

DIRECTORATE OF GOVERNEMENT EXAMINATIONS, CHENNAI-6
HSE PUBLIC EXAMINATIONS (FIRST YEAR) - MARCH/APRIL 2023
BUSINESS MATHEMATICS AND STATISTICS - ANSWER KEY

Maximum Marks - 90

General Instructions :

1. Answers written only in **BLACK** or **BLUE** should be evaluated.
2. For objective type questions, award 1 mark for "writing the correct option's code and the corresponding option's answer".
3. Award "0 marks" for one who wrote both "option's code" and "option's answer" with one of them is not correct.
4. Marks should be awarded for suitable alternative method also.
5. Mark(s) should not be reduced for the correct answer / stage, if it is written without formula / properties also, 2* means award one mark for the formula.
6. Award full mark directly, if the solution is arrived with no mistakes without giving weightage for the stages.
7. The stage mark is essential, only if the part of the solution is incorrect.
8. Award marks, if the answer is in decimal value and also approximately equal to the key answer
9. **Important Note for Part II, Part III and Part IV :**
For a particular stage in which the stage mark is greater than 1 and one who begins with correct step but reaches with incorrect solution, for such suitable credits should be given by breaking the stage marks.

PART - I

- i. Answer all the questions
 ii. Choose the most appropriate from the given four alternatives and write the option code and the corresponding answer

20×1=20

Q.No	Option	Answer
1	(c)	2
2	(b)	0
3	(c)	5
4	(b)	2^n
5	(d)	$4x - 1 = 0$
6	(a)	$\tan^{-1} \frac{\sqrt{33}}{5}$
7	(c)	$\frac{1}{4}$
8	(d)	$\sin 50^\circ$
9	(b)	(0, 0)
10	(c)	
11	(b)	2
12	(c)	Face Value
13	(a)	An endowment fund to give scholarship to the students.
14	(d)	Q_2
15	(b)	1
16	(c)	$\frac{1}{4}$
17	(d)	Two regression lines
18	(d)	Positive
19	(a)	Minimize the total project duration
20	(d)	15

PART - II

Q.No	Answer any <u>seven</u> questions. Question no. 30 is compulsory.		7x2=14
21	$x^2 - (x^2-1)$	1	2
	$= 1$	1	
22	Total number of ways = $7P_5$	1	2
	$= 2520$	1	
23	vertex $v(h,k) = v(0,0)$	1	2
	focus $f(a,0) = f(5,0)$	1	
24	$\lim_{x \rightarrow \infty} \left(\frac{x-3}{1+x+x^2} \right)$	1	2
	$= 0$	1	
25	MC $= \frac{dc}{dx} = 4 + \frac{3}{2\sqrt{x}}$	1	2
	When $x=9$, MC = ₹ 4.50	1	
26	Market value of a share = ₹ 118	1	2
	Market value of 325 shares = $118 \times 325 = ₹ 38,350$	1	
27	HM $= \frac{1}{\frac{1}{100} + \frac{1}{200} + \frac{1}{300} + \frac{1}{400}}$	1	2
	$= 192 \text{ km/hr}$	1	
28	$r = \frac{\sqrt{N\sum X^2 - (\sum X)^2} \sqrt{N\sum Y^2 - (\sum Y)^2}}{9(597) - (45)(108)}$	1	2
	$= \frac{513}{540} = + 0.95$	1	
29	<pre> graph LR 1((1)) -- A --> 2((2)) 1 -- B --> 3((3)) 2 -- C --> 4((4)) 3 -- D --> 4 4 -- E --> 5((5)) 4 -- F --> 6((6)) 5 -- G --> 7((7)) 6 -- H --> 7 </pre>		2
30	Compulsory : $\tan 150^\circ = \tan(180^\circ - 30^\circ) = -\tan 30^\circ$	1	2
	$= -\frac{1}{\sqrt{3}}$	1	

PART - III

Q.No	Answer any seven questions. Question number 40 is compulsory.		7×3=21
31	A has inverse $ A = \begin{vmatrix} 1 & 4 \\ 2 & 7 \\ 9 & 11 \end{vmatrix} = 0$ $-16 + 28 = 0$ $= \frac{7}{4}$	1 1 1	3
32	Alphabetical order are A, C, H, T Number of words starting with A = 3! = 6 Number of words starting with CA = 2! = 2 Number of words starting with CHAT = 0! = 1 \therefore Rank of the word 'CHAT' = 9	1 1 1	3
33	(i) $b-2=0$ $b=2$ (ii) $a-2=b$ $\Rightarrow a=4$ \therefore Equation of the circle is $2x^2 + 2y^2 + 4x + 4y - 1 = 0$	1 1 1	3
34	$\tan^{-1} \frac{2}{3} + \tan^{-1} \frac{1}{4} = \tan^{-1} \frac{2+3}{1-2 \times \frac{3}{4}}$ $= \tan^{-1} \left(\frac{1+2}{1-\frac{2}{2} \times \frac{11}{2}} \right)$ $= \tan^{-1} \left(\frac{3}{1-\frac{11}{2}} \right)$	2* 1	3
35	$y = \frac{10}{x+5} + 5$ $\frac{dy}{dx} = 3 \left[1 + \frac{10}{(x+5)^2} \right]$ \therefore When 'x' increases MC decreases	1 1 1	3
36	Income from 20% stock at ₹140 = $\frac{20}{100} \times 140 \times 70$ $= ₹1400$ Income from 10% stock at ₹70 = $\frac{10}{100} \times 140 \times 70$ $= ₹1400$ Both are equal investments.	1 1 1	3

37	<table border="1"> <thead> <tr> <th>Marks</th> <th>No. of students</th> <th>c.f</th> </tr> </thead> <tbody> <tr> <td>10</td> <td>4</td> <td>4</td> </tr> <tr> <td>20</td> <td>7</td> <td>11</td> </tr> <tr> <td>30</td> <td>15</td> <td>26</td> </tr> <tr> <td>40</td> <td>8</td> <td>34</td> </tr> <tr> <td>50</td> <td>7</td> <td>41</td> </tr> <tr> <td>60</td> <td>2</td> <td>43</td> </tr> </tbody> </table>	Marks	No. of students	c.f	10	4	4	20	7	11	30	15	26	40	8	34	50	7	41	60	2	43		3
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N=43 Q ₁ = Size of 11 th value = 20	1																							
D _{z = 8.8} Size of 8.8 th value = 20	1																							
F _{z = 39.6} Size Of 39.6 th value = 50	1																							
38	$d_z = 128$ $= 1 - \frac{6z^2}{N(N^2-1)} = 1 - \frac{6(128)^2}{10(10^2-1)}$ $= 0.2242$	1	3																					
39	<p>Maximize $Z = 50x_1 + 15x_2$</p> <p>Subject to the constraints</p> $x_1 + x_2 \leq 60$ $5x_1 + 2x_2 \leq 100$ $x_1, x_2 \geq 0$	1	3																					
40	<p>Compulsory :</p> $\frac{dx}{d\theta} = 3a \sec^3 \theta \tan \theta$ $\frac{dy}{d\theta} = 3b \tan^2 \theta \sec \theta$ $\frac{dy}{dx} = \frac{b \tan \theta}{a \sec \theta} \text{ (or) } \frac{dy}{dx} = \frac{b}{a} \sin \theta$	1	3																					
		1																						
		1																						

PART - IV

Q.No	Answer all the question	7x5=35
41 (a)	$AB = \frac{1}{5} \begin{bmatrix} 4 & 3 & 1 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 4 & -3 & -1 \\ -1 & -2 & 4 \end{bmatrix}$	1
	$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$	1
	$BA = \frac{1}{5} \begin{bmatrix} 4 & -2 & -1 \\ -1 & 3 & -1 \\ -1 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$	1
	$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \text{ (or) } \text{Similarly we can prove } BA=I \text{ (2 Marks)}$	1
	<p>AB = BA = I \therefore A and B are inverse of each other. (OR)</p>	1
41 (b)	$\tan(2 +) = \frac{\tan 2 + \tan}{1 - \tan 2 \cdot \tan}$	1
	$\tan 2 = \frac{3}{4}$	2*
	$\tan(2 +) = 1$	1
	$\therefore (2 +) = \frac{1}{\tan}$	1
	<p>(OR)</p>	
42 (a)	$t^{2r} (2 - 12C_r) = 12C_{r+1} t^{2(r+1)}$	2*
	$24 - 3r = 0$	1
	$r = 8$	1
	$t^{8+1} = 12C_8 2^8 = 7920$	1
	<p>(OR)</p>	
42 (b)	$\frac{\partial d}{\partial p_1} = -2 - 2p_1, \frac{\partial d}{\partial p_2} = 1 - p_1^2$	1
	$\frac{E_d}{E_{p_1}} = \frac{-2 - 2p_1}{-2} = 1 + p_1$	1
	$\frac{E_d}{E_{p_1}} = \frac{-2 - 2p_1}{-2} = 1 + p_1$	1
	$\frac{E_d}{E_{p_1}} = \frac{-2 - 2p_1}{-2} = 1 + p_1$	1
	$\frac{E_d}{E_{p_2}} = \frac{1 - p_1^2}{1 - p_1^2} = 1$	1
	$\frac{E_d}{E_{p_2}} = \frac{1 - p_1^2}{1 - p_1^2} = 1$	1

43 (a)	$P(n) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} = \frac{n^2 + 1}{4}$	1	5																																																										
	$n = 1, P(1) \text{ is true.}$	1																																																											
	$n = k, P(k) \text{ is true.}$	1																																																											
	$P(k+1) = \dots P(k+1) \text{ is true}$	1																																																											
	$P(n) \text{ is true for all } n \in \mathbb{N}$ (OR)	1																																																											
43 (b)	$P(n) \text{ is true for all } n \in \mathbb{N}$	2	1																																																										
	$L[f(x)]_{x=2} = \lim_{h \rightarrow 0} \{2 - (2 - h)\} = 0$	2																																																											
	$R[f(x)]_{x=2} = \lim_{h \rightarrow 0} \{2 - (2 - h)\} = 0$ $R[f(x)]_{x=2} = \lim_{h \rightarrow 0} \{4 + h\} = 4$	2																																																											
	Hence $f(x)$ is not continuous at $x = 2$	1																																																											
44 (a)		2	2																																																										
	<table border="1"> <thead> <tr> <th>Activity</th> <th>Duration t_{ij}</th> <th>EST</th> <th>EFT = EST + t_{ij}</th> <th>LFT</th> <th>LST = LFT - t_{ij}</th> </tr> </thead> <tbody> <tr> <td>1-2</td> <td>7</td> <td>0</td> <td>7</td> <td>7</td> <td>0</td> </tr> <tr> <td>1-6</td> <td>6</td> <td>0</td> <td>6</td> <td>7</td> <td>1</td> </tr> <tr> <td>2-3</td> <td>14</td> <td>7</td> <td>21</td> <td>21</td> <td>7</td> </tr> <tr> <td>2-4</td> <td>5</td> <td>7</td> <td>12</td> <td>25</td> <td>20</td> </tr> <tr> <td>3-5</td> <td>11</td> <td>21</td> <td>32</td> <td>32</td> <td>21</td> </tr> <tr> <td>4-5</td> <td>7</td> <td>12</td> <td>19</td> <td>32</td> <td>25</td> </tr> <tr> <td>6-7</td> <td>11</td> <td>6</td> <td>17</td> <td>18</td> <td>7</td> </tr> <tr> <td>5-8</td> <td>4</td> <td>32</td> <td>36</td> <td>36</td> <td>32</td> </tr> <tr> <td>7-8</td> <td>18</td> <td>17</td> <td>35</td> <td>36</td> <td>18</td> </tr> </tbody> </table>	Activity		Duration t_{ij}	EST	EFT = EST + t_{ij}	LFT	LST = LFT - t_{ij}	1-2	7	0	7	7	0	1-6	6	0	6	7	1	2-3	14	7	21	21	7	2-4	5	7	12	25	20	3-5	11	21	32	32	21	4-5	7	12	19	32	25	6-7	11	6	17	18	7	5-8	4	32	36	36	32	7-8	18	17	35	36
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	The critical path = 1 - 2 - 3 - 5 - 8 Duration to complete the project = 36 days (OR)	1	5																																																										
44 (b)	$\log y = \log \frac{(x-1)(x-2)}{\sqrt{(x-3)(x^2+x+1)}}$	1	1																																																										
	$\log y = \frac{1}{2} [\log(x-1) + \log(x-2) - \log(x-3) - \log(x^2+x+1)]$	1																																																											
	$\frac{1}{y} \frac{dy}{dx} = \frac{1}{2} \left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{2x+1}{x^2+x+1} \right]$	2																																																											
	$\frac{dy}{dx} = \frac{1}{2} \sqrt{\frac{(x-1)(x-2)}{(x-3)(x^2+x+1)}} \left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{2x+1}{x^2+x+1} \right]$	1																																																											

45 (a)	$r = \frac{N \sum XY - \sum X \sum Y}{\sqrt{N \sum X^2 - (\sum X)^2} \sqrt{N \sum Y^2 - (\sum Y)^2}}$ $r = \frac{10(8171) - (311 \times 257)}{\sqrt{(10 \times 9875) - (311)^2} \sqrt{(10 \times 6763) - (257)^2}} = \frac{1783}{2029 \sqrt{1581}}$ $r = 0.996$ <p>Note : Any suitable method award full marks (OR)</p>	2 1 1 1	5	
45 (b)	$EOQ = \sqrt{\frac{2CR}{C_1}} = 2000 \text{ units}$ <p>Minimum inventory cost = $\sqrt{2RC_1C_2} = 4$</p> <p>EOQ in Rupees = ₹ 40</p> <p>EOQ in years of supply = $\frac{EOQ}{R} = 2.5$</p> <p>Number of orders per year = $\frac{D}{EOQ} = 0.4$</p>	1 1 1 1 1		
46 (a)	Mere Attempt (OR)	5		
46 (b)	$f = \frac{-y}{2}, g = \frac{-gx}{2}, c = 0$ <p>The required equation of the circle is $x^2 + y^2 - x - y = 0$</p>	1 1+1+1		5
47 (a)	$A = \frac{10000}{(1 + \frac{0.08}{4})^{10 \times 4}} = 100000[(1.02)^{40} - 1] = ₹1,20,800$ <p>(OR)</p>	1 2* 1+1		
47 (b)	$4x + 3y + 2z = 320, \quad 2x + 4y + 6z = 560, \quad 6x + 2y + 3z = 380$ $ A = 50 \quad \text{Adj } A = \begin{bmatrix} 20 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$ $X = \frac{1}{ A } \text{Adj } A \cdot B = \frac{1}{50} \begin{bmatrix} 20 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 320 \\ 560 \\ 380 \end{bmatrix}$ <p>$x = 20, \quad y = 40, \quad z = 60$ cost of 1kg onion = Rs.20 cost of 1kg wheat = Rs.40 cost of 1 kg rice = Rs.60</p>	1 1+1 1 1	5	