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EXAM NO:

STD: XI MATHEMATICS MARKS: 90

A type GOVT. PUBLIC EXAM (MAR 2023) TIME: 3 hr

PART – I

Choose the correct answer **20 x 1 = 20**

1. The rule $f(x) = x^2$ is a bijection if the domain and the co-domain are given by:
   (a) $(0, \infty], R$ (b) R, R **(c) $[0, \infty), [0, \infty)$** (d) $R, (0, \infty)$
2. If $f(x) = mx + c$ and $f(0) = f'(0) = 1$ then $f(3)$ is:
   (a) 3 (b) 1 **(c) 4** (d) 2
3. There are 8 points in a plane and 4 of them are collinear. The number of straight lines joining any 2 points is:
   (a) 39 (b) 45 (c) 38 **(d) 23**
4. The points lie on the locus of $3x^2 + 3y^2 - 8x - 12y + 17 = 0$.
   (a) $(1, 2)$ (b) $(0,0)$ (c) $(0, -1)$ (d) $(-2,3)$
5. Which of the following is not true about the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}$?
   (a) an upper triangular matrix **(b) a scalar matrix**
   (c) a lower triangular matrix (d) a diagonal matrix
6. Which of the following is not true?
   (a) $\tan\theta = 25$ (b) $\sin\theta = -\frac{3}{4}$ **(c) $\sec\theta = \frac{1}{4}$** (d) $\cos\theta = -1$
7. Number of sides of a polygon having 44 diagonals is :
   (a) 11 (b) 4 (c) 22 (d) 4!
8. If $n \in N$, then $7^{2n} + 3^{3n-3} \cdot 3^{n-1}$ is always divisible by :
   (a) 45 **(b) 25** **(c) 55** **(d) 35**
9. The value of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{DA} + \overrightarrow{CD}$ is :
   (a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $-\overrightarrow{AD}$ (d) $\overrightarrow{CA}$
10. $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx =$
    (a) $-2\ \sin\sqrt{x} + c$ (b) $2\cos\sqrt{x} + c$ **(c) $-2\cos\sqrt{x} + c$** (d) $2\sin\sqrt{x} + c$
11. $\lim_{x \to 0} \frac{\sqrt{1-\cos 2x}}{x} =$
    (a) 1 (b) $\sqrt{2}$ (c) 0 **(d) None of the above**
12. $\int \sin^3 x\, dx$ is :
    (a) $-\frac{3}{4}\cos x + \frac{\cos 3x}{12} + c$ (b) $-\frac{3}{4}\cos x - \frac{\cos 3x}{12} + c$
    (c) $-\frac{3}{4}\sin x - \frac{\sin 3x}{12} + c$ (d) $\frac{3}{4}\cos x + \frac{\cos 3x}{12} + c$
13. The sequence $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}+\sqrt{2}}, \frac{1}{\sqrt{3}+2\sqrt{2}}$ . . form an:
    (a) Harmonic Progression (b) Arithmetic Progression
    (c) Arithmetico-Geometric Progression (d) Geometric Progression

14. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ are :

(a) 4 (b) 2 (c) 1 (d) 3

15. The number of relations on a set containing 3 elements is :

(a) 512 (b) 9 (c) 1024 (d) 81

16. If $|\vec{a}| = 13$, $|\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 60$ then $|\vec{a} \times \vec{b}|$ is:

(a) 45 (b) 15 **(c) 25** (d) 35

17. $\cos 1° + \cos 2° + \cos 3° + \cdots + \cos 179° =$

(a) -1 **(b) 0** (c) 89 (d) 1

18. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals to :

(a) 3 **(b) -3** (c) 1 (d) -1

19. If ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^{n}C_3$ then :

(a) $n > 7$ (b) $n > 5$ (c) $n > 4$ (d) $n > 6$

20. Ten coins are tossed. The probability of getting atleast 8 heads is :

(a) $\frac{7}{16}$ (b) $\frac{7}{64}$ **(c) $\frac{7}{128}$** (d) $\frac{7}{32}$

PART - II

Answer any SEVEN questions

Question number 30 is compulsory **7 x 2 = 14**

21. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, find $n((A \cup B) \times (A \cap B) \times (A \Delta B))$.

Solution:

We have $n(A \cup B) = 6$, $n(A \cap B) = 2$ and $n(A \triangle B) = 4$.

$$n\big((A \cup B) \times (A \cap B) \times (A \triangle B)\big) = n(A \cup B) \times n(A \cap B) \times n(A \triangle B)$$
$$= 6 \times 2 \times 4 = 48.$$

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22. (a) The odds that the event A occurs is 5 to 7, find P(A).

(b) Suppose $P(B) = \frac{2}{5}$. Express the odds that the event B occurs.

Solution:

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(a) $P(A) = \frac{5}{12}$

(b) Odds for B is 2: 3

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23. Prove $\log a + \log a^2 + \log a^3 + \cdots + \log a^n = \frac{n(n+1)}{2} \log a$.

Solution:

$$\text{L.H.S.} = \log a + \log a^2 + \log a^3 + \cdots + \log a^n$$
$$= \log a + 2\log a + 3\log a + \cdots + n\log a$$
$$= \log a\,(1 + 2 + 3 + \cdots + n)$$
$$= \log a\left(\frac{n(n+1)}{2}\right)$$
$$= \left(\frac{n(n+1)}{2}\right)\log a = \text{R.H.S.}$$

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24. Evaluate $\lim\limits_{\sqrt{x}\to 3} \frac{x^2-81}{\sqrt{x}-3}$.

Solution:

$$\lim_{\sqrt{x}\to 3} \frac{x^2-81}{\sqrt{x}-3} = \lim_{\sqrt{x}\to 3} \frac{\left(\sqrt{x}\right)^2 - 3^4}{\sqrt{x}-3} \qquad \because \lim_{x\to a} \frac{x^n-a^n}{x-a} = na^{n-1}$$

$$= 4(3)^{4-1}$$

$$= 4(3^3) = 108$$

---

25. If $A + B = 45°$, show that $(1 + \tan A)(1 + \tan B) = 2$.

Solution:

$$A + B = 45°$$

$$\tan(A+B) = \tan 45°$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\tan A + \tan B = 1 - \tan A \tan B$$

$$\text{L.H.S.} = (1+\tan A)(1+\tan B)$$

$$= 1 + \tan A + \tan B + \tan A \tan B$$

$$= 1 + 1 - \tan A \tan B + \tan A \tan B$$

$$= 2 = \text{R.H.S.}$$

---

26. If ${}^nC_4 = 495$, what is n?.

Solution:

$${}^nC_4 = 495$$

$$\frac{n!}{4!\,(n-4)!} = 495$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)!}{4!\,(n-4)!} = 495$$

$$n(n-1)(n-2)(n-3) = 495 \times 4 \times 3 \times 2 \times 1$$

$$n(n-1)(n-2)(n-3) = 12 \times 11 \times 10 \times 9$$

$$n = 12$$

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27. Find $\sqrt[3]{1001}$ approximately (two decimal places).

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Solution:

$$\sqrt[3]{1001} = (1001)^{\frac{1}{3}} = (1000+1)^{\frac{1}{3}}$$

$$= 1000^{\frac{1}{3}}\left(1 + \frac{1}{1000}\right)^{\frac{1}{3}} = (10^3)^{\frac{1}{3}}\left(1 + \frac{1}{1000}\right)^{\frac{1}{3}}$$

$$\sqrt[3]{1001} = 10\left(1 + \frac{1}{3}\left(\frac{1}{1000}\right) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{1}{1000}\right)^2 + \cdots\right)$$

$$= 10(1 + 0.0003 + \cdots) = 10.0003$$

28. Find the separate equation of the pair of straight lines $3x^2 + 2xy - y^2 = 0$.

Solution:

consider $3x^2 + 2xy - y^2 = 0$

$$3x^2 + 3xy - xy - y^2 = 0$$
$$3x(x+y) - y(x+y) = 0$$
$$(x+y)(3x-y) = 0$$
$$x + y = 0 \;;\; 3x - y = 0$$
$$y = -x \;;\; y = 3x$$

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The separate equations are $y = -x$ and $y = 3x$

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29. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is singular, find the value of x.

Solution:

$$\text{Given that A is singular} \Rightarrow |A| = 0$$
$$\begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix} = 0$$
$$1(-6-2) + 2(-3-x) + 3(2-2x) = 0$$
$$-8 - 6 - 2x + 6 - 6x = 0$$
$$-8x = 8$$
$$x = -1$$

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30. Evaluate : $\lim_{n\to\infty} [6^n + 5^n]^{\frac{1}{n}}$

Solution:

$$\lim_{n\to\infty} [6^n + 5^n]^{\frac{1}{n}} = \lim_{n\to\infty} (6^n)^{\frac{1}{n}} \left[1 + \left(\frac{5}{6}\right)^n\right]^{\frac{1}{n}}$$
$$= \lim_{n\to\infty} 6[1+0]^{\frac{1}{n}} \quad \because \left(\frac{5}{6}\right)^n \to 0$$
$$= 6(1) = 6$$

PART – III

Answer any SEVEN questions

Question number 40 is compulsory **7 x 3 = 21**

31. The probability of an event A occurring is 0.5 and B occurring is 0.3. If A and B are mutually exclusive events, then find the probability of

(i) $P(A \cup B)$ (ii) $P(A \cap \overline{B})$ (iii) $P(\overline{A} \cap B)$.

Solution:

Given $P(A) = 0.5 \,;\, P(B) = 0.3 \,;\, P(A \cap B) = 0$

i) $P(A \cup B) = P(A) + P(B)$

$= 0.5 + 0.3$

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$$= 0.8$$
$$\text{ii) } P(A \cap \bar{B}) = P(A) - P(A \cap B)$$
$$= 0.5 - 0$$
$$= 0.5$$
$$\text{iii) } P(\bar{A} \cap B) = P(B) - P(A \cap B)$$
$$= 0.3 - 0$$
$$= 0.3$$

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32. Find the integral of $\frac{1}{\sqrt{x^2-4x+5}}$.

Solution:

$$\int \frac{1}{\sqrt{x^2-4x+5}}dx = \int \frac{1}{\sqrt{x^2-4x+4+1}}dx$$
$$= \int \frac{1}{\sqrt{(x-2)^2+1^2}}dx$$
$$= \log\left|(x-2)+\sqrt{(x-2)^2+1^2}\right| + c$$
$$= \log\left|(x-2)+\sqrt{x^2-4x+5}\right| + c$$

---

33. Find the range of the function $\frac{1}{2\cos x - 1}$.

Solution:

$$-1 \le \cos x \le 1$$
$$-2 \le 2\cos x \le 2$$
$$-2-1 \le 2\cos x - 1 \le 2-1$$
$$-3 \le 2\cos x - 1 \le 1$$

By taking reciprocals, we get $\frac{1}{1-3\cos x} \le -\frac{1}{3}$ and $\frac{1}{1-3\cos x} \ge 1$

Hence the range of $\frac{1}{2\cos x - 1}$ is $\left(-\infty, -\frac{1}{3}\right] \cup [1, \infty)$

---

34. Differentiate with respect to x : $y = \frac{\cos x}{x^3}$.

Solution:

$$y = \frac{\cos x}{x^3}$$
$$\frac{dy}{dx} = \frac{x^3(-\sin x) - \cos x\,(3x^2)}{(x^3)^2}$$
$$= \frac{-x^2(x\sin x + 3\cos x)}{x^6} = -\frac{(x\sin x + 3\cos x)}{x^4}.$$

---

35. Find the constant b that makes g continuous on $(-\infty, \infty)$

$$g(x) = \begin{cases} x^2 - b^2 & \text{if } x < 4 \\ bx + 20 & \text{if } x \ge 4 \end{cases}.$$

Solution:

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$$\text{Given } g(x) \text{ is continuous on } (-\infty, \infty)$$
$$\Longrightarrow g(x) \text{ is continuous at } x = 4$$
$$\therefore g(4^-) = g(4^+) = g(4)$$
$$g(4^-) = \lim_{x\to 4^-} g(x) = \lim_{x\to 4^-} (x^2 - b^2) = 4^2 - b^2$$
$$g(4^-) = 16 - b^2$$
$$g(4^+) = \lim_{x\to 4^+} g(x) = \lim_{x\to 4^+} (bx + 20) = b(4) + 20$$
$$g(4^+) = 4b + 20$$
$$\therefore 16 - b^2 = 4b + 20$$
$$b^2 + 4b + 20 - 16 = 0$$
$$b^2 + 4b + 4 = 0$$
$$(b+2)^2 = 0$$
$$b + 2 = 0$$
$$b = -2$$

---

36. If θ is a parameter, find the equation of the locus of a moving point, whose coordinates are $x = a\cos^3\theta$, $y = a\sin^3\theta$.

Solution:

Let $P(h, k)$ be a point on the locus

$$\text{Given } h = a\cos^3\theta \; ; \; k = a\sin^3\theta$$
$$\frac{h}{a} = \cos^3\theta \; ; \frac{k}{a} = \sin^3\theta$$
$$\left(\frac{h}{a}\right)^{\frac{1}{3}} = \cos\theta \; ; \; \left(\frac{k}{a}\right)^{\frac{1}{3}} = \sin\theta$$

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$$\text{w.k.t.} \quad \cos^2\theta + \sin^2\theta = 1$$
$$\left(\left(\frac{h}{a}\right)^{\frac{1}{3}}\right)^2 + \left(\left(\frac{k}{a}\right)^{\frac{1}{3}}\right)^2 = 1$$
$$h^{\frac{2}{3}} + k^{\frac{2}{3}} = a^{\frac{2}{3}}$$
$$\therefore \text{locus of the point } P(h, k) \text{ is } x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

---

37. Find the area of the parallelogram whose adjacent sides are $\overrightarrow{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\overrightarrow{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$.

Solution:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = \hat{\imath}(1+4) - \hat{\jmath}(3-4) + \hat{k}(-3-1)$$
$$= 5\hat{\imath} + \hat{\jmath} - 4\hat{k}$$
$$|\vec{a} \times \vec{b}| = |5\hat{\imath} + \hat{\jmath} - 4\hat{k}| = \sqrt{42}$$
$$\text{Area of the parallelogram is } \sqrt{42} \text{ sq. units.}$$

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38. Prove that $\frac{\sin 4x+\sin 2x}{\cos 4x+\cos 2x} = \tan 3x$.

Solution:

$$\text{L.H.S.} = \frac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x}$$

$$= \frac{2\sin\frac{4x+2x}{2}\cos\frac{4x-2x}{2}}{2\cos\frac{4x+2x}{2}\cos\frac{4x-2x}{2}}$$

$$= \frac{\sin 3x}{\cos 3x} = \tan 3x = \text{R.H.S.}$$

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39. Solve the equation $\sqrt{6-4x-x^2} = x+4$.

Solution:

$$\sqrt{6-4x-x^2} = x+4$$

$$\sqrt{6-4x-x^2} \geq 0 \Rightarrow x+4 \geq 0 \Rightarrow x \geq -4$$

$$\sqrt{6-4x-x^2} = x+4$$

$$6-4x-x^2 = (x+4)^2$$

$$6-4x-x^2 = x^2+8x+16$$

$$2x^2+12x+10 = 0$$

$$x^2+6x+5 = 0$$

$$(x+5)(x+1) = 0$$

$$x = -5 \quad ; \quad x = -1$$

$$\text{Since } x \geq -4$$

$$\therefore x = -1$$

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40. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$ then find the value of r.

Solution:

$$\text{Given } {}^nC_{r-1} = 36,\ {}^nC_r = 84 \text{ and } {}^nC_{r+1} = 126$$

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{84}{36}$$

$$\frac{n-r+1}{r} = \frac{84}{36}$$

$$\frac{n-r+1}{r} = \frac{7}{3}$$

$$3n-3r+3 = 7r$$

$$10r-3n = 3 \rightarrow (1)$$

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{126}{84}$$

$$\frac{n-(r+1)+1}{r+1} = \frac{126}{84}$$

$$\frac{n-r}{r+1} = \frac{3}{2}$$

$$2n-2r = 3r+3$$

$$5r-2n = -3 \rightarrow (2)$$

$$20r-6n = 6 \rightarrow (1)$$

$$15r-6n = -9 \rightarrow (2)\times 2$$

$$5r = 15$$

$$r = 3$$

PART – IV

Answer ALL questions **7 x 5 = 35**

41. (a) Write the values of f at $-4, 1, -2, 7, 0$ if

$$f(x) = \begin{cases} -x + 4 & \text{if } -\infty < x \le -3 \\ x + 4 & \text{if } -3 < x < -2 \\ x^2 - x & \text{if } -2 \le x < 1 \\ x - x^2 & \text{if } 1 \le x < 7 \\ 0 & \text{otherwise} \end{cases}$$

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Solution:

$$f(-4) = -(-4) + 4 = 4 + 4 = 8$$
$$f(1) = 1 - 1^2 = 0$$
$$f(-2) = (-2)^2 - (-2) = 4 + 2 = 6$$
$$f(7) = 0$$
$$f(0) = 0^2 - 0 = 0$$

OR

(b) If θ is an acute angle, then find $\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$, when $\sin\theta = \frac{1}{25}$.

Solution:

$$\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos 2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{2}}$$
$$= \sqrt{\frac{1 - \cos\left(\frac{\pi}{2} - \theta\right)}{2}}$$
$$= \sqrt{\frac{1 - \sin\theta}{2}}$$
$$= \sqrt{\frac{1 - \frac{1}{25}}{2}} = \sqrt{\frac{24}{25} \times \frac{1}{2}} = \frac{2\sqrt{3}}{5}$$

---

42. (a) If $A + B + C = 180°$, prove that

$$\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1.$$

Solution:

$$A + B + C = 180°$$
$$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90°$$
$$\frac{A}{2} + \frac{B}{2} = 90° - \frac{C}{2}$$

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$$\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(90°-\frac{C}{2}\right)$$

$$\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\cot\frac{C}{2}$$

$$\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\frac{1}{\tan\frac{C}{2}}$$

$$\tan\frac{C}{2}\tan\frac{A}{2}+\tan\frac{B}{2}\tan\frac{C}{2}=1-\tan\frac{A}{2}\tan\frac{B}{2}$$

$$\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$$

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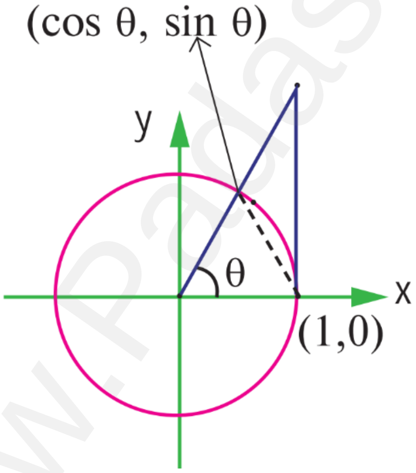
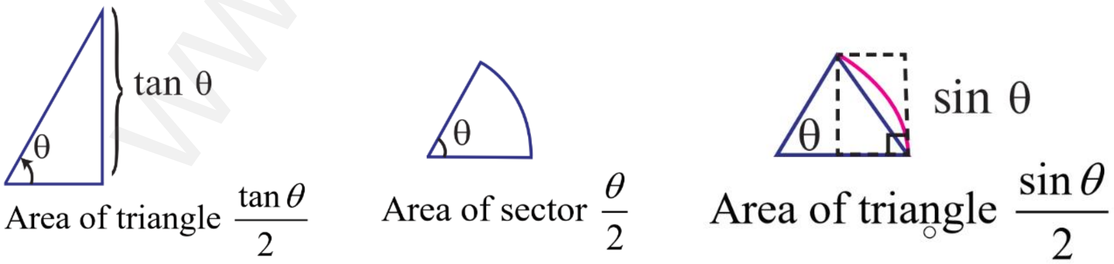
OR

(b) Prove that $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$.

Solution:

Consider the circle with centre $(0,0)$ and radius 1. Any point on this circle is $R(\cos\theta, \sin\theta)$.





By area property $\frac{\tan\theta}{2}\geq\frac{\theta}{2}\geq\frac{\sin\theta}{2}$

$\times$ by $\frac{2}{\sin\theta}$

$$\frac{1}{\cos\theta}\geq\frac{\theta}{\sin\theta}\geq 1$$

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$$\cos\theta \le \frac{\sin\theta}{\theta} \le 1$$

$$\lim_{\theta\to 0}\cos\theta = 1\,;\ \lim_{\theta\to 0}(1) = 1$$

Sandwich theorem we get

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

---

43. (a) The AM of two numbers exceeds their GM by 10 and HM by 16. Find the numbers.

Solution:

Let x, y, z be the AM, GM, HM of two numbers a and b respectively

From the given information, $x = y + 10 \rightarrow (1)$

$x = z + 16 \rightarrow (2)$

$$\text{w.k.t. } GM^2 = AM \times HM$$
$$\text{i.e., } y^2 = x \times z$$
$$(x-10)^2 = x(x-16)$$
$$x^2 - 20x + 100 = x^2 - 16x$$
$$-4x + 100 = 0$$
$$-4(x-25) = 0$$
$$x = 25$$
$$\frac{a+b}{2} = 25$$
$$a + b = 50$$

Use the above in equation (1)

$$25 = \sqrt{ab} + 10$$
$$\sqrt{ab} = 25 - 10 = 15$$
$$ab = 225$$
$$a(50-a) = 225 \quad (\text{from } a + b = 50)$$
$$50a - a^2 - 225 = 0$$
$$a^2 - 50a + 225 = 0$$
$$(a-45)(a-5) = 0$$
$$a = 5 \text{ or } a = 45$$
$$\text{if } a = 5, b = 45 \text{ (or) } a = 45, b = 5$$

Therefore numbers are 5 and 45

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OR

(b) If p_1 and p_2 are the lengths of the perpendiculars from the origin to the straight lines $x\sec\theta + y\,\mathrm{cosec}\,\theta = 2a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$, then prove that $p_1^2 + p_2^2 = a^2$.

Solution:

Given p_1 is the length of perpendicular from $(0,0)$ to

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$x \sec\theta + y \operatorname{cosec}\theta = 2a$

$$D = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

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$$p_1 = \left|\frac{0\sec\theta + 0\operatorname{cosec}\theta - 2a}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}\right|$$

$$p_1 = \frac{|-2a|}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}$$

$$p_1^2 = \frac{4a^2}{\sec^2\theta + \operatorname{cosec}^2\theta}$$

$$p_1^2 = \frac{4a^2\cos^2\theta\sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$p_1^2 = 4a^2\cos^2\theta\sin^2\theta$$

$$p_1^2 = a^2(4\cos^2\theta\sin^2\theta)$$

$$p_1^2 = a^2\sin^2 2\theta$$

p_2 is the length of perpendicular from (0, 0) to

$x\cos\theta - y\sin\theta = a\cos 2\theta$

$$p_2 = \left|\frac{0\cos\theta - 0\sin\theta - a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$$

$$p_2 = \left|\frac{-a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$$

$$p_2^2 = a^2\cos^2 2\theta$$

$$\text{LHS} = p_1^2 + p_2^2$$

$$= a^2(\sin^2 2\theta + \cos^2 2\theta)$$

$$= a^2 = \text{RHS}$$

hence proved

---

44. (a) If one root of $k(x-1)^2 = 5x - 7$ is double the other root, show that $k = 2$ or -25.

Solution:

$$k(x-1)^2 = 5x - 7$$

$$k(x^2 - 2x + 1) = 5x - 7$$

$$kx^2 - 2kx + k - 5x + 7 = 0$$

$$kx^2 - (2k+5)x + k + 7 = 0$$

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Let α be the root, then 2α is also a root

$$\text{Sum} = \alpha + 2\alpha = -\frac{-(2k+5)}{k}$$

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$$\Rightarrow 3\alpha = \frac{2k+5}{k}$$

$$\Rightarrow \alpha = \frac{2k+5}{3k}$$

$$\text{Product} = (\alpha)(2\alpha) = \frac{k+7}{k}$$

$$\Rightarrow 2\alpha^2 = \frac{k+7}{k}$$

$$2\left(\frac{2k+5}{3k}\right)^2 = \frac{k+7}{k}$$

$$\Rightarrow 2\left(\frac{4k^2+20k+25}{9k^2}\right) = \frac{k+7}{k}$$

$$8k^2 + 40k + 50 = 9k^2 + 63k$$

$$9k^2 + 63k - 8k^2 - 40k - 50 = 0$$

$$k^2 + 23k - 50 = 0$$

$$(k-2)(k+25) = 0$$

$$k = 2 \text{ or } k = -25$$

OR

(b) Express the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices.

Solution:

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A^T) = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix}$$

$$\text{Now } P^T = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} = P$$

$$\text{Thus, } P = \frac{1}{2}(A + A^T) \text{ is a symmetric matrix.}$$

$$\text{Let } Q = \frac{1}{2}(A - A^T)$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$$

$$\text{Then } Q^T = \frac{1}{2}\begin{bmatrix} 0 & -9 & -9 \\ 9 & 0 & 3 \\ 9 & -3 & 0 \end{bmatrix} = -Q$$

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Thus $Q = \frac{1}{2}(A - A^T)$ is a skew- symmetric matrix.

$$A = P + Q = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$$

Thus A is expressed as the sum of symmetric and skew-symmetric matrices.

---

45. (a) Show that the points whose position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.

Solution:

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$$\overrightarrow{OA} = 4\hat{i} + 5\hat{j} + \hat{k}$$
$$\overrightarrow{OB} = 0\hat{i} - \hat{j} - \hat{k}$$
$$\overrightarrow{OC} = 3\hat{i} + 9\hat{j} + 4\hat{k}$$
$$\overrightarrow{OD} = -4\hat{i} + 4\hat{j} + 4\hat{k}$$
$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -4\hat{i} - 6\hat{j} - 2\hat{k}$$
$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -\hat{i} + 4\hat{j} + 3\hat{k}$$
$$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = -8\hat{i} - \hat{j} + 3\hat{k}$$

Condition for coplanar is $\overrightarrow{AD} = l\overrightarrow{AB} + m\overrightarrow{AC}$

$$-8\hat{i} - \hat{j} + 3\hat{k} = l(-4\hat{i} - 6\hat{j} - 2\hat{k}) + m(-\hat{i} + 4\hat{j} + 3\hat{k})$$

Comparing $\hat{i}, \hat{j}$ and $\hat{k}$ we get

$$-4l - m = -8 \rightarrow (1)$$
$$-6l + 4m = -1 \rightarrow (2)$$
$$-2l + 3m = 3 \rightarrow (3)$$
$$-16l - 4m = -32 \rightarrow (1) \times 4$$
$$-6l + 4m = -1 \rightarrow (2)$$
$$-22l = -33 \Rightarrow l = \frac{3}{2}$$

Sub $l = \frac{3}{2}$ in (1)

$$-4\left(\frac{3}{2}\right) - m = -8 \Rightarrow m = 2$$

Sub $l = \frac{3}{2}$ & $m = 2$ in (3), we get

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$$-2\left(\frac{3}{2}\right) + 3(2) = 3$$
$$3 = 3$$

Hence the given points forms a coplanar

OR

(b) In how many ways 4 mathematics books, 3 physics books, 2 chemistry books and 1 biology book can be arranged on a shelf so that all books of

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the same subjects are together.

Solution:

Take 4 mathematics books as 1 unit ; 3 physics books as 1 unit
2 chemistry books as 1 unit ; 1 biology book as 1 unit
There are 4 units which can be arranged in 4! Ways
Now 4 mathematics can be rearranged among themselves in 4! Ways
Similarly other subjects

$$\text{Number of arrangements} = 4! \times 4! \times 3! \times 2! \times 1!$$
$$= 24 \times 24 \times 6 \times 2 \times 1 = 6912$$

46.(a) Evaluate : $\int \frac{6x+5}{\sqrt{1-4x-4x^2}}dx$.

Solution:

$$\text{Let } I = \int \frac{6x+5}{\sqrt{1-4x-4x^2}}dx$$
$$6x+5 = A\frac{d}{dx}(1-4x-4x^2)+B$$
$$6x+5 = A(-4-8x)+B$$

comparing x term	comparing constant term
$6=-8A$	$5=-4A+B$
$A=-\frac{3}{4}$	$5=-4\left(-\frac{3}{4}\right)+B$
	$B=2$

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$$\int \frac{6x+5}{\sqrt{1-4x-4x^2}}dx = \int \frac{-\frac{3}{4}(-4-8x)+2}{\sqrt{1-4x-4x^2}}dx$$
$$= -\frac{3}{4}\int \frac{-4-8x}{\sqrt{1-4x-4x^2}}dx + 2\int \frac{1}{\sqrt{1-4x-4x^2}}dx$$
$$= -\frac{3}{4}\int \frac{-4-8x}{\sqrt{1-4x-4x^2}}dx + 2\int \frac{1}{\sqrt{-4\left(x^2+x-\frac{1}{4}\right)}}dx$$
$$= -\frac{3}{4}\int \frac{-4-8x}{\sqrt{1-4x-4x^2}}dx + 2\int \frac{1}{\sqrt{-4\left(x^2+x+\frac{1}{4}-\frac{2}{4}\right)}}dx$$
$$= -\frac{3}{4}\int \frac{-4-8x}{\sqrt{1-4x-4x^2}}dx + 2\int \frac{1}{\sqrt{-4\left[\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{\sqrt{2}}\right)^2\right]}}dx$$
$$= -\frac{3}{4}\int \frac{-4-8x}{\sqrt{1-4x-4x^2}}dx + \int \frac{1}{\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2-\left(x+\frac{1}{2}\right)^2}}dx$$

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$$= -\frac{3}{4}\left(2\sqrt{1-4x-4x^2}\right) + \sin^{-1}\left(\frac{x+\frac{1}{2}}{\frac{1}{\sqrt{2}}}\right) + c$$

$$= -\frac{3}{4}\left(2\sqrt{1-4x-4x^2}\right) + \sin^{-1}\left(\frac{2x+1}{\sqrt{2}}\right) + c$$

OR

(b) If $y = \sin^{-1}\frac{1}{2}\left(\sqrt{1+x}+\sqrt{1-x}\right)$ then show that $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$.

Solution:

$$y = \sin^{-1}\frac{1}{2}\left(\sqrt{1+x}+\sqrt{1-x}\right)$$

$$\text{Put } x = \cos\theta \Rightarrow \theta = \cos^{-1}x$$

$$y = \sin^{-1}\frac{1}{2}\left(\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}\right)$$

$$y = \sin^{-1}\frac{1}{2}\left(\sqrt{2\cos^2\frac{\theta}{2}}+\sqrt{2\sin^2\frac{\theta}{2}}\right)$$

$$y = \sin^{-1}\frac{1}{2}\left(\sqrt{2}\cos\frac{\theta}{2}+\sqrt{2}\sin\frac{\theta}{2}\right)$$

$$y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\cos\frac{\theta}{2}+\frac{1}{\sqrt{2}}\sin\frac{\theta}{2}\right)$$

$$y = \sin^{-1}\left(\sin\frac{\pi}{4}\cos\frac{\theta}{2}+\cos\frac{\pi}{4}\sin\frac{\theta}{2}\right)$$

$$y = \sin^{-1}\left(\sin\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right)$$

$$y = \frac{\pi}{4}+\frac{\theta}{2}$$

$$y = \frac{\pi}{4}+\frac{\cos^{-1}x}{2}$$

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diff. w.r.t. "x"

$$\frac{dy}{dx} = 0 + \frac{1}{2}\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$$

---

47. (a) A factory has two machines I and II. Machine I produces 40% of items of the output and Machine II produces 60% of the items. Further 4% of items produced by Machine I are defective and 5% produced by Machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine II.

Solution:

Let A_1 be the event that the items are produced by Machine-I, A_2 be the event that items are produced by Machine-II. Let B be the event of drawing a defective item.

Now we are asked to find the conditional probability $P(A_2/B)$. Since A_1, A_2 are mutually exclusive and exhaustive events, by Bayes' theorem,

$$P(A_2/B) = \frac{P(A_2)P(B/A_2)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2)}$$

$$P(A_1) = 0.40, P(B/A_1) = 0.04$$

$$P(A_2) = 0.60, P(B/A_2) = 0.05$$

$$P(A_2/B) = \frac{(0.60)(0.05)}{(0.40)(0.04) + (0.60)(0.05)} = \frac{15}{23}$$

OR

(b) If $y = (\cos^{-1} x)^2$, Prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x = 0$.

Solution:

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$$y = (\cos^{-1} x)^2$$

$$\frac{dy}{dx} = 2(\cos^{-1} x)\left(-\frac{1}{\sqrt{1 - x^2}}\right)$$

$$\sqrt{1 - x^2}\frac{dy}{dx} = -2\cos^{-1} x$$

$$(1 - x^2)\left(\frac{dy}{dx}\right)^2 = 4(\cos^{-1} x)^2$$

$$(1 - x^2)\left(\frac{dy}{dx}\right)^2 = 4y$$

$$(1 - x^2)\left(2\frac{dy}{dx}\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2(0 - 2x) = 4\frac{dy}{dx}$$

$$\div 2\frac{dy}{dx}$$

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

$$\text{(or)} \quad (1 - x^2)y_2 - xy_1 - 2 = 0$$

$$\text{when } x = 0$$

$$(1 - 0)y_2 - (0)y_1 - 2 = 0 \Rightarrow y_2 = 2$$

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