

SECOND REVISION EXAMINATION - 2023

11 - STD

MATHS

Exam Time : 03:00:00 Hrs

Reg.No. :

Total Marks : 90

20 x 1 = 20

I. ANSWER THE FOLLOWING

- 1) The function $f: [0, 2\pi] \rightarrow [-1, 1]$ defined by $f(x) = \sin x$ is
 (a) one-to-one (b) on to (c) bijection (d) cannot be defined
- 2) The rule $f(x) = x^2$ is a bijection if the domain and the co-domain are given by
 (a) \mathbb{R}, \mathbb{R} (b) $\mathbb{R}, (0, \infty)$ (c) $(0, \infty), \mathbb{R}$ (d) $[0, \infty), [0, \infty)$
- 3) The solution $5x-1 < 24$ and $5x+1 > -24$ is
 (a) (4,5) (b) (-5,-4) (c) (-5,5) (d) (-5,4)
- 4) If a and b are the real roots of the equation $x^2 - kx + c = 0$, then the distance between the points $(a, 0)$ and $(b, 0)$ is
 (a) $\sqrt{k^2 - 4c}$ (b) $\sqrt{4k^2 - c}$ (c) $\sqrt{4c - k^2}$ (d) $\sqrt{k - 8c}$
- 5) $(1 + \cos \frac{\pi}{8}) (1 + \cos \frac{3\pi}{8}) (1 + \cos \frac{5\pi}{8}) (1 + \cos \frac{7\pi}{8}) =$
 (a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$
- 6) The number of 5 digit numbers all digits of which are odd is
 (a) 25 (b) 5^5 (c) 5^6 (d) 625
- 7) The n^{th} term of the sequence 1, 2, 4, 7, 11, ... is
 (a) $n^3 + 3n^2 + 2n$ (b) $n^3 \cdot 3n^2 + 3n$ (c) $\frac{n(n+1)(n+2)}{3}$ (d) $\frac{n^2 - n + 2}{2}$
- 8) Which of the following point lie on the locus of $3x^2 + 3y^2 - 8x - 12y + 17 = 0$
 (a) (0, 0) (b) (-2, 3) (c) (1, 2) (d) (0, -1)
- 9) The line $(p + 2q)x + (p - 3q)y = p - q$ for different values of p and q passes through the point
 (a) $(\frac{3}{5}, \frac{5}{2})$ (b) $(\frac{2}{5}, \frac{2}{5})$ (c) $(\frac{3}{5}, \frac{3}{5})$ (d) $(\frac{2}{5}, \frac{3}{5})$
- 10) What must be the matrix X , if $2x + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$?
 (a) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -6 \\ 4 & -2 \end{bmatrix}$
- 11) If $A + I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$, then $(A + I)(A - I)$ is equal to
 (a) $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & 4 \\ -8 & 9 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 4 \\ 8 & 9 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -4 \\ -8 & -9 \end{bmatrix}$
- 12) The value of $\vec{AB} + \vec{BC} + \vec{DA} + \vec{CD}$ is
 (a) \vec{AD} (b) \vec{CA} (c) $\vec{0}$ (d) $-\vec{AD}$
- 13) If \vec{a} and \vec{b} are two vectors of magnitude 2 and inclined at an angle 60° , then the angle between \vec{a} and $\vec{a} + \vec{b}$ is
 (a) 30° (b) 60° (c) 45° (d) 90°
- 14) $\lim_{x \rightarrow 0} \frac{xe^x - \sin x}{x}$ is
 (a) 1 (b) 2 (c) 3 (d) 0
- 15) If $y = \frac{1}{a-z}$, then $\frac{dz}{dy}$ is
 (a) $(a-z)^2$ (b) $-(z-a)^2$ (c) $(z+a)^2$ (d) $-(z+a)^2$

16) If $y = \frac{(1-x)^2}{x^2}$, then $\frac{dy}{dx}$ is

- (a) $\frac{2}{x^2} + \frac{2}{x^3}$ (b) $-\frac{2}{x^2} + \frac{2}{x^3}$ (c) $-\frac{2}{x^2} - \frac{2}{x^3}$ (d) $-\frac{2}{x^2} + \frac{2}{x^3}$

17) If $\int f(x)dx = g(x) + c$, then $\int f(x)g'(x)dx$

- (a) $\int (f(x))^2 dx$ (b) $\int f(x)g(x)dx$ (c) $\int f'(x)g(x)dx$ (d) $\int (g(x))^2 dx$

18) $\int x^2 e^{\frac{x}{2}} dx$ is

- (a) $x^2 e^{\frac{x}{2}} - 4x e^{\frac{x}{2}} - 8e^{\frac{x}{2}} + c$ (b) $2x^2 e^{\frac{x}{2}} - 8x e^{\frac{x}{2}} - 16e^{\frac{x}{2}} + c$ (c) $2x^2 e^{\frac{x}{2}} - 8x e^{\frac{x}{2}} + 16e^{\frac{x}{2}} + c$

- (d) $x^2 e^{\frac{x}{2}} - \frac{4x e^{\frac{x}{2}}}{2} + \frac{8e^{\frac{x}{2}}}{4} + c$

19) A, B, and C try to hit a target simultaneously but independently. Their respective probabilities of hitting the target are $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}$. The probability that the target is hit by A or B but not by C is

- (a) $\frac{21}{64}$ (b) $\frac{7}{32}$ (c) $\frac{9}{64}$ (d) $\frac{7}{8}$

20) If m is a number such that $m \leq 5$, then the probability that quadratic equation $2x^2 + 2mx + m + 1 = 0$ has real roots is

- (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

II. ANSWER ANY SEVEN (QUESTION NUMBER 30 COMPULSORY)

7 x 2 = 14

21) Write the following in roster form.

$\{x \in \mathbb{N} : x^2 < 121 \text{ and } x \text{ is a prime}\}$

22) Solve $|2x - 17| = 3$ for x.

23) Write the n^{th} term of the following sequences

$\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$

24) If p (r, c) is mid - point of a line segment between the axes, then show that $\frac{x}{r} + \frac{y}{c} = 2$

25) If the area of the triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 square units, find the values of k.

26) Find the value λ for which the vectors \vec{a} and \vec{b} are perpendicular, where $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

27) Find the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

28) Calculate $\lim_{x \rightarrow \infty} \frac{x^2 + 2x + 3}{(5x^2 + 1)}$.

29) Find $\frac{dy}{dx}$ if $x = a(t - \sin t)$, $y = a(1 - \cos t)$.

30) If for two events A and B, $P(A) = \frac{3}{4}$, $P(B) = \frac{2}{5}$ and $A \cup B = S$ (sample space), find the conditional probability $P(A/B)$.

Handwritten calculations for question 30:

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

III. ANSWER ANY SEVEN (QUESTION NUMBER 40 COMPULSORY)

7 x 3 = 21

31) Find the domain of $f(x) = \frac{1}{1 - 2\cos x}$.

32) Prove $\log \frac{a^2}{bc} + \log \frac{b^2}{ca} + \log \frac{c^2}{ab} = 0$

33) Find the value of cosec (-1410°).

34) If ${}^n P_r = 720$. If ${}^n C_r = 120$, find n, r = ?

35) Find the equations of a parallel line and a perpendicular line passing through the point (1, 2) to the line $3x + 4y = 7$.

36) For what value of x, the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{bmatrix}$ is skew-symmetric.

37) Verify that $|AB| = |A| |B|$ if $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

38) Find the angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$.

39) Find the derivatives of the following functions with respect to corresponding independent variables : $y = e^x \sin x$

40) Find the integrals of the following : $\frac{1}{(x+1)^2 - 25}$

Handwritten calculations for question 36:

$6 \times 5 \times 4 \times 3 \times 2 \times 1$

$\frac{120 \times 6}{720}$

IV. ANSWER THE FOLLOWING

7 x 5 = 35

- 41) a) Let $f, g: R \rightarrow R$ be defined as $f(x) = 2x - |x|$ and $g(x) = 2x + |x|$. find $f \circ g$.

(OR)

b) Prove that
$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right).$$

- 42) a) By the principle of mathematical induction, prove that, for all integers $n \geq 1$, $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$.

(OR)

b) Integrate the following with respect to x : $\int \frac{2x+4}{x^2+4x+6} dx$

- 43) a) Compute the sum of first n terms of the following series $6 + 66 + 666 + \dots$

(OR)

b) Prove by vector method, The medians of a triangle are concurrent.

- 44) a) Show that the equation $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines. Find the distance between them

(OR)

b) Show that the vectors $5\hat{i} + 6\hat{j} + 7\hat{k}, 7\hat{i} - 8\hat{j} + 9\hat{k}, 3\hat{i} + 20\hat{j} + 5\hat{k}$ are coplanar.

- 45) a) If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, Show that $(1-x^2)y_2 - 3xy_1 - y = 0$.

(OR)

b) A construction company employs 2 executive engineers. Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer-2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work?

- 46) a) If $A + B + C = \frac{\pi}{2}$, prove the following $\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cos B \cos C$

(OR)

b) Show that
$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x).$$
 Using factor method.

- 47) a) Resolve into partial fractions: $\frac{x}{(x+3)(x-4)}$

(OR)

b) Evaluate the following limits:

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$$

Handwritten calculations for partial fractions and limits:

Partial fractions: $\frac{x}{(x+3)(x-4)} = \frac{A}{x+3} + \frac{B}{x-4}$

Limit calculation: $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4} = \frac{0}{0}$ (L'Hôpital's rule)

Derivatives: $\frac{d}{dx}(\sqrt{x^2+1}-1) = \frac{x}{\sqrt{x^2+1}}$, $\frac{d}{dx}(\sqrt{x^2+16}-4) = \frac{x}{\sqrt{x^2+16}}$

Final result: $\lim_{x \rightarrow 0} \frac{\frac{x}{\sqrt{x^2+1}}}{\frac{x}{\sqrt{x^2+16}}} = \frac{\sqrt{x^2+16}}{\sqrt{x^2+1}} = \frac{4}{1} = 4$