HSE FIRST YEAR PUBLIC EXAMINATION – MARCH 2023 MATHEMATICS

TYPE – A PART – I

1	(c) $[0,\infty), [0,\infty)$
2	(c) 4
3	(d) 23
4	(a) (1,2)
5	(b) a scalar matrix
6	(c) $\sec\theta = \frac{1}{4}$
7	(a) 11
8	(b) 25
9	(a) $\vec{0}$
10	(c) $-2\cos\sqrt{x} + c$
11	(d) None of the above
12	(a) $-\frac{3}{4}\cos x + \frac{\cos 3x}{12} + c$
13	(a) Harmonic Progression
14	(a) 4
15	(a) 512
16	(c) 25
17	(b) 0
18	(b) −3
19	(a) $n > 7$
20	(c) $\frac{7}{128}$

TYPE – B PART – I

1	(a) 4
2	(b) a scalar matrix
3	(c) $[0,\infty), [0,\infty)$
4	(d) 23
5	(a) 512
6	(c) $\frac{7}{128}$
7	(c) 25
8	(c) 25
9	(a) $n > 7$
10	(c) $\sec\theta = \frac{1}{4}$
11	(a) 11
12	(b) −3
13	(a) $-\frac{3}{4}\cos x + \frac{\cos 3x}{12} + c$
14	(a) (1,2)
15	(a) Harmonic Progression
16	(b) 25
17	(c) $-2\cos\sqrt{x} + c$
18	(b) 0
19	(d) None of the above
20	(a) $\vec{0}$

PART – II & PART – III

21	Example 1.8	pg-7
22	Exercise 12.1 (10)	pg-247
23	Exercise 2.12 (9)	pg-80
24	Exercise 9.2 (3)	pg-102
25	Exercise 3.5 (6)	pg-118
26	Example 4.46	pg-182
27	Exercise 5.4(2)	pg-231
28	Exercise 6.4 (7)(i)	pg-282
29	Creative	$x = -1$
30	Creative	6
31	Exercise 12.2 (4)	pg-250
32	Exercise 11.10 (3)(ii)	pg-219
33	Exercise 1.3 (8)	pg-37
34	Example 10.7 (vi)	pg-159
35	Exercise 9.5 (12)	pg-128
36	Exercise 6.1 (3)	pg-243
37	Example 8.24	pg-79
38	Exercise 3.6 (8)	pg-121
39	Example 2.15	pg-63
40	Creative	$r = 3$

PART – IV

41	(a) Exercise 1.3 (2)	pg-37
	(b) Exercise 3.5(2)(i)	pg-118
42	(a) Exercise 3.7(1)(v)	pg-124
	(b) Result 9.1 (a)	pg-113
43	(a) Exercise 5.2 (8)	pg-218
	(b) Exercise 6.3 (11)	pg-272
44	(a) Exercise 2.4 (4)	pg-62
	(b) Example 7.13	pg-17
45	(a) Exercise 8.2 (10)	pg-68
	(b) Exercise 4.2 (11)	pg-178
46	(a) Creative	$I = \frac{-3}{2}\sqrt{1-4x-4x^2} + \sin^{-1}\left(\frac{2x+1}{\sqrt{2}}\right)$
	(b) Creative	
47	(a) Example 12.26	pg-261
	(b) Exercise 10.4 (28)	pg-176

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(29) Given

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$$

$\because$ Matrix A is singular then,

$|A| = 0$

$$\Rightarrow \begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix} = 0$$

$1(-6-2)+2(-3-x)+3(2-2x)=0$

$1(-8) - \cancel{6} - 2x + \cancel{6} - 6x = 0$

$-8 - 2x - 6x = 0$

$-8 - 8x = 0$

$-8x = 8$

$x = \frac{8}{-8}$

$\boxed{x = -1}$

(30) $\lim_{n\to\infty} [6^n + 5^n]^{1/n}$

$= \lim_{n\to\infty} \left[6^n\left(1+\frac{5^n}{6^n}\right)\right]^{1/n}$

$= \lim_{n\to\infty} 6^{n\times 1/n}\left(1+\left(\frac{5}{6}\right)^n\right)^{1/n}$

$= \lim_{n\to\infty} 6\left(1+\left(\frac{5}{6}\right)^n\right)^{1/n}$

As $\lim_{n\to\infty} \Rightarrow (5/6)^n \to 0$ and $\frac{1}{n} \to 0$

$\therefore = \lim_{n\to\infty} 6[1+0]^0$

$= 6(1)$

$= 6$

(40) $\frac{^nC_r}{^nC_{r-1}} = \frac{84}{36}$

$\Rightarrow \dfrac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = \frac{84}{36}$

$\Rightarrow \frac{\cancel{n!}}{r\cancel{(r-1)!}\cancel{(n-r)!}} \times \frac{\cancel{(r-1)!}(n-r+1)\cancel{(n-r)!}}{\cancel{n!}} = \frac{7}{3}$

$\frac{n-r+1}{r} = \frac{7}{3}$

$3(n-r+1) = 7r$

$3n-3r+3 = 7r$

$3n+3 = 7r+3r$

$3n+3 = 10r \longrightarrow (1)$

$\frac{^nC_{r+1}}{^nC_r} = \frac{126}{84}$

$\dfrac{\frac{n!}{(r+1)!(n-r-1)!}}{\frac{n!}{r!(n-r)!}} = \frac{3}{2}$

$\frac{\cancel{n!}}{(r+1)\cancel{r!}\cdot\cancel{(n-r-1)!}} \times \frac{\cancel{r!}\cdot\cancel{(n-r-1)!}(n-r)}{\cancel{n!}} = \frac{3}{2}$

$\frac{n-r}{r+1} = \frac{3}{2}$

$2(n-r) = 3(r+1)$

$2n-2r = 3r+3$

$2n-3 = 3r+2r$

$2n-3 = 5r \longrightarrow (2)$

$\frac{(1)}{(2)} \Rightarrow \frac{3n+3}{2n-3} = \frac{10r}{5r}$

$\frac{3n+3}{2n-3} = 2$

$3n+3 = 2(2n-3)$

$3n+3 = 4n-6$

$6+3 = 4n-3n$

$9 = n$

$\boxed{n=9}$

$(1) \Rightarrow 3(9)+3 = 10r$

$27+3 = 10r$

$30 = 10r$

$\frac{30}{10} = r$

$3 = r$

$\boxed{r = 3}$

46 (a) $\int \frac{6x+5}{\sqrt{1-4x-4x^2}}\,dx$

$$6x+5 = A\frac{d}{dx}(1-4x-4x^2)+B$$

$$6x+5 = A(-4-8x)+B$$

Comparing the coefficients

$-8A = 6$

$A = \frac{-6}{8}$

$\boxed{A = \frac{-3}{4}}$

$-4A+B = 5$

$-4\left(\frac{-3}{4}\right)+B = 5$

$3+B = 5$

$B = 5-3$

$\boxed{B=2}$

$$I = \int \frac{\frac{-3}{4}(-4-8x)+2}{\sqrt{1-4x-4x^2}}\,dx$$

$$= \frac{-3}{4}\int \frac{(-4-8x)}{\sqrt{1-4x-4x^2}}\,dx + 2\int \frac{1}{\sqrt{1-4x-4x^2}}\,dx$$

$$= \frac{-3}{4}(2)\sqrt{1-4x-4x^2} + 2\int \frac{1}{\sqrt{4\left(\frac{1}{4}-x-x^2\right)}}\,dx$$

$$= \frac{-3}{2}\sqrt{1-4x-4x^2} + 2\int \frac{1}{2\sqrt{\left(\frac{1}{\sqrt2}\right)^2-\left(x+\frac{1}{2}\right)^2}}\,dx$$

$$= \frac{-3}{2}\sqrt{1-4x-4x^2} + \int \frac{1}{\sqrt{\left(\frac{1}{\sqrt2}\right)^2-\left(x+\frac{1}{2}\right)^2}}\,dx$$

$$= \frac{-3}{2}\sqrt{1-4x-4x^2} + \sin^{-1}\left(\frac{x+\frac{1}{2}}{\frac{1}{\sqrt2}}\right)$$

$$= \frac{-3}{2}\sqrt{1-4x-4x^2} + \sin^{-1}\left(\frac{\frac{2x+1}{2}}{\frac{1}{\sqrt2}}\right)$$

$$= \frac{-3}{2}\sqrt{1-4x-4x^2} + \sin^{-1}\left(\frac{\frac{2x+1}{\sqrt2\times\sqrt2}}{\frac{1}{\sqrt2}}\right)$$

$$= \boxed{\frac{-3}{2}\sqrt{1-4x-4x^2} + \sin^{-1}\left(\frac{2x+1}{\sqrt2}\right)}$$

46 (b)

$$y = \sin^{-1}\frac{1}{2}\left(\sqrt{1+x}+\sqrt{1-x}\right)$$

Let $x = \cos 2\theta$

$$y = \sin^{-1}\frac{1}{2}\left[\sqrt{1+\cos2\theta}+\sqrt{1-\cos2\theta}\right]$$

$\cos2\theta = 1-2\sin^2\theta$
$= 2\cos^2\theta - 1$

$$y = \sin^{-1}\frac{1}{2}\left[\sqrt{1+2\cos^2\theta-1}+\sqrt{1-(1-2\sin^2\theta)}\right]$$

$$y = \sin^{-1}\frac{1}{2}\left[\sqrt{2\cos^2\theta}+\sqrt{1-1+2\sin^2\theta}\right]$$

$$y = \sin^{-1}\frac{1}{2}\left[\sqrt2\cos\theta+\sqrt{2\sin^2\theta}\right]$$

$$y = \sin^{-1}\frac{1}{2}\left[\sqrt2\cos\theta+\sqrt2\sin\theta\right]$$

$$y = \sin^{-1}\frac{1}{2}\left[\sqrt2(\cos\theta+\sin\theta)\right]$$

$$y = \sin^{-1}\frac{1}{\sqrt2}(\cos\theta+\sin\theta)$$

$$y = \sin^{-1}\left[\frac{1}{\sqrt2}\cos\theta+\sin\theta\frac{1}{\sqrt2}\right]$$

$$y = \sin^{-1}\left[\sin\frac{\pi}{4}\cos\theta+\sin\theta\cos\frac{\pi}{4}\right]$$

$\sin(A+B) = \sin A\cos B+\cos A\sin B$

$$y = \sin^{-1}\left[\sin\left(\frac{\pi}{4}+\theta\right)\right]$$

$$y = \frac{\pi}{4}+\theta$$

$x = \cos2\theta$

$2\theta = \cos^{-1}x$

$\theta = \frac{1}{2}\cos^{-1}x$

$$y = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$$

$$\frac{dy}{dx} = 0+\frac{1}{2}\left[\frac{-1}{\sqrt{1-x^2}}\right]$$

$$\boxed{\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}}$$