ORATHANADU

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LONDON KRISHNAMOORTHY MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

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MATHS

VOLUME 2

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CHAPTER 7 MATRICES AND DETERMINANTS

2 MARKS

SET 1

1. Suppose that a matrix has 12 elements. What are the possible orders it can have? What if it has 7 elements?**(EG 7.1)**

2. Construct a 2×3 matrix whose $(i, j)^{th}$ element is given by $a_{ij} = \frac{\sqrt{3}}{2}|2i - 3j|$.**(EG 7.2)**

3. Find $x, y, a,$ and b if $\begin{bmatrix} 3x + 4y & 6 & x - 2y \\ a + b & 2a - b & -3 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 4 \\ 5 & -5 & -3 \end{bmatrix}$.**(EG 7.3)**

4. Compute $A + B$ and $A - B$ if $A = \begin{bmatrix} 4 & \sqrt{5} & 7 \\ -1 & 0 & 0.5 \end{bmatrix}, B = \begin{bmatrix} \sqrt{3} & \sqrt{5} & 7.3 \\ 1 & \frac{1}{3} & \frac{1}{4} \end{bmatrix}$.**(EG 7.4)**

5. Simplify :$\sec\theta \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix} - \tan\theta \begin{bmatrix} \tan\theta & \sec\theta \\ \sec\theta & \tan\theta \end{bmatrix}$.**(EG 7.7)**

SET 2

6. Solve for x if $\begin{bmatrix} x & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ -1 & -4 & 1 \\ -1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ 2 \\ 1 \end{bmatrix} = [0]$**(EG 7.9)**

7. If $A = \begin{bmatrix} 1 & a \\ 1 & 0 \end{bmatrix}$, then compute A^4.**(EX 7.1 - 5)**

8. Consider the matrix $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ (i) Show that $A_\alpha A_\beta = A_{(\alpha+\beta)}$ **(EX 7.1 - 6)**

9. If A is a square matrix such that $A^2 = A$, find the value of $7A - (I + A)^3$.**(EX 7.1 - 12)**

10. If A is a 3×4 matrix and B is a matrix such that both $A^T B$ and BA^T are defined, what is the order of the matrix B?**(EX7.1 - 16)**

11. Let A and B be two symmetric matrices. Prove that $AB = BA$ if and only if AB is a symmetric matrix.**(EX7.1 - 22)**

SET 3

12. Compute $|A|$ using Sarrus rule if $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$.**(EG 7.17)**

13. If A is a square matrix and $|A| = 2$, find the value of $|AA^T|$.**(EX7.2 - 16)**

14. If the area of the triangle with vertices $(-3, 0), (3, 0)$ and $(0, k)$ is 9 square units, find the values of k .**(EG 7.32)**

15. Find the area of the triangle whose vertices are $(-2, -3), (3, 2),$ and $(-1, -8)$.**(EG 7.33)**

3 MARKS

SET 1

1. Determine the value of $x + y$ if $\begin{bmatrix} 2x + y & 4x \\ 5x - 7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y - 3 \\ y & x + 6 \end{bmatrix}$.**(EX 7.1 - 3)**

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2. If $A = \begin{bmatrix} 4 & 2 \\ -1 & x \end{bmatrix}$ and such that $(A - 2I)(A - 3I) = O$, find the value of x. **(EX 7.1 - 7)**

3. Find the matrix A which satisfies the matrix relation $A \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$. **(EX 7.1 - 14)**

4. (i) For what value of x, the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{bmatrix}$ is skew-symmetric. **(EX7.1 - 20)**

5. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x - y)(y - z)(z - x)$. **(EG 7.22)**

SET 2

6. Show that $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0$. **(EX7.2 - 2)**

7. Find the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ if $x, y, z \neq 1$. **(EX7.2 - 13)**

8. If $A = \begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{bmatrix}$, prove that $\sum_{k=1}^{n} \det(A^k) = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$. **(EX7.2 - 14)**

9. Determine the roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$. **(EX7.2 – 19)**

10. Solve $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$. **(EX7.3 - 3)**

SET 3

11. Solve $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$. **(EX7.3 - 5)**

12. In a triangle ABC, if $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A(1+\sin A) & \sin B(1+\sin B) & \sin C(1+\sin C) \end{vmatrix} = 0$, prove that ΔABC is an isosceles triangle. **(EG 7.26)**

13. Show that $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}$. **(EG 7.28)**

14. If $\cos 2\theta = 0$, determine $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$. **(EX7.4 - 5)**

15. Find the value of the product: $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$. **(EX7.4 - 6)**

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5 MARKS

SET 1

1. Express the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices.**(EG 7.13)**

2. Verify the property $A(B + C) = AB + AC$, when the matrices A, B, and C are given by
$A = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{bmatrix}, C = \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$.**(EX7.1 - 13)**

3. Prove that $\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2 b^2 c^2$.**(EX7.2 - 3)**

4. Prove that $\begin{vmatrix} 1 + a & 1 & 1 \\ 1 & 1 + b & 1 \\ 1 & 1 & 1 + c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$.**(EX7.2 - 4)**

SET 2

5. If a, b, c are all positive, and are p^{th}, q^{th} and r^{th} terms of a $G.P.$, show that
$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$.**(EX7.2 - 12)**

6. Using Factor Theorem, prove that $\begin{vmatrix} x + 1 & 3 & 5 \\ 2 & x + 2 & 5 \\ 2 & 3 & x + 4 \end{vmatrix} = (x - 1)^2 (x + 9)$.**(EG 7.23)**

7. Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx)$.**(EG 7.24)**

8. Prove that $|A| = \begin{vmatrix} (q + r)^2 & p^2 & p^2 \\ q^2 & (r + p)^2 & q^2 \\ r^2 & r^2 & (p + q)^2 \end{vmatrix} = 2pqr(p + q + r)^3$.**(EG 7.25)**

SET 3

9. Show that using Factor Theorem $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x - a)^2 (x + 2a)$.**(EX7.3 - 1)**

10. Show that $\begin{vmatrix} b + c & a - c & a - b \\ b - c & c + a & b - a \\ c - b & c - a & a + b \end{vmatrix} = 8abc$.**(EX7.3 - 2)**

11. Show that $\begin{vmatrix} b + c & a & a^2 \\ c + a & b & b^2 \\ a + b & c & c^2 \end{vmatrix} = (a + b + c)(a - b)(b - c)(c - a)$.**(EX7.3 - 4)**

12. Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x - y)(y - z)(z - x)$.**(EX7.3 - 6)**

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CHAPTER 8 VECTOR ALGEBRA – I

2 MARKS

SET 1

1. Represent graphically the displacement of (i) 45 cm 30°north of east. (ii) 80km 60°south of west**(EX8.1 - 1)**
2. Prove that the relation R defined on the set V of all vectors by '$\vec{a}R\vec{b}$ if $\vec{a} = \vec{b}$' is an equivalence relation on V.**(EX8.1 - 2)**
3. Let $\vec{a}$ and $\vec{b}$ be the position vectors of the points A and B. Prove that the position vectors of the points which trisects the line segment AB are $\frac{\vec{a}+2\vec{b}}{3}$ and $\frac{\vec{b}+2\vec{a}}{3}$.**(EX8.1 - 3)**
4. Let A and B be two points with position vectors $2\vec{a} + 4\vec{b}$ and $2\vec{a} - 8\vec{b}$ .Find the position vectors of the points which divide the line segment joining A and B in the ratio $1:3$ internally and externally.**(EG 8.3)**
5. If D is the midpoint of the side BC of a triangle ABC, prove that $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$.**(EX8.1 - 9)**

SET 2

6. Find a unit vector along the direction of the vector $5\vec{i} - 3\vec{j} + 4\vec{k}$.**(EG 8.4)**
7. Find a direction ratio and direction cosines of the following vectors.$(i)3\vec{i} + 4\vec{j} - 6\vec{k}, (ii)\ 3\vec{i} - 4\vec{k}$**(EG 8.5)**
8. (i) Find the direction cosines of a vector whose direction ratios are $2, 3, -6$. **(EG 8.6)**
9. Can a vector have direction angles 30° ,45°, 60°?**(EG 8.6)**
10. Find the direction cosines of $\overrightarrow{AB}$ where A is $(2, 3, 1)$ and B is $(3, -1, 2)$. **(EG 8.6)**

SET 3

11. Find the direction cosines of the line joining $(2, 3, 1)$ and $(3, -1, 2)$. **(EG 8.6)**
12. The direction ratios of a vector are $2, 3, 6$ and it's magnitude is 5. Find the vector.**(EG 8.6)**
13. If $\frac{1}{2}, \frac{1}{\sqrt{2}}, a$ are the direction cosines of some vector, then find a.**(EX8.2 - 5)**
14. Show that the following vectors are coplanar $\vec{i} - 2\vec{j} + 3\vec{k},\ -2\vec{i} + 3\vec{j} - 4\vec{k}$ and $-\vec{j} + 2\vec{k}$**(EX8.2 - 9)**
15. The position vectors $\vec{a}, \vec{b}, \vec{c}$ of three points satisfy the relation $2\vec{a} - 7\vec{b} + 5\vec{c} = \vec{0}$ Are these points collinear?**(EX8.2 - 14)**

SET 4

16. Find the value or values of m for which $m\ (\vec{i} + \vec{j} + \vec{k})$ is a unit vector.**(EX8.2 - 16)**
17. Find $(\vec{a} + 3\vec{b}).(2\vec{a} - \vec{b})$ if $\vec{a} = \vec{i} + \vec{j} + 2\vec{k}$ and $\vec{b} = 3\vec{i} + 2\vec{j} - \vec{k}$.**(EG 8.12)**

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18. If $\vec{a} = 2\vec{\imath} + 2\vec{\jmath} + 3\vec{k}, \vec{b} = -\vec{\imath} + 2\vec{\jmath} + \vec{k}$ and $\vec{c} = 3\vec{\imath} + \vec{\jmath}$ be such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ then find λ.(EG 8.13)

19. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ prove that and $\vec{a}$ and $\vec{b}$ are perpendicular.(EG 8.14)

20. For any vector $\vec{r}$ prove that $\vec{r} = (\vec{r}.\vec{\imath})\vec{\imath} + (\vec{r}.\vec{\jmath})\vec{\jmath} + (\vec{r}.\vec{k})\vec{k}$.(EG 8.15)

SET 5

21. Find the angle between the vectors $5\vec{\imath} + 3\vec{\jmath} + 4\vec{k}$ and $6\vec{\imath} - 8\vec{\jmath} - \vec{k}$.(EG 8.16)

22. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 10, |\vec{b}| = 15$ and $\vec{a}.\vec{b} = 75\sqrt{2}$ Find the angle between $\vec{a}$ and $\vec{b}$.(EX8.3 - 31)

23. Show that the points $(2, -1, 3), (4, 3, 1)$ and $(3, 1, 2)$ are collinear.(EX8.3 - 9)

24. Find the projection of the vector $\vec{\imath} + 3\vec{\jmath} + 7\vec{k}$ on the vector $2\vec{\imath} + 6\vec{\jmath} + 3\vec{k}$.(EX8.3 - 12)

25. Find λ when the projection of $\vec{a} = \lambda\vec{\imath} + \vec{\jmath} + 4\vec{k}$ on $\vec{b} = 2\vec{\imath} + 6\vec{\jmath} + 3\vec{k}$ is 4 units.(EX8.3 - 13)

SET 6

26. Find $|\vec{a} \times \vec{b}|$ where $\vec{a} = 3\vec{\imath} + 4\vec{\jmath}$ and $\vec{b} = \vec{\imath} + \vec{\jmath} + \vec{k}$.(EG 8.20)

27. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $|\vec{a} \times \vec{b}|^2 + (\vec{a}.\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$.(EG 8.25)

28. Find the magnitude of $\vec{a} \times \vec{b}$ if $\vec{a} = 2\vec{\imath} + \vec{\jmath} + 3\vec{k}$ and $\vec{b} = 3\vec{\imath} + 5\vec{\jmath} - 2\vec{k}$.(EX8.4 - 1)

29. Show that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$.(EX8.4 - 2)

30. Find $\vec{a}.\vec{b}$ when (i) $\vec{a} = \vec{\imath} - \vec{\jmath} + 5\vec{k}$ and $\vec{b} = 3\vec{\imath} - 2\vec{k}$

3 MARKS

SET 1

1. Vector addition is commutative.(RE8.5)

2. If $\vec{a}$ and $\vec{b}$ are vectors represented by two adjacent sides of a regular hexagon, then find the vectors represented by other sides.(EG 8.2)

3. A quadrilateral is a parallelogram if and only if its diagonals bisect each other.(TH8.4)

4. If D and E are the midpoints of the sides AB and AC of a triangle ABC, prove that $\overrightarrow{BE} + \overrightarrow{DC} = \frac{3}{2}\overrightarrow{BC}$.(EX8.1 - 4)

5. Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side whose length is half of the length of the third side.(EX8.1 - 5)

SET 2

6. Prove that the line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.(EX8.1 - 6)

7. If $\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{QO} + \overrightarrow{OR}$, prove that the points P, Q, R are collinear.(EX8.1 - 8)

8. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$.(EX8.1 - 10)

9. Let $A, B,$ and C be the vertices of a triangle. Let $D, E,$ and F be the midpoints of the sides BC, CA and AB respectively. Show that $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = 0$.(EX8.1 - 11)

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10. Show that the points whose position vectors are $2\vec{i} + 3\vec{j} - 5\vec{k}$, $3\vec{i} + \vec{j} - 2\vec{k}$ and $6\vec{i} - 5\vec{j} + 7\vec{k}$ are collinear.**(EG 8.7)**

SET 3

11. Show that the vectors $5\vec{i} + 6\vec{j} + 7\vec{k}$, $7\vec{i} - 8\vec{j} + 9\vec{k}$ and $3\vec{i} + 20\vec{j} + 5\vec{k}$ are coplanar.**(EG 8.10)**

12. Show that the vectors $2\vec{i} - \vec{j} + \vec{k}$, $3\vec{i} - 4\vec{j} - 4\vec{k}$ and $\vec{i} - 3\vec{j} - 5\vec{k}$ form a right angled triangle.**(EX8.2 - 7)**

13. Find the value of λ for which the vectors $\vec{a} = 3\vec{i} + 2\vec{j} + 9\vec{k}^{\wedge}$ and $\vec{b} = \vec{i} + \lambda\vec{j} + 3\vec{k}$ are parallel.**(EX8.2 - 8)**

14. If $\vec{a} = 2\vec{i} + 3\vec{j} - 4\vec{k}, \vec{b} = 3\vec{i} - 4\vec{j} - 5\vec{k}$ and $\vec{c} = -3\vec{i} + 2\vec{j} + 3\vec{k}$ find the magnitude and direction cosines of (ii) $3\vec{a} - 2\vec{b} + 5\vec{c}$.**(EX8.2 - 11)**

15. Find the unit vector parallel to $3\vec{a} - 2\vec{b} + 4\vec{c}$ if $\vec{a} = 3\vec{i} - \vec{j} - 4\vec{k}, \vec{b} = -2\vec{i} + 4\vec{j} - 3\vec{k}$ and $\vec{c} = \vec{i} + 2\vec{j} - \vec{k}$.**(EX8.2 - 13)**

SET 4

16. Show that the points A $(1, 1, 1), B(1, 2, 3)$ and $C(2, -1, 1)$ are vertices of an isosceles triangle.**(EX8.2 - 17)**

17. Find the projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ where A, B, C, D are the points $(4, -3, 0), (7, -5, -1), (-2, 1, 3), (0, 2, 5)$.**(EG 8.17)**

18. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors satisfying $\vec{a} - \sqrt{3}\vec{b} + \vec{c} = \vec{0}$ then find the angle between $\vec{a}$ and $\vec{c}$.**(EG 8.18)**

19. Show that the vectors $\vec{a} = 2\vec{i} + 3\vec{j} + 6\vec{k}, \vec{b} = 6\vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{c} = 3\vec{i} - 6\vec{j} + 2\vec{k}$ are mutually orthogonal.**(EX8.3 - 6)**

20. If $|\vec{a}| = 5, |\vec{b}| = 6, |\vec{c}| = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.**(EX8.3 - 8)**

SET 5

21. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.**(EX8.3 - 11)**

22. Find the vectors of magnitude 6 which are perpendicular to both vectors $\vec{a} = 4\vec{i} - \vec{j} + 3\vec{k}$ and $\vec{b} = -2\vec{i} + \vec{j} - 2\vec{k}$.**(EG 8.22)**

23. Find the cosine and sine angle between the vectors $\vec{a} = 2\vec{i} + \vec{j} + 3\vec{k}$ and $\vec{b} = 4\vec{i} - 2\vec{j} + 2\vec{k}$.**(EG 8.23)**

24. Find the area of the parallelogram whose adjacent sides are $\vec{a} = 3\vec{i} + \vec{j} + 4\vec{k}$ and $\vec{b} = \vec{i} - \vec{j} + \vec{k}$.**(EG 8.24)**

25. Find the area of a triangle having the points $A(1,0,0), B(0,1,0)$, and $C(0,0,1)$ as its vertices.**(EG 8.26)**

SET 6

26. Find the vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane which contains $\vec{i} + 2\vec{j} + \vec{k}$ and $\vec{i} + 3\vec{j} + 4\vec{k}$.**(EX8.4 - 3)**

27. Find the unit vectors perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + 2\vec{j} + 3\vec{k}$.**(EX8.4 - 4)**

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28. Find the area of the parallelogram whose two adjacent sides are determined by the vectors $\vec{i} + 2\vec{j} + 3\vec{k}$ and $3\vec{i} - 2\vec{j} + \vec{k}$.**(EX8.4 - 5)**

29. For any vector $\vec{a}$ prove that $|\vec{a} \times \vec{i}|^2 + |\vec{a} \times \vec{j}|^2 + |\vec{a} \times \vec{k}|^2 = 2|\vec{a}|^2$.**(EX8.4 - 8)**

30. Find the angle between the vectors $2\vec{i} + \vec{j} - \vec{k}$ and $\vec{i} + 2\vec{j} + \vec{k}$ by using cross product.**(EX8.4 - 10)**

5 MARKS

SET 1

1. The medians of a triangle are concurrent.**(TH8.3)**
2. If $ABCD$ is a quadrilateral and E and F are the mid-points of AC and BD respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$.**(EX8.1 - 12)**
3. Find a point whose position vector has magnitude 5 and parallel to the vector $4\vec{i} - 3\vec{j} + 10\vec{k}$**(EG 8.8)**
4. Prove that the points whose position vectors $2\vec{i} + 4\vec{j} + 3\vec{k}$, $4\vec{i} + \vec{j} + 9\vec{k}$ and $10\vec{i} - \vec{j} + 6\vec{k}$ form a right angled triangle.**(EG 8.9)**
5. A triangle is formed by joining the points $(1, 0, 0), (0, 1, 0)$ and $(0, 0, 1)$. Find the direction cosines of the medians.**(EX8.2 - 4)**

SET 2

6. Show that the points whose position vectors $4\vec{i} + 5\vec{j} + \vec{k}$, $-\vec{j} - \vec{k}$, $3\vec{i} + 9\vec{j} + 4\vec{k}$ and $-4\vec{i} + 4\vec{j} + 4\vec{k}$ are coplanar.**(EX8.2 - 10)**
7. The position vectors of the vertices of a triangle are $\vec{i} + 2\vec{j} + 3\vec{k}$, $3\vec{i} - 4\vec{j} + 5\vec{k}$ and $-2\vec{i} + 3\vec{j} - 7\vec{k}$ Find the perimeter of the triangle.**(EX8.2 - 12)**
8. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $|\vec{a}| = 2, |\vec{b}| = 3, |\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Find $4\vec{a}.\vec{b} + 3\vec{b}.\vec{c} + 3\vec{c}.\vec{a}$.**(EX8.3 - 14)**
9. The position vectors of the points P, Q, R, S are $\vec{i} + \vec{j} + \vec{k}$, $2\vec{i} + 5\vec{j}, 3\vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{i} - 6\vec{j} - \vec{k}$ respectively. Prove that the line PQ and RS are parallel.**(EX8.2 - 15)**
10. Show that the points $(4, -3, 1), (2, -4, 5)$ and $(1, -1, 0)$ form a right angled triangle.**(EG 8.19)**

SET 3

11. If $\vec{a}, \vec{b}$ are unit vectors and θ is the angle between them, show that $(i) \sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$

    $(ii) \cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$ $(ii) \tan\frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}$.**(EX8.3 - 10)**
12. If $\vec{a} = -3\vec{i} + 4\vec{j} - 7\vec{k}$ and $\vec{b} = 6\vec{i} + 2\vec{j} - 3\vec{k}$, verify (i) $\vec{a}$ and $\vec{a} \times \vec{b}$ are perpendicular to each other. (ii) $\vec{b}$ and $\vec{a} \times \vec{b}$ are perpendicular to each other.**(EG 8.21)**
13. Find the area of the triangle whose vertices are $A(3, -1, 2), B(1, -1, -3)$ and $C(4, -3, 1)$.**(EX8.4 - 6)**
14. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of a triangle ABC, then prove that the area of triangle ABC is $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$. Also deduce the condition for collinearity of the points A, B and C.**(EX8.4 - 7)**

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15. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. Prove that $\vec{a} = \pm \frac{2}{\sqrt{3}} (\vec{b} \times \vec{c})$. **(EX8.4 - 9)**

CHAPTER 9 LIMITS AND CONTINUITY

2 MARKS

SET 1

1. Evaluate $\lim_{x \to 2^-} \lfloor x \rfloor$ and $\lim_{x \to 2^+} \lfloor x \rfloor$. **(EG 9.3)**

2. Check if $\lim_{x \to -5} f(x)$ exists or not, where $f(x) = \begin{cases} \frac{|x+5|}{x+5}, & for\ x \neq -5 \\ 0, & for\ x = -5 \end{cases}$. **(EG 9.5)**

3. Test the existence of the limit, $\lim_{x \to 1} \frac{4|x-1|+x-1}{|x-1|}, x \neq 1$. **(EG 9.6)**

4. Write a brief description of the meaning of the notation $\lim_{x \to 8} f(x) = 25$. **(EX9.1 - 19)**

5. If $f(2) = 4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? **(EX9.1 - 20)**

SET 2

6. Evaluate: $\lim_{x \to 3} \frac{x^2-9}{x-3}$ if it exists by finding $f(3^-)$ and $f(3^+)$. **(EX9.1 - 22)**

7. Verify the existence of $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} \frac{|x-1|}{x-1}, & for\ x \neq 1 \\ 0, & for\ x = 1 \end{cases}$. **(EX9.1 - 23)**

8. Compute $\lim_{x \to 0} \left(\frac{x^2+x}{x} + 4x^3 + 3 \right)$. **(EG 9.10)**

9. Compute $\lim_{x \to 1} \frac{\sqrt{x}-1}{x-1}$. **(EG 9.14)**

10. Find $\lim_{t \to 1} \frac{\sqrt{t}-1}{t-1}$. **(EG 9.17)**

SET 3

11. Find $\lim_{x \to 0} \frac{(2+x)^5-2^5}{x}$ **(EG 9.18)**

12. Find the positive integer n so that $\lim_{x \to 3} \frac{x^n-3^n}{x-3} = 27$. **(EG 9.19)**

13. Find the relation between a and b if $\lim_{x \to 3} f(x)$ exists where $f(x) = \begin{cases} ax+b & ,\ if\ x > 3 \\ 3ax-4b+1 & ,\ if\ x < 3 \end{cases}$. **(EG 9.20)**

14. $\lim_{x \to 3} \frac{x^4-16}{x-2}$. **(EX9.2 - 1)**

15. $\lim_{x \to 1} \frac{x^m-1}{x^n-1}, m$ and n are integers. **(EX9.2 - 2)**

SET 4

16. $\lim_{\sqrt{x} \to 3} \frac{x^2-81}{\sqrt{x}-3}$.   **(EX9.2 - 3)**

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17. $\lim\limits_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}, x > 0$. **(EX9.2 - 4)**

18. $\lim\limits_{x \to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2}$. **(EX9.2 - 6)**

19. $\lim\limits_{x \to \infty} \frac{x^3+x}{x^4-3x^2+1}$. **(EX9.3 - 4)**

20. A tank contains 5000 litres of pure water. Brine (very salty water) that contains 30 grams of salt per litre of water is pumped into the tank at a rate of 25 litres per minute. The concentration of salt water after t minutes (in grams per litre) is $C(t) = \frac{30t}{200+t}$. What happens to the concentration as $t \to \infty$? **(EX9.3 - 10)**

SET 4

21. Prove that $\lim\limits_{x \to 0} \sin x = 0$. **(EG 9.30)**

22. Evaluate $\lim\limits_{x \to 0} (1 + \sin x)^{2cosec\,x}$. **(EG 9.32)**

23. Do the limits of following functions exist as $x \to 0$? State reasons for your answer. $(i) \frac{\sin |x|}{x}$. **(EG 9.35)**

24. $\lim\limits_{x \to \infty} \left(1 + \frac{3}{x}\right)^{x+2}$. **(EX9.4 - 5)**

25. Evaluate $\lim\limits_{x \to 0} \frac{\sin \alpha x}{\sin \beta x}$. **(EX9.4 - 7)**

SET 4

26. $\lim\limits_{x \to 0} \frac{\tan 2x}{\sin 5x}$. **(EX9.4 - 8)**

27. $\lim\limits_{\alpha \to 0} \frac{\sin(\alpha^n)}{(\sin \alpha)^m}$. **(EX9.4 - 9)**

28. $\lim\limits_{x \to 0} \frac{2 \arcsin x}{3x}$. **(EX9.4 - 12)**

29. Calculate $\lim\limits_{x \to 3} \frac{(x^2-6x+5)}{x^3-8x+7}$. **(EG 9.13)**

3 MARKS

SET 1

1. $\lim\limits_{x \to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$. **(EX9.2 - 8)**

2. $\lim\limits_{x \to 1} \frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$. **(EX9.2 - 10)**

3. $\lim\limits_{x \to a} \frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}, (a > b)$. **(EX9.2 - 15)**

4. Show that $(i) \lim\limits_{n \to \infty} \frac{1+2+3+\ldots\ldots+n}{3n^2+7n+2} = \frac{1}{6}$, **(EX9.3 - 8)**

SET 2

5. Show that $(ii) \lim\limits_{n \to \infty} \frac{1^2+2^2+3^2+\ldots\ldots+(3n)^2}{(1+2+\ldots\ldots+5n)(2n+3)} = \frac{9}{25}$, **(EX9.3 - 8)**

6. Show that $(iii) \lim\limits_{n \to \infty} \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots\ldots + \frac{1}{n(n+1)} = 1$. **(EX9.3 - 8)**

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7. Evaluate $\lim\limits_{x \to \infty} \left(\frac{x+2}{x-2}\right)^x$.**(EG 9.33)**

8. Do the limits of functions exist as $x \to 0$? State reasons for your answer. $\frac{\sin(x-\lfloor x \rfloor)}{x-\lfloor x \rfloor}$.**(EG 9.35)**

SET 3

9. Evaluate $\lim\limits_{x \to 0} \frac{\sin^3\left(\frac{x}{2}\right)}{x^3}$.**(EX9.4 - 6)**

10. $\lim\limits_{x \to 0} \frac{3^x - 1}{\sqrt{x+1} - 1}$.**(EX9.4 - 16)**

11. $\lim\limits_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$.**(EX9.4 - 22)**

12. $\lim\limits_{x \to 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{\tan x}$.**(EX9.4 - 23)**

13. If f and g are continuous functions with $f(3) = 5$ and $\lim\limits_{x \to 3}[2f(x) - g(x)] = 4$, find $g(3)$.**(EX9.5 - 8)**

5 MARKS

1. $(a)\ \lim\limits_{\theta \to 0} \frac{\sin \theta}{\theta} = 1,\ (b)\ \lim\limits_{\theta \to 0} \frac{1 - \cos \theta}{\theta} = 0$.**(RE9.1)**

2. Show that $\lim\limits_{x \to 0^+} x\left[\left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \dots \dots + \left\lfloor \frac{15}{x} \right\rfloor\right] = 120$.**(EG 9.31)**

3. For what value of α is this $f(x) = \begin{cases} \frac{x^4 - 1}{x - 1}, & if\ x \neq 1 \\ \alpha, & if\ x = 1 \end{cases}$, continuous at $x = 1$?**(EX9.5 - 6)**

4. A function f is defined as follows: $f(x) = \begin{cases} 0, & for\ x < 0 \\ x, & for\ 0 \leq x < 1 \\ -x^2 + 4x - 2, & for\ 1 \leq x < 3 \\ 4 - x, & for\ x \geq 3 \end{cases}$ Is the function continuous?**(EX9.5 - 10)**

CHAPTER 10 – DIFFERENTIAL CALCULUS - DIFFERENTIABILITY AND METHODS OF DIFFERENTIATION

2 MARKS

SET 1

1. Find the slope of the tangent line to the graph of $f(x) = 7x + 5$ at any point $(x_0, f(x_0))$.**(EG 10.1)**

2. Find the derivatives of the functions using first principle. $f(x) = -x^2 + 2$**(EX10.1 - 1)**

3. Examine the differentiability of functions in R by drawing the diagrams. $(i)|\sin x|$ **EX10.1 - 7)**

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4. Differentiate the following with respect to x: ($viii$) Find $f'(3)$ and $f'(5)$ if $f(x) = |x - 4|$.(**EG 10.7**)
5. $g(t) = 4 \sec t + \tan t$(**EX10.2 - 6**)

SET 2

6. Find the derivatives $y = e^x \sin x$(**EX10.2 - 7**)
7. Find the derivatives $y = \frac{\sin x}{1 + \cos x}$(**EX10.2 - 9**)
8. $y = \frac{\sin x}{x^2}$(**EX10.2 - 12**)
9. $y = e^{-x} \cdot \log x$(**EX10.2 - 16**)
10. $y = \sin x°$(**EX10.2 - 18**)

SET 3

11. $y = \log_{10} x$(**EX10.2 - 19**)
12. Differentiate: $y = (x^3 - 1)^{100}$.(**EG 10.10**)
13. $y = cosec\ x \cdot \cot x$(**EX10.2 - 14**)
14. Differentiate: $y = e^{\sin x}$.(**EG 10.14**)
15. Differentiate 2^x.(**EG 10.15**)

SET 4

16. $y = e^{\sqrt{x}}$(**EX10.3 - 5**)
17. $y = xe^{-x^2}$(**EX10.3 - 15**)
18. $y = 5^{\frac{-1}{x}}$(**EX10.3 - 20**)
19. $y = e^{x \cos x}$(**EX10.3 - 27**)
20. Differentiate: $= x^{\sqrt{x}}$.(**EG 10.23**)

SET 5

21. Find $\frac{dy}{dx}$ if $x^2 + y^2 = 1$.(**EG 10.17**)
22. Find $\frac{dy}{dx}$ if $x = at^2, y = 2at, t \neq 0$.(**EG 10.26**)
23. Find y', y'' and y''' if $y = x^3 - 6x^2 - 5x + 3$.(**EG 10.31**)
24. Find y''' if $y = \frac{1}{x}$.(**EG 10.32**)
25. Find f'' if $f(x) = x \cos x$.(**EG 10.33**)

3 MARKS

SET 1

1. Determine whether the function is differentiable at the indicated values. (iii)f(x) = |x| + |x − 1| at x = 0, 1(**EX10.1 - 3**)
2. Show that the functions are not differentiable at the indicated value of x. $f(x) = \begin{cases} -x + 2, x \leq 2 \\ 2x - 4, x > 2 \end{cases}, x = 2$ (**EX10.1 - 4**)
3. Draw the function $f'(x)$ if $f(x) = 2x^2 - 5x + 3$(**EX10.2 - 20**)
4. Differentiate $(2x + 1)^5 (x^3 - x + 1)^4$.(**EG 10.13**)

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5. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, find y'.(EG 10.16)

SET 2

6. $y = \sin^2(\cos kx)$(EX10.3 - 23)

7. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$(EX10.3 - 28)

8. Find $\frac{dy}{dx}$ if $x^4 + x^2y^3 - y^5 = 2x + 1$.(EG 10.19)

9. Find $\frac{dy}{dx}$ if $\sin y = y\cos 2x$.(EG 10.20)

10. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, find y'.(EG 10.24)

SET 3

11. Find $\frac{dy}{dx}$ if $x = a(t - \sin t), y = a(1 - \cos t)$.(EG 10.27)

12. Find the derivative of $\tan^{-1}(1 + x^2)$ with respect to $x^2 + x + 1$.(EG 10.29)

13. Differentiate $\sin(ax^2 + bx + c)$ with respect to $\cos(lx^2 + mx + n)$.(EG 10.30)

14. Find f'' if $x^4 + y^4 = 16$.(EG 10.34)

15. Find the second order derivative if x and y are given by $x = a\cos t, y = a\sin t$.(EG 10.35)

SET 1

16. Find $\frac{d^2y}{dx^2}$ if $x^2 + y^2 = 4$.(EG 10.36)

17. $x = a\cos^3 t, y = a\sin^3 t$(EX10.4 - 13)

18. $x = \frac{1-t^2}{1+t^2}, \ y = \frac{2t}{1+t^2}$.(EX10.4 - 15)

19. Find the derivative of $\sin x^2$ with respect to x^2.(EX10.4 - 19)

20. If $y = \sin^{-1} x$ then find y''.(EX10.4 - 23)

5 MARKS

SET 1

1. Differentiate: $y = \frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}$.(EG 10.22)

2. Find the derivative with $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ with respect to $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.(EX10.4 - 22)

3. If $y = e^{\tan^{-1} x}$ show that $(1 + x^2)y'' + (2x - 1)y' = 0$.(EX10.4 - 24)

4. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1 - x^2)y_2 - 3xy_1 - y = 0$.(EX10.4 - 25)

5. If $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ then prove that at, $\theta = \frac{\pi}{2}, y'' = \frac{1}{a}$.(EX10.4 - 26)

6. If $\sin y = x\sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}, a \neq n\pi$.(EX10.4 - 27)

7. If $y = (\cos^{-1} x)^2$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x = 0$.(EX10.4 - 28)

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CHAPTER 11 – INTEGRAL CALCULUS

2 MARKS

SET 1

1. Integrate the with respect to x: (iii) $\frac{\sin x}{\cos^2 x}$ **EG 11.2)**
2. Integrate the functions with respect to x: (ii) $\frac{1}{(2-3x)^4}$ **(EX 11.2 - 1)**
3. Evaluate $30 \sec(2 - 15x) \tan(2 - 15x)$ **(EX 11.2 - 4)**
4. Evaluate the with respect to x: (ii) $\int \sqrt{15 - 2x} \, dx$ **(EG 11.4)**
5. Integrate $\frac{1}{\sqrt{1-25x^2}}$ **(EG 11.7)**

SET 2

6. If $f'(x) = 3x^2 - 4x + 5$ and $f(1) = 3$, then find $f(x)$. **(EG 11.10)**
7. If $f'(x) = 4x - 5$ and $f(2) = 1$, find $f(x)$. **(EX 11.4 - 1)**
8. If $f'(x) = 9x^2 - 6x$ and $f(0) = -3$, find $f(x)$. **(EX 11.4 - 2)**
9. Integrate the following with respect to x: (i) $\cos 5x \sin 3x$ **(EG 11.16)**
10. Evaluate: $\int \sqrt{1 + \cos 2x} \, dx$. **(EG 11.20)**

SET 3

11. Evaluate: (ii) $\int e^{x \log 2} e^x dx$. **(EG 11.26)**
12. Evaluate: $e^{x \log a} e^x$ **(EX 11.5 - 13)**
13. Evaluate $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ **(EX 11.6 - 3)**
14. Evaluate $\frac{\sin \sqrt{x}}{\sqrt{x}}$ **(EX 11.6 - 5)**
15. Evaluate $\frac{\sin^{-1} x}{\sqrt{1-x^2}}$ **(EX 11.6 - 9)**

SET 4

16. Evaluate $\alpha \beta x^{\alpha-1} e^{-\beta x^\alpha}$ **(EX 11.6 - 12)**
17. Evaluate the integrals (ii) $\int e^{-5x} \sin 3x \, dx$ **(EG 11.36)**
18. Integrate the with respect to x: $e^x(\tan x + \log \sec x)$ **(EX 11.9 - 1)**
19. Find the integrals of the : (ii) $\frac{1}{25-4x^2}$ **(EX 11.10 - 1)**
20. Evaluate the following x: (i) $\int \sqrt{4 - x^2} dx$ **(EG 11.41)**

3 MARKS

SET 1

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1. Evaluate the following integrals:$(i) \frac{12}{(4x-5)^3} + \frac{6}{3x+2} + 16e^{4x+3}$ **(EG 11.9)**
2. A train started from Madurai Junction towards Coimbatore at $3pm$ (time $t = 0$) with velocity $v(t) = 20t + 50$ kilometer per hour, where t is measured in hours. Find the distance covered by the train at $5pm$. **(EG 11.11)**
3. Evaluate: $\int (\tan x + cotx)^2 dx$ **(EG 11.22)**
4. Evaluate: $\int \frac{1}{\sqrt{x+1}+\sqrt{x}} dx$. **(EG 11.28)**
5. Evaluate: $(ii) \int \frac{x+3}{(x+2)^2(x+1)} dx$ **(EG 11.29)**

SET 2

6. $(3x + 4)\sqrt{3x + 7}$ **(EX 11.5 - 14)**
7. $\frac{1}{(x-1)(x+2)^2}$ **(EX 11.5 - 18)**
8. $\frac{x^3}{(x-1)(x-2)}$. **(EX 11.5 - 20)**
9. $\sin^5 x \cos^3 x$ **(EX 11.6 - 15)**
10. $\frac{x \sin^{-1} x}{\sqrt{1-x^2}}$ **(EX 11.7 − 3)**

SET 3

11. $e^x \left(\frac{2+\sin 2x}{1+\cos 2x} \right)$ **(EX 11.9 - 4)**
12. $\frac{\log x}{(1+\log x)^2}$ **(EX 11.9 - 6)**
13. Evaluate the integrals $(iii) \int \frac{1}{\sqrt{12+4x-x^2}} dx$ **(EG 11.39)**
14. $\frac{1}{\sqrt{x^2+4x+2}}$ **(EX 11.10 - 2)**
15. Integrate the following functions with respect to x: $(ii) \sqrt{x^2 - 2x - 3}$ **EX 11.12 - 1)**

5 MARKS

1. At a particular moment, a student needs to stop his speedy bike to avoid a collision with the barrier ahead at a distance 40 metres away from him. Immediately he slows (retardation) the bike under braking at a rate of 8 $metre/second^2$. If the bike is moving at a speed of $24m/s$, when the brakes are applied, would it stop before collision? **(EG 11.14)**
2. Evaluate the following integrals
   $(i) \int \frac{3x+5}{x^2+4x+7} dx$ $(ii) \int \frac{x+1}{x^2-3x+1} dx$
   $(iii) \int \frac{2x+3}{\sqrt{x^2+x+1}} dx$ $(iv) \int \frac{5x-7}{\sqrt{3x-x^2-2}} dx$ **(EG 11.40)**
3. Integrate the following with respect to x:
   $(1) (i) \frac{2x-3}{x^2+4x-12}$ $(ii) \frac{5x-2}{2+2x+x^2}$ $(iii) \frac{3x+1}{2x^2-2x+3}$ **(EX 11.11 - 1)**

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(2) (i) $\frac{2x+1}{\sqrt{9+4x-x^2}}$ (ii) $\frac{x+2}{\sqrt{x^2-1}}$ (iii) $\frac{2x+3}{\sqrt{x^2+4x+1}}$ **(EX 11.11 - 2)**

CHAPTER 12 PROBABILITY THEORY

2 MARKS

SET 1

1. An integer is chosen at random from the first ten positive integers. Find the probability that it is (i) an even number (ii) multiple of three.**(EG 12.2)**
2. Suppose a fair die is rolled. Find the probability of getting (i) an even number (ii) multiple of three.**(EG 12.5)**
3. An experiment has the four possible mutually exclusive and exhaustive outcomes A, B, C, and D. Check whether the assignments of probability are permissible.(i) $P(A) = 0.15, P(B) = 0.30, P(C) = 0.43, P(D) = 0.12$**(EX 12.1 - 1)**
4. If two coins are tossed simultaneously, then find the probability of getting (i)one head and one tail (ii) at most two tails**(EX 12.1 - 2)**
5. What is the chance that (i) non-leap year (ii) leap year should have fifty three Sundays?**(EX 12.1 - 4)**

SET 2

6. (i) The odds that the event A occurs is 5 to 7, find $P(A)$.(ii) Suppose $P(B) = \frac{2}{5}$. Express the odds that the event Boccurs.**(EX 12.1 - 10)**
7. If $\bar{A}$is the complementary event of A, then $P(\bar{A}) = 1 - P(A)$.**(TH12.4)**
8. Find the probability of getting the number 7, when a usual die is rolled.**(EG 12.12)**
9. Nine coins are tossed once, find the probability to get at least two heads.**(EG 12.13)**
10. If A and $B$are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.**(TH12.6)**

SET 3

11. If $A$and $B$are any two events and $\bar{B}$is the complementary events of B , then$P(A \cap \bar{B}) = P(A) - P(A \cap B)$.**(TH12.5)**
12. A die is thrown twice. Let Abe the event, '$First\ die\ shows$ 5' and Bbe the event, '$second\ die\ shows$ 5'.Find $(A \cup B)$.**(EX 12.2 - 3)**
13. If$P(A) = 0.6, P(B) = 0.5$, and $P(A \cap B) = 0.2$,Find (i) $P(A/B)$ (ii) $P(\bar{A}/B)$ (iii) $P(A/\bar{B})$.**(EG 12.16)**
14. A die is rolled. If it shows an odd number, then find the probability of getting 5.**(EG 12.17)**
15. If A and $B$are two independent events such that $P(A) = 0.4$ and $P(A \cup B) = 0.9$. Find $P(B)$.**(EG 12.20)**

SET 4

16. Can two events be mutually exclusive and independent simultaneously?**(EX 12.3 - 1)**
17. If $A$and B are two events such that $P(A \cup B) = 0.7, P(A \cap B) = 0.2$,and $P(B) = 0.5$, then show that $A$and $B$are independent.**(EX 12.3 - 2)**
18. If $P(A) = 0.5, P(B) = 0.8$ and $P(B/A) = 0.8$, find $P(A / B)$ and $P(A \cup B)$.**(EX 12.3 - 4)**

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3 MARKS

SET 1

1. Three coins are tossed simultaneously, what is the probability of getting (i) exactly one head(ii) at least one head (iii) at most one head?**(EG 12.3)**
2. Suppose ten coins are tossed. Find the probability to get (i) exactly two heads (ii) at most two heads (iii) at least two heads**(EG 12.4)**
3. When a pair of fair dice is rolled, what are the probabilities of getting the sum(i) $7$$(ii)$ 7 or $9$$(iii)$ 7 or 12?**(EG 12.6)**
4. Three candidates X, Y and $Z$are going to play in a chess competition to win $FIDE$ (World Chess Federation) cup this year. $X$is thrice as likely to win as $Y$and $Y$is twice as likely as to win Z. Find the respective probability of $X, Y$and Z to win the cup.**(EG 12.7)**
5. Three letters are written to three different persons and addresses on three envelopes are also written. Without looking at the addresses, what is the probability that (i) exactly one letter goes to the right envelopes (ii)none of the letters go into the right envelopes?**(EG 12.8)**

SET 2

6. Let the matrix $M = \begin{bmatrix} x & y \\ z & 1 \end{bmatrix}$. If x, y and $z$are chosen at random from the set $\{1, 2, 3\}$,and repetition is allowed$(i.e., x = y = z)$, what is the probability that the given matrix $M$is a singular matrix?**(EG 12.9)**
7. Five mangoes and 4 apples are in a box. If two fruits are chosen at random, find the probability that (i) one is a mango and the other is an apple (ii) both are of the same variety.**(EX 12.1 - 3)**
8. Eight coins are tossed once, find the probability of getting(i)exactly two tails (ii) at least two tails (iii) at most two tails**(EX 12.1 - 5)**
9. Given that $P(A) = 0.52, P(B) = 0.43$, and $P(A \cap B) = 0.24$, find (i) $P(A \cap \bar{B})$$(ii)$ $P(A \cup B)$$(iii)$ $P(\bar{A} \cap \bar{B})$$(iv)$ $P(\bar{A} \cup \bar{B})$**(EG 12.14)**
10. The probability that a girl, preparing for competitive examination will get a State Government service is 0.12, the probability that she will get a Central Government job is 0.25,and the probability that she will get both is 0.07. Find the probability that (i) she will get atleast one of the two jobs (ii) she will get only one of the two jobs.**(EG 12.15)**

SET 3

11. If A and B are mutually exclusive events $P(A) = \frac{3}{8}$ and $P(B) = \frac{1}{8}$,then find(i) $P(\bar{A})$ (ii) $P(A \cup B)$ (iii) $P(\bar{A} \cap B)$ (iv) $P(\bar{A} \cup \bar{B})$**(EX 12.2 - 1)**
12. If $A$and $B$are two events associated with a random experiment for which$P(A) = 0.35, P(A\ or\ B) = 0.85$,and$P(A\ and\ B) = 0.15$.Find (i) $P(only\ B)$ (ii) $P(\bar{B})$ (iii) $P(only\ A)$**(EX 12.2 - 2)**
13. A town has 2 fire engines operating independently. The probability that a fire engine is available when needed is 0.96. (i) What is the probability that a fire engine is available when needed? (ii) What is the probability that neither is available when needed?**(EX 12.2 - 5)**
14. The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it will get both awards is 0.2. What is the probability, that (i) it will get at least one of the two awards (ii) it will get only one of the awards.**(EX 12.2 - 6)**

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15. If A and B are independent then $(i)\bar{A}$ and $\bar{B}$ are independent. (ii) A and $\bar{B}$ are independent. $(iii)\bar{A}$ and B are also independent.**(TH12.8)**

SET 4

16. Two cards are drawn from a pack of 52 cards in succession. Find the probability that both are Jack when the first drawn card is (i) replaced (ii) not replaced**(EG 12.18)**
17. If A and B are two independent events such that $P(A \cup B) = 0.6, P(A) = 0.2$, find $P(B)$.**(EX 12.3 - 3)**
18. If for two events A and B, $P(A) = \frac{3}{4}, P(B) = \frac{2}{5}$ and $A \cup B = S$ (sample space), find the conditional probability $P(A/B)$.**(EX 12.3 - 5)**
19. Two thirds of students in a class are boys and rest girls. It is known that the probability of a girl getting a first grade is 0.85 and that of boys is 0.70. Find the probability that a student chosen at random will get first grade marks.**(EX 12.3 - 9)**
20. A year is selected at random. What is the probability that(i) it contains 53 Sundays (ii) it is a leap year which contains 53 Sundays**(EX 12.3 - 11)**

SET 5

21. Suppose the chances of hitting a target by a person X is 3 times in 4 shots, by Y is 4 times in 5 shots, and by Z is 2 times in 3 shots. They fire simultaneously exactly one time. What is the probability that the target is damaged by exactly 2 hits?**(EX 12.3 - 12)**
22. Urn$-I$ contains 8 red and 4 blue balls and urn$-II$ contains 5 red and 10 blue balls. One urn is chosen at random and two balls are drawn from it. Find the probability that both balls are red.**(EG 12.24)**

5 MARKS

SET 1

1. A coin is tossed twice. Events E and F are defined as follows $E =$Head on first toss, $F =$ Head on second toss. Find$(i)P(E \cup F)$
   (ii) $P(E/F)$ (iii) $P(\bar{E}/F)$ (iv) Are the events E and F independent?**(EG 12.19)**
2. X speaks truth in 70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?**(EG 12.22)**
3. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ (i)What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it? **(EX 12.3 - 6)**
4. The probability that a car being filled with petrol will also need an oil change is 0.30, the probability that it needs a new oil filter is 0.40, and the probability that both the oil and filter need changing is 0.15. (i)If the oil had to be changed, what is the probability that a new oil filter is needed? (ii) If a new oil filter is needed, what is the probability that the oil has to be changed? **(EX 12.3 - 7)**
5. A factory has two machines I and II. Machine-I produces40% of items of the output and Machine$-II$ produces 60% of the items. Further 4% of items produced by Machine$-I$ are defective and 5% produced by Machine$-II$ are defective. If an item is drawn at random, find the probability that it is a defective item.**(EG 12.25)**

SET 2

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6. A factory has two machines I and II. Machine I produces 40% of items of the output and Machine II produces 60% of the items. Further 4% of items produced by Machine I are defective and 5% produced by Machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine II. (See the previous example, compare the questions).**(EG 12.26)**

7. A construction company employs 2 executive engineers. Engineer—1 does the work for 60% of jobs of the company. Engineer—2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer—1 does the work is 0.03, whereas the probability of an error in the work of engineer—2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work?**(EG 12.27)**

8. The chances of X, Y and Z becoming managers of a certain company are $4 : 2 : 3$. The probabilities that bonus scheme will be introduced if X, Y and Z become managers are $0.3, 0.5$ and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that Z was appointed as the manager?**(EG 12.28)**

9. A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions (i) what is the probability that the firm will get a car in good condition? (ii) if a car is in good condition, what is probability that it has come from agency N?**(EG 12.29)**

10. A factory has two Machines—I and II. Machine—I produces 60% of items and Machine—II produces 40% of the items of the total output. Further 2% of the items produced by Machine—I are defective whereas 4% produced by Machine—II are defective. If an item is drawn at random what is the probability that it is defective?**(EX 12.4 - 1)**

SET 3

11. There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?**(EX 12.4 - 2)**

12. A firm manufactures PVC pipes in three plants viz, X, Y and Z. The daily production volumes from the three firms X, Y and Z are respectively 2000 units, 3000 units and 5000 units. It is known from the past experience that 3% of the output from plant X, 4% from plant Y and 2% from plant Z are defective. A pipe is selected at random from a day's total production, (i) find the probability that the selected pipe is a defective one. (ii) if the selected pipe is a defective, then what is the probability that it was produced by plant Y ?**(EX 12.4 - 3)**

13. The chances of A, B and C becoming manager of a certain company are $5 : 3 : 2$. The probabilities that the office canteen will be improved if A, B, and C become managers are $0.4, 0.5$ and 0.3 respectively. If the office canteen has been improved, what is the probability that B was appointed as the manager?**(EX 12.4 - 4)**

14. An advertising executive is studying television viewing habits of married men and women during prime time hours. Based on the past viewing records he has determined that during prime time wives are watching television 60% of the time. It has also been determined that when the wife is watching television, 40% of the time the husband is also watching. When the wife is not watching the television, 30% of the time the husband is watching the television. Find the probability that (i) the husband is watching the television during the prime time of television (ii) if the husband is watching the television, the wife is also watching the television.**(EX 12.4 - 5)**

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