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PREFACE

The woods are lovely, dark and deep.
But I have promises to keep, and
miles to go before I sleep

\- Robert Frost

Respected Principals, Correspondents, Head Masters / Head Mistresses, Teachers, and dear Students.

From the bottom of our heart, we at SURA Publications sincerely thank you for the support and patronage that you have extended to us for more than a decade.

It is in our sincerest effort we take the pride of releasing **SURA'S Business Mathematics and Statistics** for +2 Standard. This guide has been authored and edited by qualified teachers having teaching experience for over a decade in their respective subject fields. This Guide has been reviewed by reputed Professors who are currently serving as Head of the Department in esteemed Universities and Colleges.

With due respect to Teachers, I would like to mention that this guide will serve as a teaching companion to qualified teachers. Also, this guide will be an excellent learning companion to students with exhaustive exercises, additional problems and 1 marks as per new model in addition to precise answers for exercise problems.

In complete cognizance of the dedicated role of Teachers, I completely believe that our students will learn the subject effectively with this guide and prove their excellence in Board Examinations.

I once again sincerely thank the Teachers, Parents and Students for supporting and valuing our efforts.

God Bless all.

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CONTENTS

1. Applications of Matrices and Determinants ....1 - 35

2. Integral Calculus I ....36 - 82

3. Integral Calculus II ....83 - 118

4. Differential Equations ....119 - 164

5. Numerical Methods...165 - 194

6. Random Variable and Mathematical expectation .... 195 - 226

7. Probability Distributions .... 227 - 260

8. Sampling techniques and Statistical Inference .... 261 - 280

9. Applied Statistics .... 281 - 320

10. Operations Research... 321 - 362

Instant Supplementary Exam - July 2022 Question Paper with Answers .... 363 - 378

Chapter
1

APPLICATIONS OF MATRICES AND DETERMINANTS

CHAPTER SNAPSHOT

Rank of a matrix :-

The rank of a matrix A is the order of the largest non-zero minor of A and is denoted by ρ (A).

(i) $\rho(A) \geq 0$.

(ii) If A is a matrix of order $m \times n$, then $\rho(A) \leq \min\{m, n\}$.

(iii) Rank of a zero matrix is 0.

(iv) The rank of a non - singular matrix of order $n \times n$ is "n".

Elementary transformations :

(i) Interchange any two rows (or columns)

$$R_i \leftrightarrow R_j \ (C_i \leftrightarrow C_j)$$

(ii) Multiplication of each element of a row (or column) by any non-zero scalar k.

$$R_i \rightarrow k\, R_i \ (\text{or } C_i \rightarrow k\, C_i)$$

(iii) Addition to the elements of any row (or column) the same scalar multiples of corresponding elements of any other row (or column).

$$R_i \rightarrow R_i + k\, R_j \ (\text{or } C_i \rightarrow C_i + k\, C_i)$$

Equivalent matrices:

Two matrices A and B are said to be equivalent if one is obtained from the another by applying a finite number of elementary transformations.

$A \cong b$

Echelon form :

A matrix A of order $m \times n$ is said to be in echelon form if

(i) Every row of A which has all its entries 0 occurs below every row which has a non - zero entry.

(ii) The number of zeros before the first non - zero element in a row is less than the number of such zeros in the next row.

Transition matrix :

The transition probabilities P_{jk} satisfy $P_{jk} > 0$, $\sum_{k} P_{jk} = 1$ for all j

FORMULAE TO REMEMBER

1. Linear equations can be written in matrix form AX = B, then the solution is $X = A^{-1}B$, provided $|A| \neq 0$.
2. Consistency of non homogeneous linear equations by rank method.

(i) If $\rho([A,B]) = \rho(A)$, then the equations are consistent.

(ii) If $\rho([A,B]) = \rho(A) = n$, where n is the number of variables then the equations are consistent and have unique solution.

(iii) If $\rho([A,B]) = \rho(A) < n$, then the equations are consistent and have infinitely many solutions.

(iv) If $\rho([A,B]) \neq \rho(A)$, then the equations are inconsistent and has no solution.

3. Solving non-homogeneous linear equations by Cramer's rule.

If
$$a_1x + b_1y + c_1z = d_1,$$
$$a_2x + b_2y + c_2z = d_2,$$
$$a_3x + b_3y + c_3z = d_3$$

$$\text{Then } \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0,\ \Delta x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$$

$$\Delta y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix},\quad \Delta z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$\text{Then } x = \frac{\Delta x}{\Delta},\quad y = \frac{\Delta y}{\Delta} \text{ and } z = \frac{\Delta z}{\Delta}$$

TEXTUAL QUESTIONS

EXERCISE 1.1

1. Find the rank of each of the following matrices.

(i) $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 1 & -1 \\ 3 & -6 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}$ **[FRT - 2022]** **(iv)** $\begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$

(v) $\begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$ **[July - 2022]**

(vi) $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

(vii) $\begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$

(viii) $\begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$ **[FRT - 2022]**

Sol : (i) Let A = $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

Order of A is 2 × 2

$\therefore \rho(A) \leq 2$ [Since minimum of (2, 2) is 2]

Consider the second order minor,

$$\begin{vmatrix} 5 & 6 \\ 7 & 8 \end{vmatrix} = 40 - 42$$
$$= -2 \neq 0.$$

There is a minor of order 2, which is not zero

$\therefore \rho(A) = 2$

(ii) Let A = $\begin{pmatrix} 1 & -1 \\ 3 & -6 \end{pmatrix}$

Order of A is 2 × 2

$\therefore \rho(A) \leq 2$ [Since minimum of (2, 2) is 2]

Consider the second order minor,

$$\begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix} = -6-(-3) = -6+3 = -3 \neq 0.$$

There is a minor of order 2, which is not zero

$\therefore \rho(A) = 2.$

(iii) Let A = $\begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}$ **[QY-2019]**

Order of A is 2 × 2 [Since minimum of (2,2) is 2]

Consider the second order minor $\begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix}$

$= 8 - 8 = 0.$

Since the second order minor vanishes, $\rho(A) \neq 2$

Consider a first order minor $|1| \neq 0$

There is a minor of order 1, which is not zero

$\therefore \rho(A) = 1.$

(iv) Let A = $\begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$

[PTA - 1; Aug.-2021]

The order of A is 3 × 3

∴ ρ(A) ≤ 3 [Since minimum of (3, 3) is 3]

Let us transform the matrix A to an echelon form

Matrix A	Elementary Transformation
$A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & -5 \\ 2 & -1 & 1 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & -8 \\ 0 & -3 & -1 \end{pmatrix}$	$R_2 \to R_2 - 3R_1$ $R_3 \to R_3 - 2R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -4 \\ 0 & -3 & -1 \end{pmatrix}$	$R_2 \to R_2 \div 2$
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -4 \\ 0 & 0 & 11 \end{pmatrix}$	$R_3 \to R_3 - 3R_2$

This matrix is in echelon from and number of non-zero rows is 3.

$$\therefore \rho(A) = 3.$$

(v) $\text{Let } A = \begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$

The order of A is 3 × 3

∴ ρ(A) ≤ 3 [Since minimum of (3, 3) is 3]

Let us transform the matrix to an echelon form.

Matrix A	Elementary Transformation
$A = \begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & -2 & 2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$	$R_1 \to R_1 (-1)$
$\sim \begin{pmatrix} 1 & -2 & 2 \\ 0 & 5 & -4 \\ -2 & 4 & -4 \end{pmatrix}$	$R_2 \to R_2 - 4R_1$
$\sim \begin{pmatrix} 1 & -2 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 + 2R_1$

The matrix is in echelon form and the number of non-zero rows is 2.

$$\therefore \rho(A) = 2.$$

(vi) $\text{Let } A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

The order of A is 3 × 4

∴ ρ(A) ≤ 3 [Since minimum of (3, 3) is 3]

Let us transform the matrix to an echelon form.

Matrix A	Elementary Transformation
$A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 3 & 6 & 3 & -7 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\sim \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 6 & -16 \end{pmatrix}$	$R_3 \to R_3 - 3R_1$
$\sim \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -18 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - 2R_2$

The matrix is in echelon form and the number of non- zero rows is 2.

$$\therefore \rho(A) = 2.$$

(vii) $A = \begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$

The order of A is 3 × 4

∴ ρ(A) ≤ 3 [Since minimum of (3, 4) is 3]

Let us transform the matrix A to an echelon form.

Matrix A	**Elementary Transformation**
$A = \begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & -5 & -1 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 3 & 1 & -5 & -1 \end{pmatrix}$	$R_2 \to R_2 - R_1$
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & -14 & 16 & -7 \end{pmatrix}$	$R_3 \to R_3 - 3R_1$
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & 0 & 0 & 7 \end{pmatrix}$	$R_3 \to R_3 - 2R_2$

The matrix is in echelon form and the number of non-zero matrix is 3.

$\therefore \rho(A) = 3.$

(viii) $A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$

The order of A is 3×4

$\therefore \rho(A) \leq$ minimum of $(3, 4) \Rightarrow \rho(A) \leq 3$

Let us transform the matrix A to an echelon form.

Matrix A	**Elementary Transformation**
$A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ -1 & 2 & 7 & 6 \end{pmatrix}$	$R_2 \to R_2 + 2R_1$
$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 10 & 10 \end{pmatrix}$	$R_3 \to R_3 + R_1$
$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - 2R_2$

The matrix is in echelon form and the number of non- zero rows is 2.

$\therefore \rho(A) = 2.$

2. If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ **and** $B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$, **then find the rank of AB and the rank of BA.**

Sol :

Given $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2-5 & -2+4-1 & 3-6+1 \\ 2+6+20 & -4-12+4 & 6+18-4 \\ 3+4+15 & -6-8+3 & 9+12-3 \end{pmatrix}$$

$$AB = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$$

Matrix (AB)	**Elementary Transformation**
$AB = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & -6 & -2 \\ -12 & 28 & 20 \\ -11 & 22 & 18 \end{pmatrix}$	$C_1 \leftrightarrow C_2$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ -11 & 22 & 18 \end{pmatrix}$	$R_2 \to R_2 + 12R_1$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & -44 & -4 \end{pmatrix}$	$R_3 \to R_3 + 11R_1$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - R_2$

The matrix is in echelon form and the number of non-zero rows is 2.

$\therefore \rho(AB) = 2.$

$$\text{Now, BA} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1-4+9 & 1+6-6 & -1-8+9 \\ -2+8-18 & -2-12+12 & 2+16-18 \\ 5+2-3 & 5-3+2 & -5+4-3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{pmatrix}$$

Matrix (BA)	Elementary Transformation
$\text{BA} = \begin{pmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 6 & 0 \\ -2 & -12 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	$C_1 \leftrightarrow C_2$
$\sim \begin{pmatrix} 1 & 6 & 0 \\ 0 & 0 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	$R_2 \rightarrow R_2 + 2R_1$
$\sim \begin{pmatrix} 1 & 6 & 0 \\ 0 & 0 & 0 \\ 0 & -20 & -4 \end{pmatrix}$	$R_3 \rightarrow R_3 - 4R_1$

The number of non-zero rows is 2.

$\therefore \rho(\text{BA}) = 2.$

3. **Solve the following system of equations by rank method $x + y + z = 9$, $2x + 5y + 7z = 52$, $2x + y - z = 0$**

Sol : The given equations are $x + y + z = 9$, $2x + 5y + 7z = 52$, $2x + y - z = 0$

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 52 \\ 0 \end{pmatrix}$$

$$\text{A} \quad \text{X} = \text{B}$$

Augmented matrix [AB]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -1 & -3 & -18 \end{pmatrix}$	$R_2 \rightarrow R_2 - 2R_1$ $R_3 \rightarrow R_3 - 2R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{pmatrix}$	$R_3 \rightarrow 3R_3 + R_2$
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -4 \end{pmatrix}$	$\Rightarrow$ P (A) = 3

Since augmented matrix $[A, B] \sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{pmatrix}$ has three non-zero rows, $\rho([A,B]) = 3$.

That is, $\rho(A) = \rho([A,B]) = 3$ = number of unknowns.

So the given system is consistent and has unique solution.

To find the solution, we rewrite the echelon form into the matrix form.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 34 \\ -20 \end{pmatrix}$$

$$\Rightarrow \quad x + y + z = 9 \quad \text{... (1)}$$
$$3y + 5z = 34 \quad \text{...(2)}$$
$$-4z = -20 \quad \text{...(3)}$$

$$(3) \Rightarrow \quad -4z = -20$$
$$z = \frac{-20}{-4} = 5$$
$$(2) \Rightarrow \quad 3y + 5(5) = 34$$
$$\Rightarrow \quad 3y + 25 = 34 \Rightarrow 3y = 34 - 25$$
$$\Rightarrow \quad 3y = 9 \quad \Rightarrow y = \frac{9}{3}$$
$$y = 3.$$
$$(1) \Rightarrow \quad x + 3 + 5 = 9$$
$$\Rightarrow \quad x + 8 = 9 \quad \Rightarrow x = 9 - 8 \Rightarrow x = 1$$

$\therefore x = 1,\ y = 3,\ z = 5$ is the unique solution of the given equations.

4. **Show that the equations $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ are consistent and solve them by rank method.**

Sol : Given non-homogeneous equations are

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 5 \end{pmatrix}$$

$$\text{A} \quad \text{X} = \text{B}$$

$$\text{Augmented matrix [A, B]} = \begin{pmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{pmatrix}$$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	
$\sim \begin{pmatrix} 3 & 26 & 2 & 9 \\ 5 & 3 & 7 & 4 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	$R_1 \leftrightarrow R_2$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 5 & 3 & 7 & 4 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	$R_1 \to R_1 \div 3$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-121}{3} & \frac{11}{3} & -11 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	$R_2 \to R_2 - 5R_1$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-121}{3} & \frac{11}{3} & -11 \\ 0 & \frac{-176}{3} & \frac{16}{3} & -16 \end{pmatrix}$	$R_3 \to R_3 - 7R_1$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \end{pmatrix}$	$R_2 \to R_2 \div 11$ $R_3 \to R_3 \div 16$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - R_2$

Here $\rho(A) = \rho(A, B) = 2 <$ Number of unknowns.

$\therefore$ The system is consistent with infinitely many solutions let us rewrite the above echelon form into matrix form.

$$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} \\ 0 & \frac{-11}{3} & \frac{1}{3} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$$

$$x + \frac{26}{3} y + \frac{2}{3} z = 3 \quad \text{... (1)}$$

$$\frac{-11}{3} y + \frac{1}{3} z = -1 \quad \text{...(2)}$$

let $z = k$; where $k \in$ R

$$(2) \Rightarrow \frac{-11}{3} y + \frac{k}{3} = -1$$

$$\Rightarrow \quad \frac{-11}{\cancel{3}} y = -1 - \frac{k}{3} = \frac{-3-k}{\cancel{3}}$$

$$\Rightarrow \quad -11y = -3 - k$$

$$\Rightarrow \quad 11y = 3 + k$$

$$\Rightarrow \quad y = \frac{1}{11}(3 + k)$$

Substituting $y = \frac{1}{11}(3 + k)$ and $z = k$ in (1) we get,

$$x + \frac{26}{3}\left(\frac{3+k}{11}\right) + \frac{2}{3} k = 3$$

$$x = -\frac{26}{3}\left(\frac{3+k}{11}\right) - \frac{2k}{3} + 3$$

$$= \frac{-78-26k}{33} - \frac{2k}{3} + 3 = \frac{-78-26k-22k+99}{33}$$

$$= \frac{21-48k}{33} = \frac{3(7-16k)}{33} \Rightarrow x = \frac{1}{11}(7 - 16k)$$

$\therefore$ Solution set is $\{\frac{1}{11}(7-16k), \frac{1}{11}(3+k), k\}\ k \in$ R.

Hence, for different values of k, we get infinitely many solutions.

5. **Show that the following system of equations have unique solution: $x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6$ by rank method.** [QY-2019]

Sol : Given non-homogeneous equations are

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

$$\text{A} \quad \text{X} = \text{B}$$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 6 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & 8 & 3 \end{pmatrix}$	$R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 0 \end{pmatrix}$	$R_3 \to R_3 - 3R_2$

Clearly the last equivalent matrix is in echelon form and it has three non-zero rows.

$\therefore$ $\rho(A) = 3$ and $\rho([A, B]) = 3$

$\Rightarrow \rho(A) = \rho([A, B]) = 3 =$ Number of unknowns.

$\therefore$ The given system is consistent and has unique solution.

To find the solution, let us rewrite the above echelon form into the matrix form.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \quad x + y + z &= 3 & \text{... (1)} \\ y + 2z &= 1 & \text{... (2)} \\ 2z &= 0 & \text{... (3)} \end{aligned}$$

$(3) \Rightarrow 2z = 0 \Rightarrow z = \dfrac{0}{2} = 0$

$(2) \Rightarrow y + 2(0) = 1 \Rightarrow y + 0 = 1 \Rightarrow y = 1 - 0 = 1$

$$\begin{aligned} (1) \Rightarrow \quad x + 1 + 0 &= 3 \\ \Rightarrow \quad x + 1 &= 3 \\ \Rightarrow \quad x &= 3 - 1 \\ \Rightarrow \quad x &= 2 \end{aligned}$$

$\therefore$ Solution is {2, 1, 0}

6. For what values of the parameter λ, will the following equations fail to have unique solution: $3x - y + \lambda z = 1$, $2x + y + z = 2$, $x + 2y - \lambda z = -1$ by rank method. **[July - 2022]**

Sol : Given non-homogeneous equations are

$$\begin{aligned} 3x - y + \lambda z &= 1 \\ 2x + y + z &= 2 \\ x + 2y - \lambda z &= -1 \end{aligned}$$

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 3 & -1 & \lambda \\ 2 & 1 & 1 \\ 1 & 2 & -\lambda \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$A \qquad X = B$$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 3 & -1 & \lambda & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 2 & -\lambda & -1 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 2 & 1 & 1 & 2 \\ 3 & -1 & \lambda & 1 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1+2\lambda & 4 \\ 3 & -1 & \lambda & 1 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1+2\lambda & 4 \\ 0 & -7 & 4\lambda & 4 \end{pmatrix}$	$R_3 \to R_3 - 3R_1$
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -1 & \frac{1+2\lambda}{3} & \frac{4}{3} \\ 0 & -1 & \frac{4\lambda}{7} & \frac{4}{7} \end{pmatrix}$	$R_2 \to R_2 \div 3$ $R_3 \to R_3 \div 7$
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -1 & \frac{1+2\lambda}{3} & \frac{4}{3} \\ 0 & 0 & \frac{-7-2\lambda}{21} & \frac{-16}{21} \end{pmatrix}$	$R_3 \to R_3 - R_2$

Since

$$\begin{aligned} &\frac{4\lambda}{7} - \frac{1+2\lambda}{3} \\ &= \frac{12\lambda - 7 - 14\lambda}{21} = \frac{-7-2\lambda}{21} \\ \text{and } &\frac{4}{7} - \frac{4}{3} = \frac{12-28}{21} \\ &= \frac{-16}{21} \end{aligned}$$

Additional Questions & Answers

1 MARK

I. CHOOSE THE CORRECT ANSWER :

1. **If the minor of a_{23} = the co-factor of a_{23} in $|a_{ij}|$ then the minor of a_{23} is.**

(a) 1 (b) 2 (c) 0 (d) 3

[Ans: (c) 0]

2. **If AB = BA = |A| I then the matrix B is the.**

(a) inverse of A (b) Transpose of A
(c) Adjoint of A (d) 2A

[Ans: (a) inverse of A]

3. **If A is a square matrix of order 3, then |adj A| is**

(a) $|A|^2$ (b) $|A|$ (c) $|A|^3$ (d) $|A|^4$

[Ans: (a) $|A|^2$]

4. **If |A| = 0, then |adj A| is.**

(a) 0 (b) 1 (c) –1 (d) ± 1

[Ans: (a) 0]

5. **For what value of k, the matrix A = $\begin{pmatrix} 2 & k \\ 3 & 5 \end{pmatrix}$ has no inverse?**

(a) $\frac{3}{10}$ (b) $\frac{10}{3}$ (c) 3 (d) 10

[Ans: (b) $\frac{10}{3}$]

6. **The rank of an $n \times n$ matrix each of whose elements is 2 is**

(a) 1 (b) 2 (c) n (d) n^2

[Ans: (a) 1]

7. **The value of $\begin{vmatrix} 5^2 & 5^3 & 5^4 \\ 5^3 & 5^4 & 5^5 \\ 5^4 & 5^5 & 5^6 \end{vmatrix}$ is.**

(a) 5^2 (b) 0 (c) 5^{13} (d) 5^9

[Ans: (b) 0]

8. **If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ then x =**

(a) 3 (b) ± 3 (c) ± 6 (d) 6

[Ans: (c) ± 6]

9. **If A is a singular matrix, then Adj A is.**

(a) non-singular (b) singular
(c) symmetric (d) not defined

[Ans: (b) singular]

10. **If A, B are two $n \times n$ non-singular matrices, then.**

(a) AB is non-singular (b) AB is singular
(c) $(AB)^{-1} = A^{-1} B^{-1}$
(d) $(AB)^{-1}$ does not exit

[Ans: (a) AB is non-singular]

II. FILL IN THE BLANKS :

1. **If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ then the value of |adj A| is ______** [Ans: a^6]

2. **For any 2 × 2 matrix, if A (adj A) = $\begin{vmatrix} 10 & 0 \\ 0 & 10 \end{vmatrix}$ then |A| is _______.**

[Ans: 10]

3. **If A is a square matrix of order n, then |Adj A| = _____.** [Ans: $|A|^{n-1}$]

4. **If A is a matrix of order 3 and |A| = 8 then |adj A| = ______.** [Ans: 64]

5. **If A is a square matrix such that A^2 = I, then A^{-1} = _______.** [Ans: A]

6. **The system of equation $x + y + z = 2$, $3x - y + 2z = 6$ and $3x + y - z = -18$ has _______ solution** [Ans: unique]

7. **The number of solutions of the system of equations $2x + y - z = 7$, $x - 3y + 2z = 1$, $x + 4y - 3z = 5$ is ____.** [Ans: 0]

8. **The system of linear equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has a unique solution if k is_______.** [Ans: Not equal to 0]

9. **The value of λ for which the system of equations $x + y + z = 5$, $x + 2y + 3z = 9$, $x + 3y + \lambda z = \mu$ is _____.** [Ans: $\lambda \neq 5$]

10. **A set of values of the variable $x_1, x_2, \ldots x_n$ satisfying all the equations simultaneously is called _______ of the system.** [Ans: Solution]

III. MATCH THE FOLLOWING :

1.	Rank of a matrix	i.	1
2.	If A is a matrix of order $m \times n$, then $\rho(A) \leq$	ii.	non-zero row
3.	Rank of a zero matrix is	iii.	n
4.	Rank of a non-singular matrix of order $n \times n$ is	iv.	unique solution
5.	If A is of rank 2, then adj A is of rank	v.	inconsistent
6.	A row having at least one non-zero element is	vi.	infinitely many solutions
7.	For the system of equations AX = B, the solution is $X = A^{-1} B$ provided	vii.	$\leq \min \{m, n\}$
8.	If $\rho(A, B) = \rho(A) < n$ then the system has	viii.	0
9.	If $\rho(A, B) = \rho(A) = n$, then the system has	ix.	$\|A\| \neq 0$
10.	If $\rho(A, B) \neq \rho(A)$ then the system is	x.	≥ 0

[Ans: 1 - x, 2 - vii, 3 - viii, 4 - iii, 5 - i, 6 - ii, 7 - ix, 8 - vi, 9 - iv, 10 - v]

IV. CHOOSE THE ODD ONE OUT:

1. The system of non-homogeneous equations will have.

(a) unique solution
(b) Infinitely many solutions
(c) No solution (d) Trivial solution

[Ans: (d) Trivial solution]

2. Rank of a 2 × 2 matrix may be

(a) 0 (b) 1 (c) 2 (d) 3

[Ans: (d) 3]

3. The transition probabilities P_{jk} satisfy

(a) $P_{jk} > 0$ (b) $\sum_{k}^{1} P_{jk} = 1$ for all j
(c) $P_{jk} \leq 0$ (d) $P_{jk} > 1$

[Ans: (c) $P_{jk} \leq 0$]

4. If $|A| = 0$, then

(a) A is a singular matrix
(b) System has either no solution or infinitely many solutions
(c) No solution (d) non-singular matrix

[Ans: (d) non-singular matrix]

V. WHICH IS THE FOLLOWING IS NOT CORRECT IN THE GIVEN STATEMENT:

1. (a) $|A| = |A^T|$ where $A = [a_{ij}]_{3 \times 3}$

(b) $|KA| = K^3 |A|$ where $A = [a_{ij}]_{3 \times 3}$

(c) If is a non-singular matrix, then $|A| \neq 0$

(d) $\begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix} = \begin{vmatrix} a & c \\ e & g \end{vmatrix} + \begin{vmatrix} b & d \\ f & h \end{vmatrix}$

[Ans: (d) $\begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix} = \begin{vmatrix} a & c \\ e & g \end{vmatrix} + \begin{vmatrix} b & d \\ f & h \end{vmatrix}$]

2. For the matrix $A = [a_{ij}]_{3 \times 3}$

(a) Order of minor is less than the order of |A|.

(b) Minor of an element can never be equal to co-factor of the same element.

(c) Value of a determinant is obtained by multiplying elements of a row or column by corresponding factors

(d) Order of minor and co-factors of elements of A is same.

[Ans: (b) Minor of an element can never be equal to co-factor of the same element.]

3. If A is an invertible matrix, then which of the following is not true?

(a) $(A^2)^{-1} = (A^{-1})^2$ (b) $|A^{-1}| = |A|^{-1}$
(c) $(A^T)^{-1} = (A^{-1})^T$ (d) $|A| \neq 0$

[Ans: (a) $(A^2)^{-1} = (A^{-1})^2$]

4. (a) If three planes intersect at a point, then the system has unique solution.
(b) If three planes intersect along a line then the system has infinitely many solutions lying on this line.
(c) If two planes intersect at a point then the system has unique solution.
(d) If three planes are parallel and distinct and there is no point in common, then the system has no solutions

[Ans: (c) If two planes intersect at a point then the system has unique solution]

5. Solution of the system of equations $x + 2y = 7$ and $3x + 6y = 21$ is

(a) $x = 5, y = 1$ (b) $x = 3, y = 2$
(c) $x = 0, y = 1$ (d) $x = -3, y = 5$

[Ans: (c) $x = 0, y = 1$]

2 MARKS

1. Solve: $x + 2y = 3$ and $2x + 4y = 6$ using rank method.

Sol : The non-homogeneous equations are

$x + 2y = 3, \qquad 2x + 4y = 6$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$

Here $\rho(A) = 1$ and $\rho([A, B]) = 1$

Since $\rho(A) = \rho([A, B]) = 1 <$ Number of unknowns, the given system is consistent with infinitely many solutions.

To find the solution, let us rewrite the above echelon form into the matrix form, we get

$$\begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \Rightarrow x + 2y = 3 \quad \ldots (1)$$

$$\text{let } y = k, \quad k \in \mathbb{R}$$

$$(1) \Rightarrow \quad x + 2k = 3 \quad \Rightarrow \quad x = 3 - 2k$$

$\therefore$ Solution set is $\{3 - 2k, k\}, k \in \mathbb{R}$.

For different values of k, we get infinite number of solutions.

2. Show that the equations $x + y + z = 6$, $x + 2y + 3z = 14$ and $x + 4y + 7z = 30$ are consistent.

Sol : Given non-homogeneous equations are

$x + y + z = 6, x + 2y + 3z = 14, x + 4y + 7z = 30.$

Augmented matrix	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{pmatrix}$	$R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - 3R_2$

Here $\rho(A) = 2$ and $\rho(A, B) = 2$

$\therefore \quad \rho(A) = \rho(A, B) = 2 <$ Number of unknowns.

$\therefore \quad$ The given system is consistent.

3. If A and B are non-singular matrices, prove that AB is non-singular.

Sol : Since A and B are non-singular, $|A| \neq 0, |B| \neq 0$

Consider $|AB| = |A| \cdot |B| \neq 0$

since $|A| \neq 0$ and $|B| \neq 0. \Rightarrow |AB| \neq 0$

$\therefore$ AB is non-singular.

4. For what value of x, the matrix

$$\mathbf{A} = \begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix} \text{ is singular?}$$

Sol : The matrix A is singular, if $\begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix} = 0$

$$1\begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ x & -3 \end{vmatrix} + 3\begin{vmatrix} 1 & 2 \\ x & 2 \end{vmatrix} = 0$$

$\Rightarrow 1\,(-6-2) + 2\,(-3-x) + 3\,(2-2x) = 0$

$\Rightarrow 1\,(-8) - 6 - 2x + 6 - 6x = 0$

$\Rightarrow \quad -8 - 2x - 6x = 0 \quad \Rightarrow -8 - 8x = 0$

$\Rightarrow \quad -8 = 8x \quad \Rightarrow x = \dfrac{-8}{8} = -1$

5. If $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ find x, y and z

Sol : Given $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x+0+0 \\ 0+y+0 \\ 0+0+z \end{pmatrix}$

$$= \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \Rightarrow x = 1, y = -1, z = 0$$

6. Two newspapers A and B are published in a city. Their market shares are 15% for A and 85% for B of those who bought A the previous year, 65% continue to buy it again while 35% switch over to B. Of those who bought B the previous year, 55% buy it again and 45% switch over to A. Find their market shares after one year.

Sol : Transition probability matrix $T = \begin{array}{c|cc} & A & B \\ \hline A & 0.65 & 0.35 \\ B & 0.45 & 0.55 \end{array}$

Given present market shares are 15% for A and 85% for B

$\therefore$ Market shares after one year

$$= \begin{pmatrix} 0.15 & 0.85 \end{pmatrix}\begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix}$$

$$= \begin{pmatrix} (0.15)(0.65) + (0.85)(0.45) & 0.15 \times 0.35 + 0.85 \times 0.55 \end{pmatrix}$$

$$= \begin{pmatrix} 0.0975 + 0.3825 & 0.0525 + 0.4675 \end{pmatrix} = \begin{pmatrix} 0.48 & 0.52 \end{pmatrix}$$

$\therefore$ Market shares after one year for A is 48% and for B is 52%

Chapter 2

INTEGRAL CALCULUS-I

FORMULAE TO REMEMBER

(i) Integration is the reverse process of differentiation

(ii) $\int k\, f(x)\, dx = k \int f(x)\, dx$ where k is a constant.

(iii) $\int [\, f(x) \pm g\,(x)]\, dx = \int f(x) dx \pm \int g(x)\, dx$

(iv) The following are the four principal methods of integration

- (i) Integration by decomposition
- (ii) Integration by Parts
- (iii) Integration by Substitution
- (iv) Integration by successive reduction

First fundamental theorem of integral calculus :

If $f(x)$ is a continuous function and $F\,(x) = \int_a^x f(t)dt,$ then $F'(x) = f(x)$.

Second fundamental theorem of integral calculus :

$$\int_a^b f(x)\, dx = F\,(b) - F(a)$$

$\int_a^b f(x)\, dx$ is a definite constant, whereas $\int_a^x f(t)\, dt$ is a function of the variable x

Indefinite integral :-

An integral function which is expressed without limits, and so containing an arbitrary constant.

Proper definite integral :-

An integral function which has both the limits. a and b are finite.

Improper definite integral :-

An integral function, in which the limits either a or b or both are infinite.

Gamma function :-

For $n > 0$, $\int_0^\infty x^{n-1}e^{-x}\ dx$ and is denoted by $\Gamma(n)$

1) $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$

2) $\int (ax + b)^n\ dx = \frac{(ax+b)^{n+1}}{a(n+1)}\ + c,\ n \neq -1$

3) $\int \frac{1}{x} dx = \log|x| + c$

4) $\int \frac{1}{ax+b} dx\ = \frac{1}{a}\ \log|ax + b| + c$

5) $\int e^x\ dx\ = e^x + c$

6) $\int e^{ax+b}\ dx\ = \frac{1}{a}\ e^{ax+b} + c$

7) $\int a^x dx = \frac{a^x}{\log a}\ + c,\ a > 0$ and $a \neq 1$

8) $\int a^{mx+n} dx\ = \frac{1}{m \log a}\ a^{mx+n} + c,\ a > 0$ and $a \neq 1$

9) $\int \sin x\ dx\ = -\cos x + c$

10) $\int \sin (ax + b)\ dx\ = -\ \frac{1}{a}\ \cos (ax + b) + c$

11) $\int \cos\ \ dx\ = \sin x + c$

12) $\int \cos (ax + b)\ dx\ = \frac{1}{a}\ \sin (ax + b) + c$

13) $\int \sec^2 x\ dx\ = \tan x + c$

14) $\int \sec^2 (ax + b)\ dx\ = \frac{1}{a}\ \tan (ax + b) + c$

15) $\int \text{cosec}^2\ x\ dx\ = -\cot x + c$

16) $\int \text{cosec}^2 (ax + b)\ dx\ = -\frac{1}{a}\ \cot (ax + b) + c$

17) $\int udv\ = uv - \int vdu$ where u and v are two differentiable functions of x
[Integration by parts].

18) The code word used in the above formula is
I $\rightarrow$ Inverse trigonometric function
L $\rightarrow$ Logarithmic function
A $\rightarrow$ Algebraic function
T $\rightarrow$ Trigonometric function
E $\rightarrow$ Exponential function

19) Bernoulli's formula :
$\int udv = uv - u' v_1 + u'' v_2 - u''' v_3 + ...$
When $u' u'' u'''$.... are the successive derivatives of u and $v_1 v_2 v_3$ .... are the repeated integrals of v.

20) $\int [f(x)]^n\ f'(x)\ dx = \frac{[f(x)]^{n+1}}{n+1} + c,\ n \neq -1$

21) $\int \frac{f'(x)}{f(x)}\ dx\ = \log\ |f(x)|\ + c$

22) $\int \frac{f'(x)}{\sqrt{f(x)}}\ dx\ = 2\ \sqrt{f(x)} + c$

23) $\int e^x\ [f(x) + f'(x)]\ dx = e^x f(x) + c$

24) $\int e^{ax}\ [\ a f(x) + f'(x)]\ dx = e^{ax} f(x) + c$

25) $\int \frac{dx}{a^2 - x^2}\ = \frac{1}{2a}\ \log \left|\frac{a+x}{a-x}\right| + c$

26) $\int \frac{dx}{x^2 - a^2}\ = \frac{1}{2a}\ \log \left|\frac{x-a}{x+a}\right| + c$

27) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log|x + \sqrt{x^2 - a^2}\ | + c$

28) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log|x + \sqrt{x^2 + a^2}\ | + c$

29) $\int \sqrt{x^2 - a^2}\ dx = \frac{x}{2}\ \sqrt{x^2 - a^2}\ - \frac{a^2}{2}$
$\log|x + \sqrt{x^2 - a^2}\ | + c$

30) $\int \sqrt{x^2 + a^2}\ dx = \frac{x}{2}\ \sqrt{x^2 + a^2}\ + \frac{a^2}{2}\ \log$
$|x + \sqrt{x^2 + a^2}\ | + c$

Properties of definite integral

1) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

2) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3) $\int_a^b [f(x)\pm g(x)]\,dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

4) $\int_a^b f(x)\,dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

5) $\int_o^a f(x)dx = \int_o^a f(a-x)dx$

6) $\int_{-a}^{a} f(x)dx = 2\int_o^a f(x)dx$

if $f(x)$ is an even function

7) $\int_{-a}^{a} f(x)dx = 0$ if $f(x)$ is an odd function

8) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

TEXTUAL QUESTIONS

EXERCISE 2.1

Integrate the following with respect to *x*

1. $\sqrt{3x+5}$ **[July - 2022]**

Sol : $\int\sqrt{3x+5}\;dx = \int(3x+5)^{1/2}dx$

$$\left[\because \int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)}+c\right]$$

$$= \frac{(3x+5)^{\frac12+1}}{3\left(\frac12+1\right)}+c$$

$$= \frac{(3x+5)^{3/2}}{3\left(\frac32\right)}+c = \frac{(3x+5)^{3/2}}{\frac92}+c$$

$$= \frac29(3x+5)^{3/2}+c$$

2. $\left(9x^2-\frac{4}{x^2}\right)^2$

Sol : $\int\left(9x^2-\frac{4}{x^2}\right)^2dx$

$$= \int\left[(9x^2)^2-2(9\cancel{x^2})\left(\frac{4}{\cancel{x^2}}\right)+\left(\frac{4}{x^2}\right)^2\right]dx$$

$$[\because (a-b)^2 = a^2-2ab+b^2]$$

$$= \int\left(81x^4-72+\frac{16}{x^4}\right)dx$$

$$= \int 81x^4\,dx - \int 72dx + \int\frac{16}{x^4}\,dx + c$$

$$= 81\,\frac{x^{4+1}}{4+1} - 72x + 16\,\frac{x^{-4+1}}{-4+1} + c$$

$$\left[\because \frac{16}{x^4} = 16x^{-4}\right]$$

$$= 81\,\frac{x^5}{5} - 72x + 16\,\frac{x^{-3}}{-3} + c$$

$$= \frac{81}{5}x^5 - 72x - \frac{16}{3x^3} + c$$

3. **(3 + *x*) (2 – 5*x*)** **[Aug. - 2021]**

Sol : $\int(3+x)\;(2-5x)\;dx$

$$= \int(6-15x+2x-5x^2)\,dx$$

$$= \int(6-13x-5x^2)\,dx$$

$$= \int 6dx - \int 13x\,dx - \int 5x^2\,dx$$

$$= 6x - \frac{13x^2}{2} - \frac{5x^3}{3} + c$$

4. $\sqrt{x}\;(x^3-2x+3)$

Sol : $\int\sqrt{x}\;(x^3-2x+3)\,dx$

$$= \int x^{\frac12}\,(x^3-2x+3)\,dx$$

$$= \int\left(x^{3+\frac12} - 2x^{1+\frac12} + 3x^{\frac12}\right)dx$$

$$= \int x^{\frac72}dx - \int 2x^{\frac32}dx + \int 3x^{\frac12}dx$$

$$= \frac{x^{\frac72+1}}{\frac72+1} - \frac{2x^{\frac32+1}}{\frac32+1} + \frac{3x^{\frac12+1}}{\frac12+1} + c$$

$$= \frac{x^{\frac{9}{2}}}{\frac{9}{2}} - 2\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{9}x^{\frac{9}{2}} - 2\times\frac{2}{5}x^{\frac{5}{2}} + \cancel{3}\times\frac{2}{\cancel{3}}x^{\frac{3}{2}} + c$$

$$= \frac{2}{9}x^{\frac{9}{2}} - \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$$

5. $\dfrac{\mathbf{8x+13}}{\sqrt{\mathbf{4x+7}}}$

Sol : $\displaystyle\int\frac{8x+13}{\sqrt{4x+7}}dx$

$$= \int\frac{8x+14-1}{\sqrt{4x+7}}dx = \int\frac{2(4x+7)-1}{\sqrt{4x+7}}dx$$

$$= 2\int\frac{(4x+7)}{\sqrt{4x+7}}dx - \int\frac{1}{\sqrt{4x+7}}dx$$

$$= 2\int\sqrt{4x+7}\,dx - \int\frac{1}{\sqrt{4x+7}}dx$$

$$= 2\int(4x+7)^{\frac{1}{2}}dx - \int(4x+7)^{-\frac{1}{2}}dx$$

$$= 2\frac{(4x+7)^{\frac{1}{2}+1}}{4\left(\frac{1}{2}+1\right)} - \frac{(4x+7)^{-\frac{1}{2}+1}}{4\left(\frac{-1}{2}+1\right)} + c$$

$$= 2\frac{(4x+7)^{\frac{3}{2}}}{4\left(\frac{3}{2}\right)} - \frac{(4x+7)^{\frac{1}{2}}}{4\left(\frac{1}{2}\right)} + c$$

$$= \cancel{2}\frac{(4x+7)^{\frac{3}{2}}}{\cancel{6}_{3}} - \frac{(4x+7)^{\frac{1}{2}}}{2} + c$$

$$= \frac{(4x+7)^{\frac{3}{2}}}{3} - \frac{(4x+7)^{\frac{1}{2}}}{2} + c$$

6. $\dfrac{\mathbf{1}}{\sqrt{\mathbf{x+1}}+\sqrt{\mathbf{x-1}}}$

Sol : $\displaystyle\int\frac{1}{\sqrt{x+1}+\sqrt{x-1}}dx$

Multiplying and dividing the conjugate of the denominator we get

$$= \int\frac{\sqrt{x+1}-\sqrt{x-1}\,dx}{\left(\sqrt{x+1}+\sqrt{x-1}\right)\left(\sqrt{x+1}-\sqrt{x-1}\right)}$$

$$= \int\frac{\sqrt{x+1}-\sqrt{x-1}}{(x+1)-(x-1)}dx$$

[∵ $(a+b)(a-b) = a^2-b^2$]

$$= \int\frac{\sqrt{x+1}-\sqrt{x-1}}{\cancel{x}+1-\cancel{x}+1}dx = \int\frac{\sqrt{x+1}-\sqrt{x-1}}{2}dx$$

$$= \frac{1}{2}\int\left((x+1)^{½}-(x-1)^{½}\right)dx$$

$$= \frac{1}{2}\left[\frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$$

$$= \frac{1}{\cancel{2}\times\frac{3}{\cancel{2}}}\left[(x+1)^{\frac{3}{2}} - (x-1)^{\frac{3}{2}}\right] + c$$

$$= \frac{1}{3}\left[(x+1)^{\frac{3}{2}} - (x-1)^{\frac{3}{2}}\right] + c$$

7. If $f'(x) = x + b$, $f(1) = 5$ and $f(2) = 13$, then find $f(x)$

Sol : Given $f'(x) = x + b$, $f(1) = 5$ and $f(2) = 13$

$f'(x) = x + b \Rightarrow \int f'(x)\,dx = \int (x+b)\,dx$

[∴ Integration is the reverse process of differentiation]

$\Rightarrow f(0) = \frac{x^2}{2} + bx + c$ ...(1)

Given $f(1) = 5 \Rightarrow 5 = \frac{1^2}{2} + b(1) + c$

$\Rightarrow 5 = \frac{1}{2} + b(1) + c \Rightarrow 5 - \frac{1}{2} = b + c$

$\Rightarrow \frac{10-1}{2} = b + c \Rightarrow b + c = \frac{9}{2}$

$\Rightarrow 2b + 2c = 9$ ...(2)

Also $f(2) = 13 \Rightarrow 13 = \frac{2^2}{2} + b(2) + c$

$\Rightarrow 13 = 2 + 2b + c$

$\Rightarrow 13 - 2 = 2b + c$

$\Rightarrow 2b + c = 11$ ...(3)

$(2) - (3) \rightarrow 2b + 2c = 9$

$-2b + -c = -11$ $\boxed{c = -2}$

Substituting $c = -2$ in (3) we get

$2b - 2 = 11 \Rightarrow 2b = 11 + 2 \Rightarrow 2b = 13$

$\Rightarrow \boxed{b = \frac{13}{2}}$

Substituting $b = \frac{13}{2}$, $c = -2$ in (1) we get,

$$f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2$$

8. **If $f'(x) = 8x^3 - 2x$ and $f(2)=8$, then find $f(x)$.**

Sol : Given $f'(x) = 8x^3 - 2x,\ f(2) = 8$ **[May - 2022]**

$$f'(x) = 8x^3 - 2x$$

$$\Rightarrow \int f'(x)\,dx = \int (8x^3 - 2x)\,dx$$

$$\Rightarrow f(x) = \frac{\overset{2}{\cancel{8}}x^4}{\cancel{4}} - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\Rightarrow f(x) = 2x^4 - x^2 + c \qquad ...(1)$$

$$\text{Given } f(2) = 8$$

$$\Rightarrow 8 = 2(2^4) - 2^2 + c$$

$$\Rightarrow 8 = 32 - 4 + c \Rightarrow 8 = 32 - 4 + c$$

$$\Rightarrow 8 - 28 = c$$

$$\Rightarrow c = -20$$

Substituting $c = -20$ in (1) we get.

$$f(x) = 2x^4 - x^2 - 20$$

EXERCISE 2.2

Integrate the following with respect to x.

1. $\left(\sqrt{2x} - \frac{1}{\sqrt{2x}}\right)^2$

Sol : $\int\left(\sqrt{2x} - \frac{1}{\sqrt{2x}}\right)^2 dx$

$$= \int\left[\left(\sqrt{2x}\right)^2 - 2\left(\sqrt{\cancel{2x}}\right)\left(\frac{1}{\sqrt{\cancel{2x}}}\right) + \left(\frac{1}{\sqrt{2x}}\right)^2\right] dx$$

$[\because (a-b)^2 = a^2 - 2ab + b^2]$

$$= \int\left(2x - 2 + \frac{1}{2x}\right) dx$$

$$= \cancel{2}\frac{x^2}{\cancel{2}} - 2x + \frac{1}{2}\log|x| + c = x^2 - 2x + \frac{1}{2}\log|x| + c$$

2. $\frac{x^4 - x^2 + 2}{x - 1}$

Sol : $\int\frac{x^4 - x^2 + 2}{x - 1}\,dx = \int\left(x^3 + x^2 + \frac{2}{x - 1}\right) dx$

$$= \frac{x^4}{4} + \frac{x^3}{3} + 2\log|x - 1| + c$$

3. $\frac{x^3}{x + 2}$

Sol : $\int\frac{x^3}{x + 2}\,dx = \int\left(x^2 - 2x + 4 - \frac{8}{x + 2}\right) dx$

$$= \frac{x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}} + 4x - 8\log|x + 2| + c$$

$$= \frac{x^3}{3} - x^2 + 4x - 8\log|x + 2| + c$$

$\left[\because \int\frac{1}{x}dx = \log|x| + c\right]$

$$\begin{array}{r|l} & x^2 - 2x + 4 \\ \hline x+2 & x^3 \\ & {}^{(-)}x^3 + {}^{(-)}2x^2 \\ \hline & -2x^2 \\ & {}^{(+)}_{-}2x^2 \, {}^{(+)}_{-}4x \\ \hline & 4x \\ & {}^{(-)}4x + {}^{(-)}8 \\ \hline & -8 \\ \hline \end{array}$$

4. $\frac{x^3 + 3x^2 - 7x + 11}{x + 5}$

Sol : $\int\frac{x^3 + 3x^2 - 7x + 11}{x + 5}\,dx$

$$= \int\left(x^2 - 2x + 3 - \frac{4}{x + 5}\right) dx$$

$$= \frac{x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}} + 3x - 4\log|x + 5| + c$$

$$= \frac{x^3}{3} - x^2 + 3x - 4\log|x + 5| + c$$

$$\begin{array}{r|l} & x^2 - 2x + 3 \\ \hline x+5 & x^3 + 3x^2 - 7x + 11 \\ & {}^{(-)}x^3 + {}^{(-)}5x^2 \\ \hline & -2x^2 - 7x \\ & 2x^2 + 10x \\ & (-) \quad (-) \\ \hline & 3x + 11 \\ & 3x + 15 \\ & (-) \quad (-) \\ \hline & -4 \\ \hline \end{array}$$

5. $\frac{3x + 2}{(x - 2)(x - 3)}$ **[Qy - 2019]**

Sol : $\int\frac{(3x + 2)\,dx}{(x - 2)(x - 3)} = \int\left(\frac{-8}{x - 2} + \frac{11}{x - 3}\right) dx$

$$= -8\log|x - 2| + 11\log|x - 3| + c$$

$$= 11\log|x - 3| - 8\log|x - 2| + c$$

$$\frac{3x + 2}{(x - 2)(x - 3)} = \frac{A}{x - 2} + \frac{B}{x - 3}$$

$$\Rightarrow 3x + 2 = A(x - 3) + B(x - 2)$$

Put $x = 3$

$9 + 2 = B(1) \Rightarrow B = 11$

Put $x = 2$

$8 = A(-1) \Rightarrow A = -8$

$$\frac{3x + 2}{(x - 2)(x - 3)} = \frac{-8}{x - 2} + \frac{11}{x - 3}$$

6. $\dfrac{4x^2+2x+6}{(x+1)^2(x-3)}$ **[HY - 2019]**

Sol : $\displaystyle\int\frac{4x^2+2x+6}{(x+1)^2(x-3)}\,dx$

$$= \int\left(\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-3}\right)dx$$

$$= \left(\frac{1}{x+1}+\frac{-2}{(x+1)^2}+\frac{3}{x-3}\right)dx$$

$$= \log|x+1| - 2\int(x+1)^{-2}\,dx + 3\log|x-3| + c$$

$$= \log|x+1| - 2\,\frac{(x+1)^{-2+1}}{-2+1} + 3\log|x-3| + c$$

$$= \log|x+1| + 2(x+1)^{-1} + 3\log|x-3| + c$$

$$= \log|x+1| + \frac{2}{x+1} + 3\log|x-3| + c$$

$$\frac{4x^2+2x+6}{(x+1)^2(x-3)} = \frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-3}$$

$4x^2+2x+6 = A(x+1)(x-3) + B(x-3) + C(x+1)^2$

$\Rightarrow$ Putting $x = -1$, $4-2+6 = B(-4)$

$\Rightarrow 8 = B(-4) \Rightarrow B = -2$

Putting $x = 3$

$36+6+6 = C(16)$

$\Rightarrow 48 = 16C \Rightarrow C = 3$

Putting $x = 0$,

$6 = -3A - 3B + C$

$\Rightarrow 6 = -3A + 6 + 3$

$\Rightarrow 3A = 3 \Rightarrow A = 1$

$$\begin{array}{r|l} & x^3+x^2 \\ \hline x-1 & x^4-x^2+2 \\ & \overset{(-)}{}\quad\overset{(+)}{} \\ & x^4-x^3 \\ \hline & x^3-x^2 \\ & \overset{(-)}{}\quad\overset{(+)}{} \\ & x^3-x^2 \\ \hline & 2 \\ \hline \end{array}$$

7. $\dfrac{3x^2-2x+5}{(x-1)(x^2+5)}$ **[Sep. - 2020; Aug. - 2021]**

Sol : $\displaystyle\int\frac{3x^2-2x+5}{(x-1)(x^2+5)}\,dx = \int\left(\frac{A}{x-1}+\frac{Bx+c}{x^2+5}\right)dx$

$$= \int\left(\frac{1}{x-1}+\frac{2x+0}{x^2+5}\right)dx = \int\frac{1}{x-1}\,dx + \int\frac{2x}{x^2+5}\,dx$$

$$= \log|x-1| + \log|x^2+5| + c$$

$$\left[\because \int\frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c\right]$$

$$= \log|(x^2+5)(x-1)| + c$$

$$[\because \log m + \log n = \log mn]$$

$$= \log|x^3-x^2+5x-5| + c$$

$$\frac{3x^2-2x+5}{(x-1)(x^2+5)} = \frac{A}{(x-1)}+\frac{Bx+c}{(x^2+5)}$$

$\Rightarrow 3x^2-2x+5 = A(x^2+5) + (Bx+C)(x-1)$

Putting $x = 1$,

$3-2+5 = A(1+5)$

$\Rightarrow 6 = A(6) \Rightarrow \boxed{A=1}$

Putting $x = 0$,

$5 = 5A - C$

$\Rightarrow 5 = 5 - C$ $[\because A = 1]$

$\Rightarrow C = 5-5 \Rightarrow \boxed{C=0}$

Putting $x = -1$,

$3+2+5 = A(6) + (C-B)(-2)$

$\Rightarrow 10 = 6A + 2B - 2C$

$\Rightarrow 10 = 6 + 2B + 0$

$\Rightarrow 10 - 6 = 2B \Rightarrow 4 = 2B$

$\Rightarrow \boxed{B=2}$

8. If $f'(x) = \dfrac{1}{x}$ and $f(1) = \dfrac{\pi}{4}$, then find $f(x)$.

Sol : Given $f'(x) = \dfrac{1}{x}$ **[PTA - 2]**

$$\Rightarrow \int f'(x)\,dx = \int\frac{1}{x}\,dx$$

$$\Rightarrow f(x) = \log|x| + c \qquad \text{...(1)}$$

Also, $f(1) = \dfrac{\pi}{4}$, we get

$$\Rightarrow \frac{\pi}{4} = \log|1| + c$$

$$\Rightarrow \frac{\pi}{4} = c \qquad [\because \log 1 = 0]$$

Substituting $c = \dfrac{\pi}{4}$ in (1) we get,

$$f(x) = \log|x| + \frac{\pi}{4}$$

EXERCISE 2.3

Integrate the following with respect to x

1. $e^{x\log a} + e^{a\log a} - e^{n\log x}$

Sol : $\displaystyle\int(e^{x\log a} + e^{a\log a} - e^{n\log x})\,dx$

$$= \int\left(e^{\log a^x} + e^{\log a^a} - e^{\log x^n}\right)dx$$

$$[\because m\log n = \log n^m]$$

$$= \int(a^x + a^a - x^n)\,dx \qquad [\because e^{\log x} = x]$$

$$= \int a^x\,dx + \int a^a\,dx - \int x^n dx$$

$$= \left[\frac{a^x}{\log a}\right] + a^a\,(x) - \frac{x^{n+1}}{n+1} + c$$

$$[\because \int a^x = \frac{a^x}{\log a}\ ]$$

2. $\boldsymbol{\dfrac{a^x - e^{x\log b}}{e^{x\log a}\ b^x}}$

Sol : $\displaystyle\int \frac{a^x - e^{x\log b}}{e^{x\log a}\ b^x}\,dx$

$$= \int \frac{a^x - e^{\log b^x}}{e^{\log a^x}\ b^x}\,dx \qquad [\because m\log n = \log n^m]$$

$$= \int \frac{a^x - b^x}{a^x\,.\,b^x}\,dx \qquad [\because e^{\log x} = x]$$

$$= \int \frac{a^x}{a^x b^x}\,dx - \int \frac{b^x}{a^x b^x}\,dx$$

$$= \int \frac{1}{b^x}\,dx - \int \frac{1}{a^x}\,dx$$

$$= \int b^{-x}\,dx - \int a^{-x}\,dx \quad [\because \int a^{-x}\,dx = \frac{a^{-x}}{-\log a} + c]$$

$$= \frac{b^{-x}}{-\log b} - \frac{a^{-x}}{-\log a} + c$$

$$= -\frac{b^{-x}}{\log b} + \frac{a^{-x}}{\log a} + c$$

$$= \frac{-1}{b^x \log b} + \frac{1}{\log a\,.\,a^x} + c$$

$$= \frac{1}{a^x \log a} - \frac{1}{b^x \log b} + c$$

3. $\boldsymbol{(e^x + 1)^2\,e^x}$

Sol : $\int (e^x+1)^2\,e^x\,dx = \int [(e^x)^2 + 2\,(e^x)\,(1) + 1^2]\,e^x\,dx$

$$[\because (a+b)^2 = a^2 + 2ab + b^2]$$

$$= \int (e^{2x} + 2e^x + 1)\,e^x\,dx$$

$$= \int (e^{3x} + 2e^{2x} + e^x)\,dx \qquad [a^m\,.\,a^n = a^{m+n}]$$

$$= \int e^{3x}\,dx + 2\int e^{3x}\,dx + \int e^x\,dx$$

So $\int (e^x + 1)^2\,e^x\,dx = \int e^{3x}\,dx + \int 2e^{2x}\,dx + \int e^x\,dx$

$$= \frac{e^{3x}}{3} + \cancel{2}\frac{e^{2x}}{\cancel{2}} + e^x + c = e^x + e^{2x} + \frac{e^{3x}}{3} + c$$

4. $\boldsymbol{\dfrac{e^{3x} - e^{-3x}}{e^x}}$ **[Aug. - 2021]**

Sol : $\displaystyle\int \frac{e^{3x} - e^{-3x}}{e^x}\,dx = \int \frac{e^{3x}}{e^x}\,dx - \int \frac{e^{-3x}}{e^x}\,dx$

$$= \int e^{3x-x}\,dx - \int e^{-3x-x}\,dx \qquad [\because \frac{a^m}{a^m} = a^{m-n}]$$

$$= \int e^{2x}\,dx - \int e^{-4x}\,dx$$

$$= \frac{e^{2x}}{2} - \frac{e^{-4x}}{-4} + c = \frac{e^{2x}}{2} + \frac{e^{-4x}}{4} + c$$

5. $\boldsymbol{\dfrac{e^{3x} + e^{5x}}{e^x + e^{-x}}}$

Sol : $\displaystyle\int \frac{e^{3x} + e^{5x}}{e^x + e^{-x}}\,dx = \int \frac{e^{4x-x} + e^{4x+x}}{e^x + e^{-x}}$

$$= \int \frac{e^{4x}\,\cancel{(e^{-x} + e^x)}}{\cancel{e^x + e - x}}\,dx$$

So $\displaystyle\int \frac{e^{3x} + e^{5x}}{e^x + e^{-x}} = \int e^{4x}\,dx = \frac{e^{4x}}{4} + c$

6. $\boldsymbol{\left(1 - \dfrac{1}{x^2}\right) e^{\left(x + \frac{1}{x}\right)}}$

Sol : Let I $= \displaystyle\int \left(1 - \frac{1}{x^2}\right).\,e^{\left(x + \frac{1}{x}\right)}\,dx$

put $x + \dfrac{1}{x} = t$

on differentiating we get, $\left(1 - \dfrac{1}{x^2}\right) dx = dt$

$$\therefore \quad \text{I} = \int e^t\,dt = e^t + c$$

$$= e^{x + \frac{1}{x}} + c \qquad [\because t = x + \frac{1}{x}]$$

7. $\boldsymbol{\dfrac{1}{x(\log x)^2}}$

Sol : Let I $= \displaystyle\int \frac{1}{x(\log x)^2}\,dx$

put $\log x = t$ on differentiating we get,

$$\frac{1}{x}\,dx = dt \quad \therefore \text{I} = \int \frac{1}{t^2}\,dt \qquad [\because t = \log x]$$

$$\text{I} = \int t^{-2}\,dt = \frac{t^{-2+1}}{-2+1} + c$$

$$= \frac{t^{-1}}{-1} + c \Rightarrow \frac{-1}{t} + c$$

$$= \frac{-1}{\log|x|} + c \qquad [\because t = \log|x|]$$

8. If $f'(x) = e^x$ and $f(0) = 2$, then find $f(x)$

Sol : Given $f'(x) = e^x$

$$\Rightarrow \quad \int f'(x)\,dx = \int e^x\,dx$$

[Taking integration both sides]

PTA Questions & Answers

1 MARK

1. $\Gamma(n) =$ **[PTA - 1]**

(a) $n\,\Gamma(n)$ (b) $n\,\Gamma(n),\ n>0$

(c) $(n-1)\,\Gamma(n-1),\ n>0$

(d) $(n-1)\,\Gamma(n-1),\ n>1$

[*Ans.* (d) $(n-1)\,\Gamma(n-1),\ n>1$]

2. **Which of the following is not equal to $\int \tan x \sec^2 x\, dx$** **[PTA - 2]**

(a) $\frac{1}{2}\tan^2 x$ (b) $\frac{1}{2}\sec^2 x$

(c) $\frac{1}{2\cos^2 x}$ (d) none of these

[*Ans.* (a) $\frac{1}{2}\tan^2 x$]

2 MARKS

1. **Evaluate:** $\int_0^{\infty} x^2 e^{-x^3} dx$ **[PTA - 2]**

Sol : $\int_0^{\infty} x^2 e^{-x^3} dx = \int_0^{\infty} e^{-t}\frac{dt}{3}$

$$= \frac{1}{3}\left[-e^{-t}\right]_0^{\infty} = \frac{-1}{3}[0-1] = \frac{1}{3}$$

2. **Evaluate:** $\int_0^{\infty} e^{-x^2} dx$ **[PTA - 3]**

Sol : $[\Gamma(n) = \int_0^{\infty} t^{n-1}e^{-t}dt\,]$

$$\text{Put } t = x^2 \Rightarrow dt = 2xdx$$

$$\Gamma(n) = \int_0^{\infty} e^{-x^2}\left(x^2\right)^{n-1} 2xdx$$

$$= \int_0^{\infty} e^{-x^2} x^{2n-2} 2x\,dx$$

$$= 2\int_0^{\infty} e^{-x^2} x^{2n-1}\,dx$$

Put $n = \frac{1}{2}$, we have

$$\Gamma\left(\frac{1}{2}\right) = 2\int_0^{\infty} e^{-x^2} dx \Rightarrow \sqrt{\pi} = 2\int_0^{\infty} e^{-x^2} dx$$

$$\therefore \int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

3. **State any two properties of Gamma function.**

Sol : 1. $\Gamma(n) = (n-1)\Gamma(n-1), n>1$ **[PTA - 4]**

2. $\Gamma(n+1) = n\Gamma(n), n>0$

4. **Evaluate:** $\int \cos^3 x dx$ **[PTA - 5]**

Sol : $\int \cos 3x\; dx = \frac{1}{4}\int \cos 3x\; dx + \frac{3}{4}\int \cos 3x\; dx$

$$= \frac{\sin 3x}{12} + \frac{3\sin x}{4}$$

5. **Evaluate :** $\int \frac{x}{\sqrt{x^2+1}} dx$ **[PTA - 6]**

Sol : $\int \frac{x}{\sqrt{x^2+1}} dx = \frac{1}{2}\int \frac{2x}{\sqrt{x^2+1}} dx$

$$= \frac{1}{2}\int \frac{f'(x)}{\sqrt{f(x)}} dx = \frac{1}{2}\left[2\sqrt{f(x)}\right] + c$$

$$= \sqrt{x^2+1} + c$$

3 MARKS

1. **Evaluate:** $\int_0^{\frac{\pi}{2}} x \sin x\, dx$ **[PTA - 4]**

Sol : $\int_0^{\frac{\pi}{2}} x\sin x\, dx = \int_0^{\frac{\pi}{2}} udv$

$$= [uv]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} vdu = [-x\cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x\, dx$$

$$= 0 + [\sin x]_0^{\frac{\pi}{2}} = 1$$

5 MARKS

1. **Evaluate :** $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\frac{1}{\left(1+\sqrt{(\cot x)}\right)}\right) dx$ **[PTA - 4]**

Sol : $$\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\cot x}} dx$$

$$\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\frac{\sqrt{\cos x}}{\sqrt{\sin x}}} dx$$

$$\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \qquad \text{...(1)}$$

$$\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin\left(\frac{\pi}{6}+\frac{3}{\pi}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}+\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}\,dx$$

$$\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\,dx$$

$$\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \qquad \text{...(2)}$$

(1) + (2) ⇒ 2I

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left[\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right]dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$$

$$= [x]_{\frac{\pi}{3}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{2\pi-\pi}{6} = \frac{\pi}{6}$$

$$\Rightarrow 2\text{I} = \frac{\pi}{6} \Rightarrow \text{I} = \frac{\pi}{12}$$

2. Evaluate the integral as the limit of a sum: **[PTA - 6]**

$$\int_1^2 (2x+1)\,dx$$

Sol : $$\int_a^b (2x+1)\,dx = \lim_{\substack{n\to\infty \\ h\to 0}} \sum_{r=1}^{n} h\, f(a+rh)$$

Here $a = 1,\ b = 2,\ h = \frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$ and

$f(x) = 2x + 1$

$$f(a+rh) = f\left(1+\frac{r}{n}\right) = 2\left(1+\frac{r}{n}\right)+1 = 2+\frac{2r}{n}+1$$

$$f(a + rh) = 3 + \frac{2r}{n}$$

$$\int_1^2 f(x)dx = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(3+\frac{2r}{n}\right)$$

$$= \mathop{lt}_{n\to\infty}\sum_{r=1}^{n}\left(\frac{3}{n}+\frac{2r}{n}\right) = \lim_{n\to\infty}\left[\frac{3}{n}\sum_{r=1}^{n}1+\frac{2}{n^2}\sum_{r=1}^{n}r\right]$$

$$= \lim_{n\to\infty}\left[\frac{3}{n}\cdot n+\frac{2}{n^2}\frac{n(n+1)}{2}\right] = 3+\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$$

$$\int_1^2 f(x)dx = 3 + 1 = 4$$

3. Evaluate : (i) $\displaystyle\int_0^\infty \frac{e^{\tan-1_x}}{1+x^2}\,dx$ **[PTA - 6]**

(ii) $\displaystyle\int \frac{x+2}{\left(x^2+4x-5\right)^4}\,dx$

Sol : (i) $\displaystyle\int_0^\infty \frac{e^{\tan-1_x}}{1+x^2}\,dx$

Put $t = \tan^{-1} x$

$$\frac{dt}{dx} = \frac{1}{1+x^2}$$

$$dt = \frac{1}{1+x^2}\,dx$$

$x = 0\ ;\ t = 0$

$x = \infty\ ;\ t = \frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} e^t\,dt = \left[e^t\right]_0^{\frac{\pi}{2}} = e^{\frac{\pi}{2}} - e^0 = e^{\frac{\pi}{2}} - 1$$

(ii) $$\int \frac{x+2}{\left(x^2+4x-5\right)^4}\,dx$$

$t = x^2 + 4x - 5$

$$\frac{dt}{dx} = 2x + 4 = 2(x + 2)$$

$$\frac{dt}{2} = (x + 2)\,dx$$

$$\int \frac{dt}{2t^4} = \frac{1}{2}\int t^{-4}dt = \frac{1}{2}\left[\frac{t^{-3}}{-3}\right]$$

$$= -\frac{1}{6}\left[\frac{1}{t^3}\right] = \frac{-1}{6\left(x^2+4x-5\right)^3}$$

Chapter
3

INTEGRAL CALCULUS-II

SNAPSHOT

Geometrical interpretation of definite integral is the area under a curve between the given limits.
Integration helps us to find out the total cost function and total revenue function from the marginal cost.
Consumer's surplus & producer's surplus theory was developed by the economist Marshal.

FORMULAE TO REMEMBER

1. Area of the region bounded by $y = f(x)$ between the limits $x = a$, $x = b$ and less below x axis is

$$A = \int_a^b -y dx = -\int_a^b f(x)dx$$

2. Area of the region bounded by $x = f(y)$, between the limits $y = c$, $y = d$ with Y - axis and the area lies to the right of Y-axis is

$$A = \int_c^d x dy = \int_c^d f(y)dy$$

3. Area bounded by $x = f(y)$ between the limits $y = c$, $y = d$ with Y-axis and the area lies to the left of Y-axis is

$$A = \int_c^d -x dy = -\int_c^d f(y)dy$$

4. Area between two curves from $x = a$ to $x = b$ is

$$A = \int_a^b [f(x) - g(x)]dx$$

5. If c is the cost function, marginal cost function $MC = \frac{dC}{dx}$

6. $C = \int (MC)dx + k$

7. Average cost function $AC = \frac{C}{x}, x \neq 0$

8. If R is the total revenue function, marginal revenue function $MR = \frac{dR}{dx}$

9. $R = \int (MR)dx + k$

10. Demand function $P = \frac{R}{x}, x \neq 0$

11. If 'P' denotes the profit function, then $P = \int (MR - MC)dx + k$

12. Total inventory carrying cost $= C_1 \int_0^T I(x)dx$,where
C1 → holding cost
T → time period
I(x) → Inventory on hand.

13. Amount of annuity after N payments $A = \int_0^N pe^{rt} dt$

14. Elasticity of demand $\eta_d = \frac{-p}{x}\frac{dx}{dp}$

15. $\frac{Ey}{Ex} = \frac{x}{y}\frac{dy}{dx}$

16. Consumer's surplus $CS = \int_0^{x_0} f(x)dx - x_0 p_0$
where $f(x)$ is the demand function.

17. Producer's surplus $PS = x_0 p_0 - \int_0^{x_0} g(x)dx$
where $g(x)$ is the supply function.

TEXTUAL QUESTIONS

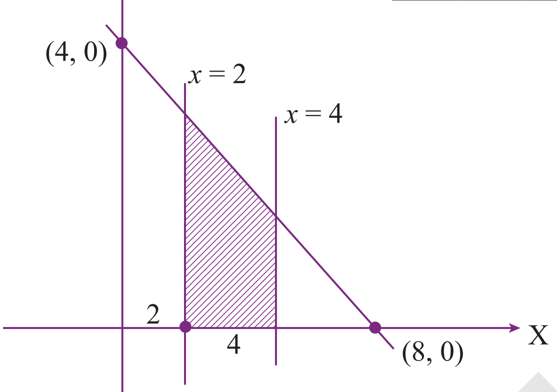
EXERCISE 3.1

1. Using Integration, find the area of the region bounded the line $2y + x = 8$, the x axis and the lines $x = 2, x = 4$. [PTA - 6]

Sol:

$2y + x = 8$

x	0	8
y	4	0



$$\text{Given} \quad 2y + x = 8$$
$$2y = 8 - x$$
$$y = \frac{1}{2}(8 - x)$$

Given limits are $x = 2$ and $x = 4$

Area of the shaded region between the given limits

$$A = \int_a^b y\,dx = \int_2^4 \frac{1}{2}(8-x)dx$$

$$\frac{1}{2}\int_2^4 (8-x)dx = \frac{1}{2}\left[8x - \frac{x^2}{x}\right]_2^4$$

$$= \frac{1}{2}\left[\left(8(4) - \frac{4^2}{2}\right) - \left(8(2) - \frac{2^2}{2}\right)\right]$$

$$= \frac{1}{2}[(32-8)-(16-21)]$$

$$= \frac{1}{2}[24-14]$$

$$= \frac{y-4}{2}(10)$$

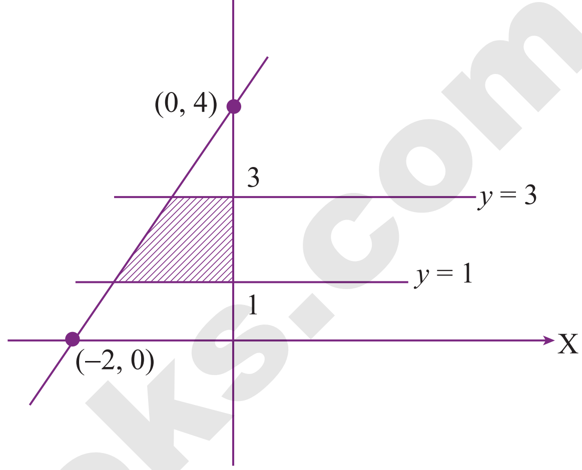
$$A = 5 \text{ sq. units.}$$

2. Find the area bounded by the lines $y - 2x - 4 = 0$, $y = 1$, $y = 3$ and the y-axis [HY-2019; FRT - 2022]

Sol:

$y - 2x - 4 = 0$

x	0	–2
y	4	0



$$\text{Given } y - 2x - 4 = 0$$
$$\Rightarrow \quad y - 4 = 2x$$
$$\Rightarrow \quad x = \frac{y-4}{2}$$

Since the area lies to the left of Y-axis, with the limits $y = 1$ & $y = 3$.

$$\text{Area} = \int_1^3 -x\,dy$$

$$= \int_1^3 -\left(\frac{1}{2}\right)(y-4)dy$$

$$= \frac{1}{2}\int_1^3 (4-y)dy = \frac{1}{2}\left[4y - \frac{y^2}{2}\right]_1^3$$

$$= \frac{1}{2}\left[\left(4(3) - \frac{3^2}{2}\right) - \left(4(1) - \frac{1^2}{2}\right)\right]$$

$$= \frac{1}{2}\left[\left(12 - \frac{9}{2}\right) - \left(4 - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{24-9}{2}\right) - \left(\frac{8-1}{2}\right)\right]$$

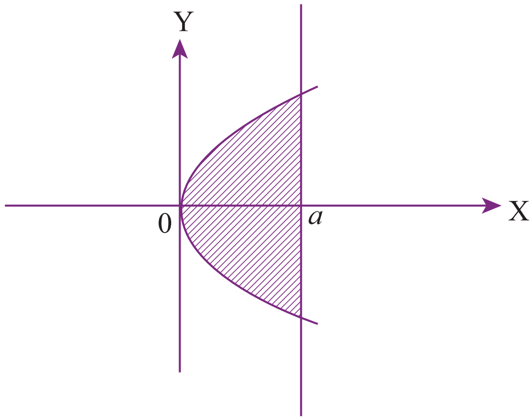
$$= \frac{1}{2}\left[\frac{15}{2} - \frac{7}{2}\right] = \frac{1}{2}\left[\frac{8}{2}\right]$$

$$= \frac{y-4}{2}$$

$$A = 2 \text{ sq. units.}$$

3. **Calculate the area bounded by the parabola $y^2 = 4\,ax$ and its latus rectum.** [Sep. - 2020]

Sol:



$$y^2 = 4\,ax \text{ is the right open}$$

$$\Rightarrow \quad y = \sqrt{4ax} \text{ parabola.}$$

The limits are from $x = 0$ to $x = a$

$$\therefore \text{Area} = 2\int_0^a y\,dx = 2\int_0^a \sqrt{4ax}\,dx$$

$$= 2\int_0^a 2\sqrt{a}\sqrt{x}\,dx$$

$$= 4\sqrt{a}\int_0^a \sqrt{x}\,dx$$

$$= 4\sqrt{a}\int_0^a x^{\frac{1}{2}}dx$$

$$= 4\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$$

$$= 4\sqrt{a}\times\frac{2}{3}\left(x^{\frac{3}{2}}\right)_0^a$$

$$= 8\frac{\sqrt{a}}{3}\left(a^{\frac{3}{2}}-0\right)$$

$$= \frac{8\sqrt{a}}{3}\left(a\sqrt{a}\right)$$

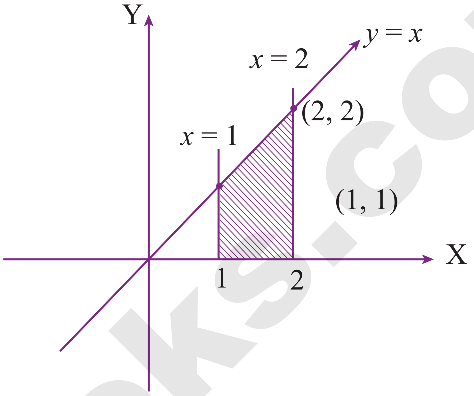
$$= \frac{8}{3}a^2 \quad \text{sq. units}$$

4. **Find the area bounded by the line $y = x$, the x-axis and the ordinates $x = 1, x = 2$.**

[PTA-4;QY-2019]

Sol: $y = x$

x	1	2
y	1	2



Given line $y = x$

The limits are $x = 1, x = 2$

Since the shaded area lies to the right of Y-axis,

$$\text{Area} = \int_1^2 y\,dx = \int_1^2 x\,dx = \left(\frac{x^2}{2}\right)_1^2$$

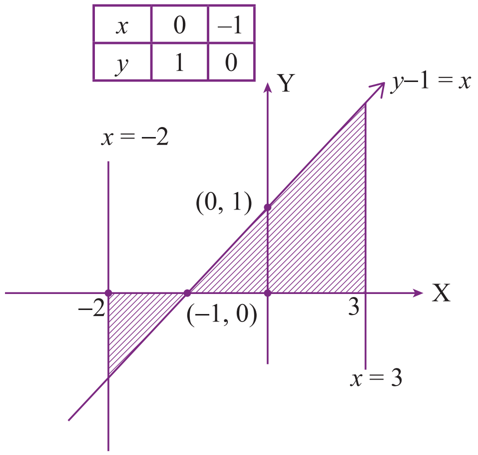
$$= \frac{2^2}{2}-\frac{1^2}{2} = 2-\frac{1}{2}$$

$$= \frac{4-1}{2} = \frac{3}{2} \text{ sq. units.}$$

5. **Using integration, find the area of the region bounded by the line $y - 1 = x$, the x-axis and the ordinates $x = –2, x = 3$.**

Sol: $y – 1 = x$

x	0	–1
y	1	0



Given line is $y – 1 = x \Rightarrow y = x + 1$

Given limits are from $x = – 2$ to 3.

In the diagram, the area from $x = -2$ to $x = -1$ lies below the X-axis and the area from $x = -1$ to $x = 3$ lies above the X-axis.

$\therefore$ Required Area

$$= \int_{-2}^{-1} -y\,dx + \int_{-1}^{3} y\,dx = -\int_{-2}^{-1} y\,dx + \int_{-1}^{3} y\,dx$$

$$= \int_{-1}^{-2} y\,dx + \int_{-1}^{3} y\,dx \left[\because \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx\right]$$

$$= \int_{-1}^{-2} (x+1)\,dx + \int_{-1}^{3} (x+1)\,dx$$

$$= \left(\frac{x^2}{2}+x\right)_{-1}^{-2} + \left(\frac{x^2}{2}+x\right)_{-1}^{3}$$

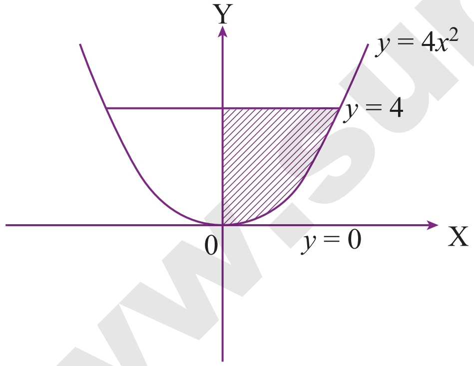
$$= \left(\frac{4}{2}-2\right) - \left(\frac{1}{2}-1\right) + \left(\frac{9}{2}+3\right) - \left(\frac{1}{2}-1\right)$$

$$= (2-2) - \left(-\frac{1}{2}\right) + \left(\frac{15}{2}\right) - \left(-\frac{1}{2}\right)$$

$$= 0 + \frac{1}{2} + \frac{15}{2} + \frac{1}{2} = \frac{17}{2} \text{ sq. units.}$$

6. Find the area of the region lying in the first quadrant bounded by the region $y = 4\ x^2$, $x = 0$, $y = 0$ and $y = 4$. **[July - 2022]**

Sol:



Given curve $y = 4x^2$ is an open upward parabola

$$\Rightarrow \quad \frac{y}{4} = x^2$$

The limits are from $y = 0$ to $y = 4$

Since the shaded region lies to the right of Y-axis,

$$\text{Required area} = \int_0^4 x\,dy = \int_0^4 \sqrt{\frac{y}{4}}\,dy$$

$$= \frac{1}{2}\int_0^4 \sqrt{y}\,dy = \frac{1}{2}\int_0^4 y^{\frac{1}{2}}\,dy$$

$$= \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3}\left[y^{\frac{3}{2}}\right]_0^4$$

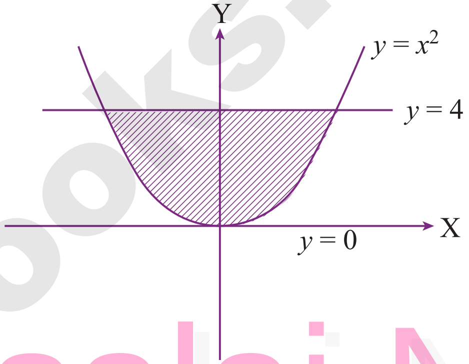
$$= \frac{1}{3}\left[4^{\frac{3}{2}} - 0\right] = \frac{1}{3}\left[4^{\frac{3}{2}}\right]$$

$$= \frac{1}{3}\,4^1\sqrt{4} = \frac{1}{3}\cdot 4(2)$$

$$A = \frac{8}{3} \text{ sq. units.}$$

7. Find the area bounded by the curve $y = x^2$ and the line $y = 4$. **[Aug. - 2021]**

Sol:



Since $y = x^2$ is symmetric about Y-axis, the required

$$\text{Area} = 2\int_0^4 x\,dy$$

When $y = x^2 \Rightarrow \quad x = \sqrt{y}$

$$\therefore \quad \text{Area} = 2\int_0^4 \sqrt{y}\,dy = 2\int_0^4 y^{\frac{1}{2}}\,dy$$

$$= 2\left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^4$$

$$= 2 \times \frac{2}{3}\left[y^{\frac{3}{2}}\right]_0^4$$

$$= \frac{4}{3}\left[4^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = \frac{4}{3}\left[4^{\frac{3}{2}}\right]$$

$$= \frac{4}{3} \times \left(2^2\right)^{\frac{3}{2}} = \frac{4}{3} \times 2^3$$

$$A = \frac{4}{3} \times 8 = \frac{32}{3} \text{ sq. units.}$$

EXERCISE 3.2

1. **The cost of over haul of an engine is ₹ 10,000 The operating cost per hour is at the rate of 2x − 240 where the engine has run x km. Find out the total cost if the engine run for 300 hours after overhaul.**

Sol: Given cost of overhaul of an engine is ₹ 10,000.

Operating cost per hour = $2x - 240$.

Total cost for the engine to run for 300 hours

$$\text{after overhaul} = 10,000+\int_0^{300}(2x-240)dx$$

$C = \int MC\ dx + k$

$\because k$ is the overhaul $\Rightarrow$ cost = 10,000

$$=10,000+\left[\frac{2x^2}{2}-240x\right]_0^{300}$$

$$=10,000+\left[x^2-240x\right]_0^{300}$$

$$=10,000+\left[(300)^2-(240)(300)\right]-0$$

=10,000 + 90,000 – 72,000 =1,00,000 – 72,000

= ₹ 28,000

2. **Elasticity of a function $\frac{Ey}{Ex}$ is given by $\frac{Ey}{Ex}=\frac{-7x}{(1-2x)(2+3x)}$. Find the function when $x = 2, y = \frac{3}{8}$.** **[PTA - 4; QY - 2019]**

Sol:

$$\frac{Ey}{Ex} = \frac{-7x}{(1-2x)(2+3x)}$$

Also, it is given that $x = 2$, when $y = \frac{3}{8}$

$$\Rightarrow \quad \frac{x}{y}\frac{dy}{dx} = \frac{-7x}{(1-2x)(2+3x)}$$

$$\Rightarrow \quad \frac{dy}{y} = \frac{-7\cancel{x}\ dx}{\cancel{x}(1-2x)(2+3x)}$$

$$= \frac{-7\,dx}{(1-2x)(2+3x)}$$

$$\Rightarrow \quad \frac{dy}{y} = \frac{7\,dx}{(2x-1)(3x+2)}$$

$$\int\frac{dy}{y} = \int\frac{7\,dx}{(2x-1)(3x+2)}$$

$$\int\frac{dy}{y} = \int\left(\frac{2}{2x-1}-\frac{3}{3x+2}\right)dx$$

$$\log y = 2\int\frac{1}{2x-1}dx - 3\int\frac{dx}{3x+2}$$

$$=2\,\frac{\log|2x-1|}{2}-3\,\frac{\log|3x+2|}{3}+\log c$$

$$= \log|2x-1| - \log|3x+2| + \log c$$

$$\log y - \log c = \log\left|\frac{2x-1}{3x+2}\right|$$

$$\Rightarrow \quad \log\left|\left(\frac{y}{c}\right)\right| = \log\left|\frac{2x-1}{3x+2}\right|$$

$$\Rightarrow \quad \frac{y}{c} = \frac{2x-1}{3x+2} \quad y = c\left(\frac{2x-1}{3x+2}\right) \quad \ldots (1)$$

When $x = 2$, $y = \frac{3}{8}$

$$\Rightarrow \quad \frac{3}{8} = c\left(\frac{4-1}{8}\right)$$

$$\frac{3}{8} = c\left(\frac{3}{8}\right) = c = 1$$

Substituting $c = 1$ in (1) we get

$$y = \left(\frac{2x-1}{3x+2}\right)$$

$$\Rightarrow \quad y = \frac{2x-1}{3x+2}$$

$$\frac{7}{(2x-1)(3x+2)} = \frac{A}{2x-1}+\frac{B}{3x+2}$$

$$7 = A(3x+2) + B(2x-1)$$

$$\text{put } x = \frac{-2}{3} \quad 7 = B\left(\frac{-4}{3}-1\right) \quad 7 = B\left(\frac{-7}{3}\right)$$

$$B = 3$$

$$\text{put } x = \frac{1}{2} \quad 7 = A\left(\frac{3}{2}+2\right) \Rightarrow 7 = A\left(\frac{7}{2}\right)$$

$$A = 2$$

$$\therefore \frac{7}{(2x-1)(3x+2)} = \frac{2}{2x-1}-\frac{3}{3x+2}$$

3. **The elasticity of demand with respect to price for a commodity is given by $\frac{(4-x)}{x}$, where p is the price when demand is x. Find the demand function when price is 4 and the demand is 2. Also find the revenue function.**

Sol: Given elasticity of demand = $\frac{(4-x)}{x}$

We know that $\eta_d = \frac{-p}{x}\frac{dx}{dp}$

$$\therefore \quad \frac{-p}{\cancel{x}}\frac{dx}{dp} = \frac{4-x}{\cancel{x}}$$

$$\Rightarrow \quad \frac{dx}{4-x} = \frac{-dp}{p}$$

$$\Rightarrow \quad \int\frac{dx}{4-x} = -\int\frac{dp}{p}$$

$$\Rightarrow \quad \log\frac{|4-x|}{-1} = -\log p + \log k$$

$$\Rightarrow -\log|4-x| = \log\left(\frac{k}{p}\right)$$

$$\Rightarrow \log(4-x)^{-1} = \log\left(\frac{k}{p}\right)$$

$$\Rightarrow \frac{1}{4-x} = \frac{k}{p} \quad \Rightarrow \quad p = k(4-x) \qquad \text{...(1)}$$

When $x = 2$ and $p = 4$

$$4 = k(4-2) \;\Rightarrow\; 4 = k(2)$$

$$\Rightarrow \quad \frac{4}{2} = k \Rightarrow k = 2$$

∴ (1) becomes

$$p = 2(4-x)$$

$$\Rightarrow \quad p = 8 - 2x$$

Also, $\quad R = px$

$$\Rightarrow \quad R = (8-2x)x$$

$$\Rightarrow \quad R = 8x - 2x^2$$

Hence, the demand function is $8 - 2x$ and the revenue function is $8x - 2x^2$.

4. A company receives a shipment of 500 scooters every 30 days. From experience it is known that the inventory on hand is related to the number of days *x*. Since the shipment, I(*x*) = 500 − 0.03*x*², the daily holding cost per scooter is ₹ 0.3. Determine the total cost for maintaining inventory for 30 days.

Sol: Given inventory on hand $I(x) = 500 - 0.03x^2$

Holding cost per scooter is $C_1 = ₹\ 0.3$

Time period $T = 30$ days

Total inventory carrying cost

$$= C_1\int_0^r I(x)\,dx$$

$$= 0.3\int_0^{30}(500 - 0.03x^2)\,dx$$

$$= 0.3\left[500x - \frac{0.03x^3}{3}\right]_0^{30}$$

$$= 0.3\left[500x - 0.01x^3\right]_0^{30}$$

$$= 0.3\left\{\left[500(30) - 0.01(30)^3\right] - 0\right\}$$

= 0.3[15000 – 0.01 (27000)]

= 0.3[15000 – 270]= 0.3 (14730)= ₹ 4419.

Hence, the total cost or maintaining inventory for 30 days = ₹ 4419.

5. An account fetches interest at the rate of 5% per annum compounded continuously An individual deposits ₹1,000 each year in his account. How much will be in the account after 5 years. ($e^{0.25} = 1.284$) .

Sol: Given P = ₹ 1000, $r = \frac{5}{100} = 0.05$ and N = 5

Amount after 5 years

$$= \int_0^5 1000\ e^{0.05t}\,dt \qquad [\because A = \int_0^N P e^{rt}\,dt\ ]$$

$$= 1000\int_0^5 e^{0.05t}\,dt$$

$$= 1000\left(\frac{e^{0.05t}}{0.05}\right)_0^5 = \frac{1000}{0.05}\left(e^{0.05t}\right)_0^5$$

$$= 20{,}000\left(e^{0.05(5)} - e^{0.05(0)}\right)$$

$$= 20{,}000(e^{0.25} - e^0)$$

$$= 20{,}000(1.284 - 1)$$

$$[\because e^{0.25} = 1.284]$$

$$= 20{,}000(.284) = ₹\ 5680$$

Hence, the amount after 5 years will be ₹ 5680.

6. The marginal cost function of a product is given by $\frac{dC}{dx}$ =100 − 10*x*+ 0.1*x*² where *x* is the output. Obtain the total and the average cost function of the firm under the assumption, that its fixed cost is ₹ 500.

Sol: Given marginal cost function

$$MC = \frac{dC}{dx} = 100 - 10x + 0.1x^2$$

$$\Rightarrow \quad C = \int(100 - 10x + 0.1x^2)\,dx$$

$$\Rightarrow \quad C = 100x - \frac{10x^2}{2} + \frac{0.1x^3}{3} + k \qquad \text{... (1)}$$

Given fixed cost is ₹ 500 ⇒ $k = 500$

∴ (1) becomes,

$$C = 100x - 5x^2 + \frac{0.1x^3}{3} + 500$$

Average cost function

$$AC = \frac{C}{x} = \frac{100x - 5x^2 + \frac{0.1x^3}{3} + 500}{x}$$

$$AC = 100 - 5x + \frac{0.1x^2 \times 10}{3 \times 10} + \frac{500}{x}$$

Average cost function

$$= 100 - 5x + \frac{x^2}{30} + \frac{500}{x}$$

7. The marginal cost function is MC = 300 $x^{\frac{2}{5}}$ and fixed cost is zero. Find out the total cost and average cost functions. **[July - 2022]**

Sol: Given MC $= 300\ x^{\frac{2}{5}}$

$$\Rightarrow \frac{dC}{dx} = 300\ x^{\frac{2}{5}}$$

$$\Rightarrow \int dC = 300 \int x^{\frac{2}{5}}\, dx$$

$$\Rightarrow C = 300 \frac{x^{\frac{2}{5}+1}}{\frac{2}{5}+1} + k$$

$$\Rightarrow C = 300 \frac{x^{\frac{7}{5}}}{\frac{7}{5}} + k$$

$$\Rightarrow C = 300 \times \frac{5}{7} x^{\frac{7}{5}} + k$$

$$\Rightarrow C = \frac{1500}{7} x^{\frac{7}{5}} + k \quad \dots (1)$$

Given fixed cost is zero ⇒ $k = 0$

∴ (1) becomes,

$$C = \frac{1500}{7} x^{\frac{7}{5}} + 0 \Rightarrow C = \frac{1500}{7} x^{\frac{7}{5}}$$

Average cost function (AC) $= \frac{C}{x}$

$$\Rightarrow AC = \frac{1500}{7} \frac{x^{\frac{7}{5}}}{x}$$

$$\Rightarrow AC = \frac{1500}{7} x^{\frac{7}{5}-1}$$

$$\Rightarrow AC = \frac{1500}{7} x^{\frac{2}{5}}$$

8. If the marginal cost function of x units of output is $\frac{a}{\sqrt{ax+b}}$ and if the cost of output is zero. Find the total cost as a function of x.

Sol: Given marginal cost function $= \frac{a}{\sqrt{ax+b}}$

$$\Rightarrow MC = \frac{a}{\sqrt{ax+b}} \Rightarrow \frac{dC}{dx} = \frac{a}{\sqrt{ax+b}}$$

$$\Rightarrow dC = \frac{a}{\sqrt{ax+b}} dx$$

$$\Rightarrow \int dC = a \int \frac{dx}{\sqrt{ax+b}}$$

$$\Rightarrow C = a \int (ax+b)^{\frac{-1}{2}} dx$$

$$\Rightarrow C = \cancel{a} \frac{(ax+b)^{\frac{-1}{2}+1}}{\left(-\frac{1}{2}+1\right)\cancel{a}} + k$$

$$\Rightarrow C = \frac{(ax+b)^{\frac{1}{2}}}{\frac{1}{2}} + k$$

$$\Rightarrow C = 2\sqrt{ax+b} + k \quad \dots (1)$$

Since the cost of output is zero.

C = 0, when $x = 0$

$$\therefore (1) \to 0 = 2\sqrt{0+b} + k$$

$$\Rightarrow 0 = 2\sqrt{b} + k$$

$$\Rightarrow k = -2\sqrt{b}$$

∴ (1) becomes

$$C = 2\sqrt{ax+b} - 2\sqrt{b}$$

9. Determine the cost of producing 200 air conditioners if the marginal cost (is per unit) is $C'(x) = \frac{x^2}{200} + 4$

Sol: Given marginal cost MC = C'(x) $= \frac{x^2}{200} + 4$

$$\Rightarrow \int MC = \int C'(x) = \int \left(\frac{x^2}{200} + 4\right) dx$$

To find the cost of producing 200 air conditioners

$$C = \int_0^{200} \left(\frac{x^2}{200} + 4\right) dx$$

$$= \left[\frac{1}{200} \times \frac{x^3}{3} + 4x\right]_0^{200} = \left[\frac{x^3}{600} + 4x\right]_0^{200}$$

$$= \left[\frac{(200)^3}{600} + 4(200)\right] - 0$$

$$= \frac{8000000}{600} + 800 = 13333.33 + 800$$

= ₹ 14133.33

Hence, the cost of producing 200 air conditioners is ₹ 14133.33

10. The marginal revenue (in thousands of Rupees) functions for a particular commodity is $5 + 3e^{-0\ 03\ x}$ where x denotes the number of units sold. Determine the total revenue from the sale of 100 units. (Given $e^{-3} = 0.05$ approximately)

Sol: Given marginal revenue MR = $5 + 3e^{-0.03x}$

$$\Rightarrow \quad \frac{dR}{dx} = 5 + 3e^{-0.03x}$$

$$\Rightarrow \quad dR = \left(5 + 3e^{-0.03x}\right)dx$$

Total revenue from sale of 100 units is

$$R = \int_0^{100} \left(5 + 3e^{-0.03x}\right)dx$$

$$= \left[5x + 3\frac{e^{-0.03x}}{-0.03}\right]_0^{100}$$

$$= \left[5x - \frac{e^{-0.03x}}{0.01}\right]_0^{100}$$

$$= \left(5(100) - \frac{e^{-0.03(100)}}{0.01}\right) - \left(0 - \frac{e^{-0.03(0)}}{0.01}\right)$$

$$= 500 - \frac{e^{-3}}{0.01} + \frac{e^{0}}{0.01}$$

$$= 500 - \frac{.05}{0.01} + \frac{1}{0.01}$$

$$= 500 - 5 + 100$$

$$= 600 - 5 = 595$$

∴ Total revenue = 595 × 1000 = ₹ 595000

[∵ Revenue is given in thousands]

11. If the marginal revenue function for a commodity is MR = 9 – 4x^2. Find the demand function.

Sol: Given marginal revenue

$$MR = 9 - 4x^2$$

$$\Rightarrow \quad \frac{dR}{dx} = 9 - 4x^2$$

$$\Rightarrow \quad dR = (9 - 4x^2)\,dx$$

$$\Rightarrow \quad \int dR = \int (9 - 4x^2)\,dx$$

$$\Rightarrow \quad R = 9x - \frac{4x^3}{3} + k \qquad \dots (1)$$

When no product is sold, revenue is zero

∴ When $x = 0$, $R = 0 \Rightarrow 0 = 0 - 0 + k \Rightarrow k = 0$

∴ (1) becomes $R = 9x - \dfrac{4x^3}{3}$

We know that R = Px where P is the demand function

$$\Rightarrow \quad P\cancel{x} = \cancel{x}\left(9 - \frac{4x^2}{3}\right) \Rightarrow P = 9 - \frac{4x^2}{3}$$

Hence, the demand function is $9 - \dfrac{4x^2}{3}$.

12. Given the marginal revenue function $\dfrac{4}{(2x+3)^2} - 1$, show that the average revenue function is $P = \dfrac{4}{(6x+9)} - 1$.

Sol: Given marginal revenue function $= \dfrac{4}{(2x+3)^2} - 1$

$$\Rightarrow \quad MR = \frac{dR}{dx} = 4\,(2x+3)^{-2} - 1$$

$$\Rightarrow \quad dR = [4\,(2x+3)^{-2} - 1]\,dx$$

$$\Rightarrow \quad \int dR = 4\int (2x+3)^{-2}\,dx - \int dx$$

$$\Rightarrow \quad R = \frac{\overset{2}{\cancel{4}}(2x+3)^{-2+1}}{\cancel{2}\ -2+1} - x$$

$$\Rightarrow \quad R = \frac{2(2x+3)^{-1}}{-1}$$

$$\Rightarrow \quad R = \frac{-2}{(2x+3)} - x + k \qquad \dots (1)$$

When $x = 0$, $R = 0 \Rightarrow 0 = \dfrac{-2}{3} - 0 + k \Rightarrow k = \dfrac{2}{3}$

∴ (1) becomes $R = \dfrac{-2}{2x+3} - x + \dfrac{2}{3}$

We know that $R = Px \Rightarrow P = \dfrac{R}{x}$

∴ Average revenue function

$$P = \frac{R}{x} = \frac{-2}{x(2x+3)} - \frac{x}{x} + \frac{2}{3x}$$

$$= \frac{-2}{x(2x+3)} - 1 + \frac{2}{3x}$$

$$= \frac{-2}{x(2x+3)} + \frac{2}{3x} - 1$$

$$= \frac{-6+2(2x+3)}{3x(2x+3)} - 1$$

$$= \frac{-\cancel{6}+4x+\cancel{6}}{3x(2x+3)} - 1$$

$$= \frac{4\cancel{x}}{3\cancel{x}(2x+3)} - 1 = \frac{4}{3(2x+3)} - 1$$

$$P = \frac{4}{6x+9} - 1$$

Hence Proved.

13. A firm's marginal revenue function is $MR = 20e^{\frac{-x}{10}}\left(1-\frac{x}{10}\right)$. Find the corresponding demand function.

Sol: Given marginal revenue function

$$MR = \frac{dR}{dx} = 20e^{\frac{-x}{10}}\left(1-\frac{x}{10}\right)$$

$$dR = 20e^{\frac{-x}{10}}\left(1-\frac{x}{10}\right)dx + k$$

$$R = \int 20e^{\frac{-x}{10}}\left(1-\frac{x}{10}\right)dx$$

We know that $\int e^{ax}\,[af(x)+f'(x)]dx = e^{ax}f(x)+c$

Here $a = \frac{-1}{10}$, $f(x) = x$, $f'(x) = 1$

$$R = 20\int e^{\frac{-x}{10}}\left[-\frac{1}{10}x+1\right]dx = 20e^{\frac{-x}{10}}\ x + k$$

$$\Rightarrow \quad R = 20e^{\frac{-x}{10}}\ x + k \qquad ...(1)$$

When $x = 0$, $R = 0 \Rightarrow 0 = 0 + k \Rightarrow k = 0$

(1) becomes $R = 20\,x\ e^{\frac{-x}{10}}$

$$\text{Demand function P} = \frac{R}{x} = \frac{20xe^{\frac{-x}{10}}}{x}$$

$$\Rightarrow \quad P = 20\ e^{\frac{-x}{10}}$$

14. The marginal cost of production of a firm is given by $C'(x) = 5 + 0.13x$, the marginal revenue is given by $R'(x) = 18$ and the fixed cost is ₹ 120. Find the profit function.

Sol: Given $C'(x) = 5 + 0.\ 13x$

$R'(x) = 18$

Fixed cost is ₹ 120

$$C'(x) = 5 + 0.13x$$

$$\Rightarrow \quad \int C'(x) = \int (5 + 0.13x)\,dx$$

$$\Rightarrow \quad C(x) = 5x + \frac{0.13x^2}{2} + k_1$$

Since fixed cost is ₹ 120 $\Rightarrow k_1 = 120$

$$\therefore C(x) = 5x + \frac{0.13x^2}{2} + 120 \qquad ... (1)$$

Also $R'(x) = 18 \Rightarrow \int R'(x) = \int 18\,dx$

$\Rightarrow$ $R(x) = 18x + k_2$

When $x = 0$, $R = 0 \Rightarrow k_2 = 0$

$\therefore$ $R(x) = 18x$ ...(2)

Profit function

$$\Rightarrow \quad P(x) = R(x) - C(x)$$

$$= 18x - 5x - \frac{0.13x^2}{2} - 120$$

[from (1) and (2)]

$$P(x) = 13x - 0.065\ x^2 - 120.$$

15. If the marginal revenue function is $R'(x) = 1500 - 4x - 3x^2$. Find the revenue function and average revenue function.

Sol: Given

$$MR = R'(x) = 1500 - 4x - 3x^2$$

$$\Rightarrow \quad \int R'(x) = \int (1500 - 4x - 3x^2)\,dx$$

$$\Rightarrow \quad R(x) = 1500x - \frac{4x^2}{2} - \frac{3x^3}{3} + k$$

$$\Rightarrow \quad R(x) = 1500x - 2x^2 - x^3 + k$$

When $x = 0$, $R = 0 \Rightarrow k = 0$

$$\Rightarrow \quad R(x) = 1500x - 2x^2 - x^3$$

$$\text{Average revenue function} = \frac{R(x)}{x}$$

$$= \frac{1500x - 2x^2 - x^3}{x}$$

$$AR = 1500 - 2x - x^2$$

16. Find the revenue function and the demand function if the marginal revenue for x units is $MR = 10+3x - x^2$.

Sol: Given $MR = 10 + 3x - x^2$

$$\frac{dR}{dx} = 10 + 3x - x^2$$

$$\Rightarrow \quad dR = (10 + 3x - x^2)\,dx + k$$

$$\Rightarrow \quad \int dR = \int (10 + 3x - x^2)\,dx$$

Chapter

8

SAMPLING TECHNIQUES AND STATISTICAL INFERENCE

MUST KNOW DEFINITIONS

Sampling	➢ **Sampling** is the process of selecting a sample from a population. ➢ **Types of sampling :** **1.** Non-Random sampling or Non-probability sampling. **2.** Random Sampling or Probability sampling.
Stratified Random Sampling	➢ First divide the population into sub-populations, which are called strata. The collection of all the samples from all strata gives the stratified random samples.
Systematic Sampling	➢ Randomly select the first sample from the first k units. Then every k^{th} member, starting with the first selected sample, is included in the sample.
Sampling distribution	➢ It is a frequency distribution which is formed with various values of a statistic computed from different samples of the same size drawn from the same population.
Estimation	➢ The method of obtaining the most likely value of the population parameter using statistic is called estimation.
Estimator	➢ Sample statistic used to estimate a population parameter.
Estimate	➢ An estimate is a specific observed value of a statistic.
Point Estimation	➢ When a single value is used as an estimate.
Interval Estimation	➢ Finding limits within which the parameter would be expected to lie.
Null Hypothesis	➢ A hypothesis which is tested for possible rejection under the assumption that it is true.
Alternative Hypothesis	➢ Any hypothesis which is complementary to the null hypothesis.
Critical value	➢ The value of test statistic which separates the critical region and the acceptance region. **Type I Error :** The error of rejecting H_0 when it is true. **Type II Error :** The error of accepting H_0 when it is false.

FORMULAE TO REMEMBER

Merits and Demerits of simple random sampling :

Merits ➢
1. Personal bias is completely eliminated.
2. It is economical as it saves time, money and labour.
3. It requires minimum knowledge about the population in advance.

Demerits ➢
1. This requires a complete list of the population but such up-to-date lists are not available in many enquiries.
2. If the size of the sample is small, then it will not be a representative of the population.

Merits and Demerits of Systematic Sampling :

Merits ➢
1. Simple and convenient method.
2. It distributes the sample more evenly over the entire listed population.
3. Time and work is reduced much.

Demerits ➢
1. They are not random samples.
2. If N is not a multiple of n, then the sampling interval (k) cannot be an integer, thus sample selection becomes difficult.

Sampling Errors ➢
1. Faulty selection of the sample instead of correct sample by defective sampling technique.
2. The investigator substitutes a convenient sample if the original sample is not available while investigation.
3. In area surveys, while dealing with border lines it depends upon the investigator whether to include them in the sample or not. This is known as Faulty demarcation of sampling units.

Non-Sampling Errors ➢
1. Due to negligence and carelessness of the part of either investigator or respondents.
2. Due to lack of trained and qualified investigators.
3. Due to framing of a wrong questionnaire.
4. Due to apply wrong statistical measure
5. Due to incomplete investigation and sample survey.

Standard Error ➢

S.No	Statistic	Standard Error
1.	Sample mean	$\sigma/\sqrt{n}$
2.	Observed sample proportion	$\sqrt{PQ/n}$
3.	Sample standard deviation	$\sqrt{\sigma^2/2n}$
4.	Sample variance	$\sigma^2\sqrt{2/n}$
5.	Sample quartiles	$1.36263\,\sigma/\sqrt{n}$
6.	Sample median	$1.25331\,\sigma/\sqrt{n}$
7.	Sample correlation coefficient	$(1-\rho^2)/\sqrt{n}$

Characteristic of a good estimator ➢ **1.** Unbiasedness **2.** Consistency **3.** Efficiency **4.** Sufficiency

Normal Probability Table

Critical Values Z_α	Level of significance (α)			
	1%	2%	5%	10%
Two-tailed test	$\lvert Z_\alpha\rvert = 2.58$	$\lvert Z_\alpha\rvert = 2.33$	$\lvert Z_\alpha\rvert = 1.96$	$\lvert Z_\alpha\rvert = 1.645$
Right tailed test	$Z_\alpha = 2.33$	$Z_\alpha = 2.055$	$Z_\alpha = 1.645$	$Z_\alpha = 1.28$
Left tailed test	$Z_\alpha = -2.33$	$Z_\alpha = -2.055$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

Test of significance for single mean ➢

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

TEXTUAL QUESTIONS

EXERCISE 8.1

1. What is population?

Ans. The group of individuals considered under study is called as population.

2. What is sample?

Ans. A selection of a group of individuals from a population in such a way that it represents the population is called as sample.

3. What is statistic?

Ans. Any statistical measure computed from sample is known as statistic.

4. Define parameter.

Ans. The statistical constants of the population like mean (μ), variance (σ^2) are referred as population parameters.

5. What is sampling distribution of a statistic?

Ans. Sampling distribution of a statistic is the frequency distribution which is formed with various values of a statistic computed from different samples of the same size drawn from the same population.

6. What is standard error? **[PTA-1]**

Ans. The standard deviation of the sampling distribution of a statistic is known as its Standard Error.

7. Explain in detail about simple random sampling with a suitable example.

Ans. In this technique the samples are selected in such a way that each and every unit in the population has an equal and independent chance of being selected as a sample.

Simple random sampling without replacement is followed. The following two methods are generally used.

(A) Lottery method : This is the most popular and simplest method when the population is finite. In this method, all the items of the population are numbered on separate slips of paper of same size, shape and colour. They are folded and placed in a container and shuffled thoroughly. Then the required numbers of slips are selected.

(B) Table of Random number : The random number table has been so constructed that each of the digits 0,1,2,…,9 will appear approximately with the same frequency and independently of each other.

The various random number tables available are

a. L.H.C. Tippett random number series

b. Fisher and Yates random number series

c. Kendall and Smith random number series

d. Rand Corporation random number series.

Example : Tippett's table of random numbers is 20 items out of 6000.

Here we consider row wise selection of random numbers.

~~2952~~	6641	~~3992~~	9792	7969	~~5911~~	~~3170~~	~~5624~~
~~4167~~	9524	~~1545~~	~~1396~~	7203	~~5356~~	~~1300~~	~~2693~~
~~2670~~	7483	~~3408~~	~~2762~~	~~3563~~	~~1089~~	6913	7991
~~0560~~	~~5246~~	~~1112~~	6107	6008	8125	~~4233~~	8776
2754	9143	1405	9025	7002	6111	8816	6446

8. Explain the stratified random sampling with a suitable example.

Ans. In stratified random sampling, first divide the population into sub-populations, which are called strata. Then, the samples are selected from each of the strata through random techniques. The collection of all the samples from all strata gives the stratified random samples.

Example :

From the following data, select 68 random samples from the population of heterogeneous group with size of 500 through stratified random sampling, considering the following categories as strata.

Category1: Lower income class -39%

Category2: Middle income class - 38%

Category3: Upper income class- 23%

Sol :

Stratum	Homogenous group	Percentage from population	Number of people in each strata	Random Samples
Category1	Lower income class	39	$\frac{39}{100} \times 500 = 195$	$195 \times \frac{68}{500} = 26.5 \sim 26$
Category2	Middle income class	38	$\frac{38}{100} \times 500 = 190$	$190 \times \frac{68}{500} = 26.5 \sim 26$
Category3	Upper income class	23	$\frac{23}{100} \times 500 = 115$	$115 \times \frac{68}{500} = 15.6 \sim 16$
Total		**100**	**500**	**68**

9. Explain in detail about systematic random sampling with example.

Ans. In systematic sampling, randomly select the first sample from the first k units. Then every k^{th} member, starting with the first selected sample, is included in the sample.

Procedure for selection of samples by systematic sampling method.

(i) If we want to select a sample of 10 students from a class of 100 students, then $k = \frac{N}{n} = \frac{100}{10} = 10.$

Thus, sampling interval = 10 denotes that for every 10 samples one sample has to be selected.

(ii) The first sample is selected from the first 10 (sampling interval) samples through random selection procedures.

(iii) If the selected first random sample is 5, then the rest of the samples are automatically selected as 5, 15 , 25, 35, 45, 55, 65, 75, 85, 95. $[\because k = 10]$

10. Explain in detail about sampling error.

Ans. Errors, which arise in the normal course of investigation or enumeration on account of chance, are called sampling errors. Sampling Errors arise due to the following reasons:

(a) Faulty selection of the sample instead of correct sample by defective sampling technique.

(b) The investigator substitutes a convenient sample if the original sample is not available while investigation.

(c) In area surveys, while dealing with border lines it depends upon the investigator whether to include them in the sample or not. This is known as Faulty demarcation of sampling units.

11. Explain in detail about non-sampling error.

Ans. The errors that arise due to human factors which always vary from one investigator to another in selecting, estimating or using measuring instruments are called Non-Sampling errors.

It may arise in the following ways:

(a) Due to negligence and carelessness of the part of either investigator or respondents.

(b) Due to lack of trained and qualified investigators.

(c) Due to framing of a wrong questionnaire.

(d) Due to apply wrong statistical measure.

(e) Due to incomplete investigation and sample survey.

12. State any two merits of simple random sampling. **[Sep. - 2020]**

Ans. 1. This method is economical as it saves time, money and labour.

2. Personal bias is completely eliminated.

13. State any three merits of stratified random sampling. **[HY-2019]**

Ans. 1. A stratified random sample can be kept small in size without losing its accuracy.

2. It is easy to administer,if the population under study is sub-divided.

3. It reduces the time and expenses in dividing the strata into geographical divisions, since the government itself had divided the geographical areas.

14. State any two demerits of systematic random sampling. **[PTA-3]**

Ans. 1. Systematic samples are not random samples.

2. If N is not a multiple of n, then the sampling interval (k) cannot be an integer, thus sample selection becomes difficult.

15. State any two merits for systematic random sampling.

Ans. 1. This is simple and convenient method.

2. This method distributes the sample more evenly over the entire listed population.

16. Using the following Tippet's random number table

2952	6641	3992	9792	7969	5911	3170	5624
4167	9524	1545	1396	7203	5356	1300	2693
2670	7483	3408	2762	3563	1089	6913	7991
0560	5246	1112	6107	6008	8125	4233	8776
2754	9143	1405	9025	7002	6111	8816	6446

Draw a sample of 10 three digit numbers which are even numbers.

Sol :

2952	6641	3992	9792	7969	5911	3170	5624
4167	9254	1545	1396	7203	5356	1300	2693
2670	7483	3408	2762	3563	1089	6913	7991
0560	5246	1112	6107	6008	8125	4233	8776
2754	9143	1405	9025	7002	6111	8816	6446

As Tippett's random numbers are available only in four digits. Thus, we can select the first three digits which are even numbers from the four digit random sample number. The required random samples are underlined. Hence, we consider row wise selection of random numbers.

17. A wholesaler in apples claims that only 4% of the apples supplied by him are defective. A random sample of 600 apples contained 36 defective apples. Calculate the standard error concerning of good apples. **[July - 2022]**

Sol : Sample size N = 600

Population proportion P = 4% = 0.04

Q = 1 – P = 1– 0.04

Q = 0.96

∴ The standard error for sample proportion is given by

$$\text{S.E} = \sqrt{\frac{PQ}{N}} = \sqrt{\frac{(0.04)(0.96)}{600}}$$

$$= \sqrt{\frac{0.0384}{600}} = \sqrt{.000064}$$

$$\text{S.E} = 0.008$$

18. A sample of 1000 students whose mean weight is 119 lbs(pounds) from a school in Tamil Nadu State was taken and their average weight was found to be 120 lbs with a standard deviation of 30 lbs. Calculate standard error of mean.

Sol : Sample size n = 1000

S.D (σ) = 30

$$\text{Standard error of mean} = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{30}{\sqrt{1000}} = \frac{30}{31.62}$$

$$\text{S.E} = 0.9487$$

19. A random sample of 60 observations was drawn from a large population and its standard deviation was found to be 2.5. Calculate the suitable standard error that this sample is taken from a population with standard deviation 3? ***[PTA-3] [HY-2019]***

Sol : Sample size $n = 60$

Population standard deviation $\sigma = 3$

The standard error for sample standard deviation

$$= \sqrt{\frac{\sigma^2}{2n}}$$

$$= \sqrt{\frac{3^2}{2(60)}} = \sqrt{\frac{9}{120}} = \sqrt{0.075}$$

S.E. of sample standard deviation = 0.2739.

20. In a sample of 400 population from a village 230 are found to be eaters of vegetarian items and the rest non-vegetarian items. Compute the standard error assuming that both vegetarian and non-vegetarian foods are equally popular in that village?

Sol : Sample size $n = 400$

$$\text{Probability of vegetarian} = \frac{230}{400} = 0.575$$

∴ P = 0.575

Probability of non-vegetarian $q = 1 - p$

$= 1 - 0.575 \Rightarrow q = 0.425$

$$\text{Standard error for sample proportion} = \sqrt{\frac{pq}{n}}$$

$$= \sqrt{\frac{(0.575)(0.425)}{400}}$$

$$= \sqrt{0.000610} = 0.0246$$

$$\text{S.E} = 0.025 \text{ (app)}$$

EXERCISE 8.2

1. Mention two branches of statistical inference? [March - 2020]

Ans. The two branches of statistical inference are Estimation and testing of hypothesis

2. What is an estimator?

Ans. An estimator is a sample statistic used to estimate a population parameter.

3. What is an estimate?

Ans. An estimate is a specific observed value of a statistic.

4. What is point estimation? [Govt.MQP-2019]

Ans. An estimate of a population parameter given by a single number is called as point estimation.

5. What is interval estimation?

Ans. Generally, there are situations where point estimation is not desirable and we are interested in finding limits within which the parameter would be expected to lie is called an interval estimation.

6. What is confidence interval?

Ans. The interval $[c_1, c_2]$ within which the unknown value of the population parameter is expected to lie is known as Confidence Interval.

7. What is null hypothesis? Give an example.

Ans. Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true and it is denoted by H_0.

Eg : If we want to find the population mean has a specific value μ_0, then the null hypothesis H_0 is $H_0 : \mu = \mu_0$

8. Define alternative hypothesis. [PTA-2]

Ans. Any hypothesis which is complementary to the null hypothesis is called as the alternative hypothesis and it is usually denoted by H_1.

9. Define critical region.

Ans. A region corresponding to a test statistic in the sample space which tends to rejection of H_0 is called critical region.

10. Define critical value.

Ans. The value of test statistic which separates the critical region and the acceptance region is called the critical value or significant value.

11. Define level of significance.

Ans. The probability of type I error is known as level of significance.

12. What is type I error?

Ans. The error of rejecting H_0 when it is true is type I error.

13. What is single tailed test?

Ans. When the hypothesis about the population parameter is rejected only for the value of sample statistic falling into one of the tails of the sampling distribution, then it is known as one-tailed test.

14. A sample of 100 items, draw from a universe with mean value 4 and S.D 3, has a mean value 63.5. Is the difference in the mean significant at 0.05 level of significance? [PTA-3]

Sol : Sample size $n = 100$,

Sample mean $\bar{X} = 3.5$

Population mean $\mu = 4$

Population standard deviation $\sigma = 3$

Null Hypotheses : There is no significant difference in the mean. i.e., $H_0 : \mu = 4$

Alternative Hypotheses : There is significant difference in the mean.

i.e., $H_1 : \mu \neq H$

The level of significance $\alpha = 5\% = 0.05$

Applying the test statistic,

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\Rightarrow \quad Z = \frac{3.5 - 4}{\frac{3}{\sqrt{100}}} = \frac{-0.5}{0.3} = -1.667$$

$$\Rightarrow \quad |Z| = 1.667$$

The table value $Z_{\frac{\alpha}{2}} = 1.96$

Here $Z < Z_{\frac{\alpha}{2}}$ i.e., $1.667 < 1.96$.

Inference : Since $Z < Z_{\frac{\alpha}{2}}$ at 5% level of significance, the null hypothesis H_0 is accepted. Hence there is no significant difference in the mean.

14. If probability $P\left[\left|\hat{\theta}-\theta\right|<\varepsilon\right]\to 1$ as $n\to\infty$, for any positive ε then $\hat{\theta}$ is said to ………….. estimator of θ.

(a) efficient (b) sufficient
(c) unbiased (d) consistent

[Ans: (d) consistent]

15. An estimator is said to be ………… if it contains all the information in the data about the parameter it estimates. [PTA - 6]

(a) efficient (b) sufficient
(c) unbiased (d) consistent

[Ans: (b) sufficient]

16. An estimate of a population parameter given by two numbers between which the parameter would be expected to lie is called an …………….. interval estimate of the parameter.

(a) point estimate
(b) interval estimation
(c) standard error (d) confidence

[Ans: (b) interval estimation]

17. A …………….. is a statement or an assertion about the population parameter. [PTA-3]

(a) hypothesis (b) statistic
(c) sample (d) census

[Ans: (a) hypothesis]

18. Type I error is [PTA-2; Sep-2020]

(a) Accept H_0 when it is true
(b) Accept H_0 when it is false
(c) Reject H_0 when it is true
(d) Reject H_0 when it is false.

[Ans: (c) Reject H_0 when it is true]

19. Type II error is [March - 2020]

(a) Accept H_0 when it is wrong
(b) Accept H_0 when it is true
(c) Reject H_0 when it is true
(d) Reject H_0 when it is false

[Ans: (a) Accept H_0 when it is wrong]

20. The standard error of sample mean is

[PTA-2 & 4;Govt.MQP-2019; May - 2022]

(a) $\dfrac{\sigma}{\sqrt{2n}}$ (b) $\dfrac{\sigma}{n}$

(c) $\dfrac{\sigma}{\sqrt{n}}$ (d) $\dfrac{\sigma^2}{\sqrt{n}}$

[Ans: (c) $\dfrac{\sigma}{\sqrt{n}}$]

Miscellaneous problems

1. Explain the types of sampling.

Ans. **The types of sampling are**

(i) Non-Random sampling or Non-probability sampling.
(ii) Random Sampling or Probability sampling.

Different types of probability sampling are [PTA-6]

(i) Simple random sampling
(ii) Stratified random sampling
(iii) Systematic sampling

Simple random sampling has

(a) Lottery method and
(b) Table of random number method

2. Write short note on sampling distribution and standard error.

Ans. **Sampling distribution :** Sampling distribution of a statistic is the frequency distribution which is formed with various values of a statistic computed from different samples of the same size drawn from the same population.

Standard Error :

The standard deviation of the sampling distribution of a statistic is known as its Standard Error.

S.No	Statistic	Standard Error
1.	Sample mean	$\sigma/\sqrt{n}$
2.	Observed sample proportion	$\sqrt{PQ/n}$
3.	Sample standard deviation	$\sqrt{\sigma^2/2n}$
4.	Sample variance	$\sigma^2\sqrt{2/n}$
5.	Sample quartiles	$1.36263\,\sigma/\sqrt{n}$
6.	Sample median	$1.25331\,\sigma/\sqrt{n}$
7.	Sample correlation coefficient	$\left(1-\rho^2\right)/\sqrt{n}$

3. Explain the procedures of testing of hypothesis

Ans. Hypothesis testing is also referred as "Statistical Decision Making".

There are two types of statistical hypothesis

(i) **Null hypothesis :** which is tested for possible rejection under the assumption that it is true
(ii) **Alternative hypothesis :** which is complementary to the null hypothesis

Alternative Hypotheses H_1 : $\mu \neq 100$

Level of significance $\alpha = 0.05$

Applying the test statistic,

$$Z = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \Rightarrow Z = \frac{99-100}{\frac{15}{\sqrt{1600}}} = \frac{-1}{\frac{15}{40}} = -\frac{40}{15} = -2.667$$

$$\therefore |Z| = 2.667$$

Critical value at 5% level of significance is $Z_{\frac{\alpha}{2}} = 1.96$

Here $|Z| > Z_{\alpha/2}$ as 2.667> 1.96.

Inference : Since $|Z| > Z_{\alpha/2}$ at 5% level of significance, the null hypothesis is rejected.

Hence, we can conclude that the sample has not been taken from the population with mean $\mu = 100$.

PTA Questions & Answers

1 MARK

1. **An estimate of a population parameter given by a singe number is called as** **[PTA - 1]**
(a) confidence interval
(b) interval estimation
(c) point estimation (d) estimation
[Ans: (c) point estimation**]**

2 MARKS

1. **What are the properties of mathematical exceptions.** **[PTA - 5]**

Sol : There are two types of error, They are

Type I error: The error of rejecting H_0 when it is true.

Type II error: The error of accepting when H_0 it is false.

3 MARKS

1. **The standard deviation of a sample of size 50 is 6.3. Determine the standard error whose population standard deviation is 6?** **[PTA - 1 & 4; Govt. MQP - 2019; May - 2022]**

Sol : Sample size $n = 50$

Sample S.D $s = 6.3$

Population S.D $\sigma = 6$

The standard error for sample S.D is given by

$$\text{S.E.} = \sqrt{\frac{\sigma^2}{2n}} = \frac{6}{\sqrt{2(50)}} = \frac{6}{\sqrt{100}} = 1.8974$$

Thus standard error for sample S.D = 1.8974

2. **Mention the merits and demerits of the systematic random sampling.** **[PTA-4]**

Sol : Merits

1. Personal bias is completely eliminated.
2. This method is economical as it saves time, money and labour.
3. The method requires minimum knowledge about the population in advance.

Demerits

1. This requires a complete list of the population but such up-to-date lists are not available in many enquiries.
2. If the size of the sample is small, then it will not be a representative of the population.

3. **The mean life time of 50 electric bulbs produced by a manufacturing company is estimated to be 825 hours with the S.D. of 110 hours. If μ is the mean life time of all the bulbs produced by the company, test the hypothesis that μ = 900 hours at 5% level of significance.** **[PTA-2]**

Sol : Given sample size n = 50
Sample mean $\bar{x}$ = 825
Population mean μ = 900
Population S.D. σ = 110
Null hypotheses : $H_0 : \mu$ = 900

Alternative hypotheses:$H_1 : \mu \neq 900$

$$\text{The test statistic, } z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{825-900}{\frac{110}{\sqrt{50}}} = -4.82 \quad \therefore |z| = -4.82$$

As the significance level is $\alpha = 0.05$, $Z_{\frac{\alpha}{2}} = 1.96$

Here $|z| > Z_{\frac{\alpha}{2}}$ as 4.82 > 1.96

Inference : As $|z| > Z_{\frac{\alpha}{2}}$, H_0 is rejected. Hence, we can conclude that mean life time of the population of electric bulbs cannot be taken as 900 hours.

Inference: Since, the calculated value is much greater than table value i.e.,$Z > Z_\alpha$, it is highly significant at 5% level of significance. Hence we reject the null hypothesis H_0 and conclude that the advertising campaign was definitely successful in promoting sales.

Govt. Exam Question & Answers

1 MARK

1. For testing $H_0 : \mu = \mu_0$: against $H_1\ \mu < \mu_0$, what is the critical value at $\alpha = 0.01$? **[March - 2020]**

(a) –1.645 (b) –2.33

(c) 2.33 (d) 1.645

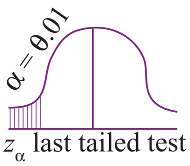
[Ans: (b) –2.33]

Hint: $H_0 : \mu = \mu_0$, $H_1\ \mu < \mu_0$

The null hypothesis $H_0 = \mu = \mu_0$ against $\mu_1\ \mu < \mu_0$ is last tailed test.

The critical value at $\alpha = 0.01$

(i.e.) $= z_\alpha = -2.33$



2 MARKS

1. A die is thrown 900 times and a throw of 5 or 6 is observed 324 times. Find the standard error of the proportion for an unbiased die. **[Sep. - 2020]**

Sol : If the occurrence of 5 or 6 on the die is called a success, then

Sample size = 900;

Number of the success = 324

$$\text{Sample proportion} = P = \frac{324}{900} = 0.36$$

Population proportion (P) = Prob (getting 5 or 6 when a die is thrown)

$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$= 0.3333$$

Thus P = 0.3333 and

Q = 1 – P = 1 – 0.3333

= 0.6667

The S.E for sample proportion is given by

$$\text{S.E} = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{(0.3333)(0.6667)}{900}} = 0.0157$$

Hence the standard error for sample proportion is S.E. = 0.0157.

2. A sample of 100 students is chosen from a large group of students. The average height of these students is 162 cm and standard deviation (S.D) is 8 cm. Obtain the standard error for the average height of large group of students of 160 cm. **[Aug. - 2021]**

Sol : Given $n = 100$, $\overline{x} = 162$ cm, $s = 8$ cm is known in this problem

Since σ is unknown, so we consider $\hat{\sigma} = s$ and $\phi = 160$ cm

$$\text{S.E.} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{8}{\sqrt{100}} = 0.8$$

Therefore the standard error for the average height of large group of students of 160 cm is 0.8.

3. Find the sample size for the given standard deviation 10 and the standard error with respect of sample mean is 3. **[July - 2022]**

Sol : Refer Text Book Example 8.7

3 MARKS

1. A random sample of 500 apples was taken from large consignment and 45 of them were found to be bad. Find the limits at which the bad apples lie at 99% confidence level. **[HY-2019]**

Sol : Given $n = 500$, $p = \frac{45}{500} = 0.9$, $q = 1 - 0.09 = 0.91$

99% Confidence limits for the bad apples:

$$p \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{pq}{n}} \Rightarrow 0.09 \pm 2.58\sqrt{\frac{(0.09)(0.91)}{500}}$$

$\Rightarrow$ (0.057, 0.123)

∴ The bad apples in the consignment lie between 5.7% and 12.3%

2. A die is thrown 9000 times and a throw of 3 or 4 observed 3240 times. Find the standard error of the proportion for an unbiased die. **[March - 2020]**

Sol : If the occurrence of 3 or 4 on the die is called a success, then

Sample size = 9000; Number of Success = 3240

$$\text{Sample proportion} = p = \frac{3240}{9000} = 0.36.$$

Population proportion (P) = Prob(getting 3 or 4 when a die is thrown)

$$\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3} = 0.3333$$

Thus P = 0.3333 and Q = 1 – P = 1– 0.3333 = 0.6667.

The S.E for sample proportion is given by

$$S.E = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{(0.3333)(0.6667)}{9000}} = 0.00496$$

Hence the standard error for sample proportion is S.E = 0.00496.

5 MARKS

1. **A sample of 100 measurements at breaking strength of cotton thread gave a mean of 7.4 gram and a standard deviation of 1.2 gram. Find 95% confidence limits for the mean breaking strength of cotton thread.** **[Govt.MQP-2019; Sep. - 2020; Aug. - 2021]**

Sol : Given, sample size = 100, $\bar{x}$ = 7.4, since σ is unknown but s = 1.2 is known.

In this problem, we consider $\sigma = s$, $Z_{\alpha/2} = 1.96$

$$S.E. = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{1.2}{\sqrt{100}} = 0.12$$

Hence 95% confidence limits for the population mean are

$$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$7.4 - (1.96 \times 0.12) \le \mu \le 7.4 + (1.96 \times 0.12)$$

$$7.4 - 0.2352 \le \mu \le 7.4 + 0.2352$$

$$7.165 \le \mu \le 7.635$$

This implies that the probability that the true value of the population mean breaking strength of the cotton threads will fall in this interval (7.165,7.635) at 95%

2. **An auto company decided to introduce a new six cylinder car whose mean petrol consumption is claimed to be lower than that of the existing auto engine. It was found that the mean petrol consumption for the 50 cars was 10 km per litre with a standard deviation of 3.5 km per litre. Test at 5% level of significance, whether the claim of the new car petrol consumption is 9.5 km per litre on the average is acceptable.** **[HY-2019]**

Sol : Sample size n = 50 Sample mean x = 10 km Sample standard deviation s = 3.5 km

Population mean μ =9.5 km

Since population SD is unknown we consider σ = s

The sample is a large sample and so we apply Z-test

Null Hypothesis: There is no significant difference between the sample average and the company's claim, i.e., $H_0 : \mu = 9.5$

Alternative Hypothesis: There is significant difference between the sample average and the company's claim, i.e., $H_1 : \mu \neq 9.5$ (two tailed test)

The level of significance α= 5% = 0.05

Applying the test statistic

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1); Z = \frac{10-9.5}{\frac{3.5}{\sqrt{50}}} \sim N(0,1)$$

$$= \frac{0.5}{0.495} = 1.01$$

Thus the calculated value 1.01 and the significant value or table value $Z_{\frac{\alpha}{2}} = 1.96$

Comparing the calculated and table value ,Here

$Z < Z_{\frac{\alpha}{2}}$ i.e., 1.01 < 1.96.

Inference: Since the calculated value is less than table value i.e., $Z < Z_{\frac{\alpha}{2}}$ at 5% level of significance, the null hypothesis H_0 is accepted. Hence we conclude that the company's claim that the new car petrol consumption is 9.5 km per litre is acceptable.

3. **Wages of the factor workers are assumed to be normally distributed with variance 25. A random sample of 50 workers gives the total wages equal to ₹ 2,550. Test the hypothesis μ = 52, against the alternative hypothesis μ = 49 at 1% level of significance.** **[March-2020]**

Sol : Sample size n = 50 workers ... (1)

Total wages $\Sigma x = 2550$ ... (1)

$$\text{Sample mean } \bar{x} = \frac{\text{total wages}}{n} - \frac{\Sigma x}{n}$$

$$= \frac{2550}{50} = 51 \text{ units}$$

Population mean μ = 52

Population variance $\sigma^2 = 52$

Population SD σ = 5

Under the null hypothesis $H_0 : \mu = 52$ (Two tail)

Against the alternative hypothesis $H_1 : \mu \neq 52$ (Two tail)

Level of significance μ = 0.01

$$\text{Test statistic } Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0.1)$$

$$Z = \frac{51-52}{\frac{5}{\sqrt{50}}} = \frac{-1}{0.7071} = -1.4142$$

Since alternative hypothesis is of two tailed test we can take |Z|=1.4142

Critical value at 1% level of significance is $Z_{\frac{\alpha}{2}} = 2.58$

Inference : Since the calculated value is less than table value i.e., $Z < Z_{\frac{\alpha}{2}}$ at 1% level of significance, the null hypothesis H_0 is accepted. Therefore, we conclude that there is no significant difference between the sample mean and population mean μ = 52 and SD σ = 5. Therefore μ = 49 is rejected.

4. An ambulance service claims that it takes on the average 8.9 minutes to reach its destination in emergency calls. To check on this claim the agency which licenses ambulance services has them timed on 50 emergency calls getting a mean of 9.3 minutes with a standard deviation of 1.6 minutes. What can they conclude at 5% level of significance? **[Sep. - 2020]**

Sol :
Sample size n = 50
Sample mean $\bar{x}$ = 9.3 minutes
Sample S.D s = 1.6 minutes
Population mean μ = 8.9 minutes
Null hypothesis $H_0 : \mu$ = 8.9
Alternative hypothesis $H_1 : \mu$ = 8.9(Two tail)
Level of significance μ = 0.05

Test statistic $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$

$$Z = \frac{9.3-8.9}{\frac{1.6}{\sqrt{50}}} = \frac{0.4}{0.2263} = 1.7676$$

Calculated value Z = 1.7676

Critical value at 5% level of significance is $Z_{\frac{\alpha}{2}} = 1.96$.

Inference : Since the calculated value is less than table value i.e., $Z < Z_{\frac{\alpha}{2}}$ at 5% level of significance, the null hypothesis is accepted. Therefore we conclude that an ambulance service claims on the average 8.9 minutes to reach its destination in emergency calls.

5. A machine produces a component of a product with a standard deviation of 1.6 cm in length. A random sample of 64 components was selected from the output and this sample has a mean length of 90 cm. The customer will reject the part if it is either less than 88 cm or more than 92 cm. Does the 95% confidence interval for the true mean length of all the components produced ensure acceptance by the customer? **[May & July - 2022]**

1%	2%	5%	10%
$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 2.33$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$

Sol : Refer Text Book Example 8.11

ADDITIONAL QUESTIONS & ANSWERS

1 MARK

1. The central limit theorem states that the sampling distribution of the mean will approach normal distribution
(a) as the size of the population increases
(b) as the sample size increase and becomes larger
(c) as the number of samples gets larger
(d) as the sample size decreases.

[Ans: (b) as the sample size increase and becomes larger]

2. Probability of rejecting null hypothesis, when it is true is ________
(a) Type I error (b) Type II error
(c) Sampling error (d) Standard error

[Ans: (a) Type I error]

Hint: $10C_2 = \frac{10\times 9}{1\times 2} = 45$

3. The number of ways in which one can select 2 customers out of 10 customers is
(a) 90 (b) 60 (c) 45 (d) 50

[Ans: (c) 45]

4. The standard error of the sample mean is
(a) Type I error (b) Type II error
(c) Standard deviation of the sampling distribution of the mean
(d) Variance of the sampling distribution of the mean.

[Ans: (c) Standard deviation of the sampling distribution of the mean]

5. Which of the following statements is true?
(a) point estimate gives a range of value
(b) sampling is done only to estimate a statistic
(c) sampling is done to estimate the population parameter
(d) sampling is not possible for an infinite population

[Ans: (c) sampling is done to estimate the population parameter]

6. The Z-value that is used to establish a 95% confidence interval for the estimation of a population parameter is
(a) 1.28 (b) 1.65 (c) 1.96 (d) 2.58

[Ans: (c) 1.96]

7. If a random sample of size 64 is taken from a population whose standard deviation is 32, then the standard error of the mean is
(a) 0.5 (b) 2 (c) 4 (d) 32

Hint: Standard error = $\frac{\sigma}{\sqrt{n}} = \frac{32}{\sqrt{64}} = 4$ **[Ans: (c) 4]**

8. The mean I.Q. of a sample of 1600 children was 99. It is likely that this was a r.sample from a population with mean I.Q. 100 and S.D 15^2. Then the value of Z is
(a) –2.667 (b) 2.667
(c) 1.96 (d) 2.58

Hint: $n = 1600, \bar{x} = 99, \sigma^2 = 15^2$ **[Ans: (a) –2.667]**

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{99-100}{\frac{15}{\sqrt{1600}}} = \frac{-1\times 40}{15} = -2.667$$

9. Out of 1000 T.V viewers, 320 watched a particular programme. Then the standard error is
(a) –0.147 (b) 0.147
(c) 0.0147 (d) –0.0147

[Ans: (c) 0.0147]

10. A random sample of 500 apples was taken from large consignment and 45 of them were found to be bad. Then the standard error is
(a) 0.033 (b) 0.0128
(c) 0.0128 (d) 0.00128

Hint: $p = \frac{45}{500}$, **[Ans: (b) 0.0128]**

$$q = 1 - \frac{45}{500} = \frac{455}{500}$$

$$S.E = \sqrt{\frac{pq}{n}} = \sqrt{\frac{\frac{450}{500}\times\frac{455}{500}}{500}} = 0.0128$$

1.

A		B	
1.	Null hypothesis	a)	H_1
2.	Alternate hypothesis	b)	Error of accepting H_0 when it is false
3.	Type I error	c)	H_0
4.	Type II error	d)	The error of rejecting H_0 when it is true

The correct match is
(a) 1 – c, 2 – a, 3 – d, 4 – b
(b) 1 – b, 2 – d, 3 – a, 4 – c
(c) 1 – a, 2 – d, 3 – b, 4 – c
(d) 1 – d, 2 – c, 3 – b, 4 – a

[Ans: (a) 1 – c, 2 – a, 3 – d, 4 – b]

2.

A		B	
1.	Sample	a)	Process of selecting a sample
2.	population	b)	Number of individual included in a sample
3.	Sampling	c)	Selection of a group of individuals from a population
4.	Sample size	d)	Group of individual considered under study

The correct match is
(a) 1 – d, 2 – a, 3 – b, 4 – c
(b) 1 – c, 2 – d, 3 – a, 4 – b
(c) 1 – a, 2 – b, 3 – c, 4 – d
(d) 1 – d, 2 – b, 3 – c, 4 – a

[Ans: (b) 1 – c, 2 – d, 3 – a, 4 – b]

1. A sample of 100 students are drawn from 1550 student of a school. The mean weight and variance of the sample are 67.45 kg and 9 kg. Then the standard error is ________.
(a) 0.3 (b) 0.9
(c) 0.6745 (d) 6.745

Hint: $n = 100,$ **[Ans: (a) 0.3]**

$$\bar{x} = 67.45, s = \sqrt{9} = 3$$

σ is unknown $\breve{\sigma} = s$

$$S.E = \frac{\breve{\sigma}}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{3}{\sqrt{100}} = 0.3$$

2. **The point estimate mean of the following data is ________.**

21.1, 25.0, 20.0, 16.0, 12.0, 10.0, 17.0, 18.0, 13.0, 11.0

(a) 16.3 (b) 13.6
(c) 21.21 (d) 212.10 **[Ans: (a) 16.3]**

Hint:

$$\bar{x} = \frac{21.1+25.0+20.0+16.0+12.0+10.0+17.0+18.0+13.0+11.0}{10}$$

$$\bar{x} = \frac{163.1}{10} = 16.31$$

3. **The point estimate variance of 21, 25, 20, 16, 12, 10, 17, 18, 13 and 11 is ________.**

(a) 23.5 (b) 2.35 (c) 4.85 (d) 48.5

Hint: $\bar{x} = 16.3$ **[Ans: (a) 23.5]**

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2 = \frac{1}{9}.\sum_{i=1}^{10}(x_i-16.3)^2 = 23.5$$

4. **The point estimate means of 6.33, 6.37, 6.36, 6.32. 6.37 is ________**

(a) 6.33 (b) 6.36 (c) 6.35 (d) 6.37

[Ans: (c) 6.35]

Hint:

$$\bar{x} = \frac{6.33+6.37+6.36+6.32+6.37}{5}$$

$$= 6.35$$

5. **The point estimate variance of 6.33, 6.37, 6.36, 6.32, 6.37 is**

(a) 0.0022 (b) 0.00055
(c) 0.0055 (d) 0.055

Hint: $\bar{x} = 6.35$ **[Ans: (b) 0.00055]**

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

$$= \frac{1}{4}\sum_{i=1}^{5}(x_i-6.35)^2 = 0.00055$$

6. **There are ________ branches of statistical inference**

(a) 1 (b) 2 (c) 3 (d) 4

[Ans: (b) 2]

7. **An ______ is a specific observed value of a statistic**

(a) Estimation (b) Estimator
(c) Estimate
(d) Testing of hypothesis **[Ans: (c) Estimate]**

8. **If 55 is the mean mark obtained by a sample of 5 students randomly drawn from a class of 100 students is considered to the means marks of the entire class. This single value 55 is a**

(a) estimation (b) estimate
(c) point estimate (d) estimator

[Ans: (c) point estimate]

9. **If α is the level of significance, then the confidence Co-efficient is**

(a) α (b) 1
(c) 1 – α (d) 1 + α

[Ans: (c) 1 – α]

10. **Any hypothesis which is complementary to the null hypothesis is ______ hypothesis.**

(a) Null (b) Alternative
(c) Statistical (d) testing

[Ans: (b) Alternative]

1.
(a) L.H.C Tippett random number series
(b) Fisher and Yates random number series
(c) Kendall and Smith random number series
(d) Harper random number series

[Ans: (d) Harper random number series]

[∵ a, b, c are random number series of simple random sampling]

2.
(a) Personal bias is eliminated
(b) It is economical
(c) It requires a complete list of the population
(d) It requires minimum knowledge

[Ans: (c) It requires a complete list of the population]

[∵ a, b, d are merits of simple random sampling]

3.
(a) Simple and convenient method
(b) If N is not a multiple of *n*, then the sample selection becomes difficult
(c) time and work is reduced
(d) If distributes the sample evenly over the entire listed population

[Ans: (b) If N is not a multiple of *n*, then the sample selection becomes difficult]

[∵ *a*, *c*, *d* are merits of systematic sampling]

4. **The confidence interval for population mean (μ) when the S.D is σ is**

(a) $\bar{X} \pm Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$

(b) $\bar{X} - Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$

(c) $\bar{X} \pm Z_{\frac{\alpha}{2}}(S.E)$ (d) $\bar{X} \pm (S.E)$

[Ans: (d) $\bar{X} \pm (S.E)$]

[∵ *a*, *b*, *c* are the correct formula for confidence interval]

5. The test statistic (for large samples) is

(a) $\frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(1,0)$ (b) $\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

(c) $\frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N\,(0,1)$ (d) $Z = \frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$

[Ans: (a) $\frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(1,0)$]

[∵ *b*, *c*, *d* are the list of significance for single mean]

1. The characteristics of good estimate are

(a) sufficiency (b) efficiency
(c) consistency (d) biasedness

[Ans: (d) biasedness]

2. Sampling is the

(a) procedure of selecting a sample from a population
(b) Process of selecting a sample
(c) number of individuals included in a sample
(d) selection of a group of individuals from a population **[Ans: (c) number of individuals included in a sample]**

3. The population parameters are

(a) mean (b) median
(c) variance
(d) standard deviation **[Ans: (b) median]**

4. Different types of probability sampling are

(a) Non-random sampling
(b) Simple random sampling
(c) Stratified random sampling
(d) Systematic sampling

[Ans: (a) Non-random sampling]

5. If the selected random samples are 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95, then the sampling is done by

(a) $K = \frac{N}{n}$
(b) Systematic sampling
(c) Lottery method
(d) Sampling interval = 10

[Ans: (c) Lottery method]

2 MARKS

1. A random sample of size 50 with mean 67.9 is drawn from a normal population. If it is known that the standard error of the sample mean is $\sqrt{0.7}$, find 95% confidence interval for the population mean.

Sol : Give sample size $n = 50$

$$\text{Sample mean } \bar{x} = 67.9$$
$$\text{S.E.} = \sqrt{0.7}$$

95% confidence interval for population mean μ are $\bar{x} - z_{\frac{\alpha}{2}}(\text{S.E}) \le \mu \le \bar{x} + z_{\frac{\alpha}{2}}(\text{S.E})$

As the level of significance is $\alpha = 0.05, Z_{\frac{\alpha}{2}} = 1.96$

$\therefore\ 67.9-(1.96)\,(\sqrt{0.7}\,) \le \mu \le 67.9+(1.96)\ \sqrt{0.7}$

$\Rightarrow 67.9 - 1.64 \le \mu \le 67.9 + 1.64$

$\Rightarrow 66.2 \le \mu \le 69.54$

Thus, the 95% confidence intervals for estimating μ is given by (66.2, 69.54).

2. Out of 1000 T.V. viewers, 320 watched a particular programme. Calculate the standard error .

Sol : Sample size $n = 1000$

Sample proportion of T.V. viewers $p = \frac{320}{1000} = 0.32$

$\therefore\ q = 1 - p = 1 - 0.32 = 0.68$

Standard error $= \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.32)(0.68)}{1000}}$

S.E = 0.0147

3. Out of 1500 school students, a sample of 150 selected to test the accuracy of solving a problem in B.M. and of them 10 did a mistake. Calculate the standard error of sample proportion.

Sol : Given population size N = 1500

Sample size n = 150

Sample proportion $p = \frac{10}{150} = 0.07$

$\therefore\ q = 1 - p = 1 - 0.07 = 0.93$

Standard error of sample proportion

$$= \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.07)(0.93)}{150}}$$

S.E (p) = 0.02

12th STD

Reg. No.

INSTANT SUPPLEMENTARY EXAM - JULY 2022

Part - III

Business Mathematics and Statistics

(with answers)

Time Allowed : 3.00 Hours] **[Maximum Marks : 90**

Instructions :

1. Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
2. Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer **all** the questions. **[20 × 1 = 20]**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. If $A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ then, the rank of AA^T is :

(a) 0 (b) 1 (c) 2 (d) 3

2. In a transition probability matrix all the entries are greater than or equal to :

(a) 2 (b) 1 (c) 0 (d) 3

3. $\int \frac{\sin 5x - \sin x}{\cos 3x}\, dx$ is

(a) $-\cos 2x + C$ (b) $\cos 2x + C$

(c) $-\frac{1}{4}\cos 2x + C$ (d) $-4\cos 2x + C$

4. $\int_0^4 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$ is :

(a) $\frac{20}{3}$ (b) $\frac{21}{3}$ (c) $\frac{28}{3}$ (d) $\frac{1}{3}$

5. If the marginal revenue function of a firm is $MR = e^{-\frac{x}{10}}$ then the revenue is :

(a) $-10\, e^{-\frac{x}{10}}$ (b) $1 - e^{-\frac{x}{10}}$

(c) $10\left(1 - e^{-\frac{x}{10}}\right)$ (d) $e^{-\frac{x}{10}} + 10$

6. The marginal cost function is $MC = 100\sqrt{x}$. Find AC, given that TC = 0 when the output is zero is :

(a) $\frac{200}{3}x^{\frac{1}{2}}$ (b) $\frac{200}{3}x^{\frac{3}{2}}$

(c) $\frac{200}{3x^{3/2}}$ (d) $\frac{200}{3x^{1/2}}$

7. The differential equation $\left(\frac{dx}{dy}\right)^3 + 2y^{1/2} = x$ is

(a) of order 2 and degree 1

(b) of order 1 and degree 3

(c) of order 1 and degree 6

(d) of order 1 and degree 2

8. The variable separable form of $\frac{dy}{dx} = \frac{y(x-y)}{x(x+y)}$ by taking $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

(a) $\frac{2v^2}{1+v}dv = \frac{dx}{x}$ (b) $\frac{2v^2}{1+v}dv = -\frac{dx}{x}$

(c) $\frac{2v^2}{1-v}dv = \frac{dx}{x}$ (d) $\frac{1+v}{2v^2}dv = -\frac{dx}{x}$

9. If $h = 1$ then $\Delta\,(x^2) =$:

(a) $2x$ (b) $2x - 1$

(c) $2x + 1$ (d) 1

10. If n is a positive integer then $\Delta^n[\Delta^{-n} f(x)]$ is :

(a) $f(2x)$ (b) $f(x+h)$

(c) $f(x)$ (d) $\Delta f(x)$

11. A variable that can assume any possible value between two points is called :

(a) Discrete random variable
(b) Continuous random variable
(c) Discrete sample space
(d) Random variable

12. If C is a constant then, E(C) is :

(a) 0 (b) 1 (c) $Cf(C)$ (d) C

13. In turning out of certain toys in a manufacturing company the average number of defectives is 1%. The probability that the sample of 100 toys there will be 3 defectives is :

(a) 0.0613 (b) 0.613
(c) 0.00613 (d) 0.3913

14. Capetown is estimated to have 21% of homes whose owners subscribe to the satellite service, DSTV. If a random sample of four home is taken, what is the probability that all four homes subscribe to DSTV?

(a) 0.2100 (b) 0.5000
(c) 0.8791 (d) 0.0019

15. Which one of the following is probability sampling?

(a) Purposive sampling
(b) Judgement sampling
(c) Simple random sampling
(d) Convenience sampling

16. An estimator is a sample statistic used to estimate a :

(a) Population parameter
(b) Biased estimate
(c) Sample size
(d) Census

17. A time series consists of :

(a) Five components
(b) Four components
(c) Three components
(d) Two components

18. Least square method of fitting a trend is :

(a) Most exact
(b) Least exact
(c) Full of subjectivity
(d) Mathematically unsolved

19. In a degenerarte solution number of allocations is :

(a) equal to $m + n - 1$
(b) not equal to $m + n + 1$
(c) less than $m + n - 1$
(d) greater than $m + n + 1$

20. A type of decision - making environment is :

(a) certainty (b) uncertainty
(c) risk (d) all of the above

PART - II

Note : Answer **any seven** questions. Question number **30** is **compulsory.** **7 × 2 = 14**

21. Find the rank of the matrix $\begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$.

22. Integrate with respect to x $\int \sqrt{3x+5}\; dx$.

23. The marginal cost function MC = $300\,x^{\frac{2}{5}}$ and fixed cost is zero. Find out the total cost and average cost functions.

24. Find the differential equation : $xy = C^2$.

25. Evaluate $\Delta^2\left(\frac{1}{x}\right)$ by taking $h = 1$.

26. In an investment a man can make a profit of ₹5,000 with a probability of 0.62 or a loss of ₹8,000 with a probability of 0.38. Find the expected gain.

27. Mention the properties of poisson distribution.

28. Find the sample size for the given standard deviation 10 and the standard error with respect of sample mean is 3.

29. What do you mean by balanced transportation problem?

30. Fit a trend line by the method of free hand method for the given data.

Year	2000	2001	2002	2003	2004	2005	2006	2007
Sales (in tones)	30	46	25	59	40	60	38	65

PART - III

Note : Answer **any seven** questions. Question number **40** is **compulsory.** **7 × 3 = 21**

31. Show that the equations $2x + y = 5$, $4x + 2y = 10$ are consistent and solve them.

32. Evaluate : $\int x^3 e^x dx$

33. Form the differential equation having for its general solution $y = ax^2 + bx$.

34. Given $y_3 = 2, y_4 = -6, y_5 = 8, y_6 = 9$ and $y_7 = 17$. Calculate $\Delta^4 y_3$.

35. Find the expected value for the random variable of an unbiased die.

36. The mortality rate for a certain disease is 7 in 1000. What is the probability for just 2 deaths on account of this disease in a group of 400? [Given $e^{-2.8} = 0.06$]

37. A wholesaler in apple claims that only 4% of apples supplied by him are defective. A random sample of 600 apples contained 36 defective apples. Calculate the standard error concerning of good apples.

38. Fit a trend line by the method of semi-averages for the given data.

Year	2000	2001	2002	2003	2004	2005	2006
Production ('000)	105	115	120	100	110	125	135

39. Consider the following pay-off matrix.

Alternative	PAY - OFF (Conditional events)			
	A_1	A_2	A_3	A_4
E_1	7	12	20	27
E_2	10	9	10	25
E_3	23	20	14	23
E_4	32	24	21	17

Using min-max principle determine the best alternative.

40. Find the area bounded by $y = 4x + 3$ with x - axis between the lines $x = 1$ and $x = 4$.

PART - IV

Note : Answer ***all*** the questions. **7 × 5 = 35**

41. (a) Two products A and B currently share the market with shares 50% and 50% each respectively. Each week some brand switching takes place. Of those who bought A the previous week, 60% buy it again whereas 40% switch over to B. Of those who bought B the previous week, 80% buy it again whereas 20% switch over to A. Find their shares after one week and after two weeks. If the price war continues, when is the equilibrium reached?

(OR)

(b) Evaluate $\int \frac{3x+2}{(x-2)^2(x-3)}\,dx$.

42. (a) Evaluate $\int_0^2 f(x)\,dx$ where

$$f(x) = \begin{cases} 3-2x-x^2, & x \le 1 \\ x^2+2x-3 & 1 < x \le 2 \end{cases}$$

(OR)

(b) The demand and supply functions under perfect competition are $Pd = 25 - 3x$ and $Ps = 5 + 2x$ respectively. Find the consumer surplus and produce4r surplus.

43. (a) The area A of circle of diameter 'D' is given for the following values.

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the approximate values for the area of circle of diameter 82.

(OR)

(b) Out of 750 families with 4 children each, how many families would be expected to have (i) atleast one boy (ii) atmost 2 girls (iii) children of both sexes ? Assume equal probabilities for boys and girls.

44. (a) Find the area of the region lying in the first quadrant bounded by the region $y = 4x^2$, $x = 0$, $y = 0$ and $y = 4$.

(OR)

(b) Given below are the data relating to the production of sugarcane in a district. Fit a straight line trend by the method of least squares and tabulate the trend values.

Year	2000	2001	2002	2003	2004	2005	2006
Production of Sugarcane (in tons)	40	45	46	42	47	50	46

45. (a) The probability density function of a random variable X is $f(x) = ke^{-|x|}$, $-\infty < x < \infty$. Find the value of k and also fine mean and variable for the random variable.

(OR)