SECOND REVISION EXAM - 2023

STD - XII

TIME : 3.00 Hrs

MATHS

2101

MARKS : 90

PART - I

I. Answer all the questions : 20 x 1 = 20

1. A Zero of $x^3 + 64$ is

   a) 0 b) 4 c) 4i d) -4

2. A random variable x has binomial distribution with n = 25 and p =0.8, then the standard deviation of x is

   a) 2 b) 6 c) 4 d) 3

3. The value of $\int_0^{\pi} Sin^4 x dx$ is

   a) $\frac{3\pi}{2}$ b) $\frac{3\pi}{10}$ c) $\frac{3\pi}{8}$ d) $\frac{3\pi}{4}$

4. arg(o) is :

   a) ∞ b) 0 c) π d) undefined

5. Which one of the following is not true in the case of discrete random variable x?

   a) $\lim_{x\to\infty} F(x) = F(\infty) = 1$ b) $0 \le F(x) \le 1$

   c) F(x) is real valued decreasing function d) $\lim_{x\to-\infty} F(x) = F(-\infty) = 0$

6. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ then $(A^T)^{-1}$ =

   a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

7. if P(A) = P ([A / B]) then the system Ax = B of linear equations is

   a) inconsistent b) consistend and unique solution

   c) consistent d) consistant and has infinitely many solutions

8. If Z = x + iy is a complex number such that | Z + Z | = | Z - Z |, then the locus of Z is .....

   a) reax axis b) imaginary axis c) ellipse d) circle

9. The area between $y^2 = 4x$ and its latus rectum is

   a) 2 / 3 b) 4 / 3 c) 8 / 3 d) 5 / 3

10. The angle between the lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ is

    a) $\pi / 3$ b) $\pi / 4$ c) $2\pi / 3$ d) $\pi / 3$

11. The distance from the origin to the plane 3x - 6y + 2z +7 = 0 is

    a) 0 b) 1 c) 2 d) 3

12. The value of $Cos^{-1}\left(\frac{1}{2}\right) + Sin^{-1}\left(\frac{1}{2}\right)$ is

    a) 0 b) $\pi / 2$ c) $\pi / 3$ d) π

13. Subtraction is not a binary operation in

a) Q b) R c) Z d) N

14. The solution of $\frac{dy}{dx}+P(x)y=0$ is

a) $x=ce^{-\int pdx}$ b) $y=ce^{\int pdx}$ c) $x=ce^{\int pdy}$ d) $y=ce^{-\int pdy}$

15. The differential equation representing the family of curves y = A Cos (x + B), where A and B are parameters is

a) $\frac{d^2y}{dx^2}-y=0$ b) $\frac{d^2y}{dx^2}+y=0$ c) $\frac{d^2y}{dx^2}=0$ d) $\frac{d^2x}{dy^2}=0$

16. The angle between the parabolas $y^2 = x$ and $x^2 = y$ at the origin is

a) $\pi/4$ b) $\pi/6$ c) $\pi/2$ d) 0

17. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

a) t = 3 b) t = 0 c) t = 1 / 3 d) t = 1

18. If $u(x,y)=e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}=$

a) y^2u b) $e^{x^2+y^2}$ c) 2xy d) x^2u

19. The ecentricity of the hyperbola $\frac{x^2}{16}-\frac{(y-3)^2}{4}=1$ is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{5}}{2}$ c) $\sqrt{5}$ d) 1 / 2

20. The vertex of the parabola is $x^2 = 8y - 1$ is

a) $\left(0,\frac{-1}{8}\right)$ b) $\left(\frac{-1}{8},0\right)$ c) $\left(\frac{1}{8},0\right)$ d) $\left(0,\frac{1}{8}\right)$

PART - II

II. Answer any seven questions. Q.No. 30 is compulsory. 7 x 2 = 14

21. If α and β are the roots of $x^2 + 5x + 6 = 0$, then show that $\alpha^2 + \beta^2 = 13$.

22. Find the vertices and foci of the hyperbola $9x^2 - 6y^2 = 144$.

23. Evaluate : $\int_0^\infty x^5e^{-3x}dx$

24. A random variable X has the following probability mass function. Find the value k.

x	1	2	3	4	5
f(x)	k^2	$2k^2$	$3k^2$	2k	3k

25. Find the tangent to the curve $y = x^2 - x^4$ at (1, 0).

26. Find the value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$.

27. Find the intervals of monotonicity for the function $f(x) = x^{2/3}$

28. Find the magnitude and the direction cosines of the torque about the point (2, 0, -1) of a force $2\hat{i}+\hat{j}-\hat{k}$, whose line of action passes through the origin.

29. Show that $y = ae^x + be^{-x}$ is a solution of the differential equation $y'' - y = 0$.

30. Show that $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right) = -2i$.

PART - III

III. Answer any seven questions. Q.No. 40 is compulsory 7 x 3 = 21

31. Solve the system of linear equations, using matrix inversion method. 5x + 2y =3, 3x + 2y =5

32. A circular plate expends uniformly under the infulence of heat. If its radius increases from 10.5 cm to 10.75 cm ; then find an approximate change in the area.

33. A concrete bridge is designed as a parabolic arch. The road over bridge is 40m long and the maximum height of the arch is 15m. Write the equation of the parabolic arch. Take (0, 0) as the vertex.

34. Verify i) closure property ii) commutative property of the following operation on the given set $a*b=a^b, \forall a,b\in N$

35. Let x be a continuous random variable and f(x) is define as $f(x)=\begin{cases} kx(1-x)^{10}, & 0<x<1 \\ 0, & otherwise \end{cases}$ find the value of k.

36. Show that $\lim_{x\to 0^+} x\log x$ is 0.

37. If $A=\begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$ then verify that $(A^T)^{-1}=(A^{-1})^T$

38. Find the local extremium of the function $f(x) = x^4 + 32x$

39. For any three vectors, $\vec{a}, \vec{b}, \vec{c}$ prove that $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right]=2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

40. Which one of the points 10 - 8i, 11 + 6i is closest to 1 + i.

PART - IV

IV. Answer all the questions. 7 x 5 = 35

41. a) Investigate the value of λ and μ the system linear equations 2x + 3y + 5z = 9 ; 7x + 3y + 5z = 8, 2x + 3y + λz = μ have i) no solution ii) unique solution iii) infinitely many solution. **(OR)**

b) Let W (x, y, z) = $\frac{1}{\sqrt{x^2+y^2+z^2}}$, (x, y, z) ≠ (0, 0, 0). Show that $\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}=0$.

42. a) Find all cube roots of $\sqrt{3}+i$ **(OR)**

b) At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. if the path of water is a parabola, find the height of water at a horizontal distance of 0.75m from the point of origin.

43. a) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and 0 < x, y, z < 1 show that $x^2 + y^2 + z^2 + 2xy = 1$ **(OR)**

b) If $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=\hat{i}-\hat{j}-4\hat{k}$, $\vec{c}=3\hat{j}-\hat{k}$ and $\vec{d}=2\hat{i}+5\hat{j}+\hat{k}$ then verify that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a}\ \vec{b}\ \vec{d}]\vec{c}-[\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$

44. a) Find the centre, vertices and the foci of the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$. Also prove that the lengh of the latus rectum is 2. **(OR)**

b) The cumulative distribution function of a discrete random variable is given by

$$f(x) = \begin{cases} 0, & -\infty < x < 1 \\ 0.15, & -1 \le x < 0 \\ 0.35, & 0 \le x < 1 \\ 0.60, & 1 \le x < 2 \\ 0.85, & 2 \le x < 3 \\ 1, & 3 \le x < \infty \end{cases}$$

Find (i) the probability mass function (ii) P (X < 1) (iii) P ($X \ge 2$)

45. a) Find the area of the region bounded between the parabole $x^2 = y$ and the curve $y = |x|$ **(OR)**

b) Water at temperature 100^0 C cools in 10 minutes to 80^0 C in a room termperature of 25^0C. Find i) the temperature of water after 20 minutes ii) the time when the temperature is 40^0C.

$$\left[\log_e^{\frac{11}{15}} = -0.3101; \log_e^5 = 1.6094\right]$$

46. a) Sketch the curve $y = \dfrac{x^2 - 3x}{x - 1}$ **(OR)**

b) Find the parametric form of vector equation and Cartesian equations of the plane through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane 2x + 6y + 6z = 9

47. a) Define an operation * on Q as follows : $a * b = \left(\dfrac{a+b}{2}\right)$; $a, b \in Q$. Examine the closure, commutative ; associative properties existence of identify and existence of inverse for the operation * and Q. **(OR)**

b) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally, then show that $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{c} - \dfrac{1}{d}$.