St. Anne's Academy

THE TRUSTED NAME FOR SPECIALIZED TUTORING

Holy Cross College Road, I Floor - Jafro Dental Clinic, Punnai Nagar, Nagercoil - 629 004.

KANYAKUMARI DISTRICT - COMMON HALF YEARLY EXAM 2022 (MODEL QUESTION)

MARKS: 90 **TIME: 3.00 HRS**

PART - I

I CHOOSE THE BEST ANSWER: **20X1=20**

1. If $A=\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$, then $9I_2 - A =$

(1) A^{1} (2) $\frac{A^{1}}{2}$ (3) $3A^{1}$ (4) $2A^{1}$

2. If $A \quad A^{-1}$ is symmetric, then $A^2 =$

(1) A^{1} (2) $(A \quad)^2$ (3) A^T (4) $(A^{1})^2$

3. If $\left|z-\frac{3}{z}\right|=2$, then the least value of $|z|$ is

(1) 1 (2) 2 (3) 3 (4) 5

4 The principal argument of $\frac{3}{-1+i}$ is

(1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

5. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

(1) $|k| \le 6$ (2) $k = 0$ (3) $|k| > 6$ (4) $|k| \ge 6$

6. The number of positive zeros of the polynomial $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$ is

(1) 0 (2) n (3) $< n$ (4) r

7 $\sin^{-1}\left(2\cos^2 x - 1\right)+\cos^{-1}\left(1-2\sin^2 x\right)=$

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

8. $\sin(\tan^{-1}x), |x|<1$ is equal to

(1) $\frac{x}{\sqrt{1 \quad x}}$ (2) $\frac{1}{\sqrt{1-x}}$ (3) $\frac{1}{\sqrt{1+x}}$ (4) $\frac{x}{\sqrt{1+x^2}}$

9. The length of the diameter of the circle which touches the x -axis at the point $(1,0)$ and passes through the point $(2,3)$

(1) $\frac{6}{5}$ (2) $\frac{5}{3}$ (3) $\frac{10}{3}$ (4) $\frac{3}{5}$

10. The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$ is

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

11. If the line $\frac{x-2}{3}=\frac{y-1}{5}=\frac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta=0$, then (α,β) is

(1) $(\ 5,5)$ – (2) $(\ 6,7)$ (3) $(5,\ 5)$ (4) $(6,\ 7)$

12. If the planes $\vec{r}.(2\hat{i}-\lambda\hat{j}+\hat{k})=3$ and $\vec{r}.(4\hat{i}+\hat{j}-\mu\hat{k})=5$ are parallel, then the value of λ and μ are

(1) $\frac{1}{2},\ 2$ (2) $\frac{1}{2},2$ (3) $\frac{1}{2},\ 2$ (4) $\frac{1}{2},2$

13. Angle between $y^2=x$ and $x^2=y$ at the origin is

(1) $\tan^{-1}\frac{3}{4}$ (2) $\tan^{1}\left(\frac{4}{3}\right)$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

14. The maximum value of the function $x^2 e^{2x}, x>0$ is

(1) $\frac{1}{e}$ (2) $\frac{1}{2e}$ (3) $\frac{1}{e^2}$ (4) $\frac{4}{e^4}$

15. If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(1) 0.4 cu.cm (2) 0.45 cu.cm (3) 2 cu.cm (4) 4.8 cu.cm

16. If $w(x,y,z)=x^2(y-z)+y^2(z-x)+z^2(x-y)$, then $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}$ is

(1) $xy+yz+zx$ (2) $x(y+z)$ (3) $y(z+x)$ (4) 0

17. The value of $\int_{-4}^{4}\left[\tan^{-1}\left(\frac{x^2}{x^4+1}\right)+\tan^{1}\left(\frac{x^4+1}{x^2}\right)\right]dx$ is

(1) π (2) 2π (3) 3π (4) 4π

18. The value of $\int_{0}^{\frac{\pi}{6}}\cos^3 3x\,dx$ is

(1) $\frac{2}{3}$ (2) $\frac{2}{9}$ (3) $\frac{1}{9}$ (4) $\frac{1}{3}$

19. The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/3}+x^{1/4}=0$ are respectively

(1) 2, 3 (2) 3 , 3 (3) 2, 6 (4) 2, 4

20. The integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{1+y}{\lambda}$ is

(1) $\frac{x}{e^{\lambda}}$ (2) $\frac{e^{\lambda}}{}$ (3) λe (4) e^x

PART - B

II ANSWER ANY 7 QUESTIONS (Q.NO 30 IS COMPLUSORY): 7X2=14

21. If A is a non-singular matrix of odd order, prove that $|\text{adj}\, A|$ is positive.

22. If $z_1 = \; -i$ and $z_2 = -4+3i$, find the inverse of $z\,z$ and $\frac{z_1}{z_2}$.

23. Find a polynomial equation of minimum degree with rational coefficients, having $2-\sqrt{3}$ as a root.

24. Find the value of $\tan\left(\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(-\frac{1}{2}\right)\right)$

25. Examine the position of the point $(2,3)$ with respect to the circle $x^2+y^2-6x-8y+12=0$.

26. Show that the vectors $\hat{i}+2\hat{j}-3\hat{k},\ 2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ are coplanar.

27. Evaluate: $\lim\limits_{x\to\frac{\pi}{2}} \frac{\sec x}{\tan x}$ (if necessary use l'Hôpital Rule)

28. Evaluate the following integral as the limits of sums: $\int_0^1 (5x+4)dx$

29. Show that $y = e^{x}+mx+n$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right)-1=0$.

30. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number

PART - C

III ANSWER ANY 7 QUESTIONS (Q.NO 40 IS COMPULSORY): 7X3=21

31. Solve the following systems of linear equation by Gaussian elimination method:
$2x-2y+3z=2,\quad x+2y-z=3,\quad 3x-y+2z=1$

32. If $|z|=2$ show that $3\le|z+3+4i|\le 7$

33. If α,β,γ, and δ are the roots of the polynomial equation $2x^4+5x^3-7x^2+8=0$, find a quadratic equation with integer coefficients whose roots are $\alpha+\beta+\gamma+\delta$ and $\alpha\beta\gamma\delta$.

34. Prove that $\tan^{-1}x+\tan^{-1}\frac{2x}{1-x^2}=\tan^{-1}\frac{3x-x^3}{1-3x^2},\ |x|<\frac{1}{\sqrt{3}}$.

35. If the normal at the point 't_1' on the parabola $y^2=4ax$ meets the parabola again at the point 't_2', then prove that $t_2=-\left(t_1+\frac{2}{t_1}\right)$.

36. Find the parametric form of vector equation and Cartesian equations of the straight line passing through the point $(-2,3,4)$ and parallel to the straight line $\frac{x-1}{-4}=\frac{y+3}{5}=\frac{8-z}{6}$.

37. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

38. Evaluate $\int_0^{2a} x^2\sqrt{2ax-x^2}\,dx$.

39. Solve: $\frac{dy}{dx}=e^{x+y}+x^3e^y$

40. Evaluate : $\lim\limits_{x\to\infty}(1+2x)^{\frac{1}{2\log x}}$.

PART - D

IV ANSWER THE FOLLOWING QUESTIONS: 7X5=35

41 (a). Investigate the values of λ and μ the system of linear equations $2x+3y+5z=9$, $7x+3y-5z=8$, $2x+3y+\lambda z=\mu$, have

(i) no solution (ii) a unique solution (iii) an infinite number of solutions.

Or

(b). If we blow air into a balloon of spherical shape at a rate of 1000 cm^3 per second, at what rate the radius of the baloon changes when the radius is 7cm? Also compute the rate at which the surface area changes.

42 (a). Solve the equation $z^3+8i=0$, where $z\in\mathbb{C}$.

Or

(b) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points $(3,6,\ 2),(\ 1,\ 2,6)$, and $(6,4,-2)$.

43 (a). Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution.

Or

(b) A boy is walking along the path $y=ax^2+bx+c$ through the points $(\ 6,8),(\ 2,\ 12)$, and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.)

44 (a) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :
$9x^2 - y^2 - 36x - 6y + 18 = 0$

Or

(b) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d,
prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$.

45 (a). State and prove Apollonius's theorem

Or

(b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground $12m$ away from the starting point. Find the angle of projection.

46 (a). Find the intervals of monotonicities and hence find the local extremum for the following functions: $f(x) = \frac{e^x}{1-e^x}$

Or

(b) If $v(x,y,z) = x^3 + y^3 + z^3 + 3xyz$, show that $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$.

47 (a). The equation of electromotive force for an electric circuit containing resistance and self-inductance is $E = Ri + L\frac{di}{dt}$, where E is the electromotive force is given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when $E = 0$.

Or

(b) The curve $y = (x-2)^2 + 1$ has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2. Find the area bounded by the curve and the chord PQ.

St. Anne's Academy
Holy Cross College Road,
I Floor - Jafro Dental Clinic,
Punnai Nagar,
Nagercoil - 4
Ph: 948 99 00 886