12th STD

PUBLIC EXAM - MARCH 2023

Reg. No.

Part - III

Mathematics (with answers)

TIME ALLOWED : 3.00 Hours] [MAXIMUM MARKS : 90

Instructions :

(1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams

PART - I

Note : (i) **All** questions are **Compulsory.**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer. **20 × 1 = 20**

1. A square matrix A of order n has inverse if and only if :

(a) $\rho(A) > n$ (b) $\rho(A) = n$

(c) $\rho(A) \neq n$ (d) $\rho(A) < n$

2. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

(a) 2 (b) 0 (c) 3 (d) 1

3. If $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$,

(a) $x \in \left(\frac{1}{2}, 1\right)$ (b) $x \in \left[\frac{1}{2}, 1\right]$

(c) $x \in (-\infty, 1)$ (d) $x \in \left[\frac{1}{2}, \infty\right)$

4. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is :

(a) $y = kx$ (b) $xy = k$

(c) $\log y = kx$ (d) $y = k \log x$

5. The number of normals that can be drawn from a point to the parabola $y^2 = 4ax$ is :

(a) 3 (b) 2 (c) 0 (d) 1

6. If $\vec{a}$ and $\vec{b}$ are parallel vectors then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to :

(a) 1 (b) 2 (c) 0 (d) –1

7. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is :

(a) 1 (b) 2 (c) ∞ (d) 4

8. Suppose that X takes on one of the values 0, 1, 2. If for some constant k, $P(X = i) = kP(X = i - 1)$ for $i = 1, 2$ and $P(X = 0) = \frac{1}{7}$, then the value of k is :

(a) 3 (b) 1 (c) 4 (d) 2

9. The maximum value of the function $x^2 e^{-2x}$, $x > 0$ is :

(a) $\frac{1}{e^2}$ (b) $\frac{1}{e}$ (c) $\frac{4}{e^4}$ (d) $\frac{1}{2e}$

10. The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on :

(a) R (b) Q^+ (c) C (d) Z

11. The area between $y^2 = 4x$ and its latus rectum is :

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{5}{3}$ (d) $\frac{4}{3}$

12. Angle between the curves $y^2 = x$ and $x^2 = y$ at the origin is :

(a) $\frac{\pi}{2}$ (b) $\tan^{-1}\left(\frac{3}{4}\right)$

(c) $\frac{\pi}{4}$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$

13. $|\text{adj}(\text{adj}A)| = |A|^{16}$, then the order of the square matrix A is :

(a) 2 (b) 3 (c) 5 (d) 4

14. The value of $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$ is :

(a) 8 (b) 4 (c) 2 (d) 6

15. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is :

(a) $\frac{1}{z}$ (b) z (c) 1 (d) $\bar{z}$

16. The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is – 0.25?

(a) –2 (b) 8 (c) 0 (d) –4

17. The value of $\int_0^{\frac{\pi}{3}} \tan x \, dx$ is :

(a) $-\log 2$ (b) $\log 2$
(c) $-\log 3$ (d) $\log 3$

18. The number of positive zeros of the polynomial $\sum_{r=0}^{n} {}^nC_r(-1)^r x^r$:

(a) $< n$ (b) 0 (c) r (d) n

19. The Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is :

(a) $\frac{-\pi}{6}$ (b) 0 (c) $\frac{-\pi}{2}$ (d) $\frac{\pi}{2}$

20. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is :

(a) $\sqrt{ab}$ (b) $2ab$ (c) $\frac{a}{b}$ (d) ab

PART - II

Note : Answer **any seven** questions. Question No. **30** is **Compulsory.** **7 × 2 = 14**

21. If $|z| = 2$, show that $3 \le |z + 3 + 4i| \le 7$

22. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}}=0$

23. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c.

24. If the radius of a sphere with radius 10 cm, has to decrease by 0.1 cm, approximately how much, will its volume decrease?

25. Evaluate : $\int_b^{\infty} \frac{1}{a^2+x^2}dx$, $a > 0$, $b \in R$.

26. Find the vector equation of a plane which is at a distance of 7 units from the origin having 3, –4, 5 are direction ratios of a normal to it.

27. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

28. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

29. Find the equation of tangent to the curve $y = x^2 + 3x - 2$ at the point (1, 2).

30. Express $e^{\cos\theta + i\sin\theta}$ in $a + ib$ form.

PART - III

Note : Answer **any seven** questions. Question No. **40** is **Compulsory.** **7 × 3 = 21**

31. Find the equation of the parabola with vertex (–1, –2), axis parallel to y - axis and passing through (3, 6) .

32. The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

33. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds?

34. Find the angle made by the straight line $\frac{x+3}{2}=\frac{y-1}{2}=-z$ with coordinate axes.

35. Use the linear approximation to find an approximate value of $(123)^{\frac{2}{3}}$.

36. Solve : $x \cos y \, dy = e^x (x \log x + 1) \, dx$

37. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$

38. Show that $p \to q$ and $q \to p$ are not equivalent.

39. If $z = (2 + 3i)(1 - i)$, then find z^{-1}.

40. If $a + b + c = 0$ and a, b, c are rational numbers then, prove that the roots of the equation $(b + c - a) x^2 + (c + a - b) x + (a + b - c) = 0$ are rational numbers.

PART - IV

Note : Answer **all** the questions. **7 × 5 = 35**

41. (a) Solve the equation $z^3 + 8i = 0$, where $z \in C$.

OR

(b) Solve : $(1 + x + xy^2)\frac{dy}{dx} + (y + y^3) = 0$

42. (a) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

OR

(b) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function of random variable X is $f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$

(i) the value of k
(ii) the distribution function
(iii) the probability that daily sales will fall between 300 litres and 500 litres.

43. (a) Identify the type of conic and find centre, foci and vertices of $18x^2 + 12y^2 - 144x + 48y + 120 = 0$

OR

(b) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$.

44. (a) A boy is walking along the path $y = ax^2 + bx + c$ through the point (–6, 8), (–2, –12) and (3,8). He wants to meet his friend at P(7, 60). Will he meet his friend? (Use Gaussian Elimination method).

OR

(b) Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

45. (a) Find the parametric form of Vector equation and Cartesian equations of the plane containing the line $\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + t(2\hat{i} - \hat{j} + 4\hat{k})$ and perpendicular to the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = 8$.

OR

(b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

46. (a) Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

OR

(b) Suppose a person deposits ₹ 10,000 in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

47. (a) Find the maximum value of $\frac{\log x}{x}$.

OR

(b) Find the area of the region common to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$.

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Answers

PART - I

1. (b) $\rho(A) = n$

2. (d) 1

3. (b) $x \in \left[\frac{1}{2}, 1\right]$

4. (a) $y = kx$

5. (d) 1

6. (c) 0

7. (b) 2

8. (d) 2

9. (a) $\frac{1}{e^2}$

10. (d) Z

11. (a) $\frac{8}{3}$

12. (a) $\frac{\pi}{2}$

13. (c) 5

14. (c) 2

15. (b) z

16. (d) –4

17. (b) log 2

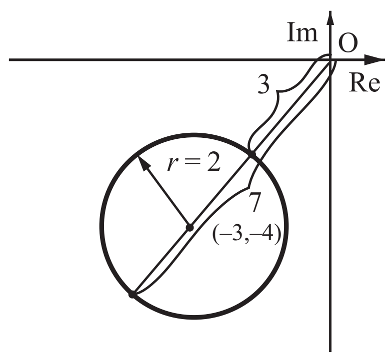
18. (d) n

19. (a) $\frac{-\pi}{6}$

20. (b) $2ab$

PART - II

21.



$|z+3+4i| \le |z| + |3+4i| = 2+5 = 7$

$|z+3+4i| \le 7$ ... (1)

$|z+3+4i| \ge ||z| - |3+4i|| = |2-5| = 3$

$|z+3+4i| \ge 3$ ...(2)

From (1) and (2), we get $3 \le |z+3+4i| \le 7$

22. Given p, q are the roots of $lx^2 + nx + n = 0$

$$p + q = -\frac{n}{l} \text{ and } pq = \frac{n}{l}$$

Consider $\dfrac{(p+q)^2}{pq} = \dfrac{\left(\frac{-n}{l}\right)^2}{\left(\frac{n}{l}\right)} = \dfrac{n^2}{l^2} \times \dfrac{l}{n} = \dfrac{n}{l}$

Taking square root on both sides

$$\sqrt{\frac{(p+q)^2}{pq}} = \sqrt{\frac{n}{l}}$$

$$\Rightarrow \pm\frac{(p+q)}{\sqrt{pq}} = \sqrt{\frac{n}{l}}$$

Consider $-\dfrac{(p+q)}{\sqrt{pq}} = \sqrt{\dfrac{n}{l}}$

$$\Rightarrow \frac{(p+q)}{\sqrt{pq}} = -\sqrt{\frac{n}{l}}$$

$$\frac{p}{\sqrt{pq}} + \frac{q}{\sqrt{pq}} = -\sqrt{\frac{n}{l}}$$

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$$

Hence proved.

23. The condition for the line $y = mx + c$ to be a tangent to the circle $x^2 + y^2 = a^2$ = is $c^2 = a^2(1 + m^2)$.

Then, $c = \pm\sqrt{9(1+6)}$

$c = \pm 3\sqrt{17}$

24. We know that volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, where $r > 0$ is the radius. So the differential $dV = 4\pi r^2\, dr$ and hence

$\Delta V \approx dV = 4\pi\,(10)^2(9.9 - 10)\text{cm}^3$

$= 4\pi\, 10^2(-0.1)\text{cm}^3 = -40\pi\text{cm}^3.$

Note that we have used $dr = (9.9 - 10)$ cm, because radius decreases from 10 to 9.9. Again the negative sign in the answer indicates that the volume of the sphere decreases about 40π cm^3.

25. We have $\displaystyle\int_b^\infty \frac{1}{a^2+x^2}dx = \left[\frac{1}{a}\tan^{-1}\frac{x}{a}\right]_b^\infty$

$$= \frac{1}{a}\tan^{-1}\infty - \frac{1}{a}\tan^{-1}\frac{b}{a} = \frac{1}{a}\left[\frac{\pi}{2} - \tan^{-1}\frac{b}{a}\right]$$

26. Given $p = 7$

$$\hat{d} = \frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{\sqrt{3^2 + (-4)^2 + 5^2}} = \frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{\sqrt{9+16+25}}$$

$$= \frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{\sqrt{50}} = \frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{5\sqrt{2}}$$

[∵ 3, –4, 5 are direction ratios]

The equation of the plane at a distance p from the origin and perpendicular to the unit normal vector $\hat{d}$ is $\vec{r} \cdot \hat{d} = p$.

Equation of the required plane is $r \cdot \left(\dfrac{3\hat{i} - 4\hat{j} + 5\hat{k}}{5\sqrt{2}}\right) = 7$

27. Then $A \vee B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0\vee 1 & 1\vee 1 \\ 1\vee 0 & 1\vee 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$A \wedge B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0\wedge 1 & 1\wedge 1 \\ 1\wedge 0 & 1\wedge 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

28. Let $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Then, $A^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

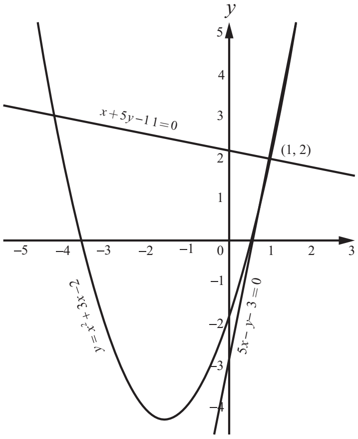
$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2.$$

Similarly, we get $A^TA = I_2$.

Hence $AA^T = A^TA = I_2 \Rightarrow$ A is orthogonal.

29. We have, $\frac{dy}{dx} = 2x + 3$ Hence, $\left(\frac{dy}{dx}\right)_{(1,2)} = 5$



Therefore, the required equation of tangent is $(y-2) = 5(x-1) \Rightarrow 5x - y - 3 = 0$.

The slope of the normal at the point (1, 2) is $-\frac{1}{5}$

Therefore, the required equation of normal is

$(y-2) = -\frac{1}{5}(x-1) \Rightarrow x + 5 - 11 = 0$.

30. $e^{\cos\theta + i\sin\theta} = e^{\cos\theta} \cdot e^{i\sin\theta}$

$= e^{\cos\theta}\,[\cos(\sin\theta) + i\sin(\sin\theta)]$

$[\because e^{i\theta} = \cos\theta + i\sin\theta]$

$= e^{\cos\theta}\cos(\sin\theta) + i\,e^{\cos\theta}\sin(\sin\theta)]$

↓

This is of $a + ib$ form

Hence proved.

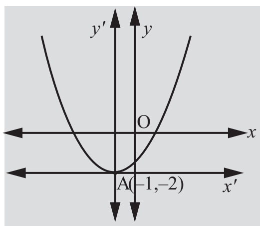
PART - III

31. Since axis is parallel to y-axis the required equation of the parabola is

$(x+1)^2 = 4a\,(y+2)$.

Since this passes through (3,6), we get

$(3+1)^2 = 4a\,(6+2) \Rightarrow a = \frac{1}{2}$



Then the equation of parabola is $(x+1)^2 = 2\,(y+2)$ which on simplifying yields,

$x^2 + 2x - 2y - 3 = 0$.

32. AS = 94.5 × 106 km, SA′ = 152×10^6 km

$a + c = 152 \times 10^6$

$a - c = 94.5 \times 10^6$

Subtracting $2c = 57.5 \times 10^6 = 575 \times 10^5$ km

Distance of the Sun from the other focus is SS′=×575×10^5 km.

33. Given $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$

$\cos\frac{\pi}{2} < 3x - 1 < \cos\pi \Rightarrow \quad 0 < 3x - 1 < -1$

$0 + 1 < 3x < -1 + 1$

$1 < 3x < 0$

$\frac{1}{3} < x < 0$

This inequality is true, only when $0 < x < \frac{1}{3}$.

34. If $\hat{b}$ is a unit vector parallel to the given line, then $\hat{b} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{|2\hat{i} + 2\hat{j} - \hat{k}|} = \frac{1}{3}(2\hat{i} + 2\hat{j} - \hat{k})$. Therefore, from the definition of direction cosines of $\hat{b}$, we have

$\alpha = \cos\alpha = \frac{2}{3}$, $\cos\beta = \frac{2}{3}$, $\cos\gamma = -\frac{1}{3}$,

where α, β, γ are the angles made by $\hat{b}$ with the positive x - axis, positive y - axis, and positive z - axis, respectively. As the angle between the given straight line with the coordinate axes are same as the angles made by $\hat{b}$ with the coordinate axes, we have

$\alpha = \cos^{-1}\left(\frac{2}{3}\right), \beta = \cos^{-1}\left(\frac{2}{3}\right), \gamma = \cos^{-1}\left(\frac{-1}{3}\right)$

respectively.

35. Let $f(x) = x^{\frac{2}{3}}$, $x_0 = 125$, $\Delta x = -2$

$\therefore (123)^{\frac{2}{3}} = f(125) + f'(125)\,(-2)$ ... (1)

$f(125) = (125)^{\frac{2}{3}} = (5^3)^{\frac{2}{3}} = 5^2 = 25$

$f'(x) = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3x^{\frac{1}{3}}}$

$$f'(125) = \frac{2}{3(125)^{\frac{1}{3}}} = \frac{2}{3(5^3)^{\frac{1}{3}}} = \frac{2}{3(5)} = \frac{2}{15}$$

$\therefore$ (1) becomes

$$(123)^{\frac{2}{3}} = 25 + \frac{2}{15}(-2) = 25 - \frac{4}{15} = 25 - 0.27$$

$$(123)^{\frac{2}{3}} = 24.73.$$

36.

$$x \cos y\, dy = e^x (x \log x + 1)\, dx$$

$$\Rightarrow \cos y\, dy = e^x \frac{(x \log x + 1)}{x} dx$$

$$= \left[e^x\left(\log x + \frac{1}{x}\right)\right] dx$$

$$\Rightarrow \int \cos y\, dy = \int e^x \left(\log x + \frac{1}{x}\right) dx$$

$$\Rightarrow \sin y = \int e^x (f(x) + f'(x)) dx \qquad \text{[where } f(x) = \log x\text{]}$$

$$\Rightarrow \sin y = e^x f(x) + c$$

$$\Rightarrow \sin y = e^x \log x + C.$$

37. Given that $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$.

Expanding along R_1 we get,

$$|F(\alpha)| = \cos\alpha \begin{vmatrix} 1 & 0 \\ 0 & \cos\alpha \end{vmatrix} - 0 + \sin\alpha \begin{vmatrix} 0 & 1 \\ -\sin\alpha & 0 \end{vmatrix}$$

$$= \cos\alpha(\cos - 0) + \sin\alpha(0 + \sin\alpha)$$

$$= \cos^2 + \sin^2\alpha = 1 \neq 0$$

Since $F(\alpha)$ is a non-singular matrix, $[F(\alpha)]^{-1}$ exists.

$$\text{Now, adj } (F(\alpha)) = \begin{bmatrix} +\begin{vmatrix} 1 & 0 \\ 0 & \cos\alpha \end{vmatrix} & -\begin{vmatrix} 0 & 0 \\ \sin\alpha & \cos\alpha \end{vmatrix} & +\begin{vmatrix} 0 & 1 \\ -\sin\alpha & 0 \end{vmatrix} \\ -\begin{vmatrix} 0 & \sin\alpha \\ 0 & \cos\alpha \end{vmatrix} & +\begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix} & -\begin{vmatrix} \cos\alpha & 0 \\ -\sin\alpha & 0 \end{vmatrix} \\ +\begin{vmatrix} 0 & \sin\alpha \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} \cos\alpha & \sin\alpha \\ 0 & 0 \end{vmatrix} & +\begin{vmatrix} \cos\alpha & 0 \\ 0 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} +(\cos\alpha - 0) & -(0) & +(0 + \sin\alpha) \\ -(0) & +(\cos^2\alpha + \sin^2\alpha) & -(0) \\ +(0 - \sin\alpha) & -(0) & +(\cos - 0) \end{bmatrix}^T$$

$$= \begin{bmatrix} \cos\alpha & 0 & +\sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}^T = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$\therefore F(\alpha)^{-1} = \frac{1}{|F(\alpha)|} \text{adj } (F(\alpha))$$

$$[F(\alpha)]^{-1} = \frac{1}{1}\begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad \text{...(1)}$$

$$\text{Now, } F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & 0 & \sin(-\alpha) \\ 0 & 1 & 0 \\ -\sin(-\alpha) & 0 & \cos(-\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad \text{... (2)}$$

[$\because$ cosα is an even function, cos $(-\alpha)$ = cos α and sin α is an odd function, sin $(-\alpha) = -\sin\alpha$]

From (1) and (2),

$$[F(\alpha)]^{-1} = F(-\alpha)$$

Hence proved.

38.

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

The entries in column (3) and column (4) are not identical.

39. We have $z = (2 + 3i)(1 - i) = (2 + 3) + (3 - 2)i = 5 + i$

$\Rightarrow \quad z^{-1} = \frac{1}{z} = \frac{1}{5+i}$

Multiplying the numerator and denominator by the conjugate of the denominator, we get

$$z^{-1} = \frac{(5-i)}{(5+i)(5-i)} = \frac{5-i}{5^2+1^2} = \frac{5}{26} - i\frac{1}{26} \Rightarrow z^{-1} = \frac{5}{26} - i\frac{1}{26}$$

40. Given $a + b + c = 0$

On comparing the given equation with $Ax^2 + Bx + C = 0$

We get, $A = (b + c - a)$, $B = (c + a - b)$, $C = (a + b - c)$

We know, $\Delta = B^2 - 4AC$

$\Delta = (c + a - b)^2 - 4(b + c - a)(a + b - c)$

$= (a + b + c - 2b)^2 - 4(a + b + c - 2a)(a + b + c - 2c)$

where $a + b + c = 0 = (-2b)^2 - 4(-2a)(-2c) = 4b^2 - 4(4ac) = 4b^2 - 16ac$

$= 4(b^2 - 4ac) = 4[(-a - c)^2 - 4ac] = 4(a - c)^2$

$= [2(a - c)]^2 \Rightarrow$ perfect square

$\therefore$ They are rational numbers. Hence proved.

PART - IV

41. (a)

$$\text{Let } z^3 + 8i = 0. \text{ Then, we get}$$
$$z^3 = -8i$$
$$= 8(-i) = 8\left(\cos\left(-\frac{\pi}{2}+2k\pi\right)+i\sin\left(-\frac{\pi}{2}+2k\pi\right)\right), k = \mathbb{Z}.$$
$$\text{Therefore, } z = \sqrt[3]{8}\left(\cos\left(\frac{-\pi+4k\pi}{6}\right)+i\sin\left(\frac{-\pi+4k\pi}{6}\right)\right), k = 0, 1, 2.$$

Taking $k = 0, 1, 2$, we get,

$$k=0, \quad z = 2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)=2\left(\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)=\sqrt{3}-i$$
$$k=1, \quad z = 2\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)=2=2\,(0+i)=0+2i=2i$$
$$k=2, \quad z = 2\left(\cos\left(\frac{7\pi}{6}\right)+i\sin\left(\frac{7\pi}{6}\right)\right)=2\left(\cos\left(\pi+\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)$$
$$= 2\left(-\cos\left(\frac{\pi}{6}\right)-i\sin\left(\frac{\pi}{6}\right)\right)=2\left(-\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)=-\sqrt{3}-i$$

The values of z are $\sqrt{3}-i$, $2i$, and $\sqrt{3}-i$

OR

(b)

$$(1+x+xy^2)\frac{dy}{dx}+(y+y^3) = 0$$
$$\Rightarrow (1+x+xy^2)dy+(y+y^3)dx = 0$$
$$\Rightarrow (1+x+xy^2)+(y+y^3)\frac{dx}{dy} = 0$$
$$\Rightarrow (y+y^3)\frac{dx}{dy}+(1+x+xy^2) = 0$$

Dividing by $y+y^3$, we get,

$$\frac{dx}{dy}+\frac{1+x+xy^2}{y+y^3} = 0$$
$$\frac{dx}{dy}+\frac{1+x\left(1+y^2\right)}{y+y^3} = 0$$
$$\Rightarrow \frac{dx}{dy}+\frac{1}{y+y^3}+\frac{x\left(1+y^2\right)}{y+y^3} = 0$$
$$\Rightarrow \frac{dx}{dy}+\frac{x\left(1+y^2\right)}{y\left(1+y^2\right)} = \frac{-1}{y\left(1+y^2\right)}$$
$$\Rightarrow \frac{dx}{dy}+\frac{1}{y}.x = \frac{-1}{y\left(1+y^2\right)}$$

Here $P = \frac{1}{y}$ and $Q = \frac{-1}{y\left(1+y^2\right)}$

$\therefore \int p\,dy = \int \frac{1}{y}dy = \log y$

$\therefore$ I.F $= e^{\int p\,dy} = e^{\log y} = y$

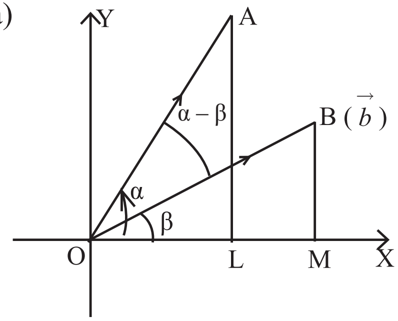
$\therefore$ The solution is $\quad x\,e^{\int p\,dy} = \int Qe^{\int p\,dy}dy + c$

$\Rightarrow \quad xy = \int \frac{-1}{y(1+y^2)}y\,dy + c = -\int \frac{dy}{1+y^2} + c$

$\Rightarrow \quad xy = -\tan^{-1}(y) + c$

$\Rightarrow \quad xy + \tan^{-1} y = C$

42. (a)



Let $\hat{a} = \overrightarrow{OA}$ and $\hat{b} = \overrightarrow{OB}$ be the unit vectors and which make angles α, β respectively with positive x-axis.
Draw AL and BM $\perp^r$ to x-axis

Then $|\overrightarrow{OL}| = |\overrightarrow{OA}| \cos\alpha = \cos\alpha$

$|\overrightarrow{LA}| = |\overrightarrow{OA}| \sin\alpha = \sin\alpha$ Q[$\because$ ΔOAL, $\sin\alpha = \frac{\text{opp}}{\text{hyp}} = \cos\alpha = \frac{\text{adj}}{\text{hyp}}$]

$\therefore \overrightarrow{OL} = |\overrightarrow{OL}|\hat{i} = \cos\alpha\,\hat{i}$

$\overrightarrow{LA} = |\overrightarrow{LA}|\hat{j} = \sin\alpha\,\hat{j}$

$\therefore \hat{a} = \overrightarrow{OA} = \overrightarrow{OL} + \overrightarrow{LA}$ [Using Δ law of addition]

$\hat{a} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$ ...(1)

Similarly $\hat{b} = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$ ...(2)

The angle between $\hat{a}$ and $\hat{b}$ is (α – β),

$\therefore \hat{a}\cdot\hat{b} = |\vec{a}||\vec{b}|\cos(\alpha-\beta) = \cos(\alpha-\beta)$ ...(3)

Also $\vec{a}.\vec{b} = (\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j})(\cos\beta\,\hat{i} + \sin\beta\,\hat{j})$ [$\because |\vec{a}| = |\vec{b}| = 1$]

$\hat{a}\cdot\hat{b} = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ ...(4)

From (3) and (4), we get,

cos (α – β) = cos α cos β + sin α sin β.

OR

(b) Given $f(x) = \begin{cases} k, & 200 \le x \le 600 \\ 0, & \text{otherwise} \end{cases}$

(i) Since $f(x)$ is a probability density function,

$$\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_{200}^{600} k\,dx = 1$$

$$\Rightarrow \quad k[x]_{200}^{600} = 1 \Rightarrow k(600-200) = 1$$

$$\Rightarrow \quad 400k = 1 \Rightarrow k = \frac{1}{400}$$

(ii) The distribution function $F(x) = p(X \le x) = \int_{-\infty}^{x} f(x)dx$

When $x < 200$, $F(x) = \int_{-\infty}^{200} f(x)dx = \int_{-\infty}^{200} 0\, dx = 0$

When $200 \le x \le 600$,

$$F(x) = \int_{-\infty}^{200} f(x)dx + \int_{200}^{x} f(x)dx = \int_{-\infty}^{200} 0\, dx + \int_{200}^{x} k\, dx = 0 + [kx]_{200}^{x}$$

$$= kx - k(200) = \frac{x}{400} - \frac{200}{400} = \frac{x}{400} - \frac{1}{2}$$

When $x > 600$, $F(x) = \int_{200}^{600} \frac{1}{400} dx + \int_{600}^{x} 0\, dx = \int_{200}^{600} \frac{1}{400} dx = 1$

$\therefore$ Distribution function is $f(x) = \begin{cases} 0, & x < 200 \\ \frac{x}{400} - \frac{1}{2}, & 200 \le x \le 600 \\ 1, & x > 600 \end{cases}$

(iii) $P(300 < X < 500) = \int_{300}^{500} f(x)dx$

$$= \int_{300}^{500} k\, dx = \frac{1}{400}[x]_{300}^{500}$$

$$= \frac{1}{400}[500-300] = \frac{200}{400} = \frac{1}{2}$$

43. (a) Given equation is

$$18x^2 + 12y^2 - 144x + 48y + 120 = 0$$
$$18x^2 - 144x + 12y^2 + 48y = -120$$
$$\Rightarrow \quad 18(x^2 - 8x) + 12(y^2 + 4y) = -120$$
$$\Rightarrow \quad 18(x^2 - 8x + 16 - 16) + 12(y^2 + 4y + 4 - 4) = -120$$
$$18(x-4)^2 - 288 + 12(y+2)^2 - 48 = -120$$
$$\Rightarrow \quad 18(x-4)^2 + 12(y+2)^2 = -120 + 288 + 48$$
$$\Rightarrow \quad 18(x-4)^2 + 12(y+2)^2 = 216$$

Dividing by 216 we get,

$$\frac{18(x-4)^2}{216} + \frac{12(y+2)^2}{216} = 1$$

$$\Rightarrow \quad \frac{(x-4)^2}{12} + \frac{(y+2)^2}{18} = 1$$

This is an equation of the ellipse with major axis parallel to y-axis.

$\therefore\ a^2 = 18,\ b^2 = 12$

$\therefore\ c^2 = a^2 - b^2 = 18 - 12 = 6 \Rightarrow c = \sqrt{6}$

$$e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{12}{18}} = \sqrt{\frac{18-12}{18}}$$

$$= \sqrt{\frac{\not{6}}{\underset{3}{\not{18}}}} = \sqrt{\frac{1}{3}}$$

(a) Center is $(4, -2)$

$\Rightarrow h = 4, k = -2$

(b) Foci are $(h, k - c), (h, k + c)$

$\Rightarrow (4, -2 - \sqrt{6}), (4, -2 + \sqrt{6})$

(c) Vertices are $(h, k - a), (h, k + a)$

$\Rightarrow (4, -2 - 3\sqrt{2}), (4, -2 + 3\sqrt{2})$ $[\because a^2 = 18 \Rightarrow a = \sqrt{18} = 3\sqrt{2}]$

OR

(b) Let $\cos^{-1} x = \alpha$ and $\cos-1\ y = \beta$. Then, $x = \cos\alpha$ and $y = \beta$.

$\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$

gives $\alpha + \beta = \pi - \cos^{-1} z$. ... (1)

Now, $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = xy - \sqrt{1-x^2}\sqrt{1-y^2}$.

From (1), we get $\cos(\pi - \cos^{-1} z) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$.

$-\cos(\cos^{-1} z) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

So, $-z = xy - \sqrt{1-x^2}\sqrt{1-y^2}$, which gives $-xy - z = -\sqrt{1-x^2}\sqrt{1-y^2}$.

Squaring on both sides and simplifying, we get $-x^2 + y^2 + z^2 + 2xyz = 1$.

44. (a) Given $y = ax^2 + bx + c$ ...(1)

$(-6, 8)$ lies on (1)

$\Rightarrow 8 = a(-6)^2 + b(-6) + c$

$\Rightarrow 8 = 36a - 6b + c$ ...(2)

$(-2, -12)$ lies on (1)

$\Rightarrow -12 = a(-2)^2 + b(-2) + c$

$\Rightarrow -12 = 4a - 2b + c$ ...(3)

Also $(3, 8)$ lies on (1)

$\Rightarrow 8 = a(3)^2 + b(3) + c$

$\Rightarrow 8 = 9a + 3b + c$ ...(4)

Reducing the augment matrix to an equivalent row-echelon form by using elementary row operations, we get,

$$\left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{array}\right] \xrightarrow[R_3 \to 4R_3 - R_1]{R_2 \to 9R_2 - R_1} \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & -12 & 8 & -116 \\ 0 & 18 & 3 & 24 \end{array}\right] \xrightarrow[R_3 \to R_3 \div 3]{R_2 \to R_2 \div 4} \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & -3 & 2 & -29 \\ 0 & 6 & 1 & 8 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + 2R_2} \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & -3 & 2 & -29 \\ 0 & 0 & 5 & -50 \end{array}\right]$$

Writing the equivalent equation from the row echelon matrix, we get $36a - 6b + c = 8$ ...(1)

$$-3b + 2c = -29 \quad ...(2)$$

$$5c = -50$$

$$\Rightarrow \quad c = \frac{-50}{5} = -10$$

Substituting $c = -10$ in (2) we get,

$$-3b + 2(-10) = -29$$
$$\Rightarrow \quad -3b - 20 = -29$$
$$\Rightarrow \quad -3b = -29 + 20$$
$$\Rightarrow \quad -3b = -9$$
$$\Rightarrow \quad b = \frac{-9}{-3} = 3$$

Substituting $b = 3$ and $c = -10$ in (1) we get,

$$36a - 6(3) - 10 = 8$$
$$\Rightarrow \quad 36a - 18 - 10 = 8$$
$$\Rightarrow \quad 36a - 28 = 8$$
$$\Rightarrow \quad 36a = 8 + 28 = 36$$
$$\Rightarrow \quad a = \frac{36}{36} = 1$$
$$\therefore a = 1, b = 3, c = -10$$

Hence the path of the boy is

$$y = 1(x^2) + 3(x) - 10$$
$$\Rightarrow \quad y = x^2 + 3x - 10$$

Since his friend is at P(7, 60),

$$60 = (7)^2 + 3(7) - 10$$
$$\Rightarrow \quad 60 = 49 + 21 - 10$$
$$\Rightarrow \quad 60 = 70 - 10 = 60$$
$$\Rightarrow \quad 60 = 60$$

Since (7, 60) satisfies his path , he can meet his friend who is at P(7, 60)

OR

(b) Let the point of intersection of the two curves be (a,b). Hence,

$$a^2 + b^2 = 8 \text{ and } a^2 - 2b^2 = 4 \quad ... (1)$$

It is enough to show that the product of the slopes of the two curves evaluated at (a,b) is -1.

Differentiation of $x^2 + 4y^2 = 8$ with respect x, gives

$$2x + 8y\frac{dy}{dx} = 0$$

Therefore $\dfrac{dy}{dx} = -\dfrac{x}{4y}$.

Differentiation of $x^2 - 2y^2 = 4$ with respect x, gives

$$2x - 4y\frac{dy}{dx} = 0$$

Therefore $\dfrac{dy}{dx} = \dfrac{x}{2y}$.

Then $\dfrac{dy}{dx}$ at $(a, b) = m_2 = \dfrac{a}{2b}$.

Therefore $m_1 \times m_2 = \left(-\dfrac{a}{4b}\right)\times\left(\dfrac{a}{2b}\right) = \dfrac{a^2}{8b^2}$ ... (2)

Applying the ratio of proportions in (1), we get

$$\frac{a^2}{-16-16} = \frac{b^2}{-8+4} = \frac{1}{-2-4}$$

Therefore $\dfrac{a^2}{b^2} = \dfrac{32}{4} = 8$. Substituting in (2), we get $m_1 \times m_2 = -1$. Hence, the curves cut orthogonally.

45. (a) The plane containing the line $\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + t(2\hat{i} - \hat{j} + 4\hat{k})$.

$\therefore$ The required plane is passing through the point $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and parallel to a vector $\vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}$.

Also, the plane is perpendicular to the plane $\vec{r}\cdot(\hat{i} + 2\hat{j} + \hat{k}) = 8$.

$\therefore$ The required plane is parallel to the vector $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$.

(i) The parametric form of vector equation of the plane passing through one point ($\vec{a}$) and parallel to two vectors $\vec{b}$ and $\vec{c}$ is

$$\vec{r} = \vec{a} + s\vec{b} + t\vec{c} \text{ where } s, t \in \mathbb{R}$$

$$\Rightarrow \vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + s(2\hat{i} - \hat{j} + 4\hat{k}) + t(\hat{i} + 2\hat{j} + \hat{k}), s, t \in \mathbb{R}$$

(ii) Cartesian equation is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-1 & y+1 & z-3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$$\Rightarrow (x-1)(-1-8) - (y+1)(2-4) + (z-3)(4+1) = 0$$
$$\Rightarrow (x-1)(-9) - (y+1)(-2) + (z-3)5 = 0$$
$$\Rightarrow 9x + 9 + 2y + 2 + 5z - 15 = 0$$
$$\Rightarrow -9x + 2y + 5z - 4 = 0$$
$$\Rightarrow 9x - 2y - 5z + 4 = 0$$

OR

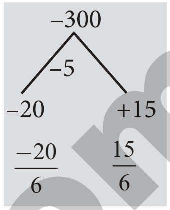
(b) This equation is Type - 2 even degree reciprocal equation. Hence, it can be rewritten as

$$6\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) - 38 = 0 \quad ...(1)$$

put $x + \dfrac{1}{x} = y \Rightarrow \left(x + \dfrac{1}{x}\right)^2 = y^2$

$\Rightarrow \quad x^2+\frac{1}{x^2}+2 = y^2 \Rightarrow x^2+\frac{1}{x^2} = y^2-2$

$\therefore$ (1) becomes as,



$6(y^2-2)-5y-38 = 0$

$\Rightarrow \quad 6y^2-12-5y-38 = 0$

$\Rightarrow \quad 6y^2-5y-50 = 0$

$\Rightarrow \quad (3y-10)(2y+5) = 0$

$\Rightarrow \quad y = \frac{10}{3}, \frac{-5}{2}$

Case (i)

When $y = \frac{+10}{3},\ x+\frac{1}{x} = \frac{+10}{3}$

$\frac{x^2+1}{x} = \frac{+10}{3} \Rightarrow 3x^2+3 = 10x$

$3x^2-10x+3 = 0$

$(x-3)(x-\frac{1}{3}) = 0 \Rightarrow x = 3, \frac{1}{3}$

Case (ii)

When $y = -\frac{5}{2} \Rightarrow x+\frac{1}{x} = -\frac{5}{2} \Rightarrow \frac{x^2+1}{x} = \frac{-5}{2}$

$\Rightarrow \quad 2x^2+2+5x = 0$

$\Rightarrow \quad 2x^2+5x+2 = 0$

$\Rightarrow \quad (x+2)(2x+1) = 0$

$\Rightarrow \quad x = -2, \frac{-1}{2}$

$\therefore$ The roots are $3, \frac{1}{3}, -2, \frac{-1}{2}$.

46. (a)

p	q	r	$\neg q$	$\sim q \vee r$	$p \to (\neg q \vee r)$	$\neg p$	$\neg p \vee (\neg q \vee r)$
T	T	T	F	T	T	F	T
T	T	F	F	F	F	F	F
T	F	T	T	T	T	F	T
T	F	F	T	T	T	F	T
F	T	T	F	T	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

The entries in column (6) and column (8) are identical.

$\therefore p \to (\neg q \vee \neg r) \equiv \neg p \vee (\neg q \vee r)$

OR

(b) Let P be the Principal at time t.

Given ratio = 5%

$$\therefore \frac{dP}{dt} = P\left(\frac{5}{100}\right) = 0.05P$$

$$\Rightarrow \frac{dP}{P} = 0.05dt \qquad \text{[separating the variable]}$$

Integrating, $$\int \frac{dP}{P} = 0.05 \int dt$$

$$\Rightarrow \log P = 0.05\,t + \log C$$

$$\Rightarrow \log P - \log C = 0.05\,t$$

$$\Rightarrow \log\left(\frac{P}{C}\right) = 0.05\,t$$

$$\Rightarrow \frac{P}{C} = e^{0.05\,t}$$

$$\Rightarrow P = Ce^{0.05\,t} \qquad \ldots(1)$$

Given when $t = 0$, P = ₹10,000

Substituting in (1) we get,

$$\Rightarrow 10{,}000 = Ce^0$$

$$\Rightarrow C = 10{,}000$$

$\therefore$ (1) becomes, $$P = 10{,}000\, e^{0.05t}$$

When t = 18 months = 1½ years = $\frac{3}{2}$ years we get

$$P = 10{,}000\, e^{0.05\left(\frac{3}{2}\right)}$$

$$\therefore P = 10{,}000\, e^{0.075}$$

47. (a) Given : $f(x) = \dfrac{\log x}{x} \qquad \Rightarrow f'(x) = \dfrac{1-\log x}{x^2}$

For a local maxima or a local minima, we must have

$$f'(x) = 0$$

$$\Rightarrow \frac{1-\log x}{x^2} = 0$$

$$\Rightarrow 1 - \log x = 0$$

$$\Rightarrow \log x = 1$$

$$\Rightarrow \log x = \log e$$

$$x = e$$

Now, $$f''x = \frac{-x-2x(1-\log x)}{x^4} = \frac{-3x-2x\log x}{x^4}$$

At $x = e$:

$$f''(e) = \frac{-3e-2e\log e}{e^4} = \frac{-5}{e^3} < 0$$

So, $x = e$ is a point of local maximum.

Thus, the local maximum value is given by

$$f(e) = \frac{\log e}{e} = \frac{1}{e}$$

OR

(b) Given the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ ... (1)

and equation of line $\frac{x}{a}+\frac{y}{b} = 1$ ... (2)

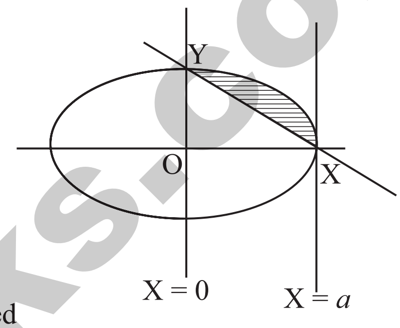
On putting the value of $\frac{x}{a}$ from equation (2) in equation (1) we get

$$\left(1-\frac{y}{b}\right)^2+\frac{y^2}{b^2} = 1$$

$$\Rightarrow \quad 1+2\left(\frac{y^2}{b^2}\right)-\frac{2y}{b} = 1$$

$$\Rightarrow \quad \frac{2y}{b}\left(\frac{y}{b}-1\right) = 0$$

$$\Rightarrow \quad y = 0, y = b \text{ then } x = a, x = 0$$



i.e., the intersection points are A(a, 0) and B (0, b). The required area is shown in the shaded figure. For the ellipse.

$$\frac{y^2}{b^2} = 1-\frac{x^2}{a^2} \Rightarrow |y| = \frac{b}{a}\sqrt{a^2-x^2}$$

Now, area of $\Delta AOB = \frac{1}{2}|OA|.|OB| = \frac{1}{2}ab$ sq. unit

Also, area under the ellipse in the first quadrant

$$= \int_0^a y\,dx = \int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

$$= \frac{b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{b}{2a}\left[(0+a^2\sin^{-1}(1))-(0+a^2\sin^{-1}(0))\right]$$

$$= \frac{b}{2a}\left[a^2\cdot\frac{\pi}{2}-a^2\cdot 0\right] = \frac{\pi ab}{4} \text{ sq. unit}$$

∴ Required area , Area of shaded region = Area of curve ΔABO – Area of ΔOAB

$$= \frac{\pi ab}{4}-\frac{1}{2}ab = \frac{(\pi-2)ab}{4}$$

$$= \frac{ab}{4}(\pi-2) \text{ sq. unit}$$

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