DIRECTORATE OF GOVERNMENT EXAMINATIONS, CHENNAI – 600 006

HSE FIRST YEAR EXAMINATION – MARCH/APRIL 2023

MATHEMATICS MARKING SCHEME – ENGLISH MEDIUM

Maximum Marks: 90

GENERAL INSTRUCTIONS

1. The answers given in the marking scheme are Text book bound.
2. If a student has given any answer which is different from one given in the marking scheme, but carries the prescribed content meaning (rigorous) such answers should be given full credit with suitable distribution.
3. Follow the footnotes which are given under certain answer schemes.
4. If a particular stage is wrong and if the candidate writes the appropriate formula then award 1 mark for the formula (for the stage mark 2*). This mark (*) is attached with that stage. This is done with the aim that a student who did the problem correctly without writing the formula should not be penalized.
5. In the case of Part II, Part III and Part IV, if the solution is correct then award full mark directly. The stage mark is essential only if the part of the solution is incorrect.
6. Answers written only in **Black or Blue** ink should be evaluated.

PART – I

1. **One mark to write the correct option and the corresponding answer.**
2. **If one of them (answer or option) is wrong, then award zero mark only.**

20×1=20

TYPE A			TYPE B		
Q. NO	**OPTION**	**ANSWER**	**Q.NO**	**OPTION**	**ANSWER**
1.	(c)	$[0,\infty)$, $[0,\infty)$	**1.**	(a)	4
2.	(c)	4	**2.**	(b)	a scalar matrix
3.	(d)	23	**3.**	(c)	$[0,\infty)$, $[0,\infty)$
4.	(a)	(1,2)	**4.**	(d)	23
5.	(b)	a scalar matrix	**5.**	(a)	512
6.	(c)	$\sec\theta = \frac{1}{4}$	**6.**	(c)	$\frac{7}{128}$
7.	(a)	11	**7.**	(c)	25
8.		Mere Attempt	**8.**	(c)	4
9.	(a)	$\vec{0}$	**9.**	(a)	$n > 7$
10.	(c)	$-2\cos\sqrt{x} + c$	**10.**	(c)	$\sec\theta = \frac{1}{4}$
11.	(d)	None of the above	**11.**	(a)	11
12.	(a)	$-\frac{3}{4}\cos x + \frac{\cos 3x}{12} + c$	**12.**	(b)	-3
13.	(a)	Harmonic Progression	**13.**	(a)	$-\frac{3}{4}\cos x + \frac{\cos 3x}{12} + c$
14.	(a)	4	**14.**	(a)	(1,2)
15.	(a)	512	**15.**	(a)	Harmonic Progression
16.	(c)	25	**16.**		Mere Attempt
17.	(b)	0	**17.**	(c)	$-2\cos\sqrt{x} + c$
18.	(b)	-3	**18.**	(b)	0
19.	(a)	$n > 7$	**19.**	(d)	None of the above
20.	(c)	$\frac{7}{128}$	**20.**	(a)	$\vec{0}$

Important Note for Part-II, Part – III and Part – IV

In an answer to a question, between any two particular stages of marks greater than one) if a student starts from a stage with correct step but reaches the next stage with a wrong result then suitable credits should be given to the related steps instead of denying the entire marks meant for the stage.

PART – II

Answer **any Seven** questions: **Question No. 30 is compulsory.** **7×2=14**

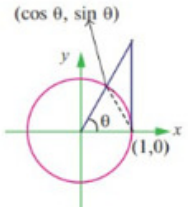
Q.NO	CONTENT	MARKS
21	$n(A \cup B) = 6 \quad n(A \cap B) = 2 \quad n(A \Delta B) = 4$ $n(A \cup B) \times n(A \cap B) \times n(A \Delta B) = 48$	1 1
22	$P(A) = \frac{5}{12}$ The odds that the event B occurs is 2 to 3	1 1
23	$\log a + 2\log a + 3\log a + \cdots + n\log a$ $\frac{n(n+1)}{2}\log a$	1 1
24	$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n\,a^{n-1}$ (or) $\lim_{\sqrt{x} \to 3} \frac{(\sqrt{x})^4 - 3^4}{\sqrt{x} - 3}$ 108	1 1
25	$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$ $(1 + \tan A)(1 + \tan B) = 2$	1 1
26	$\frac{n(n-1)(n-2)(n-3)}{4 \times 3 \times 2 \times 1} = 495$ $n = 12$	1 1
27	$(1000)^{\frac{1}{3}}\left(1 + \frac{1}{1000}\right)^{\frac{1}{3}}$ $\cong 10.0033$	1 1
28	$(x + y)(3x - y) = 0$ $x + y = 0\,, 3x - y = 0$	1 1
29	$\begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix} = 0$ $x = -1$	1 1
30	$\lim_{n \to \infty} \left[6^n \left(1 + \left(\frac{5}{6}\right)^n\right)\right]^{1/n}$ 6	1 1

PART –III

Answer **any Seven** questions: **Question No. 40 is compulsory.** 7×3=21

Q.NO	CONTENT	MARKS
31	$P(A \cup B) = 0.8$ $P(A \cap \overline{B}) = 0.5$ $P(\overline{A} \cap B) = 0.3$	1 1 1
32	$\int \frac{dx}{\sqrt{(x-2)^2+1}}$ Formula : $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left\|x+\sqrt{x^2+a^2}\right\| + c$ $\log\left\|x-2+\sqrt{x^2-4x+5}\right\| + c$	1 2*
33	$-1 \le \cos x \le 1$ $-\frac{1}{3} \ge \frac{1}{2\cos x - 1} \ge 1$ Range is $\left(-\infty, -\frac{1}{3}\right] \cup [1, \infty)$	1 1 1
34	$\frac{x^3(-\sin x) - \cos x\,(3x^2)}{(x^3)^2}$ $\frac{-(x\sin x + 3\cos x)}{x^4}$	2* 1
35	$\lim_{x\to 4^-} g(x) = \lim_{x\to 4^+} g(x) = g(4)$ $16 - b^2 = 4b + 20$ $b = -2$	1 1 1
36	$\cos\theta = \left(\frac{x}{a}\right)^{\frac{1}{3}}$, $\sin\theta = \left(\frac{y}{a}\right)^{\frac{1}{3}}$ $\cos^2\theta + \sin^2\theta = 1$ $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$	1 1 1
37	$\vec{a} \times \vec{b} = 5\hat{i} + \hat{j} - 4\hat{k}$ Area of the parallelogram = $\left\|\vec{a} \times \vec{b}\right\| = \sqrt{42}$	1 2*
38	$\frac{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)}{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)}$ $\tan 3x$	2* 1
39	$x + 4 \ge 0$ $6 - 4x - x^2 = (x+4)^2$ $x = -1$	1 1 1
40.	$nc_r = \frac{n!}{(n-r)!\,r!}$ $3n - 10r = -3$ *and* $2n - 5r = 3$ $r = 3$	1 1 1

PART –IV **Answer all the questions** **7×5=35**

Q.NO	CONTENT	MARKS
41.(a)	$f(-4) = 8$	1
	$f(1) = 0$	1
	$f(-2) = 6$	1
	$f(7) = 0$	1
	$f(0) = 0$	1
	OR	
41.(b)	$\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)$	2*
	$\sin^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \frac{1}{2}(1 - \sin\theta)$	2
	$\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \sqrt{\frac{12}{25}} \quad (or) \quad \frac{2\sqrt{3}}{5}$	1
42.(a)	$A + B + C = 180^0$	1
	$\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(90 - \frac{C}{2}\right) = \cot C$	1
	$\frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \frac{1}{\tan\frac{C}{2}}$	1
	$\tan\frac{A}{2}\tan\frac{C}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{A}{2}\tan\frac{B}{2} = 1$	2
	OR	
42.(b)	 (Rough Diagram)	2
	$\frac{\tan\theta}{2} \geq \frac{\theta}{2} \geq \frac{\sin\theta}{2}$	1
	$\cos\theta \leq \frac{\sin\theta}{\theta} \leq 1$	1
	$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$	1

43.(a)	$AM = \frac{a+b}{2}$ $GM = \sqrt{ab}$ $HM = \frac{2ab}{a+b}$ $AM - GM = 10$ $AM - HM = 16$ $The\ numbers\ are\ 5, 45\ (or)\ 45, 5$	1 1 1 2
	OR	
43.(b)	Formula : Length of perpedicular from (x_1, y_1) $to\ the\ line$ $ax + by + c = 0\ is\ p = \pm\left(\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right)$ $p_1 = 2\,a\ \sin\theta\cos\theta$ $p_2 = a\ \cos 2\theta$ $p_1^2 + p_2^2 = a^2$	 2* 1 2
44.(a)	$kx^2 + x(-2k-5) + (k+7) = 0$ Let the roots be $\alpha, 2\alpha$ $3\alpha = \frac{2k+5}{k}$ $2\alpha^2 = \frac{k+7}{k}$ $k = 2\ or\ -25$	2 1 1 1
	OR	
44.(b)	$P = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix}$ $P^T = P$ $Q = \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$ $Q^T = -Q$ $A = P + Q$	1 1 1 1 1

45.(a)	$\vec{a} = -4\hat{i} - 6\hat{j} - 2\hat{k}$	1
	$\vec{b} = -\hat{i} + 4\hat{j} + 3\hat{k}$	1
	$\vec{c} = -8\hat{i} - \hat{j} + 3\hat{k}$	1
	$-4\hat{i} - 6\hat{j} - 2\hat{k} = (-s - 8t)\hat{i} + (4s - t)\hat{j} + (3s + 3t)\hat{k}$	1
	The points are coplanar	1
	(**Note**: Any other method)	
	OR	
45.(b)	Mathematics books can be arranged in 4! ways	1
	Physics books can be arranged in 3! ways	1
	Chemistry books can be arranged in 2! ways	1
	Biology book can be arranged in 1! way	1
	Total number of arrangements = $4! \times 4! \times 3! \times 2! \times 1! = 6912$	1
46.(a)	$6x + 5 = A\frac{d}{dx}(1 - 4x - 4x^2) + B$	1
	$6x + 5 = \frac{-3}{4}(-4 - 8x) + 2$	2
	$\int \frac{6x+5}{\sqrt{1-4x-4x^2}}dx = \frac{-3}{4}\int \frac{-4-8x}{\sqrt{1-4x-4x^2}}dx + 2\int \frac{dx}{\sqrt{1-4x-4x^2}}$	1
	$\frac{-3}{2}\sqrt{1-4x-4x^2} + \sin^{-1}\left(\frac{2x+1}{\sqrt{2}}\right) + C$	1
	OR	
46.(b)	Put $x = \cos\theta$	1
	$y = \sin^{-1}\left[\frac{1}{2}\left(\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}\right)\right]$	1
	$y = \frac{\theta}{2} + \frac{\pi}{4}$	1
	$y = \frac{1}{2}\cos^{-1}x + \frac{\pi}{4}$	
	$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$	2
	Note : Use any other method	

47.(a)	$P(A_1) = 0 \cdot 4 \quad P(B/A_1) = 0.04$ $P(A_2) = 0 \cdot 6 \quad P(B / A_2) = 0.05$ $P(A_2 / B) = \frac{P(A_2)\ P(B / A_2)}{P(A_1)\ P(B / A_1) + P(A_2)\ P(B / A_2)} = \frac{15}{23}$	1 1 3*
	OR	
47.(b)	$y' = 2\ \cos^{-1} x \frac{-1}{\sqrt{1-x^2}}$ $(y')^2\ (1-x^2) = 4\ y$ $(-2x)\ (y')^2 + 2(1-x^2)y'y'' = 4\ y'$ $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$ When $x = 0,\ y_2 = 2$	1 1 2 1