value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ is equal to

(1) $\tan^2\theta$

(2) 1

(3) $\cot^2\theta$

(4) 0

2. $\tan\theta \operatorname{cosec}^2\theta - \tan\theta$ is equal to

(1) $\sec\theta$

(2) $\cot^2\theta$

(3) $\sin\theta$

(4) $\cot\theta$

Solution:

$\tan\theta \operatorname{coseo}^2\theta - \tan\theta = \tan\theta(\operatorname{cosec}^2\theta - 1)$

$= \tan\theta(\cot^2\theta)$

$= \cancel{\tan\theta} \times \cancel{\cot\theta} \times \cot\theta = \cot\theta$

3. If $(\sin a + \operatorname{cosec} a)^2 + (\cos a + \sec a)^2 = k + \tan^2 a + \cot^2 a$, then the value of k is equal to

(1) 9

(2) 7

(3) 5

(4) 3

Solution:

$\sin\alpha + \text{cosec}^2\alpha + \cancel{2\sin\alpha}\ \cancel{\text{cosec}\alpha} + \cos^2\alpha + \sec^2\alpha + 2\cos\alpha\sec\alpha = k + \tan^2\alpha + \cot^2\alpha$

$1 + 2 + 2 + 1 + \cos^2\alpha + 1 + \tan^2\alpha = k + \tan^2\alpha + \cot^2\alpha$

$7 + \cot^2\alpha + \tan^2\alpha = k + \tan^2\alpha + \cot^2\alpha \Rightarrow k = 7$

4. If $\sin\theta + \cos\theta = a$ and $\sec\theta + \text{cosec}\theta = b$, then the value of $b(a^2 - 1)$ is equal to

(1) 2*a*

(2) 3*a*

(3) 0

(4) 2*ab*

Solution:

$b(a^2 - 1) = (\sec\theta + \text{cosec}\,\theta)\,([\sin\theta + \cos\theta]^2 - 1)$

$= (\sec\theta + \text{cosec}\,\theta)\,(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1)$

$= (\sec\theta + \text{cosec}\,\theta)\,(\cancel{1} + 2\sin\theta\cos\theta - \cancel{1})$

$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$

$= \left(\frac{\sin\theta + \cos\theta}{\cancel{\sin\theta}\,\cancel{\cos\theta}}\right)(2\cancel{\sin\theta}\,\cancel{\cos\theta})$

$= 2\,(a) = 2a$

5. If $5x = \sec\theta$ and $5/x = \tan\theta$, then $x^2 - (1/x^2)$ is equal to

(1) 25

(2) 1/25

(3) 5

(4) 1

Solution:

$$x = \frac{\sec\theta}{5}\,\frac{1}{x} = \frac{\tan\theta}{5}$$

$$x^2 - \frac{1}{x^2} = \frac{\sec^2\theta}{25} - \frac{\tan^2\theta}{25} = \frac{\sec^2\theta - \tan^2\theta}{25} = \frac{1}{25}$$

6. If $\sin\theta = \cos\theta$, then $2\tan^2\theta + \sin^2\theta - 1$ is equal to

(1) -3/2

(2) 3/2

(3) 2/3

(4) -2/3

Solution:

$$\frac{\sin\theta}{\cos\theta} = 1,\ \theta = 45^\circ,\ 2\tan^2\theta + \sin^2\theta - 1$$

$$= 2\tan^2 45^\circ + \sin^2 45 - 1$$

$$= 2(1)^2 + (\tfrac{1}{\sqrt{2}})^2 - 1$$

$$= 2 + \tfrac{1}{2} - 1 = \frac{4 + 1 - 2}{2} = \frac{3}{2}$$

7. If $x = a\tan\theta$ and $y = b\sec\theta$ then

(1) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ (2) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(3) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (4) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

Ans: (1)

Solution:

$\frac{x}{y} = \tan\theta, \ \frac{y}{b} = \sec\theta$

$\sec^2\theta - \tan^2\theta = 1$

$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

8. $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\theta)$ is equal to

(1) 0

(2) 1

(3) 2

(4) -1

Solution:

$$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$= \frac{(\cos\theta+\sin\theta)^2-1^2}{\sin\theta\cos\theta}$$

$$= \frac{\cancel{\cos^2\theta}+\cancel{\sin^2\theta}+2\sin\theta\cos\theta\cancel{-1}}{\sin\theta\cos\theta}$$

$$= \frac{2\cancel{\sin\theta}\ \cancel{\cos\theta}}{\cancel{\sin\theta}\ \cancel{\cos\theta}} = 2$$

9. $a \cot\theta + b \text{cosec}\theta = p$ and $b \cot\theta + a \text{cosec}\theta = q$ then $p^2 - q^2$ is equal to

(1) $a^2 - b^2$

(2) $b^2 - a^2$

(3) $a^2 + b^2$

(4) $b - a$

Solution:

$$p^2 - q^2 = (a^2 \cot^2\theta + b^2 \operatorname{cosec}^2\theta + 2ab \cot\theta \operatorname{cosec}\theta$$
$$-(b^2\cot^2\theta + a^2\operatorname{cosec}^2\theta + 2ab \cot\theta \operatorname{casec}\theta)$$
$$= -a^2\cot^2 + b^2\operatorname{cosec}^2\theta + 2ab \cot\theta \operatorname{casec}\theta$$
$$-b^2\cot^2\theta - a^2\operatorname{cosec}^2\theta - 2ab \cot\theta \operatorname{casec}\theta$$
$$= a^2(\cot^2\theta - \operatorname{cosec}^2\theta) + b^2(\operatorname{cosec}^2\theta - \cot^2\theta)$$
$$= a^2(-1) + b^2(1) = -a^2 + b^2 = b^2 - a^2$$

10. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3} : 1$ then the angle of elevation of the sun has measure

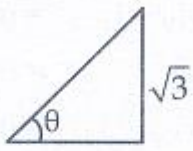
(1) 45°

(2) 30°

(3) 90°

(4) 60°

Solution:



$$\tan\theta = \frac{opp}{adj} = \frac{\sqrt{3}}{1}$$

$$\tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60^\circ$$

$$\theta = 60^\circ$$

11. The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point '*b*' metres above the first, the depression of the foot of the tower is 60° . The height of the tower (in metres) is equal to

(1) $\sqrt{3}b$

(2) *b*/3

(3) $b/2$

(4) $b/\sqrt{3}$

Solution:

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30^\circ = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = \sqrt{}h \text{ --------(1)}$$

$$\tan 60^\circ = \frac{b}{x}$$

$$\sqrt{3} = \frac{b}{x}$$

$$x = \frac{b}{\sqrt{3}} \text{ --------(2)}$$

from (1) & (2)

$$\sqrt{3}h = \frac{b}{\sqrt{3}}$$

$$h = \frac{b}{\sqrt{3} \times \sqrt{3}} = \frac{b}{3}$$

12. A tower is 60 m height. Its shadow is x metres shorter when the sun's altitude is 45° than when it has been 30° , then x is equal to

(1) 41.92 m

(2) 43.92 m

(3) 43 m

(4) 45.6 m

Solution:

$$\tan\theta = \frac{opp}{adj}$$

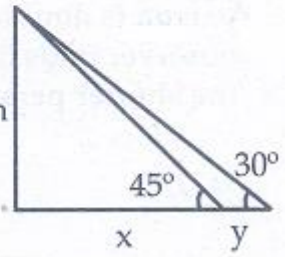
$$\tan 45^\circ = \frac{60}{x}$$

$$1 = \frac{60}{x} \Rightarrow \boxed{x = 60}$$

$$\tan 30^\circ = \frac{60}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{60}{x+y}$$

$$x + y = 60\sqrt{3}$$



$$y = 60\sqrt{3} - 60 = 103.92 - 60 = 43.92 \text{ m}$$

13. The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30° and 60° respectively. The height of the multistoried building and the distance between two buildings (in metres) is

(1) 20, $10\sqrt{3}$

(2) 30, $5\sqrt{3}$

(3) 20, 10

(4) 30, $10\sqrt{3}$

Solution:

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30 = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = h\sqrt{3}$$

$$\tan 60 = \frac{h+20}{x}$$

$$x = \frac{h+20}{\sqrt{3}}$$

$$h\sqrt{3} = \frac{h+20}{\sqrt{3}}$$

$$3h = h + 20$$

$$2h = 20$$

$$h = 10$$

$$x = h\sqrt{3}$$

$$= 10\sqrt{3}$$

Total height = 20 + 10 = 30m

14. Two persons are standing 'x' metres apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is

(1) $\sqrt{2}\,x$

(2) $x / 2\sqrt{2}$

(3) $x / \sqrt{2}$

(4) $2x$

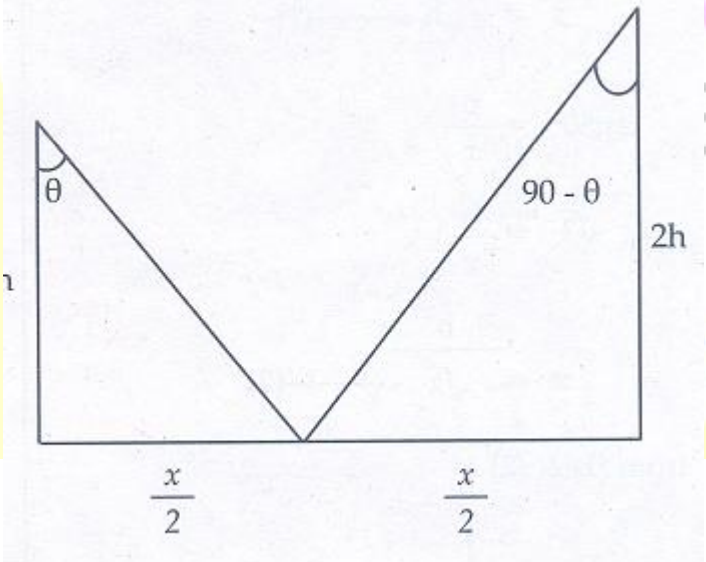
Solution:

$$\tan\theta = \frac{h}{x} = \frac{\frac{x}{2}}{h} = \frac{x}{2h}$$

$$\cot\theta = \frac{\frac{x}{2}}{2h} = \frac{x}{4h} \Rightarrow \tan\theta = \frac{4h}{x}$$

$$\frac{x}{2h} = \frac{4h}{x} \Rightarrow x^2 = 8h^2 \quad x = 2\sqrt{2}\,h$$

$$h = \frac{x}{2\sqrt{2}}$$



15. The angle of elevation of a cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45° . The height of location of the cloud from the lake is

(1) $\frac{h(1+\tan\beta)}{1-\tan\beta}$

(2) $\frac{h(1-\tan\beta)}{1+\tan\beta}$

(3) $h\tan(45°-\beta)$

(4) none of these

Ans: (1)

Solution:

height $= \frac{h(\tan\theta_1 + \tan\theta_2)}{\tan\theta_1 - \tan\theta_2}$

$= \frac{h(\tan 45° + \tan\beta)}{\tan 45° - \tan\beta}$

$= \frac{h(1+\tan\beta)}{1-\tan\beta}$

Unit Exercise

1. Prove that

(i) $\cot^2 A\left(\frac{\sec A-1}{1+\sin A}\right)+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)=0$

(ii) $\frac{\tan^2\theta-1}{\tan^2\theta+1}=1-2\cos^2\theta$

Solution: *LHS*

$$= \cot^2 A\left[\frac{\sec A-1}{1+\sin A}\right]+\sec^2 A\left[\frac{\sin A-1}{1+\sec A}\right]$$

$$= \frac{\cot^2 A(\sec A-1)(\sec A+1)+\sec^2 A(\sin A-1)(\sin A+1)}{(\sin A+1)(\sec A+1)}$$

$$= \frac{\cot^2 A(\sec^2 A-1)+(\sec^2 A)(\sin^2 A-1)}{(\sin A+1)(\sec A+1)} = \cot^2 A\times\tan^2 A+\sec^2 A\times(-\cos^2 A)$$

$$= \frac{1-1}{(\sin A+1)(\sec A+1)} = \frac{0}{(\sin A+1)(\sec A+1)} = 0 = \text{RHs}$$

Solution: *LHS*

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}-1}{\frac{\sin^2\theta}{\cos^2\theta}+1} = \frac{\frac{\sin^2\theta-\cos^2\theta}{\cancel{\cos^2\theta}}}{\frac{\sin^2\theta+\cos^2\theta}{\cancel{\cos^2\theta}}}$$

$$= \frac{\sin^2\theta-\cos^2\theta}{1}$$

$$= 1 - \cos^2\theta - \cos^2\theta$$

$$= 1 - 2\cos^2\theta = RHS$$

2. Prove that $\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Solution: *LHS*

$$= \left[\frac{(1+\sin\theta)-\cos\theta}{(1+\sin\theta)+\cos\theta}\right]^2$$

$$= \frac{(1+\sin\theta)^2+\cos^2\theta-2(1+\sin\theta)\cos\theta}{(1+\sin\theta)^2+\cos^2\theta+2(1+\sin\theta)\cos\theta}$$

$$= \frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta-2\cos\theta-2\cos\theta\sin\theta}{1+2\sin\theta+\sin^2\theta+\cos^2\theta+2\cos\theta+2\sin\theta\cos\theta}$$

$$= \frac{1+2\sin\theta+1-2\cos\theta-2\cos\theta\sin\theta}{1+2\sin\theta+1+2\cos\theta-2\sin\theta\cos\theta}$$

$$= \frac{2+2\sin\theta-2\cos\theta-2\sin\theta\cos\theta}{2+2\sin\theta+2\cos\theta+2\sin\theta\cos\theta}$$

$$= \frac{\cancel{2}(1+\sin\theta-\cos\theta-\sin\theta\cos\theta)}{\cancel{2}(1+\sin\theta+\cos\theta+\sin\theta\cos\theta)}$$

$$= \frac{(1+\sin\theta)-\cos\theta(1+\sin\theta)}{(1+\sin\theta)+\cos\theta(1+\sin\theta)}$$

$$= \frac{(1+\cancel{\sin\theta})(1-\cos\theta)}{(1+\cancel{\sin\theta})(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = \text{RHS}$$

3. If $x \sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then prove that $x^2 + y^2 = 1$.

Solution:

$\Rightarrow x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow y\cos\theta(1-\cos^2\theta) + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow y\cos\theta - y\cancel{\cos^3}\theta + y\cancel{\cos^3}\theta = \sin\theta\cos\theta$

$\boxed{y = \sin\theta}$ $\boxed{x = \cos\theta}$

$LHS = x^2+y^2 = \cos^2\theta + \sin^2\theta = 1 = \text{RHS}$

4. If $a\cos\theta - b\sin\theta = c$, then prove that $(a\sin\theta + b\cos\theta) = \pm\sqrt{[a^2 + b^2 - c^2]}$

Solution:

RHS $= a^2 + b^2 - c^2$

$= a^2 + b^2 - [a\cos\theta - b\sin\theta]^2$

$= a^2 + b^2 - [a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta$

$= a^2[1-\cos^2\theta] + b^2[1-\sin^2\theta] + 2ab\sin\theta\cos\theta$

$= a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta$

$= [a\sin\theta + b\cos\theta]^2$

$\Rightarrow [a\sin\theta + b\cos\theta]^2 = a^2 + b^2 - c^2$

Take square root on both sides

$a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$ hence proved.

5. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45° . The bird flies away horizontally in such away that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30° . Determine the speed at which the bird flies. ($\sqrt{3} = 1.732$)

Solution:

$\tan\theta = \frac{opp}{adj}$

$\tan 45° = \frac{80}{x}$

$x = 80$

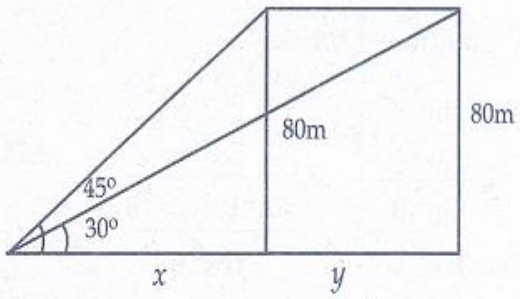
$\tan 30° = \frac{80}{x+y}$

$\frac{1}{\sqrt{3}} = \frac{80}{x+y}$

$x + y = 80\sqrt{3}$

$y = 80\sqrt{3} - x = 80.\sqrt{3} - 80$

$y = (\sqrt{3} - 1)80 = (1.732-1)80 = 0.732 \times 8 = 58.56$



Time × speed = distance

2 × speed = 58.56

speed = 29.28 m/s

6. An aeroplane is flying parallel to the Earth's surface at a speed of 175 m/sec and at a height of 600 m. The angle of elevation of the aeroplane from a point on the Earth's surface is 37° at a given point. After what period of time does the angle of elevation increase to 53° ? (tan 53° = 1.3270, tan 37° = 0. 7536)

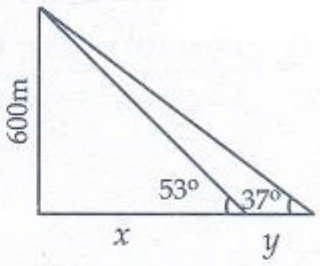
Solution:

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 37^\circ = \frac{600}{x+y}$$

$$0.7536 = \frac{600}{x+y}$$

$$(0.7536)\,(x+y) = 600$$



$$\tan 53^\circ = \frac{600}{x}$$

$$1.3270 = \frac{600}{x}$$

$$x = \frac{600}{1.3270}$$

$$x = 452.15$$

$$0.7536x + 7536y = 600$$

$$0.7536 \times 452.15 + 0.7536y = 600$$

$$0.75.36y = 600 - 340.74$$

$$0.7356y = 259.26$$

$$y = \frac{259.26}{0.7356}$$

$$y = 344.03$$

Distance = 344.03

Time × speed = 344.03

$$\text{Time} = \frac{344.03}{\text{speed}} = \frac{344.03}{175} = 1.97 \text{ sec}$$

Time = 1.97 sec

7. A bird is flying from *A* towards *B* at an angle of 35° , a point 30 km away from *A*. At *B* it changes its course of flight and heads towards *C* on a bearing of 48° and distance 32 km away.

(i) How far is B to the North of A?

(ii) How far is B to the West of A?

(iii) How far is C to the North of B?

(iv) How far is C to the East of B?

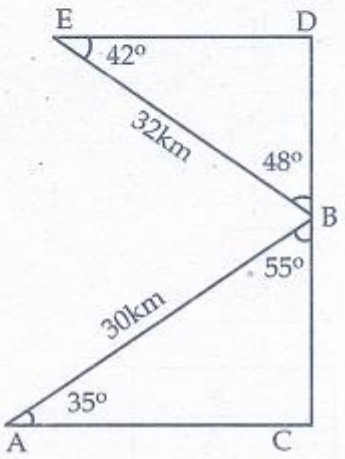
($\sin 55° = 0.8192$, $\cos 55° = 0.5736$, $\sin 42° = 0.6691$, $\cos 42° = 0.7431$)

Solution:

$$\sin\theta = \frac{opp}{hyp}, \cos\theta = \frac{adj}{hyp}$$

$$\Rightarrow \sin 55^\circ = \frac{AC}{30}$$

$$AC = 0.8192 \times 30 = 24.58\text{km}.$$



$$\Rightarrow \cos 55^\circ = \frac{BC}{30}$$

$$BC = 0.5736 \times 30 = 17.21\text{km}$$

$$\Rightarrow \sin 42^\circ = \frac{BD}{32}$$

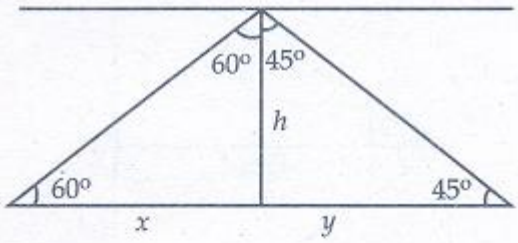
$$BD = 0.6691 \times 32 = 21.41\text{km}$$

$$\Rightarrow \cos 42^\circ = \frac{DE}{32}$$

$$DE = 0.7431 \times 32 = 23.78\text{km}$$

8. Two ships are sailing in the sea on either side of the lighthouse. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45° respectively. If the distance between the ships is 200 (($\sqrt{3}$+ 1) / $\sqrt{3}$) metres, find the height of the lighthouse.

Solution:



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 45^\circ = \frac{h}{y}$$

$$1 = \frac{h}{y}$$

$$\boxed{y = h}$$

$$\tan 60^\circ = \frac{h}{y}$$

$$\sqrt{3} = \frac{h}{y}$$

$$\boxed{x = \frac{h}{\sqrt{3}}}$$

$$x = y = h + \frac{h}{\sqrt{3}} = \frac{\sqrt{3}h + h}{\sqrt{3}}$$

$$= h\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) = 200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$$

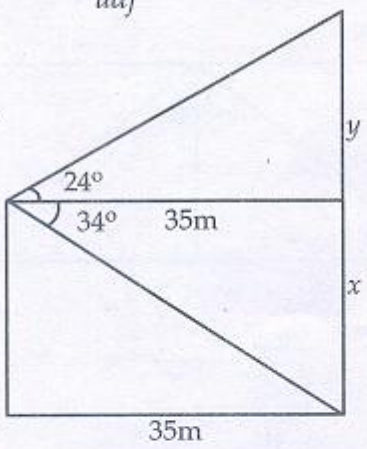
$$\boxed{h = 200m}$$

9. A building and a statue are in opposite side of a street from each other 35 m apart. From a point on the roof of building the angle of elevation of the top of

statue is 24° and the angle of depression of base of the statue is 34° . Find the height of the statue. (tan 24° = 0.4452, tan 34° = 0.6745)

Solution:

$$\tan\theta = \frac{opp}{adj}$$



$$\tan 24° = \frac{h}{35}$$

$$h = 0.4452 \times 35$$

$$\boxed{h = 15.58m}$$

$$\tan 34° = \frac{x}{35}$$

$$x = 0.6745 \times 35$$

$$\boxed{x = 23.61m}$$

Height of the statne = x + h = 23.61 + 15.58m

= 39.19 m

Points to Remember

- An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle.
- Trigonometric identities

(i) $\sin^2 \theta + \cos^2 \theta = 1$

(ii) $1 + \tan^2 \theta = \sec^2 \theta$

(iii) $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

- The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.
- The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level.
- The angle of depression of an object viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level.
- The height or length of an object or distance between two distant objects can be determined with the help of trigonometric ratios.