12th STD

Reg. No. ☐☐☐☐☐☐

PUBLIC EXAMINATION - MARCH 2023

Part - III

Business Mathematics and Statistics

(with answers)

Time Allowed : 3.00 Hours] [Maximum Marks : 90

Instructions :

1. Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
2. Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer **all** the questions. **20 × 1 = 20**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. The rank of $m \times n$ matrix whose elements are unity is :

(a) m (b) 0 (c) n (d) 1

2. If $|A_{n \times n}| = 3$ and $|\text{adj } A| = 243$ then the value of 'n' is :

(a) 6 (b) 4 (c) 7 (d) 5

3. The value of $\int \frac{\sin 2x}{2 \sin x} dx$ is :

(a) $\cos x + c$ (b) $\sin x + c$

(c) $\frac{1}{2} \cos x + c$ (d) $\frac{1}{2} \sin x + c$

4. The value of $\int \sqrt{e^x}\, dx$ is :

(a) $\frac{1}{2}\sqrt{e^x} + c$ (b) $\sqrt{e^x} + c$

(c) $\frac{1}{2\sqrt{e^x}} + c$ (d) $2\sqrt{e^x} + c$

5. The profit of a function $p(x)$ is maximum when :

(a) MR = 0 (b) MC – MR = 0

(c) MC + MR = 0 (d) MC = 0

6. The demand function for the marginal function $MR = 100 - 9x^2$ is :

(a) $100x - 9x^2$ (b) $100 - 3x^2$

(c) $100 + 9x^2$ (d) $100x - 3x^2$

7. The differential equation formed by eliminating A and B from $y = e^{-2x}(A \cos x + B \sin x)$ is :

(a) $y_2 - 4y_1 - 5 = 0$ (b) $y_2 - 4y_1 + 5 = 0$

(c) $y_2 + 4y_1 + 5 = 0$ (d) $y_2 + 4y_1 - 5 = 0$

8. The Particular Integral of $(3D^2 + D - 14)\, y = 13e^{2x}$ is :

(a) $\frac{x^2}{2} e^{2x}$ (b) $\frac{x}{2} e^{2x}$

(c) $13xe^{2x}$ (d) xe^{2x}

9. $E \equiv$

(a) $1 + \nabla$ (b) $1 + \Delta$

(c) $1 - \nabla$ (d) $1 - \Delta$

10. $E\,(Ey_0) =$

(a) y_2 (b) y_0 (c) y_3 (d) y_1

11. E [X – E (X)] is equal to :

(a) 0 (b) E(X)

(c) E(X) – X (d) V(X)

12. If $p(x) = \frac{1}{10}$, $x = 10$ then E(X) is :

(a) 1 (b) zero (c) –1 (d) $\frac{6}{8}$

13. Normal distribution was invented by :

(a) Gauss (b) Laplace

(c) James Bernoulli (d) De-Moivre

14. In a parametric distribution the mean is equal to variance is :

(a) normal (b) poisson

(c) binomial (d) all of the above

15. A finite subset of statistical individuals in a population is called ________.
(a) universe (b) a sample
(c) census (d) a population

16. An estimator is sample statistics used to estimate a :
(a) sample size (b) population parameter
(c) census (d) biased estimate

17. Fisher's price index number is the ______ between Laspeyre's and Paasche's price index number.
(a) Arithmetic mean
(b) Geometric mean
(c) Harmonic mean
(d) both (a) and (c)

18. The seasonal variation means the variations occurring within :
(a) a month (b) some years
(c) a week (d) a year

19. The transportation problem is said to be unbalanced if ________.
(a) $m = n$
(b) Total supply ≠ Total demand
(c) $m + n - 1$
(d) Total supply = Total demand

20. North-West corner refers to ________.
(a) bottom right corner
(b) top left corner
(c) bottom left corner
(d) top right corner

PART - II

Note : Answer **any seven** questions. Question No. **30** is **compulsory.** **7 × 2 = 14**

21. Find the rank of the matrix $\begin{bmatrix} 0 & -1 & 5 \\ 2 & 4 & -6 \\ 1 & 1 & 5 \end{bmatrix}$.

22. Evaluate : $\int(3 + x)(2 - 5x)\, dx$.

23. Find the area bounded by the curve $y = 4x + 3$ with x - axis between the lines $x = 1$ and $x = 4$.

24. Evaluate : $\Delta(\log ax)$

25. The following information is the probability distribution of successes.

No. of successes X $=x$	0	1	2
Probability P(x)	$\frac{6}{11}$	$\frac{9}{22}$	$\frac{1}{22}$

Determine the expected number of success.

26. Define Binomial distribution.

27. State any two merits of simple random sampling.

28. Solve : $\frac{dy}{dx} = xy + x + y + 1$.

29. Give mathematical form of Assignment problem.

30. For $\Sigma p_0 q_0 = 1974$, $\Sigma p_1 q_0 = 3140$, $\Sigma p_1 q_1 = 2005$ find the Cost of Living Index by Aggregate Expenditure Method.

PART - III

Note : Answer **any seven** questions. Question No. **40** is **compulsory.** **7 × 3 = 21**

31. Find k, if the equations $x + 2y - 3z = -2$, $3x - y - 2z = 1$, $2x + 3y - 5z = k$ are consistent.

32. If MR $= 20 - 5x + 3x^2$, find total revenue function.

33. Solve : $9y'' - 12y' + 4y = 0$.

34. Find the missing entry in the following table.

x	0	1	2	3	4
y_x	1	3	9	-	81

35. Consider a random variable X with probability density function, $f(x) = \begin{cases} 4x^3, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$
Find E(X) and V(X).

36. In a book of 520 pages, 390 typo-graphical errors occur. Assuming Poisson law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error.

37. Find the sample size for the given standard deviation 10 and the standard error with respect of sample mean is 3.

38. Construct the cost of living index number for 2011 on the basis of 2007 from the given data using Family Budget method.

Commodities	**Price**		**Weights**
	2007	**2011**	
A	350	400	40
B	175	250	35
C	100	115	15
D	75	105	20
E	60	80	25

39. From the following pay-off matrix, find the optimal decision under each of the following rule (i) maxmin (ii) minimax.

Act	States of nature			
	S_1	S_2	S_3	S_4
A_1	14	9	10	5
A_2	11	10	8	7
A_3	9	10	10	11
A_4	8	10	11	13

40. Evaluate : $\int \frac{1}{\sqrt{x+2}-\sqrt{x+3}}dx$.

PART - IV

Note : Answer ***all*** questions. **7 × 5 = 35**

41. (a) Solve by Cramer's rule, $x + y + z = 4$; $2x - y + 3z = 1$; $3x + 2y - z = 1$.

(OR)

(b) The population of a certain town is as follows.

Year : X	1941	1951	1961	1971	1981	1991
Population in lakhs : Y	20	24	29	36	46	51

Using appropriate interpolation formula, estimate the population during the period 1946.

42. (a) Evaluate the integral as the limit of a sum $\int_1^2 (2x+1)dx$.

(OR)

(b) Find the probability of guessing correctly atleast six of the ten answers in a TRUE / FALSE objective test.

43. (a) Find the consumer's surplus and producer's surplus for the demand function $P_d = 25 - 3x$ and supply function $P_s = 5 + 2x$.

(OR)

(b) X is normally distributed with mean 12 and S.D. 4.
Find P $(X \leq 20)$ and $P(0 \leq X \leq 12)$
$[P(0 < Z < 2) = 0.4772]$

44. (a) Solve : $(D^2 - 2D + 1)\, y = e^{2x} + e^x$.

(OR)

(b) An ambulance service claims that it takes on an average 8.9 minutes to reach its destination in emergency calls. To check on this claim, the agency which licenses ambulance services, has then timed on 50 emergency calls, getting a mean of 9.3 minutes with a standard deviation of 1.6 minutes. What can they conclude at 5% level of significance?

45. (a) Using Lagrange's interpolation formula find y (10) from the following table.

X	5	6	9	11
Y	12	13	14	16

(OR)

(b) Solve : $\frac{dy}{dx} + \frac{y}{x} = x^3$

46. (a) Compute (i) Laspeyre's (ii) Paasche's (iii) Fisher's Index numbers for the year 2010 from the following data.

Commodity	Price		Quantity	
	2000	2010	2000	2010
A	12	14	18	16
B	15	16	20	15
C	14	15	24	20
D	12	12	29	23

(OR)

(b) The probability function of a random variable X is given by
$$p(x) = \begin{cases} \frac{1}{4}, & \text{for } x = -2 \\ \frac{1}{4}, & \text{for } x = 0 \\ \frac{1}{2}, & \text{for } x = 10 \\ 0, & \text{elsewhere} \end{cases}$$

Evaluate the following probabilities.
(i) $P(X \leq 0)$, (ii) $P(X < 0)$,
(iii) $P(|X| \leq 2)$ and (iv) $P(0 \leq X \leq 10$

47. (a) Given below are the values of sample mean $(\overline{X})$ and the range (R) for ten samples of size 5 each. Draw mean chart and comment on the state of control of the process.

Sample Number	1	2	3	4	5	6	7	8	9	10
$(\overline{X})$	43	49	37	44	45	37	51	46	43	47
R	5	6	5	7	7	4	8	6	4	6

Given the following control chart constraint for : $n = 5$, $A_2 = 0.58$, $D_3 = 0$ and $D_4 = 2.115$.

(OR)

(b) Obtain an initial basic feasible solution to the following transportation problem using Vogel's approximation method.

Warehouses	Stores				Availability
↓	I	II	III	IV	(a_i)
A	5	1	3	3	34
B	3	3	5	4	15
C	6	4	4	3	12
D	4	1	4	5	19
Requirement (b_j)	21	25	17	17	

* * *

ANSWERS

PART - I

1. (d) 1
2. (a) 6
3. (b) $\sin x + c$
4. (d) $2\sqrt{e^x} + c$
5. (b) MC – MR = 0
6. (b) $100 - 3x^2$
7. (c) $y_2 + 4y_1 + 5 = 0$
8. (d) xe^{2x}
9. (b) $1 + \Delta$
10. (a) y_2
11. (a) 0
12. (a) 1
13. (d) De-Moivre
14. (b) poisson
15. (b) a sample
16. (b) population parameter
17. (b) Geometric mean
18. (d) a year
19. (b) Total supply ≠ Total demand
20. (b) top left corner

PART - II

21. Let $A = \begin{pmatrix} 0 & -1 & 5 \\ 2 & 4 & -6 \\ 1 & 1 & 5 \end{pmatrix}$

Order of A is 3×3.

$\therefore p(A) \leq 3$

Consider the third order minor

$$\begin{vmatrix} 0 & -1 & 5 \\ 2 & 4 & -6 \\ 1 & 1 & 5 \end{vmatrix} = 6 \neq 0$$

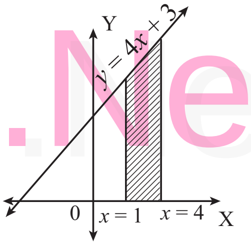
There is a minor of order 3, which is not zero

$\therefore p(A) = 3$.

22. $\int (3+x)\ (2-5\,x)\ dx$

$$= \int (6 - 15x + 2x - 5x^2)\ dx$$
$$= \int (6 - 13x - 5x^2)\ dx$$
$$= \int 6dx - \int 13x\ dx - \int 5x^2\ dx$$
$$= 6x - \frac{13x^2}{2} - \frac{5x^3}{3} + c$$

23. Area $= \int_1^4 ydx$



$$= \int_1^4 (4x+3)\ dx$$
$$= \left[2x^2 + 3x\right]_1^4$$
$$= 32 + 12 - 2 - 3$$
$$= 39 \text{ sq. units}$$

24. $\Delta\ (\log ax) = \log a\ \ (x + h) - \log ax$

$$= \log\left[\frac{a(x+h)}{ax}\right] = \log\left[1 + \frac{h}{x}\right].$$

25. Expected number of success is

$$E(X) = \sum_x x P_X\ (x)$$
$$= \left(0 \times \frac{6}{11}\right) + \left(1 \times \frac{9}{22}\right) + \left(2 \times \frac{1}{22}\right) = \frac{11}{22} = 0.5$$

Therefore, the expected number of success is 0.5.Approximately one success.

26. A random variable X is said to follow binomial distribution with parameter n and p, if it assumes only non-negative value and its probability mass function is given by

$$P(X = x) = p(x) = \begin{cases} {}^nC_x p^x q^{n-x}, x = 0,1,2,.....n; \\ \qquad q = 1 - p \\ 0, \qquad \text{otherwise} \end{cases}$$

27. **1.** This method is economical as it saves time, money and labour.

2. Personal bias is completely eliminated.

28.
$$\frac{dy}{dx} = xy + x + y + 1$$
$$\Rightarrow \frac{dy}{dx} = x(y+1) + 1(y+1)$$
$$\Rightarrow \frac{dy}{dx} = (y+1)(x+1)$$

Separating the variables we get,
$$\frac{dy}{y+1} = (x+1)\,dx$$

Integrating both sides we get,
$$\int \frac{dy}{y+1} = \int (x+1)\,dx$$
$$\log(y+1) = \frac{x^2}{2} + x + c.$$

29. The mathematical form of assignment problem is

$$\text{Minimize Z} = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}\, x_{ij} \text{ subject to the}$$

Constraints $\sum_{i=1}^{n} x_{ij} = 1, j = 1,2,.... n$

$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, ...n$ and $x_{ij} = 0$ or 1 for all i, j

30. Cost of living Index Number
$$= \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$
$$= \frac{3140}{1974} \times 100$$
$$= 159.0679$$

PART - III

31. The matrix equation corresponding to the given system is
$$\begin{pmatrix} 1 & 2 & -3 \\ 3 & -1 & -2 \\ 2 & 3 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ k \end{pmatrix}$$
A X = B

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 1 & 2 & -3 & -2 \\ 3 & -1 & -2 & 1 \\ 2 & 3 & -5 & k \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & 7 \\ 0 & -1 & 1 & 4+k \end{pmatrix}$	$R_2 \to R_2 - 3R_1$ $R_3 \to R_3 - 2R_1$
$\sim \begin{pmatrix} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & 7 \\ 0 & 0 & 0 & 21+7k \end{pmatrix}$	$R_3 \to 7R_3 - R_2$
$\rho(A) = 2, \rho([A, B]) = 2$ or 3	

For the equations to be consistent,
$$\rho([A, B]) = \rho(A) = 2$$
$$\therefore 21 + 7k = 0$$
$$7k = -21$$
$$k = -3$$

32. Given $\text{MR} = 20 - 5x + 3x^2$
$$\Rightarrow \frac{dR}{dx} = 20 - 5x + 3x^2$$
$$\Rightarrow dR = (20 - 5x + 3x^2)\,dx$$
$$\Rightarrow \int dR = \int(20 - 5x + 3x^2)\,dx$$
$$\Rightarrow R = 20x - \frac{5x^2}{2} + \frac{3x^3}{3} + k$$

When $x = 0$, $R = 0 \Rightarrow k = 0$
$$\therefore R = 20x - \frac{5x^2}{2} + x^3$$

33. Given $(9D^2 - 12D + 4)y = 0$

The auxiliary equation is $(3m - 2)^2 = 0$

$(3m-2)(3m-2) = 0 \Rightarrow m = \frac{2}{3}, \frac{2}{3}$

Roots are real and equal.

The C.F. is $(Ax + B)e^{\frac{2}{3}x}$

The general solution is $y = (Ax + B)e^{\frac{2}{3}x}$

34. Since only four values of $f(x)$ are given, the polynomial which fits the data is of degree 3. Hence fourth differences are zero.
$$\therefore \Delta^4(y_0) = 0$$
$$\Rightarrow (E-1)^4(y_0) = 0$$
$$\Rightarrow (E^4 - 4E^3 + 6E^2 - 4E + 1)\,y_0 = 0$$
$$\Rightarrow y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$\Rightarrow 81 - 4(y_3) + 6(9) - 4(3) + 1 = 0$$
$$\Rightarrow \quad 81 - 4y_3 + 54 - 12 + 1 = 0$$
$$\Rightarrow \quad 81 + 54 - 11 = 4y_3$$
$$\Rightarrow \quad 124 = 4y_3$$
$$\Rightarrow \quad y_3 = \frac{124}{4} = 31.$$
$$\Rightarrow \quad y_3 = 31$$

35. We know that

$$\text{E (X)} = \int_{-\infty}^{\infty} x\,f(x)dx = \int_0^1 x4x^3\,dx = 4\left[\frac{x^5}{5}\right]_0^1$$

$$\text{E(X)} = \frac{4}{5}$$

$$\text{E(X}^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_0^1 x^2\,4x^3\,dx$$

$$= 4\left[\frac{x^6}{6}\right]_0^1 = \frac{4}{6} = \frac{2}{3}$$

$$\text{V(X)} = \text{E(X}^2) - [\text{E(X)}]^2$$

$$= \frac{4}{6} - \left[\frac{4}{5}\right]^2 = \frac{2}{75}$$

36. The average number of typographical errors per page in the book is given by

$\lambda = (390/520) = 0.75$.

Hence using Poisson probability law, the probability of x errors per page is given by

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} = e^{-0.75}\frac{(0.75)^x}{x!},\ x = 0,1,2,\ldots$$

The required probability that a random sample of 5 pages will contain no error is given by :

$$[P(X = 0)]^5 = (e^{-0.75})^5 = e^{-3.75}$$

37. Given $\sigma = 10$, S.E. $\overline{X} = 3$ We know that $\text{S.E} = \dfrac{\sigma}{\sqrt{n}}$

$$\text{Therefore, } 3 = \frac{10}{\sqrt{n}} \Rightarrow \sqrt{n} = \frac{10}{3}$$

Taking Squaring on both sides we get

$$n = \left(\frac{10}{3}\right)^2 = \frac{100}{9} = 11.11 \cong 11,$$

The required sample size is 11.

38.

Commodity	Price 2012 (p_0)	Price 2015 (p_0)	Weights (V)	$P = \frac{p_1}{p_0} \times 100$	PV
A	350	400	40	114.285	4571.4
B	175	250	35	142.857	4999.99
C	100	115	15	115	1725
D	75	105	20	114	2280
E	60	80	25	133.333	3333.33
		Total	135		16909.7315

$$\text{Cost of Living Index Number} = \frac{\sum PV}{\sum V}$$

$$= \frac{16909.7315}{135} = 125.2572$$

Hence, the Cost of Living Index Number for a particular class of people for the year 2011 is increased by 25.25 % as compared to the year 2007.

39.

Act	States of Nature S$_1$	S$_2$	S$_3$	S$_4$	Minimum pay off	Maximum Pay off
A$_1$	14	9	10	5	5	14
A$_2$	11	10	8	7	7	11
A$_3$	9	10	10	11	9	11
A$_4$	8	10	11	13	8	13

(i) Max (5, 7, 9, 8) = 9

$\therefore$ A$_3$ is the optimal decision according to maximin principle

(ii) Min (14, 11, 11, 13) = 11

$\therefore$ A$_2$ and A$_3$ are optimal decisions according to minimax principle

40.

$$I = \int \frac{1}{\sqrt{x+2}-\sqrt{x+3}}\,dx$$

Multiply and divide with the conjugate of the denominator,

$$\text{We get I} = \int \frac{\sqrt{x+2}+\sqrt{x+3}}{\left(\sqrt{x+2}-\sqrt{x+3}\right)\left(\sqrt{x+2}+\sqrt{x+3}\right)}$$

$$= \int \frac{\sqrt{x+2}+\sqrt{x+3}}{(x+2)-(x+3)}\,dx \; [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \int \frac{\sqrt{x+2}+\sqrt{x+3}}{\cancel{x}+2-\cancel{x}-3}\,dx = -\int\left(\sqrt{x+2}+\sqrt{x+3}\right)dx$$

$$= \left[-\frac{(x+2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{(x+3)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]+c$$

$$= -\frac{2}{3}\left[(x+2)^{\frac{3}{2}}+(x+3)^{\frac{3}{2}}\right]+c$$

PART - IV

41. (a) Here $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{vmatrix} = 13 \neq 0$

$\therefore$ We can apply Cramer's Rule and the system is consistent and it has unique solution.

$$\Delta_x = \begin{vmatrix} 4 & 1 & 1 \\ 1 & -1 & 3 \\ 1 & 2 & -1 \end{vmatrix} = -13$$

$$\Delta_y = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & -1 \end{vmatrix} = 39$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 4 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 26$$

$\therefore$ By Cramer's rule, $x = \frac{\Delta_x}{\Delta} = \frac{-13}{13} = -1$

$y = \frac{\Delta_y}{\Delta} = \frac{39}{13} = 3$

$z = \frac{\Delta_z}{\Delta} = \frac{26}{13} = 2$

$\therefore$ The solution is $(x, y, z) = (-1, 3, 2)$

(OR)

(b)

x	1941	1951	1961	1971	1981	1991
y	20	24	29	36	46	51

Here we find the population for year1946. (i.e) the value of y at x =1946. Since the value of y is required near the beginning of the table, we use the Newton's forward interpolation formula.

$$y_{(x = x_0 + nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \ldots$$

To find y at x = 1946

$\therefore x_0 + nh$ = 1946, $x_0 = 1941$, $h = 10$

$1941 + n(10) = 1946 \Rightarrow n = 0.5$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	**20**					
		4				
1951	24		**1**			
		5		**1**		
1961	29		2		**0**	
		7		1		**–9**
1971	36		3		– 9	
		10		– 8		
1981	46		– 5			
		5				
1991	51					

$$y_{(x = 1946)} = 20 + \frac{0.5}{1!}(4) + \frac{0.5(0.5-1)}{2!}(1) + \frac{0.5(0.5-1)(0.5-2)}{3!}(1) + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!}(0) + \frac{0.5(0.5-1)(0.5-2)(0.5-3)(0.5-4)}{5!}(-9)$$

$$= 20 + 2 - 0.125 + 0.0625 - 0.24609$$

$$= 21.69 \text{ lakhs}$$

42. (a)

$$\int_a^b (2x+1)\,dx = \lim_{\substack{n\to\infty\\ h\to 0}} \sum_{r=1}^{n} h\, f(a+rh)$$

Here $a = 1,\ b = 2,\ h = \frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$ and $f(x) = 2x + 1$

$$f(a+rh) = f\left(1+\frac{r}{n}\right) = 2\left(1+\frac{r}{n}\right)+1 = 2 + \frac{2r}{n} + 1$$

$$f(a+rh) = 3 + \frac{2r}{n}$$

$$\int_1^2 f(x)\,dx = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(3+\frac{2r}{n}\right) = \operatorname*{lt}_{n\to\infty} \sum_{r=1}^{n} \left(\frac{3}{n}+\frac{2r}{n}\right) = \lim_{n\to\infty}\left[\frac{3}{n}\sum_{r=1}^{n} 1 + \frac{2}{n^2}\sum_{r=1}^{n} r\right]$$

$$= \lim_{n\to\infty}\left[\frac{3}{n}\cdot n + \frac{2}{n^2}\,\frac{n(n+1)}{2}\right] = 3 + \lim_{n\to\infty}\left(1+\frac{1}{n}\right)$$

$$\int_1^2 f(x)\,dx = 3 + 1 = 4$$

(OR)

(b) Probability p of guessing an answer correctly is $p = \frac{1}{2}$

$$\Rightarrow \quad q = \frac{1}{2}$$

Probability of guessing correctly x answers in 10 questions

$$\text{P}(\text{X} = x) = p(x) = {}^{n}\text{C}_{x} p^{x} q^{n-x}$$

$$= 10\text{C}x \left(\frac{1}{2}\right)^{x} \left(\frac{1}{2}\right)^{10-x}$$

The required probability $\text{P}(\text{X} \geq 6)$

$$= \text{P}(6) + \text{P}(7) + \text{P}(8) + \text{P}(9) + \text{P}(10)$$

$$= \left(\frac{1}{2}\right)^{10} [10\text{C}_6 + 10\text{C}_7 + 10\text{C}_8 + 10\text{C}_9 + 10\text{C}_{10}]$$

$$= \left[\frac{1}{1024}\right][210 + 120 + 45 + 10 + 1] = \frac{193}{512}$$

43. (a) Given demand function $p_d = 25 - 3x$ and supply function $p_s = 5 + 2x$

At market equilibrium, $p_d = p_s$

$$\Rightarrow \quad 25 - 3x = 5 + 2x$$

$$\Rightarrow \quad 25 - 5 = 2x + 3x$$

$$\Rightarrow \quad 20 = 5x$$

$$\Rightarrow \quad x = \frac{20}{5}$$

$$\Rightarrow \quad x_0 = 4$$

When $x_0 = 4$, $p_0 = 25 - 3(4) = 25 - 12$

$$p_0 = 13$$

$$\therefore\ p_0x_0 = 13(4) = 52$$

$\therefore$ Consumer's Surplus

$$CS = \int_0^x f(x)\,dx - p_0x_0 = \int_0^4 (25-3x)\,dx - 52$$

$$= \left[25x - \frac{3x^2}{2}\right]_0^4 - 52 = 25(4) - \frac{3(4^2)}{2} - 52$$

$$= 100 - 24 - 52 = 100 - 76$$

$$CS = 24 \text{ units}$$

Producer's Surplus

$$(PS) = p_0x_0 - \int_0^x g(x)\,dx$$

$$= 52 - \int_0^4 (5+2x)\,dx$$

$$= 52 - \left(5x + \frac{\cancel{2}x^2}{\cancel{2}}\right)_0^4$$

$$= 52 - (5(4) + 4^2)$$

$$= 52 - (20 + 16)$$

$$= 52 - 36$$

$$PS = 16 \text{ units}$$

(OR)

(b) Given $\mu = 12$ and $\sigma = 4$

(i) $P(X \leq 20)$

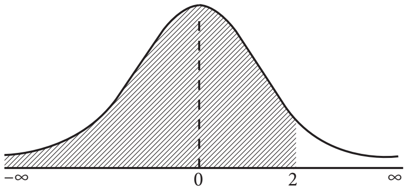
$$\text{When } X = 20,\ Z = \frac{X-\mu}{\sigma} = \frac{20-12}{4} = \frac{8}{4} = 2$$

$$\therefore\ P(X \leq 20) = P(Z \leq 2)$$

$$= P(-\infty < Z < 0) + P(0 \leq Z \leq 2)$$

$$= 0.5 + 0.4772$$

$$P(X \leq 20) = 0.9772.$$



(ii) **$P(0 \le X \le 12)$**

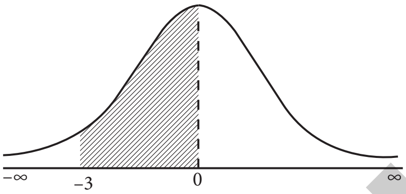
$$\text{When X} = 0, Z = \frac{X-\mu}{\sigma} = \frac{0-12}{4} = \frac{-12}{4} = -3$$

$$\text{When X} = 12, Z = \frac{X-\mu}{\sigma} = \frac{12-12}{4} = \frac{0}{4} = 0$$

$$\therefore P(0 \le X \le 12) = P(-3 \le Z \le 0)$$

$$= P(0 \le Z \le 3) \text{ (By symmetry)}$$

$$P(0 \le X \le 12) = 0.4987.$$



44. (a) $(D^2 - 2D + 1)y = e^{2x} + e^x$

The auxiliary equation is

$$m^2 - 2m + 1 = 0$$

$$\Rightarrow \quad (m-1)(m-1) = 0$$

$$m = 1, 1$$

$$\text{C.F} = (Ax + B)e^x$$

$$\text{PI} = \frac{1}{\phi(D)} f(x) = \frac{1}{D^2 - 2D + 1}(e^{2x} + e^x)$$

Now, $$\text{P.I}_1 = \frac{1}{D^2 - 2D + 1} e^{2x}$$

$$= \frac{1}{4 - 4 + 1} e^{2x} \qquad \text{(replace D by 2)}$$

$$= e^{2x}$$

$$\text{and P.I}_2 = \frac{1}{D^2 - 2D + 1} e^{2x} = \frac{1}{(D-1)^2} e^x$$

Replace D by 1. $(D - 1)^2 = 0$ when $D = 1$

$$\therefore \text{P.I}_2 = x.\frac{1}{2(D-1)} e^x$$

Replace D by 1. $(D - 1) = 0$ when $D = 1$

$$\therefore \text{P.I}_2 = x^2 \frac{1}{2} e^x$$

The general solution is

$$y = \text{C.F} + \text{P.I}_1 + \text{P.I}_2$$

$$y = (Ax + B)e^x + e^{2x} + \frac{x^2}{2} e^x$$

(OR)

(b) Sample size n = 50

Sample mean $\bar{x}$ = 9.3 minutes

Sample S.D s = 1.6 minutes

Population mean μ = 8.9 minutes

Null hypothesis $H_0 : \mu$ = 8.9

Alternative hypothesis $H_1 : \mu$ = 8.9(Two tail)

Level of significance μ = 0.05

$$\text{Test statistic } Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

$$Z = \frac{9.3-8.9}{\frac{1.6}{\sqrt{50}}} = \frac{0.4}{0.2263} = 1.7676$$

Calculated value Z = 1.7676

Critical value at 5% level of significance is $Z_{\frac{\alpha}{2}} = 1.96$.

Inference : Since the calculated value is less than table value i.e., $Z < Z_{\frac{\alpha}{2}}$ at 5% level of significance, the null hypothesis is accepted. Therefore we conclude that an ambulance service claims on the average 8.9 minutes to reach its destination in emergency calls.

45. (a) Here the intervals are unequal. By Lagrange's interpolation formula we have

$x_0 = 5, x_1 = 6, x_2 = 9, x_3 = 11$

$y_0 = 12, y_1 = 13, y_2 = 14, y_3 = 16$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times \text{y0} + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

$$= \frac{(x-6)(x-9)(x-11)}{(5-6)(5-6)(5-11)}(12) + \frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-9)}(13)$$

$$+ \frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)}(14) + \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)}(16)$$

Put $x + 10$

$$y(10) = f(10) = \frac{4(1)(-1)}{(-1)(-4)(-6)}(12) + \frac{(5)(1)(-1)}{(1)(-3)(-5)}(13) + \frac{5(4)(-1)}{4(3)(-2)}(14) + \frac{(5)(4)(1)}{6(5)(2)}(16)$$

$$= \frac{1}{6}(12) - \frac{13}{3} + \frac{5(14)}{3 \times 2} + \frac{4 \times 16}{12} = 14.6663$$

(OR)

(b) Given $\frac{dy}{dx}+\frac{1}{x}y=x^3$

It is of the form $\frac{dy}{dx}+\text{P}y = \text{Q}$

Here $\text{P} = \frac{1}{x}, \text{Q} = x^3$

$\int \text{P}dx = \int \frac{1}{x} dx = \log x$

$\text{I.F} = e^{\int pdx} = e^{\log x} = x$

The required solution is

$y\text{ (I.F.)} = \int \text{Q(I.F)}\, dx + c$

$yx = \int x^3 . x\, dx + c = \int x^4 dx + c = \frac{x^5}{5}+c$

$\therefore yx = \frac{x^5}{5}+c$

46. **(a)**

Commodity	Price		Quantity	
	2000 (p_0)	2010 (p_1)	2000 (q_0)	2010 (q_1)
A	12	14	18	16
B	15	16	20	15
C	14	15	24	20
D	12	12	29	23

p_0q_0	p_0q_1	p_1q_0	p_1q_1
216	192	252	224
300	225	320	240
336	280	360	300
348	276	348	276
1200	**973**	**1280**	**1040**

(i) Laspeyre's index number, $\text{P}_{01}^{\text{L}} = \frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times 100 = \frac{1280}{1200}\times 100 = 106.66$

(ii) Paasche's index number, $\text{P}_{01}^{\text{P}} = \frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times 100 = \frac{1040}{973}\times 100 = 106.88$

(iii) Fisher's index number, $\text{P}_{01}^{\text{F}} = \sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times\frac{\Sigma p_1q_1}{\Sigma p_0q_1}}\times 100 = \sqrt{\frac{1280\times 1040}{1200\times 973}}\times 100$

$$= \sqrt{\frac{1331200}{1167600}} \times 100 = \sqrt{1.1401} \times 100$$

$$= 1.067 \times 100$$

$$P_{01}^{F} = 106.7$$

(OR)

(b) Given probability function is $p(x) = \begin{cases} \frac{1}{4}, & \text{for } x = -2 \\ \frac{1}{4}, & \text{for } x = 0 \\ \frac{1}{2}, & \text{for } x = 10 \\ 0, & \text{elsewhere} \end{cases}$

$X = x$	-2	0	10
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

(i) $P(X \leq 0) = P(X=-2) + P(X=0) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

(ii) $P(X < 0) = P(X = -2) = \frac{1}{4}$

(iii) $P(|X| \leq 2) = P(-2 \leq X \leq 2)$

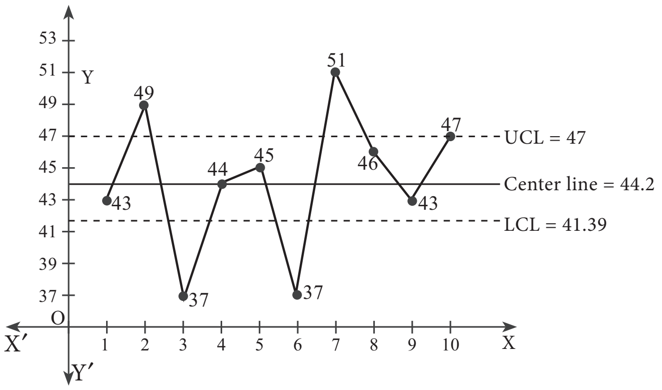
$= P(X = -2) + P(X = 0) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

(iv) $P(0 \leq X \leq 10) = P(X = 0) + P(X = 10)$

$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

47. (a) $\overline{\overline{X}} = \frac{\sum \overline{X}}{10} = \frac{442}{10} = 44.2$

$\overline{R} = \frac{\sum R}{n} = \frac{58}{10} = 5.8$



$$\text{UCL} = \overline{X} + A2\overline{R} = 44.2 + 0.483(5.8) = 47.00$$

$$\text{CL} = \overline{\overline{X}} = 44.2$$

$$\text{LCL} = \overline{\overline{X}} - A_2\overline{R} = 44.2 - 0.483(5.8) = 41.39$$

The above diagram shows all the three control lines with the data points plotted, since four points falls out of the control limits, we can say that the process is out of control.

(OR)

(b) Here $\Sigma a_i = \Sigma b_i$ =80 (i.e) Total Availability =Total Requirement

The given problem is balanced transportation problem.

Hence there exists a feasible solution to the given problem.

First Allocation:

	I	II	III	IV	a_i	Penalty
A	5	1	3	3	34	2
B	3	3	5	4	15	0
C	6	4	4	3	12	1
D	4	$^{(19)}1$	4	5	19/0	3
b_j	21	25/6	17	17		
Penalty	1	0	1	0		

Second Allocation:

	I	II	III	IV	a_i	Penalty
A	5	$^{(6)}1$	3	3	34/28	2
B	3	3	5	4	15	0
C	6	4	4	3	12	1
b_j	21	6/0	17	17		
Penalty	2	2	1	0		

Third Allocation:

	I	III	IV	a_i	Penalty
A	5	3	3	28	0
B	$^{(15)}3$	5	4	15/0	1
C	6	4	3	12	1
b_j	21/6	17	17		
Penalty	2	1	0		

Fourth Allocation:

	I	III	IV	a_i	Penalty
A	5	3	3	28	0
C	6	4	$^{(12)}3$	12/0	1
b_j	6	17	17/5		
Penalty	1	1	0		

Fifth Allocation:

	I	III	IV		Penalty
A	5	3	$^{(5)}3$	28/23	0
	6	17	5/0		
Penalty	-	-	-		

Sixth Allocation:

	I	III		Penalty
A	$^{(6)}5$	$^{(17)}3$	23/6/0	0
	6/0	17/0		
Penalty	-	-		

Thus we have the following allocations:

	I	II	III	IV	a_i
A	$^{(6)}5$	$^{(6)}1$	$^{(17)}3$	$^{(5)}3$	34
B	$^{(15)}3$	3	5	4	15
C	6	4	4	$^{(12)}3$	12
D	4	$^{(19)}1$	4	5	19/0
b_j	21	25	17	17	

Transportation schedule :

A→I, A→II, A→III, A→IV, B→I, C→IV, D→II

Total transportation cost:

$= (6 \times 5) + (6 \times 1) + (17 \times 3) + (5 \times 3) + (15 \times 3) + (12 \times 3) + (19 \times 1)$

$= 30 + 6 + 51 + 15 + 45 + 36 + 19 =$ ₹ 202

* * *