1 RELATIONS AND FUNCTIONS

FORMULAE TO REMEMBER

- **Vertical line test :**

  A curve drawn in a graph represents a functions, if every vertical line intersects the curve in at most one point.

- **Horizontal line test :**

  A function represented in a graph is one - one, if every horizontal line intersect the curve in at most one point.

- Linear functions has applications in Cryptography as well as in several branches of Science and Technology.

PUBLIC EXAM FREQUENTLY ASKED QUESTIONS

1 MARK

1. If $n(A) = p$, $n(B) = q$ then the total number of relations that exist between A and B is [PTA -1]

(A) 2^p (B) 2^q (C) 2^{p+q} (D) 2^{pq}

[Ans. (D) 2^{pq}]

Hint: $P(n(A \times B)) = 2^{n(A \times B)} = 2^{pq}$

2. Given $f(x) = (-1)^x$ is a function from $\mathbb{N}$ to $\mathbb{Z}$. Then the range of f is [PTA - 3]

(A) {1} (B) $\mathbb{N}$ (C) {1, –1} (D) $\mathbb{Z}$

[Ans. (C){1, –1}]

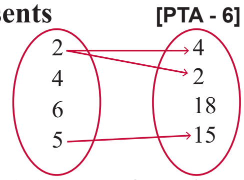
Hint:

$$f(1) = (-1)^1 = (-1)$$
$$f(2) = (-1)^2 = 1$$

$\therefore$ Range of f = {1, –1}

3. The given diagram represents [PTA - 6]

(A) an onto function
(B) constant function
(C) an one-one function
(D) not a function



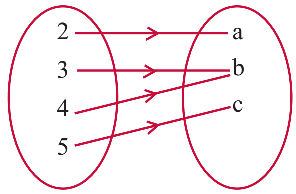
[Ans. (D) not a function]

4. $f = \{(2, a), (3, b), (4, b), (5, c)\}$ is a __________. [Govt. MQP - 2019]

(A) identity function (B) one-one function
(C) many-one function (D) constant function

[Ans. (C) many-one function]

Hint:



5. Let $f(x) = x^2 - x$, then $f(x-1) - f(x+1)$ is : [Sep. - 2020]

(A) $4x$ (B) $2 - 2x$ (C) $2 - 4x$ (D) $4x - 2$

[Ans. (C) $2 - 4x$]

Hint:

$$\begin{aligned} f(x-1) &= (x-1)^2 - (x-1) \\ &= x^2 - 2x + 1 - (x-1) \\ &= x^2 - 2x + 1 - x + 1 \\ &= x^2 - 3x + 2 \\ f(x+1) &= (x+1)^2 - (x+1) \\ &= x^2 + 2x + \cancel{1} - x - \cancel{1} \\ &= x^2 + x \end{aligned}$$

$\therefore f(x-1) - f(x+1)$

$= (x^2 - 3x + 2) - (x^2 + x)$

$= x^2 - 3x + 2 - x^2 - x$

$= -4x^2 + 2$

6. If n(A)= p and n(B)= q then n(A×B)= ______ [Qy. - 2019]

(A) $p + q$ (B) $p - q$ (C) $p \times q$ (D) $\frac{p}{q}$

[Ans. (C) $p \times q$]

Hint: $n(A \times B) = n(A) \times n(B) = p \times q$

7. For any two sets P and Q, P ∩ Q is [FRT - 2022]

(A) $\{x : x \in P \text{ or } x \in Q\}$
(B) $\{x : x \in P \text{ and } x \notin Q\}$
(C) $\{x : x \in P \text{ and } x \in Q\}$
(D) $\{x : x \notin P \text{ and } x \in Q\}$

[Ans. (C) $\{x : x \in P \text{ and } x \in Q\}$]

2 MARKS

1. A relation 'f' is defined by $f(x) = x^2 - 2$ where, $x \in \{-2, -1, 0, 3\}$ (i) List the elements of f (ii) Is f a function? [PTA - 1; Qy. - 2019]

Sol. $f(x) = x^2 - 2$ where $x \in \{-2, -1, 0, 3\}$

(i) $f(-2) = (-2)^2 - 2 = 2;$

$f(-1) = (-1)^2 - 2 = -1$

$f(0) = 0^2 - 2 = -2$

$f(3) = 3^2 - 2 = 9 - 2 = 7$

$\therefore f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$

(ii) We note that each element in the domain of f has a unique image.

Therefore f is a function.

2. A relation R is given by the set $\{(x, y)/y = x^2 + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$ Determine its domain and range. [PTA - 2]

Sol. Domain $= \{0, 1, 2, 3, 4, 5\}$

$x = 0, y = 0^2 + 3 = 3$

$x = 1, y = 1^2 + 3 = 4$

$x = 2, y = 2^2 + 3 = 7$

$x = 3, y = 3^2 + 3 = 12$

$x = 4, y = 4^2 + 3 = 19$

$x = 5, y = 5^2 + 3 = 28$

Range $= \{3, 4, 7, 12, 19, 28\}$

3. Find k, if $f(k) = 2k - 1$ and $fof(k) = 5$. [PTA - 4]

Sol. $f(k) = 2k - 1$

Consider $fof(k) = f(f(k)) = f(2k - 1)$ $[\because f(x) = 2k - 1]$

$= 2(2k - 1) - 1$

[In $f(k) = 2k - 1$, replace k by $2k - 1$]

$= 4k - 2 - 1 = 4k - 3$

$\Rightarrow 4k - 3 = 5 \Rightarrow 4k = 5 + 3 = 8$

$\Rightarrow k = \frac{8}{4} = 2$

$\therefore k = 2$

4. Let A = {1, 2, 3, ..., 100} and R be the relation defined as "is cube of" on A. Find the domain and range of R. [PTA - 4]

Sol. R $= \{(1,1) (2,8), (3,27), (4, 64)\}$

Domain $= \{1, 2, 3, 4, ...100\}$

Range $= \{1, 8, 27, 64\}$

5. Let A = {1, 2, 3, 4} and B = ℕ. Let f : A → B be defined by $f(x) = x^2$ (i) the range of f (ii) identify the type of function. [PTA - 5]

Sol. $f(1) = 1; f(2) = 4; f(3) = 9; f(4) = 16$

(i) Range $= \{1, 4, 9, 16\}$

(ii) One - one and into function

6. Let f be a function from ℝ to ℝ defined by $f(x) = 3x - 5$ Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f. [PTA - 6]

Sol. $f(x) = 3x - 5$ can be written as

$f = \{(x, 3x - 5) | x \in \mathbb{R}\}$

$(a, 4)$ mean the image of a is 4.

That is, $f(a) = 4$

$3a - 5 = 4 \Rightarrow a = 3$

$(1, b)$ means the image of 1 is b. That is,

That is, $f(1) = b \Rightarrow b = -2$

$3(1) - 5 = b \Rightarrow b = -2$

7. R = {(x, – 2), (–5, y) represents the identity function, find the values x and y. [PTA - 6]

Sol. $x = -2$

$y = -5$

8. Define a function. [Govt. MQP - 2019]

Sol. A relation f between two non-empty sets X and Y is called a function from X to Y if, for each $x \in X$ there exists only one $y \in Y$ such that $(x, y) \in f$.

That is, $f = \{(x, y) \mid \text{for all } x \in X, y \in Y\}$

9. Let f be a function f : ℕ → ℕ be defined by $f(x) = 3x + 2, x \in$ ℕ. [Govt. MQP - 2019]

(i) Find the images of 1, 2, 3
(ii) Find the pre-images of 29, 53
(iii) Identify the type of function

Sol. $f : \mathbb{N} \to \mathbb{N}$ is defined by $f(x) = 3x + 2$,

(i) $f(1) = 3(1) + 2 = 3 + 2 = 5$

$f(2) = 3(2) + 2 = 6 + 2 = 8$

$f(3) = 3(3) + 2 = 9 + 2 = 11$

The images of 1, 2, 3 are 5, 8, 11 respectively.

(ii) If x is the pre-image of 29, then $f(x) = 29$.

$\Rightarrow \quad 3x + 2 = 29$

$3x = 27 \quad \Rightarrow x = 9.$

Similarly, if x is the pre-image of 53, then $f(x) = 53. \Rightarrow 3x + 2 = 53$

$3x = 51$

$\Rightarrow \quad x = 17.$

$\therefore$ The pre-images of 29 and 53 are 9 and 17 respectively.

(iii) Since different elements of $\mathbb{N}$ have different images in the co-domain, the function f is one – one function.

The co-domain of f is $\mathbb{N}$.

But the range of $f = \{5, 8, 11, 14, 17, ...\}$ is a proper subset of $\mathbb{N}$.

$\therefore f$ is not an onto function. That is, f is an into function.

Thus f is one – one and into function.

10. Let A = {1, 2, 3, 4, 5}, B = W and f : A→B is defined by $f(x) = x^2 - 1$ find the range of f. **[Qy. - 2019]**

Sol. $f(1) = 0; f(2) = 3; f(3) = 8; f(4) = 15; f(5) = 24$

Range of $f = \{0, 3, 8, 15, 24\}$

11. Let A = {1, 2, 3, 4} and B = N. Let f : A →B be defined by $f(x) = x^3$ then, **[Hy. - 2019]**

(i) Find the range of f.

(ii) Identify the type of function

Sol.

$A = \{1, 2, 3, 4\}$

$B = \mathbb{N}$

$f : A \to B, f(x) = x^3$

(i) $f(1) = 1^3 = 1$

$f(2) = 2^3 = 8$

$f(3) = 3^3 = 27$

$f(4) = 4^3 = 64$

range of $f = \{1, 8, 27, 64, ...\}$

(ii) one - one function and into function

12. If A = {1, 3, 5} and B = {2, 3} then show that $n(A \times B) = n\{B\}$. **[Sep. - 2021]**

Sol.

$A = \{1, 3, 5\} \quad B = \{2, 3\}$

$n(A) = 3; n(B) = 2$

$\therefore \quad n(A) \times n(B) = 3 \times 2 = 6 \quad ...(1)$

$A \times B = \{(1, 2)\ (1, 3)\ (3, 2)\ (3, 3)\ (5, 2)\ (5, 3)\}$

$\therefore \quad n(A \times B) = 6 \quad ...(2)$

From (1) and (2)

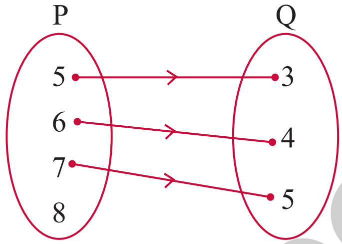
$n(A \times B) = n(A) \times n(B)$

13. For the given relation R = {(1, 3), (2,5), (4, 7), (5, 9), (3, 1)}, write the domain and range. **[FRT - 2022]**

Sol. Domain $= \{1, 2, 3, 4, 5\}$

Range $= \{1, 3, 5, 7, 9\}$

14. The arrow diagram shows a relationship between the sets P and Q. Write the releation in (i) set builder form (ii) Roster form. **[May - 2022]**



Sol. (i) Set builder form of R $= \{(x, y) \mid y = x - 2, x \in P, y \in Q\}$

(ii) Roster form R = {(5, 3),(6, 4),(7, 5)}

5 MARKS

1. $f(x) = 2x + 3, g(x) = 1 - 2x$ and $h(x) = 3x$, prove that $fo(g\ o\ h) = (f\ o\ g)\ oh$. **[PTA - 5]**

Sol.

$f(x) = 2x + 3, g(x) = 1 - 2x,$

$h(x) = 3x$

Now, $(f\ o\ g)(x) = f(g(x)) = f(1 - 2x)$

$= 2(1 - 2x) + 3 = 5 - 4x$

Then,

$(f\ o\ g)oh(x) = (f\ o\ g)h(x)) = (f\ o\ g)(3x)$

$= 5 - 4(3x) = 5 - 12x \quad ... (1)$

$(g\ o\ h)(x) = g(h(x)) = g(3x) = 1 - 2(3x)$

$= 1 - 6x$

So, $f\ o\ (g\ o\ h)(x) = f(1 - 6x) = 2(1 - 6x) + 3$

$= 5 - 12x \quad ... (2)$

From (1) and (2), we get

$(f\ o\ g)\ oh = f\ o\ (g\ o\ h)$

2. Let A = $\{x \in \mathbb{W} / 0 < x < 5\}$, B = $\{x \in \mathbb{W} / 0 \le x \le 2\}$, C = $\{x \in \mathbb{W} / x < 3\}$ then verify that A × (B ∩ C) = (A × B) ∩ (A× C) **[PTA - 3]**

Sol.

$A = \{1, 2, 3, 4\}$

$B = \{0, 1, 2\}$

$C = \{0, 1, 2\}$

$B \cap C = \{0, 1, 2\} \cap \{0, 1, 2\} = \{0, 1, 2\}$

$A \times (B \cap C) = \{1, 2, 3, 4\} \times \{0, 1, 2\}$

$= \{(1,0), (1,1), (1, 2), (2,0), (2,1), (2, 2), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4,2)\} \quad ... (1)$

$A \times B = \{1, 2, 3, 4\} \times \{0, 1, 2\}$

$= \{(1,0), (1,1), (1,2), (2,0),(2,1),(2,2), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2)$

$A \times C = \{1, 2, 3, 4\} \times \{0, 1, 2\}$

$= \{(1,0), (1,1), (1,2), (2,0),(2,1),(2,2), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2)$

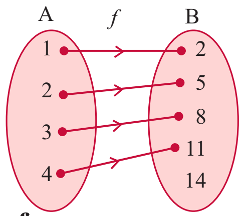
$(A \times B) \cap (A \times C) = \{(1,0), (1,1), (1,2), (2,0),(2,1),(2,2), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2) \quad ... (2)$

(1) = (2) Hence it is proved.

3. Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let f : A → B be a function given by $f(x) = 3x - 1$ Represent this function. **[PTA - 3]**
(i) by arrow diagram **[Sep. - 2020]**
(ii) in a table form
(iii) as a set of ordered pairs
(iv) in a graphical form

Sol. Let A = {1, 2, 3, 4} ; B = {2, 5, 8, 11, 14};
$f(x) = 3x - 1$
$f(1) = 3(1) - 1 = 3 - 1 = 2; f(2) = 3(2) - 1 = 6 - 1 = 5$
$f(3) = 3(3) - 1 = 9 - 1 = 8; f(4) = 4(3) - 1 = 12 - 1 = 11$

(i) Arrow diagram
Let us represent the function f :A → B by an arrow diagram

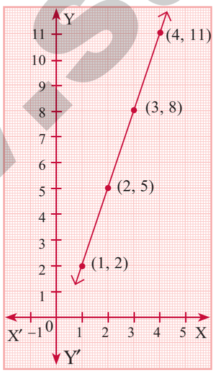


(ii) Table form
The given function f can be represented in a tabular form as given below

x	1	2	3	4
$f(x)$	2	5	8	11

(iii) Set of ordered pairs
The function f can be represented as a set of ordered pairs as
f = (1, 2),(2, 5),(3, 8),(4, 11)

(iv) Graphical form



In the adjacent xy -plane the points (1,2), (2,5), (3,8), (4,11) are plotted

4. If $f(x) = 3x-2$, $g(x) = 2x + k$ and if $fog = gof$, then find the value of k. **[Qy. - 2019]**

Sol.
$f(x) = 3x - 2, g(x) = 2x + k$
$fog(x) = f(g(x)) = f(2x + k)$
$= 3(2x + k)-2 = 6x + 3k - 2$
Thus, $fog(x) = 6x + 3k - 2$
$gof(x) = g(3x - 2) = 2(3x - 2) + k$
Thus, $gof(x) = 6x - 4 + k$
Given that $fog = gof$
$\therefore \quad 6x + 3k - 2 = 6x - 4 + k$
$6x - 6x + 3k - k = -4 + 2 \Rightarrow k = -1$

5. Let A = {$x \in \mathbb{N}$ / 1 < x < 4}, B = {$x \in \mathbb{W}$/0 ≤ x < 2} and C = {$x \in \mathbb{N}$ / x < 3}.Then verify that A × (B ∩ C) = (A × B) ∩ (A × C). **[Hy. - 2019]**

Sol.
A = {$x \in \mathbb{N}$/ 1 < x < 4} = {2, 3}
B = {$x \in \mathbb{W}$/0 ≤ x < 2} = {0, 1}
C = {$x \in \mathbb{N}$/x < 3} = {1, 2}
To prove A×(B ∩ C) = (A×B) ∩ (A×C)
B ∩ C = {0,1} ∩ {1,2}= {1}
A ×(B ∩ C) = {2,3}×{1} ={(2,1),(3,1)}
A × B = {2,3}×{0,1}
= {(2,0),(2,1),(3,0),(3,1)}
A × C = {2,3}×{1,2}
= {(2,1),(2,2),(3,1),(3,2)}
(A×B) ∩(A×C) = {(2,0),(2,1),(3,0),(3,1)} ∩{(2,1),(2,2),(3,1),(3,2)}
= {(2,1),(3,1)}
A×(B ∩C) = (A×B) ∩(A×C) is verified.

8 MARKS

1. Let X = {1, 2, 3, 4, 5}, Y = {1, 3, 5, 7, 9}, which of the following are relation from X to Y?
(i) R_1 = {(1, 3) (2,4) (3,5) (4,6) (5,7)}
(ii) R_2 = {(1, 1) (2, 1) (3,3) (4,3) (5,5)}
(iii) R_3 = {(1, 1) (1, 3) (3,5) (3,7) (5,7)}
(iv) R_4 = {(1, 3) (2, 5) (4,7) (5,9) (3,1)}
[FRT - 2022]

Sol. (i) R_1 = {(1, 3) (2, 4) (3, 5) (4, 6) (5, 7)}
Here 1 is related to 3, 3 is related to 5, 5 is related to 7.
But $4 \notin y$ and $6 \notin y$, we can say that 2 cannot be related to 4 and 4 cannot be related to 6.
Hence R_1 is a not a relation from X to Y.
(ii) R_2 = {(1, 1) (2,1) (3,3) (4, 3) (5, 5)}
R_2 is a relation since {1, 3, 5}, belongs to Y and {1, 2, 3, 4, 5} belongs to X.
(iii) R_3 = {(1, 1) (1,3) (3,5) (3, 7) (5, 7)}
R_3 is a relation
(iv) R_4 = {(1,3) (2,5) (4,7) (5, 9) (3, 1)}
R_4 is a relation