M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

EXAM NO:

STD: XII MATHEMATICS MARKS: 90

GOVT. PUBLIC EXAM (MAR 2023) TIME: 3 hr

PART – I

Choose the correct answer **20 x 1 = 20**

1. A square matrix A of order n has inverse if and only if:
   (a) $\rho(A) > n$ **(b) $\rho(A) = n$** (c) $\rho(A) \neq n$ (d) $\rho(A) < n$
2. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is
   (a) 2 (b) 0 (c) 3 **(d) 1**
3. If $3\cos^{-1}x \cos^{-1}(4x^3 - 3x)$,
   (a) $x \in \left(\frac{1}{2}, 1\right)$ **(b) $x \in \left[\frac{1}{2}, 1\right]$** (c) $x \in (-\infty, 1]$ (d) $x \in \left[\frac{1}{2}, \infty\right)$
4. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is :
   (a) $y = kx$ (b) $xy = k$ (c) $\log y = kx$ (d) $y = k \log x$
5. The number of normals that can be drawn from a point to the parabola $y^2 = 4ax$ is
   (a) 3 (b) 2 (c) 0 **(d) 1**
6. If $\vec{a}$ and $\vec{b}$ are parallel vectors then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to:
   (a) 1 (b) 2 **(c) 0** (d) -1
7. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is
   (a) 1 **(b) 2** (c) ∞ (d) 4
8. Suppose that X takes on one of the values 0,1,2. If for some constant k,
   $P(X = i) = kP(X = i - 1)$ for $i = 1,2$ and $P(X = 0) = \frac{1}{7}$, then the yalue of k is :
   (a) 3 (b) 1 (c) 4 **(d) 2**
9. The maximum value of the function $x^2 e^{-2x}, x > 0$ is :
   (a) $\frac{1}{e^2}$ (b) $\frac{1}{e}$ (c) $\frac{4}{e^4}$ (d) $\frac{1}{2e}$
10. The operation $*$defined by $a * b = \frac{ab}{7}$ is not a binary operation on :
    (a) R (b) Q^+ (c) C **(d) Z**
11. The area between $y^2 = 4x$ and its latus rectum is :
    (a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{5}{3}$ (d) $\frac{4}{3}$
12. Angle between the cu1ves $y^2 = x$ and $x^2 = y$ at the origin is :
    (a) $\frac{\pi}{2}$ (b) $\tan^{-1}\left(\frac{3}{4}\right)$ (c) $\frac{\pi}{4}$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$
13. $|adj(adjA)| = |A|^{16}$, then the order of the square matrix A is :
    (a) 2 (b) 3 **(c) 5** (d) 4
14. The value of $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$ is:
    (a) 8 (b) 4 **(c) 2** (d) 6
15. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is:
    (a) $\frac{1}{z}$ **(b) z** (c) 1 (d) $\bar{z}$

16. The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25?

(a) -2 (b) -8 (c) 0 **(d) -4**

17. The value of $\int_0^{\pi/3} \tan x \, dx$ is :

(a) $-\log 2$ **(b) $\log 2$** (c) $-\log 3$ (d) $\log 3$

18. The number of positive zeros of the polynomial $\sum_{r=0}^{n} {}^nC_r(-1)^r x^r$ is :

(a) $< n$ (b) 0 (c) r **(d) n**

19. The Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is :

(a) $\frac{-\pi}{6}$ (b) 0 (c) $\frac{-\pi}{2}$ (d) $\frac{\pi}{2}$

20. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(a) $\sqrt{ab}$ **(b) $2ab$** (c) $\frac{a}{b}$ (d) ab

M.SANKAR M.Sc., B.Ed.,
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BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
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PART - II

Answer any SEVEN questions
Question number 30 is compulsory **7 x 2 = 14**

21. If $|z| = 2$ show that $3 \le |z+3+4i| \le 7$.

Solution:

$$\text{W.K.T.} \ \big||z_1| - |z_2|\big| \le |z_1 + z_2| \le |z_1| + |z_2|$$

$$\big||z| - |3+4i|\big| \le |z+3+4i| \le |z| + |3+4i|$$

$$\text{Since } |z| = 2$$

$$\left|2 - \sqrt{9+16}\right| \le |z+3+4i| \le 2 + \sqrt{9+16}$$

$$|2-5| \le |z+3+4i| \le 2+5$$

$$|-3| \le |z+3+4i| \le 7$$

$$3 \le |z+3+4i| \le 7$$

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22. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

Solution:

G.T. p and q are the roots of the equation $lx^2 + nx + n = 0$

$$\text{Sum of the roots} = p + q = -\frac{n}{l}$$

$$\text{Product of the roots} = pq = \frac{n}{l}$$

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} = \frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} = \frac{p+q}{\sqrt{pq}}$$

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$$= -\frac{\frac{n}{l}}{\sqrt{\frac{n}{l}}} = -\frac{\sqrt{\frac{n}{l}}\sqrt{\frac{n}{l}}}{\sqrt{\frac{n}{l}}} = -\sqrt{\frac{n}{l}}$$

$$\therefore \boxed{\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0}$$

23. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c.

Solution:

$$\text{G.T.} \quad y = 4x + c \ \text{ and } x^2 + y^2 = 9$$

W.K.T. condition for the line $y = mx + c$ to be a tangent to the circle $x^2 + y^2 = a^2$ is $c^2 = a^2(1 + m^2)$

$$\text{Here } m = 4, \quad a^2 = 9$$

$$c^2 = a^2(1 + m^2)$$

$$c^2 = 9(1 + 4^2)$$

$$c^2 = 9(17)$$

$$c = \pm 3\sqrt{17}$$

24. If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

Solution:

$$\text{Given } r = 10 \text{ cm} ; dr = -0.1 \text{ cm}$$

$$\text{W.K.T. volume of a sphere is } V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = \frac{4}{3}\pi(3r^2)$$

$$dV = 4\pi r^2 dr$$

$$dV = 4\pi(10)^2(-0.1) = -40\pi \text{ cm}^3$$

volume of the sphere decreases about $40\pi cm^3$.

25. Evaluate : $\int_b^\infty \frac{1}{a^2 + x^2} dx$, $a > 0, b \in \mathbb{R}$.

Solution:

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$$\int_b^\infty \frac{1}{a^2 + x^2} dx = \left[\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]_b^\infty$$

$$= \frac{1}{a}\left[\tan^{-1}\left(\frac{x}{a}\right)\right]_b^\infty$$

$$= \frac{1}{a}\left[\tan^{-1}\infty - \tan^{-1}\left(\frac{b}{a}\right)\right] = \frac{1}{a}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{b}{a}\right)\right]$$

$$\therefore \int_b^\infty \frac{1}{a^2 + x^2} dx = \frac{1}{a}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{b}{a}\right)\right]$$

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26. Find a parametric form of vector equation of a plane which is at a distance of 7 units from the origin having $3, -4, 5$ as direction ratios of a normal to it.

Solution:

$$\text{Given } p = 7 \;\; ; \;\; \vec{n} = 3\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{\sqrt{(3)^2 + (-4)^2 + (5)^2}} = \frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{5\sqrt{2}}$$

Vector equation of plane is $\vec{r} \cdot \hat{n} = p$

$$\vec{r} \cdot \left(\frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{5\sqrt{2}}\right) = 7$$

---

27. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

Solution

$$A \vee B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \vee 1 & 1 \vee 1 \\ 1 \vee 0 & 1 \vee 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A \wedge B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \wedge 1 & 1 \wedge 1 \\ 1 \wedge 0 & 1 \wedge 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

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28. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

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Solution:

$$\text{Let } \; A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$AA^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$|||^{ly}$ $A^TA = I_2$

$$\Longrightarrow AA^T = A^TA = I_2$$

Hence, A is orthogonal

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29. Find the equation of tangent to the curve $y = x^2 + 3x - 2$ at the point $(1, 2)$.

Solution:

$$\text{Given } y = x^2 + 3x - 2 \Rightarrow \frac{dy}{dx} = 2x + 3$$

$$m = \left(\frac{dy}{dx}\right)_{(1,2)} = 2(1) + 3 = 5$$

$$\therefore \text{Equation of tangent is } y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 2 = 5(x - 1)$$

$$\Rightarrow \boxed{5x - y - 3 = 0}$$

M.SANKAR M.Sc., B.Ed.,
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BHARATHIDASAN HIGHER SECONDARY SCHOOL
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30. Express $e^{\cos\theta + i\sin\theta}$ in $a + ib$ form.

Solution:

$$e^{\cos\theta+i\sin\theta} = e^{\cos\theta}e^{i\sin\theta}$$
$$= e^{\cos\theta}(\cos(\sin\theta) + i\sin(\sin\theta))$$
$$= e^{\cos\theta}\cos(\sin\theta) + i\, e^{\cos\theta}\sin(\sin\theta)$$

PART – III

Answer any SEVEN questions

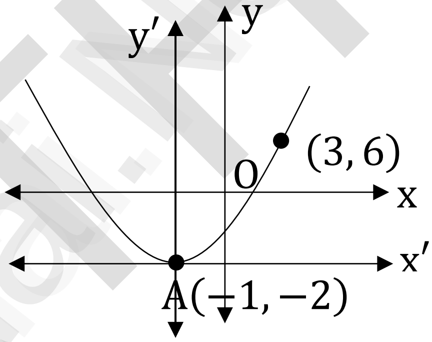
Question number 40 is compulsory **7 x 3 = 21**

31. Find the equation of the parabola with vertex $(-1,-2)$, axis parallel to y –axis and passing through $(3,6)$.

Solution:

vertex $(-1,-2)$, axis parallel to y –axis and passing through $(3,6)$

From diagram, the parabola is open up and axis of symmetry as parallel to y – axis



Then the equation of parabola is

$$(x-h)^2 = 4a(y-k)$$

Here $A(h,k) = A(-1,-2)$

$$(x+1)^2 = 4a(y+2) \rightarrow (1)$$

It passes through $(3,6)$

$$(3+1)^2 = 4a(6+2)$$
$$16 = 4a(8)$$
$$4a = 2 \Longrightarrow a = \frac{1}{2}$$

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PGT MATHEMATICS
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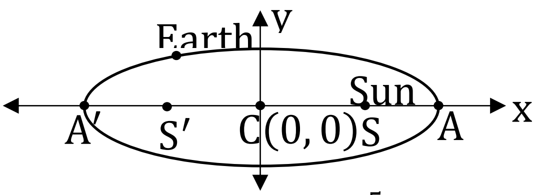
Sub $4a = 2$ in (1), weget

$$(x+1)^2 = 2(y+2)$$
$$x^2 + 2x + 1 = 2y + 4$$
$$x^2 + 2x + 1 - 2y - 4 = 0$$
$$x^2 + 2x - 2y - 3 = 0$$

32. The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

Solution:

$$SA' = a + c = 152 \times 10^6 \rightarrow (1)$$
$$SA = a - c = 94.5 \times 10^6 \rightarrow (2)$$
$$(1) - (2) \Longrightarrow 2c = 57.5 \times 10^6 = 575 \times 10^5$$



∴ The distance from the Sun to the other focus is 575×10^5 km

33. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds?

Solution:

$$\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$$

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$$\because \cos^{-1} x \text{ is decreasing function}$$
$$\cos\frac{\pi}{2} > 3x - 1 > \cos\pi$$
$$0 > 3x - 1 > -1$$
$$-1 < 3x - 1 < 0$$
$$-1 + 1 < 3x < 0 + 1$$
$$0 < 3x < 1$$
$$0 < x < \frac{1}{3}$$

34. Find the angle between the straight line $\frac{x+3}{2} = \frac{y-1}{2} = -z$ with coordinate axes.

Solution:

$$\frac{x+3}{2} = \frac{y-1}{2} = -z \Rightarrow \frac{x+3}{2} = \frac{y-1}{2} = \frac{z}{-1}$$
$$\therefore \vec{b} = 2\hat{i} + 2\hat{j} - \hat{k}$$
$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{(2)^2 + (2)^2 + (-1)^2}} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$$

Let α, β and γ are the angles made by $\hat{b}$ with $+$ ve $x-$axis, $y-$axis and $z-$axis respectively

$$\therefore \cos\alpha = \frac{2}{3} \ ; \ \cos\beta = \frac{2}{3} \ ; \ \cos\gamma = -\frac{1}{3}$$
$$\therefore \alpha = \cos^{-1}\left(\frac{2}{3}\right) \ ; \ \beta = \cos^{-1}\left(\frac{2}{3}\right) \ ; \ \gamma = \cos^{-1}\left(-\frac{1}{3}\right)$$

35. Use the linear approximation to find an approximate values of $(123)^{\frac{2}{3}}$.

Solution:

$$\text{Let } f(x) = x^{\frac{2}{3}} \Rightarrow x = 123 \ ; \ x_0 = 125$$
$$f(x_0) = f(125) = (125)^{\frac{2}{3}} = (5^3)^{\frac{2}{3}} = 25$$
$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3x^{\frac{1}{3}}} \Rightarrow f'(x_0) = f'(125) = \frac{2}{3(125)^{\frac{1}{3}}} = \frac{2}{3(5^3)^{\frac{1}{3}}} = \frac{2}{15}$$
$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$
$$L(123) = 25 + \frac{2}{15}(123 - 125) = 25 + \frac{2}{15}(-2) = 25 - \frac{4}{15}$$
$$L(123) = 25 - 0.27 = 24.73$$
$$(123)^{\frac{2}{3}} \simeq L(123) = 24.73$$

36. Solve the differential equation $x\cos y\, dy = e^x(x\log x + 1)dx$.

Solution:

$$x\cos y\, dy = e^x(x\log x + 1)dx$$
$$\cos y\, dy = e^x\left(\frac{x\log x + 1}{x}\right)dx$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
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TIRUVALLUR
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$$\cos y\,dy = e^x\left(\log x + \frac{1}{x}\right)dx$$

$$\int \cos y\,dy = \int e^x\left(\log x + \frac{1}{x}\right)dx$$

$$\sin y = e^x(\log x) + c$$

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37. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

Solution:

$$|F(\alpha)| = \cos\alpha(\cos\alpha - 0) - 0 + \sin\alpha(0 + \sin\alpha)$$

$$= \cos^2\alpha + \sin^2\alpha = 1 \neq 0 \Longrightarrow [F(\alpha)]^{-1} \text{ exists}$$

$$\text{adj}\,[F(\alpha)] = \begin{bmatrix} (\cos\alpha - 0) & -(0-0) & (0+\sin\alpha) \\ -(0-0) & (\cos^2\alpha + \sin^2\alpha) & -(0+0) \\ (0-\sin\alpha) & -(0-0) & (\cos\alpha - 0) \end{bmatrix}^T$$

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PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

$$= \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}^T$$

$$\text{adj}\,[F(\alpha)] = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$[F(\alpha)]^{-1} = \frac{1}{|F(\alpha)|}\text{adj}\,[F(\alpha)]$$

$$[F(\alpha)]^{-1} = \frac{1}{1}\begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \to (1)$$

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & 0 & \sin(-\alpha) \\ 0 & 1 & 0 \\ -\sin(-\alpha) & 0 & \cos(-\alpha) \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ +\sin\alpha & 0 & \cos\alpha \end{bmatrix} \to (2)$$

From (1) and (2), we get

$$[F(\alpha)]^{-1} = F(-\alpha)$$

---

38. Show that $p \to q$ and $q \to p$ are not equivalent.

Solution:

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T
		(1)	(2)

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From (1) and (2), we get

$$\therefore p \to q \not\equiv q \to p$$

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39. Find z^{-1}, if $z = (2+3i)(1-i)$.

Solution:

$$z = (2+3i)(1-i) = (2+3) + i(3-2) = 5 + i$$

$$\therefore z^{-1} = \frac{1}{z} = \frac{1}{5+i} = \frac{1}{5+i} \times \frac{5-i}{5-i}$$
$$= \frac{5-i}{5^2+1^2} = \frac{5-i}{25+1}$$
$$\therefore z^{-1} = \frac{5}{26} - i\frac{1}{26}$$

40. If $a+b+c=0$ and a, b, c are rational numbers then, prove that the roots of the equation $(b+c-a)x^2+(c+a-b)x+(a+b-c)=0$ are rational numbers.

Solution:

$$\text{G.T. } a+b+c=0 \Rightarrow b=-a-c$$
$$\text{G.T. } (b+c-a)x^2+(c+a-b)x+(a+b-c)=0$$
$$A=b+c-a,\quad B=c+a-b,\quad C=a+b-c$$
$$\Delta = B^2-4AC = \left(c+a-(-a-c)\right)^2-4(-a-c+c-a)(a-a-c-c)$$
$$=\left(2(c+a)\right)^2-4(-2a)(-2c)$$
$$=4(c+a)^2-4(4ac)$$
$$=4((c+a)^2-4ac)=4(c-a)^2$$
$$=\left(2(c-a)\right)^2 \text{ which is perfact square}$$

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Hence, the given equation roots are rational.

PART – IV

Answer ALL questions **7 x 5 = 35**

41. (a) Solve the equation $z^3+8i=0$, where $z \in \mathbb{C}$.

Solution:

$$z^3+8i=0 \Rightarrow z^3=-8i \Rightarrow z=\left(8(-i)\right)^{1/3} \Rightarrow z=2(-i)^{1/3}$$
$$\therefore z = 2\left[\cos\left(-\frac{\pi}{2}+2k\pi\right)+i\sin\left(-\frac{\pi}{2}+2k\pi\right)\right]^{\frac{1}{3}},\ k=0,1,2z$$
$$=2\left[\cos\left(\frac{-\pi+4k\pi}{2}\right)+i\sin\left(\frac{-\pi+4k\pi}{2}\right)\right]^{\frac{1}{3}},\ k=0,1,2$$
$$=2\left[\cos\left(\frac{-\pi+4k\pi}{6}\right)+i\sin\left(\frac{-\pi+4k\pi}{6}\right)\right],\ k=0,1,2$$
$$\text{when } k=0 \Rightarrow z=2\left[\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right]=2\left[\cos\left(\frac{\pi}{6}\right)-i\sin\left(\frac{\pi}{6}\right)\right]$$
$$=2\left[\frac{\sqrt{3}}{2}-i\frac{1}{2}\right]=\sqrt{3}-i$$
$$\text{when } k=1 \Rightarrow z=2\left[\cos\left(\frac{3\pi}{6}\right)+i\sin\left(\frac{3\pi}{6}\right)\right]=2\left[\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right]$$
$$=2[0+i(1)]=2i$$
$$\text{when } k=2 \Rightarrow z=2\left[\cos\left(\frac{7\pi}{6}\right)+i\sin\left(\frac{7\pi}{6}\right)\right]=2\left[\cos\left(\pi+\frac{\pi}{6}\right)+i\sin\left(\pi+\frac{\pi}{6}\right)\right]$$
$$=2\left[-\cos\left(\frac{\pi}{6}\right)-i\sin\left(\frac{\pi}{6}\right)\right]=2\left[-\frac{\sqrt{3}}{2}-i\frac{1}{2}\right]=-\sqrt{3}-i$$

OR

(b) Solve the Linear differential equation $(1+x+xy^2)\frac{dy}{dx}+(y+y^3)=0$.

Solution:

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

$$(1+x+xy^2)\frac{dy}{dx}+(y+y^3)=0$$
$$(y+y^3)=-(1+x+xy^2)\frac{dy}{dx}$$
$$y(1+y^2)\frac{dx}{dy}=-\left(1+x(1+y^2)\right)$$
$$y(1+y^2)\frac{dx}{dy}=-1-x(1+y^2)$$

$\div\, y(1+y^2)$

$$\frac{dx}{dy}=-\frac{1}{y(1+y^2)}-\frac{x(1+y^2)}{y(1+y^2)}$$
$$\frac{dx}{dy}+\frac{x}{y}=\frac{-1}{y(1+y^2)}$$
$$\text{Here, } P=\frac{1}{y},\ Q=\frac{-1}{y(1+y^2)}$$
$$\int P\,dy=\int\frac{1}{y}dy=\log|y|$$
$$I.F=e^{\int Pdy}=e^{\log|y|}=y$$
$$\text{The solution is } x(I.F)=\int Q(I.F)\,dy+c$$
$$x(y)=\int\frac{-1}{y(1+y^2)}\cdot y\,dy+c$$
$$xy=-\int\frac{1}{1+y^2}dy+c$$
$$xy=-\tan^{-1}x+c$$
$$xy+\tan^{-1}x=c$$

---

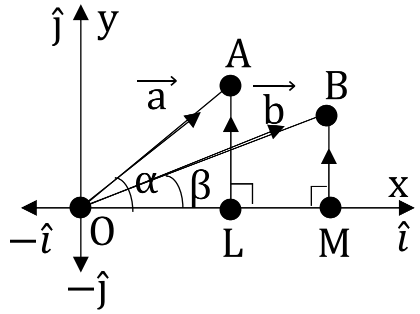
42. (a) Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

Solution:

Let $\hat{a}=\overrightarrow{OA}$ and $\hat{b}=\overrightarrow{OB}$ be the unitvectors and which makes angle α and β with positive $x-$axis respectively.

$\therefore$ The angle between $\hat{a}$ and $\hat{b}$ is $\alpha-\beta$

From the diagram



$$\cos\alpha=\frac{OL}{OA}=\frac{OL}{1}=OL \qquad \cos\beta=\frac{OM}{OB}=\frac{OM}{1}=OM$$
$$\sin\alpha=\frac{LA}{OA}=\frac{LA}{1}=LA \qquad \sin\beta=\frac{MB}{OB}=\frac{MB}{1}=MB$$

$$\hat{a} = \overrightarrow{OA} = \overrightarrow{OL} + \overrightarrow{LA} = OL\,\hat{\imath} + LA\,\hat{\jmath}$$
$$\hat{a} = \cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}$$

$$\hat{b} = \overrightarrow{OB} = \overrightarrow{OM} + \overrightarrow{MB} = OM\,\hat{\imath} + MB\,\hat{\jmath}$$
$$\hat{b} = \cos\beta\,\hat{\imath} + \sin\beta\,\hat{\jmath}$$

$$\hat{a}\,.\,\hat{b} = (\cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath})\,.\,(\cos\beta\,\hat{\imath} + \sin\beta\,\hat{\jmath})$$
$$\hat{a}\,.\,\hat{b} = \cos\alpha\cos\beta + \sin\alpha\sin\beta \quad \rightarrow \quad (1)$$
$$\hat{a}\,.\,\hat{b} = |\hat{a}||\hat{b}|\cos(\alpha-\beta) = (1)(1)\cos(\alpha-\beta) = \cos(\alpha-\beta) \rightarrow (2)$$

From (1) and (2), we get
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

OR

(b) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function $f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$ Find (i) the value of k (ii) the distribution function (iii) the probability that daily sales will fall between 300 litres and 500 litres?

Solution:

(i) Given p.d.f

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$
$$\int_{200}^{600} k\,dx = 1$$
$$k[x]_{200}^{600} = 1$$
$$k[600-200] = 1$$
$$k(400) = 1$$
$$k = \frac{1}{400}$$
$$\therefore \text{p.d.f is } f(x) = \begin{cases} \frac{1}{400}, & 200 \le x \le 600 \\ 0, & \text{Otherwise} \end{cases}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

(ii) The distribution function F(x)

$$F(x) = \int_{-\infty}^{x} f(u)\,du$$

when $x < 200$

$$F(x) = \int_{-\infty}^{x} f(u)\,du = \int_{-\infty}^{x} 0\,du = 0$$

when $200 \le x \le 600$

$$F(x) = \int_{-\infty}^{200} f(u)\,du + \int_{200}^{x} f(u)\,du$$
$$= 0 + \int_{200}^{x} \frac{1}{400}\,du$$
$$= \frac{1}{400}[u]_{200}^{x}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

$$F(x) = \frac{1}{400}[x - 200]$$

When $x > 600$

$$F(x) = \int_{-\infty}^{200} f(u)\,du + \int_{200}^{600} f(u)\,du + \int_{600}^{x} f(u)\,du$$

$$= \int_{-\infty}^{200} 0\,du + \int_{200}^{600} \frac{1}{400}\,du + \int_{600}^{x} 0\,du$$

$$= 0 + \frac{1}{400}[u]_{200}^{600} + 0$$

$$= \frac{1}{400}(600 - 200)$$

$$F(x) = 1$$

$$F(x) = \begin{cases} 0, & -\infty < x < 200 \\ \dfrac{x-200}{4}, & 200 \le x \le 600 \\ 1, & 600 < x < \infty \end{cases}$$

(iii) probability that daily sales will fall between 300 litres and 500 litres

$$P(300 < X < 500) = P(300 \le X \le 500) = F(500) - F(300)$$

$$= \frac{500-200}{400} - \frac{300-200}{400} = \frac{1}{2}$$

---

43. (a) Identify the type of conic and find centre, foci, vertices of $18x^2 + 12y^2 - 144x + 48y + 120 = 0.$

Solution:

$$18x^2 + 12y^2 - 144x + 48y + 120 = 0 \text{ which is an ellipse}$$

$$18x^2 - 144x + 12y^2 + 48y = -120$$

$$18(x^2 - 8x) + 12(y^2 + 4y) = -120$$

$$18(x^2 - 8x + 16) + 12(y^2 + 4y + 4) = -120 + 18(16) + 12(4)$$

$$18(x-4)^2 + 12(y+2)^2 = -120 + 288 + 48$$

$$18(x-4)^2 + 12(y+2)^2 = 216$$

$\div$ by 216

$$\frac{(x-4)^2}{12} + \frac{(y+2)^2}{18} = 1$$

$$a^2 = 18 \;\; ; \; b^2 = 12 \;\; ; \; c^2 = a^2 - b^2 = 18 - 12 = 6$$

$$a = 3\sqrt{2}\,; \quad b = 2\sqrt{3}\,; \quad c = \sqrt{6}$$

Here $h = 4$; $k = -2$

Major axis is parallel to $y -$ axis

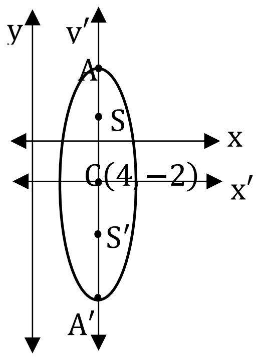
Centre : $C(h,k) = C(4,-2)$

Vertices : $A(h, k \pm a) = A\left(4, -2 \pm 3\sqrt{2}\right)$

$A\left(4, -2 + 3\sqrt{2}\right)$ and $A'\left(4, -2 - 3\sqrt{2}\right)$

Foci : $S(h, k \pm c) = S\left(4, -2 \pm \sqrt{6}\right)$

$S\left(4, -2 + \sqrt{6}\right)$ and $S'\left(4, -2 - \sqrt{6}\right)$



OR

(b) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$.

Solution:

$$\text{Let } \alpha = \cos^{-1}x \;;\; \beta = \cos^{-1}y \;;\; \gamma = \cos^{-1}z$$
$$\Rightarrow \cos\alpha = x \;;\; \cos\beta = y \;;\; \cos\gamma = z$$
$$\Rightarrow \sin\alpha = \sqrt{1-x^2} \;;\; \sin\beta = \sqrt{1-y^2} \;;\; \sin\gamma = \sqrt{1-z^2}$$
$$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi \Rightarrow \alpha+\beta+\gamma = \pi \Rightarrow \alpha+\beta = \pi-\gamma$$
$$\cos(\alpha+\beta) = \cos(\pi-\gamma)$$
$$\cos\alpha\cos\beta - \sin\alpha\sin\beta = -\cos\gamma$$
$$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$$
$$xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$$
$$(xy+z)^2 = \left(\sqrt{1-x^2}\sqrt{1-y^2}\right)^2$$
$$x^2y^2 + 2xyz + z^2 = (1-x^2)(1-y^2)$$
$$x^2y^2 + 2xyz + z^2 = 1 - x^2 - y^2 + x^2y^2$$
$$x^2 + y^2 + z^2 + 2xyz = 1$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

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44. (a) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8), (-2, -12)$, and $(3, 8)$. He wants to meet his friend at $P(7, 60)$. Will he meet his friend? (Use Gaussian elimination method).

Solution:

Given that, $y = ax^2 + bx + c$

it passes through the points $(-6, 8), (-2, -12)$ and $(3, 8)$

$36a - 6b + c = 8$

$4a - 2b + c = -12$

$9a + 3b + c = 8$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

The matrix form of the system is $AX = B$

$$\text{Where } A = \begin{bmatrix} 36 & -6 & 1 \\ 4 & -2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \;;\; X = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \;;\; B = \begin{bmatrix} 8 \\ -12 \\ 8 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_1 - 4R_3]{R_2 \to R_1 - 9R_2} \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & 12 & -8 & 116 \\ 0 & -18 & -3 & -24 \end{array}\right]$$

$$\xrightarrow{R_3 \to 2R_3 + 3R_2} \left[\begin{array}{ccc|c} 36 & -6 & 1 & 9 \\ 0 & 12 & -8 & 116 \\ 0 & 0 & -30 & 300 \end{array}\right]$$

it is a row echelon form

$36a - 6b + c = 8 \;\rightarrow$ ①

$12b - 8c = 116 \;\rightarrow$ ②

$$-30c = 300 \implies \boxed{c = -10}$$

Sub $c = -10$ in ②

$$12b - 8(-10) = 116$$
$$12b = 116 - 80$$
$$12b = 36$$
$$\boxed{b = 3}$$

Sub $b = 3,\ c = -10$ in ①

$$36a - 6(3) - 10 = 8$$
$$36a = 8 + 18 + 10$$
$$36a = 36$$
$$\boxed{a = 1}$$

∴ The given path is $y = ax^2 + bx + c$

$$\implies y = x^2 + 3x - 10$$

Since the friend at $(7,\ 60)$

$$60 = 7^2 + 3(7) - 10$$
$$60 = 49 + 21 - 10$$
$$60 = 60$$

Hence, A boy meet the friend.

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

OR

(b) Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

Solution:

Let (a, b) be the point of intersection of given curves

$$a^2 + 4b^2 = 8 \rightarrow ①$$
$$a^2 - 2b^2 = 4 \rightarrow ②$$
$$① - ② \implies 6b^2 = 4$$
$$\implies b^2 = \frac{2}{3}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

Sub $b^2 = \frac{2}{3}$ in ①

$$a^2 + 4\left(\frac{2}{3}\right) = 8 \implies a^2 = 8 - \frac{8}{3} = \frac{16}{3}$$

$$x^2 + 4y^2 = 8$$
$$2x + 8y\frac{dy}{dx} = 0$$
$$8y\frac{dy}{dx} = -2x$$
$$\frac{dy}{dx} = -\frac{x}{4y}$$
$$m_1 = \left(\frac{dy}{dx}\right)_{(a,b)} = -\frac{a}{4b}$$

$$x^2 - 2y^2 = 4$$
$$2x - 4y\frac{dy}{dx} = 0$$
$$-4y\frac{dy}{dx} = -2x$$
$$\frac{dy}{dx} = \frac{x}{2y}$$
$$m_2 = \left(\frac{dy}{dx}\right)_{(a,b)} = \frac{a}{2b}$$

$$m_1 m_2 = \left(-\frac{a}{4b}\right)\left(\frac{a}{2b}\right) = -\frac{a^2}{8b^2} = -\frac{\frac{16}{3}}{8\left(\frac{2}{3}\right)} = -1$$

$$m_1 m_2 = -1$$

∴ The given two curves cut orthogonally.

---

45. (a) Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r} = (\hat{\imath} - \hat{\jmath} + 3\hat{k}) + t(2\hat{\imath} - \hat{\jmath} + 4\hat{k})$ and perpendicular to plane $\vec{r} \cdot (\hat{\imath} + 2\hat{\jmath} + \hat{k}) = 8$.

Solution:

The requied equation of plane passing through a point $(1, -1, 3)$ and parallel to vectors $2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + \hat{k}$.

Let $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$; $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$; $\vec{c} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

Vector equation:

$$\vec{r} = \vec{a} + s\vec{b} + t\vec{c} \quad \text{where } s, t \in \mathbb{R}$$

$$\boxed{\vec{r} = (\hat{\imath} - \hat{\jmath} + 3\hat{k}) + s(2\hat{\imath} - \hat{\jmath} + 4\hat{k}) + t(\hat{\imath} + 2\hat{\jmath} + \hat{k})} \quad \text{where } s, t \in \mathbb{R}$$

Cartesian equation:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

$$(x_1, y_1, z_1) = (1, -1, 3)$$
$$(b_1, b_2, b_3) = (2, -1, 4)$$
$$(c_1, c_2, c_3) = (1, 2, 1)$$

$$\begin{vmatrix} x - 1 & y + 1 & z - 3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$(x - 1)(-1 - 8) - (y + 1)(2 - 4) + (z - 3)(4 + 1) = 0$$
$$-9(x - 1) + 2(y + 2) + 5(z - 3) = 0$$
$$-9x + 9 + 2y + 2 + 5z - 15 = 0$$
$$-9x + 2y + 5z - 4 = 0$$
$$\boxed{9x - 2y - 5z + 4 = 0}$$

OR

(b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

Solution:

$$6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0 \rightarrow (1)$$

is a even degree reciprocal equation of type I

Given that $\frac{1}{3}$ is a solution $\Longrightarrow$ 3 is also solution

$$\therefore x = \frac{1}{3} \text{ and } x = 3 \text{ are the roots}$$

	6	−5	−38	−5	6
$\frac{1}{3}$	0	2	−1	−13	−6
	6	−3	−39	−18	0
3	0	18	45	18	
	6	15	6	0	

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

$$\therefore 6x^2 + 15x + 6 = 0$$
$$\div 3 \Longrightarrow 2x^2 + 5y + 2 = 0$$
$$\left(x + \frac{4}{2}\right)\left(x + \frac{1}{2}\right) = 0$$
$$(x + 2)\left(x + \frac{1}{2}\right) = 0$$
$$x = -2, \ \ x = -\frac{1}{2}$$
$$\text{Hence, the roots are } -2, -\frac{1}{2}, \frac{1}{3} \text{ and } 3$$

---

46. (a) Prove that $p \to (\neg q \lor r) \equiv \neg p \lor (\neg q \lor r)$ using truth table.

Solution:

p	q	r	$\neg q$	$\neg q \lor r$	$p \to (\neg q \lor r)$	$\neg p$	$\neg p \lor (\neg q \lor r)$
T	T	T	F	T	T	F	T
T	T	F	F	F	F	F	F
T	F	T	T	T	T	F	T
T	F	F	T	T	T	F	T
F	T	T	F	T	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T (1)	T	T (2)

From (1) and (2), we get

$$\therefore p \to (\neg q \lor r) \equiv \neg p \lor (\neg q \lor r)$$

OR

(b) Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

Solution:

Let x be the amount deposited.

$$\frac{dx}{dt} \propto x$$
$$\frac{dx}{dt} = kx$$
$$\frac{dx}{x} = kdt$$
$$\int \frac{dx}{x} = k\int dt$$
$$\log|x| = kt + \log|C|$$
$$\frac{x}{C} = e^{kt}$$
$$x = C\,e^{kt} \quad \rightarrow \quad (1)$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

$$k = 5\% = \frac{5}{100} = 0.05$$
$$x = C\,e^{0.05t} \rightarrow (2)$$
$$\text{When } t = 0,\ x = 10{,}000$$
$$10{,}000 = C\,e^{0} = C(1)$$
$$\boxed{C = 10{,}000}$$
$$\text{Sub } C = 10{,}000 \text{ in } (2)$$
$$x = 10{,}000\,e^{0.05t} \rightarrow (3)$$
$$\text{When } t = 18 \text{ months},\ x = ?$$
$$\text{When } t = 1.5 \text{ years},\ x = ?$$
$$x = 10{,}000\,e^{0.05(1.5)}$$
$$x = 10{,}000\,e^{0.075}$$
$$\therefore \text{The amount after 18 month is } 10{,}000\,e^{0.075}$$

47. (a) Find the maximum value of $\frac{\log x}{x}$.

Solution:

$$f(x) = \frac{\log x}{x}, x \in (0, \infty)$$
$$f'(x) = \frac{x\left(\frac{1}{x}\right) - \log x\,(1)}{x^2} = \frac{1 - \log x}{x^2}$$
$$f''(x) = \frac{x^2\left(0 - \frac{1}{x}\right) - (1 - \log x)(2x)}{x^4}$$
$$= \frac{-x - 2x + 2x\log x}{x^4} = \frac{-3x + 2x\log x}{x^4} = \frac{x(2\log x - 3)}{x^4}$$
$$f''(x) = \frac{2\log x - 3}{x^3}$$
$$\text{If } f'(x) = 0, \text{then } \frac{1 - \log x}{x^2} = 0$$
$$1 - \log x = 0$$
$$\log x = 1$$
$$e^{\log x} = e^1 \Rightarrow x = e$$
$$\text{At } x = e, f''(e) = \frac{2\log e - 3}{e^3} = \frac{2-3}{e^3} = -\frac{1}{e^3} < 0$$
$$\text{Hence at } x = e, f(x) \text{ attains a local maximum.}$$
$$\text{Local Maximum Value : At } x = e, f(e) = \frac{\log e}{e} = \frac{1}{e}$$

M.SANKAR M.Sc., B.Ed.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com

OR

(b) Find the area of the region common to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$.

Solution:

Question not clear about area

PREPARED BY
M.SANKAR M.SC., B.ED.,
PGT MATHEMATICS
BHARATHIDASAN HIGHER SECONDARY SCHOOL
TIRUVALLUR
PHONE NO: 9047952772
EMAIL ID: shark.uk1986@gmail.com
