# தமிழ்நாடு பள்ளிக்கல்வித் துறை

## முதன்மைக் கல்வி அலுவலகம்

## கிருஷ்ணகிரி

# 12 – MATHEMATICS

## *SPECIAL GUIDE*

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**திருமதி கே.பி.மகேஸ்வரி**

முதன்மைக் கல்வி அலுவலர்

கிருஷ்ணகிரி மாவட்டம்

கிருஷ்ணகிரி

## வாழ்த்துச் செய்தி

**வணக்கம்,**

+2 கணிதவியல் பாடத்தில் மெல்ல கற்கும் மாணவர்கள் எளிய முறையில் 100% தேர்ச்சியுடன் அதிக மதிப்பெண்கள் பெறும் நோக்கில் இந்த "சிறப்புப் பயிற்சி கையேடு", தலைமையாசிரியர் மற்றும் முதுகலை கணித ஆசிரியர்கள் கொண்ட குழுவால் சிறப்பான முறையில் தயாரிக்கப்பட்டுள்ளது. இச்சிறப்புக் கையேட்டில் உள்ள 1 மதிப்பெண், 5 மதிப்பெண், மற்றும் 3, 2 மதிப்பெண் வினாக்களுக்கு எளிமையான முறையில் வாய்ப்பாடு மற்றும் படங்களுடன் சிறப்புப் பயிற்சி கையேடு தயாரிக்கப்பட்டுள்ளது. மேலும் கூடுதலாக கணித பாட புத்தகத்தில் உள்ள 1 மதிப்பெண் வினாக்கள் GeoGebra மென்பொருளின் உதவியோடு சரியான விடையைத் தேர்வு செய்யும் வகையில் வடிவமைக்கப்பட்டுள்ளது. இதன் கூடுதல் சிறப்பாக கையேட்டில் உள்ள 1 மதிப்பெண் வினாக்களை QR CODE - ஐ SCAN செய்து HI -Tech Lab - ல் பயிற்சி அளிக்கும் விதமாக அமைக்கப்பட்டுள்ளது. இச்சிறப்புக் கையேட்டினை SCAN செய்து PDF - ஆகவும் பதிவிறக்கம் செய்து கொள்ளலாம். எனவே, +2 மாணவ / மாணவியர்கள் கணிதவியல் பாடத்தில் 100% தேர்ச்சியுடன் அதிக மதிப்பெண்கள் பெற மகிழ்ச்சியுடன் வாழ்த்துகிறோம்.

**முதன்மைக் கல்வி அலுவலர்**

**கிருஷ்ணகிரி மாவட்டம்**

# அறிஞர் அண்ணா கல்லூரி (கலை மற்றும் அறிவியல்)

போலுப்பள்ளி - **கிருஷ்ணகிரி** - 635 115

**முனைவர் சு. தனபால்**
முதல்வர்

## வாழ்த்துச் செய்தி

கிருஷ்ணகிரி முதன்மைக் கல்வி அலுவலகம் சார்பில் பன்னிரண்டாம் வகுப்பு கணிதவியல் சிறப்பு கையேடு சிறப்பாக உள்ளது.

இக்கையேட்டில் ஒரு மதிப்பெண்கள், ஐந்து மதிப்பெண்கள், மூன்று மற்றும் இரண்டு மதிப்பெண் வினாக்கள் விடைகளுடன் எளிய முறையில் சிறப்பாக தயாரிக்கப்பட்டுள்ளது. இதில் ஒரு மதிப்பெண் வினாக்களுக்கு மென்பொருளைப் பயன்படுத்தி QR CODE உதவியுடன் ஒரு மதிப்பெண் வினாக்கள் தயாரிக்கப்பட்டுள்ளவை மிகவும் சிறப்பானது. எனவே மாணவர்கள் இக்கையேட்டினைப் பயன்படுத்தி சிறப்பு நிலையில் தேர்ச்சிபெறவும் அதிக மதிப்பெண்கள் பெற நிர்வாகம் மற்றும் பேராசிரிய பெருமக்கள் அனைவரும் மனமார வாழ்த்துகிறோம்.

**முதல்வர்**

**அறிஞர் அண்ணா கல்லூரி**

**கிருஷ்ணகிரி**

# 12-கணிதவியல்

## தலைமை ஆசிரியர், கணித ஆசிரியர்கள் குழு


**முனைவர்.பொ.ஜெ.முரளி**
தலைமை ஆசிரியர்
அரசு மேல்நிலைப்பள்ளி,பாரூர்.

**திரு.கி.ரவிகண்ணன்**

முதுகலை ஆசிரியர்

அரசு மகளிர் மேல்நிலைப்பள்ளி,

கிருஷ்ணகிரி.

**திரு. நா.காளியப்பன்**

முதுகலை ஆசிரியர்

அரசு மேல்நிலைப்பள்ளி,

மோரன அள்ளி.

**திரு. திரு.பி.முனியப்பா**

முதுகலை ஆசிரியர்

அரசு மேல்நிலைப்பள்ளி,
குருபரபள்ளி

**திரு. சு.வெங்கடேசன்**

முதுகலை ஆசிரியர்

பெரியார் இராமசாமி அரசு ஆண்கள் மேல்நிலைப்பள்ளி,நாகரசம்பட்டி.

**திரு.மு.அருண்குமார்**

முதுகலை ஆசிரியர்

அரசு மேல்நிலைப்பள்ளி,

உள்ளுகுருகை.

*12-th MATHEMATICS- ONE MARK-12-ஆம் வகுப்பு கணிதம் பாடப்புத்தகத்தில் உள்ள ஒரு மதிப்பெண் வினாக்கள் , மென்பொருளின் உதவியோடு, ஒரு வினாவிற்கு சரியான விடையை தேர்வு செய்ய , அதிகபட்சம் மூன்று வாய்ப்புகள் வழங்கி, மாணவர்களின் கற்றல் திறன் அதிகரிக்கும் வகையில் வடிவமைக்கப்பட்டுள்ளது.*

*தமிழ்வழியில் பயிற்சியை மேற்கொள்ள மேலே உள்ள QR CODE-ஐ SCAN செய்யவும்.*

*ஆங்கிலவழியில் பயிற்சியை மேற்கொள்ள மேலே உள்ள QR CODE-ஐ SCAN செய்யவும்.*

*இந்த சிறப்பு கையேடை மேலே உள்ள QR CODE-ஐ SCAN செய்து முழு PDF-ஆக பதிவிறக்கம் செய்யலாம்*

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## 1.APPLICATIONS OF MATRICES AND DETERMINANTS

1. If $adjA = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $adjB = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$, then adj (AB ) is

(a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

2. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 4 (d) 3

3. If $x^a y^b = e^m, x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then the values of $x$ and $y$ are respectively,

(a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (b) $\log(\Delta_1/\Delta_3)$ , $\log(\Delta_2/\Delta_3)$

(c) $\log(\Delta_2/\Delta_1)$ ,$\log(\Delta_3/\Delta_1)$ (d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

4. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B=adjA and C=3A , then $\frac{|adjB|}{|C|} =$

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

5. If $A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ then A=

(a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

6. If |adj (adjA )|= $|A|^9$ , then the order of the square matrix $A$ is

(a) 3 (b) 4 (c) 2 (d) 5

7. If $A$ is a 3× 3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$ , then $BB^T =$

(a) A (b) B (c) $I_3$ (d) $B^T$

8. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2 - A =$

(a) $A^{-1}$ (b) $\frac{A^{-1}}{2}$ (c) $3A^{-1}$ (d) $2A^{-1}$

9. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj(AB)| =$

(a) −40 (b) −80 (c) −60 (d) −20

10. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ & $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of $a_{23}$ is

(a) 0 (b) −2 (c) −3 (d) −1

11. If $A$, $B$ and $C$ are invertible matrices of some order, then which one of the following is not true?

(a) adjA=|A |$A^{-1}$ (b) adj(AB)=(adjA)(adjB) (c) det$A^{-1}$ =$(detA)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

12. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1}$

(a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

13. If $A^T A^{-1}$ is symmetric, then $A^2 =$

(a) $A^{-1}$ (b) $(A^T)^2$ (c) $A^T$ (d) $(A^{-1})^2$

14. If $A$ is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

(a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

15. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$, and $A^T = A^{-1}$ then the value of $x$ is

(a) $\frac{-4}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

16. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix $A$ and |A |=4 ,then $x$ is

(a) 15 (b) 12 (c) 14 (d) 11

17. If $A = \begin{bmatrix} 1 & tan\frac{\theta}{2} \\ -tan\frac{\theta}{2} & 1 \end{bmatrix}$ and AB = $I_2$ , then $B$ =

(a) $\left(cos^2\frac{\theta}{2}\right)A$ (b) $\left(cos^2\frac{\theta}{2}\right)A^T$ (c) $(cos^2\theta)I$ (d) $\left(sin^2\frac{\theta}{2}\right)A$

18. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix.

(ii) Adjoint of a diagonal matrix is also a diagonal matrix.

(iii) If $A$ is a square matrix of order $n$ and $\lambda$ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$

(iv) A(adjA )= (adjA )A= |A|I

(a) Only (i) (b) (ii) and (iii) (c) (iii) and (iv) (d) (i), (ii) and (iv)

19.If $\rho(A) = \rho[(A|B)]$ then the system $AX = B$ of linear equations is

(a) consistent and has a unique solution (b) consistent

(c) consistent and has infinitely many solution (d) inconsistent

20. If $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ and A(adjA)= $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$,then $k$ =

(a) 0 (b) sinθ (c) cosθ (d) 1

21. If $0 \le \theta \le \pi$ and the system of equations $x + (sin\theta)y - (cos\theta)z = 0$, $(cos\theta)x - y + z = 0$, $(sin\theta)x + y - z = 0$ has non-trivial solution then $\theta$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

22. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$ then $\lambda$ is

(a) 17 (b) 14 (c) 19 (d) 21

23. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then adj(adjA ) is

(a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

24. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$ . The system has infinitely many solutions if

(a) $\lambda = 7, \mu \neq -5$ (b) $\lambda = -7, \mu = 5$ (c) $\lambda \neq 7, \mu \neq -5$ (d) $\lambda = 7, \mu = -5$

25. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and 4B = $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If $B$ is the inverse of $A$, then the value of $x$ is

(a) 2 (b) 4 (c) 3 (d) 1

## 2.COMPLEX NUMBERS

1. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$ then |z | is equal to

(a) 0 (b) 1 (c) 2 (d) 3

2. If $z$ is a non-zero complex number, such that $2iz^2 = \bar{z}$ , then |z | is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

3. If | z-2+i| $\leq$ 2 then the greatest value of |z | is

(a) $\sqrt{3}-2$ (b) $\sqrt{3}+2$ (c) $\sqrt{5}-2$ (d) $\sqrt{5}+2$

4. The solution of the equation |z|-z=1+2i, is

(a) $\frac{3}{2}-2i$ (b) $-\frac{3}{2}+2i$ (c) $2-\frac{3}{2}i$ (d) $2+\frac{3}{2}i$

5. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

(a) 0 (b) 1 (c) $-1$ (d) $i$

6. The value of $\sum_{i=1}^{13}(i^n + i^{n-1})$,is

(a) 1+ $i$ (b) $i$ (c) 1 (d) 0

7. The area of the triangle formed by the complex numbers z,iz and z+iz in the Argand's diagram is

(a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$

8. The conjugate of a complex number is $\frac{1}{i-2}$,Then, the complex number is

(a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$

9.If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then $k$=

(a) 1 (b) $-1$ (c) $\sqrt{3}i$ (d) $-\sqrt{3}i$

10. If α and β are the roots of $x^2 + x + 1 = 0$ then $\alpha^{2020} + \beta^{2020}$ is

(a) $-2$ (b) $-1$ (c) 1 (d) 2

11. The product of all four values of $\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

(a) $-2$ (b) $-1$ (c) 1 (d) 2

12. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(a) cis$\frac{2\pi}{3}$ (b) cis$\frac{4\pi}{3}$ (c) -cis$\frac{2\pi}{3}$ (d) -cis$\frac{4\pi}{3}$

13. If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

(a) 1 (b) 2 (c) 3 (d) 4

14. If z is a complex number s.t $z \in C\backslash R$ and $z + \frac{1}{z} \in R$, then |z| is

(a) 0 (b) 1 (c) 2 (d) 3

15. $z_1, z_2 \& z_3$ be three complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1|=|z_2|=|z_3|=1$ , then $z_1{}^2+z_2{}^2+z_3{}^2$ is

(a) 3 (b) 2 (c) 1 (d) 0

16. If $\frac{z-1}{z+1}$ is purely imaginary, then |z | is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

17. If $\left|z-\frac{3}{z}\right|=2$, then the least value of |z | is

(a) 1 (b) 2 (c) 3 (d) 5

18. If |z|=1 , then the value of $\frac{1+z}{1+\bar{z}}$ is

(a) $z$ (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) 1

19.If $z=x+iy$ is a complex number such that |z+2|=|z-2 |,then the locus of $z$ is

(a) real axis (b) imaginary axis (c) ellipse (d) circle

20. The principal argument of $\frac{3}{-1+i}$ is

(a) $\frac{-5\pi}{6}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{-3\pi}{4}$ (d) $\frac{-\pi}{2}$

21. The principal argument of $(sin40° + icos40°)^5$ is

(a) -110° (b)-70° (c) 70° (d)110°

22. If $(1+i)(1+2i)(1+3i)\ldots\ldots(1+ni)=(x+iy)$, then 2..5.10 … ….$(1+n^2)$ is

(a) 1 (b) $i$ (c) $x^2+y^2$ (d) $1+n^2$

23. If $\omega \neq 1$is a cubic root of unity and $(1+\omega)^7 = A + B\omega$ , then (A, B)=

(a) (1,0) (b) (-1,1) (c) (0,1) (d) (1,1)

24. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$

25. If | $z_1$|=1, |$z_2$|=2, |$z_3$|=3 , and |$9z_1z_2+4z_1z_3+z_2z_3$ |=12 then the value of $|z_1+z_2+z_3|$ is

(a) 1 (b) 2 (c) 3 (d) 4

## 3.THEORY OF EQUATIONS

1. The polynomial $x^3+2x+3$ has

(a) 1 -ve and 2 imaginary zeros (b) 1 +ve and 2 imaginary zeros

(c) 3 real zeros (d) no zeros

2. The number of +ve roots of the polynomial $\sum_{j=0}^{n} n_{C_r}(-1)^r x^r$ is

(a) 0 (b) $n$ (c) < $n$ (d) r

3. If $f$ and $g$ are polynomials of degrees $m$ and $n$ respectively, and if $h(x)=(f\bullet g)(x)$, then the degree of $h$ is

(a) mn (b)m+n (c) $m^n$ (d) $n^m$

4. A polynomial equation in $x$ of degree $n$ always has

(a) $n$ distinct roots (b) $n$ real roots (c) n complex roots (d) at most one root.

5. If $\alpha$ , $\beta$ and γ are the roots of $x^3+px^2+qx+r$, then $\Sigma\frac{1}{\alpha}$ is

(a) $\frac{-q}{r}$ (b) $\frac{-p}{r}$ (c) $\frac{q}{r}$ (d) $\frac{-q}{p}$

6. A zero of $x^3+64$ is

(a) 0 (b) 4 (c) 4i (d) -4

7. The polynomial $x^3-kx^2+9x$ has three real zeros if and only if, $k$ satisfies

(a) |k |≤ 6 (b) $k$ = 0 (c) | k| > 6 (d) |k | ≥6

8.The number of real numbers [0,2$\pi$]in satisfying $sin^4x-2sin^2x+1$ is

(a) 2 (b) 4 (c)1 (d)∞

9. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

(a) $a \geq 0$ (b) $a > 0$ (c) $a < 0$ (d) $a \leq 0$

10. According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$?

(a) $-1$ (b) $\frac{5}{4}$ (c) $\frac{4}{5}$ (d) 5

## 4.INVERSE TRIGNOMETRIC FUNCTIONS

1. If $sin^{-1} x = 2sin^{-1} \alpha$ has a solution, then

(a) $|\alpha| \leq \frac{1}{\sqrt{2}}$ (b) $|\alpha| \geq \frac{1}{\sqrt{2}}$ (c) $|\alpha| < \frac{1}{\sqrt{2}}$ (d) $|\alpha| > \frac{1}{\sqrt{2}}$

2. $sin^{-1}(cosx) = \frac{\pi}{2} - x$ is valid for

(a) $-\pi \leq x \leq 0$ (b) $0 \leq x \leq \pi$ (c) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ (d) $-\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$

3. The value of $sin^{-1}(cosx)$ $0 \leq x \leq \pi$ is

(a) $\pi - x$ (b) $x - \frac{\pi}{2}$ (c) $\frac{\pi}{2} - x$ (d) $x - \pi$

4. If $sin^{-1}x + sin^{-1}y = \frac{2\pi}{3}$ then $cos^{-1}x + cos^{-1}y$ is equal to

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\pi$

5. If $sin^{-1} x + sin^{-1} y + sin^{-1} z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is

(a) 0 (b) 1 (c) 2 (d) 3

6. If $cot^{-1} x = \frac{2\pi}{5}$ for some $\in R$ , the value of $tan^{-1} x$ is

(a) $\frac{-\pi}{10}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{-\pi}{5}$

7. The domain of the function defined by $f(x) = sin^{-1}\sqrt{x-1}$ is

(a)[1,2] (b) [-1,1] (c)[0,1] (d)[-1,0]

8. $sin^{-1} \frac{3}{5} - cos^{-1} \frac{12}{13} + sec^{-1} \frac{5}{3} - cosec^{-1} \frac{13}{2}$ is equal to

(a) $2\pi$ (b) $\pi$ (c) 0 (d) $tan^{-1} \frac{12}{65}$

9. If $|x| \leq 1$ , then $2tan^{-1}x - sin^{-1} \frac{2x}{1+x^2}$ is equal to

(a) $tan^{-1}x$ (b) $sin^{-1}x$ (c) 0 (d) $\pi$

10. The equation $tan^{-1}x - cot^{-1}x = tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(a) no solution (b) unique solution (c) two solutions (d) infinite number of solutions

11. If $sin^{-1}x + cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$ ,then $x$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

12. If $sin^{-1}\frac{x}{5} + cosec^{-1}\frac{5}{4} = \frac{\pi}{2}$, then the value of $x$ is

(a) 4 (b) 5 (c) 2 (d) 3

13. $sin(tan^{-1}x), |x| < 1$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

14. If $x=\frac{1}{5}$, the value of cos $(cos^{-1}x+2sin^{-1}x)$ is

(a) $-\sqrt{\frac{24}{25}}$ (b) $\sqrt{\frac{24}{25}}$ (c) $\frac{1}{5}$ (d) $\frac{-1}{5}$

15. $tan^{-1}\left(\frac{1}{4}\right)+tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(a) $\frac{1}{2}cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2}tan^{-1}\left(\frac{3}{5}\right)$ (d) $tan^{-1}\left(\frac{1}{2}\right)$

16. If the function $f(x)=sin^{-1}(x^2-3)$, then $x$ belongs to

(a) [-1,1] (b) $[\sqrt{2},2]$ (c) $[-2,-\sqrt{2}]\cup[\sqrt{2},2]$ (d) $[-2,-\sqrt{2}]$

17. If $cot^{-1}2$ and $cot^{-1}3$ are two angles of a triangle, then the 3rd angle is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

18. $sin^{-1}\left(tan\frac{\pi}{4}\right)-sin^{-1}\left(\sqrt{\frac{3}{x}}\right)=\frac{\pi}{6}$, Then $x$ is a root of the equation

(a) $x^2-x-6=0$ (b) $x^2-x-12=0$ (c) $x^2+x-12=0$ (d) $x^2+x-6=0$

19. $sin^{-1}(2cos^2x-1)+cos^{-1}(1-2sin^2x)=$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

20. If $cot^{-1}(\sqrt{sin\alpha})+tan^{-1}(\sqrt{sin\alpha})=u$, then $cos2u$ is equal to

(a) $tan^2\alpha$ (b)0 (c)-1 (d) $tan2\alpha$

## 5.TWO DIMENSIONAL ANALYTICAL GEOMETRY

1. If the normal of the parabola $y^2=4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2+(y+2)^2=r^2$ ,then the value of $r^2$ is

(a) 2 (b) 3 (c) 1 (d) 4

2. If $x+y=k$ is a normal to the parabola$y^2=12x$ , then the value of $k$ is

(a) 3 (b) -1 (c) 1 (d) 9

3. The ellipse $E_1:\frac{x^2}{9}+\frac{y^2}{4}=1$is inscribed in a rectangle $R$ whose sides are parallel to the coordinate axes. Another ellipse $E_2$ passing through the point (0,4) circumscribes the rectangle $R$,. The eccentricity of the ellipse is

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

4. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ parallel to the straight line $-y=1$ . One of the points of contact of tangents on the hyperbola is

(a) $\left(\frac{9}{2\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$ (b) $\left(\frac{-9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (c) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (d) $(3\sqrt{3},-2\sqrt{2})$

5. The eqn of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ having centre at (0,3)is

(a) $x^2+y^2-6y-7=0$ (b) $x^2+y^2-6y+7=0$ (c) $x^2+y^2-6y-5=0$ (d) $x^2+y^2-6y+5=0$

6. Let $C$ be the circle with centre at (1,1) and radius =1. If $T$ is the circle centred at (0,y) passing through the origin and touching the circle $C$ externally, then the radius of $T$ is equal to

(a) $\frac{\sqrt{3}}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

7. Consider an ellipse whose centre is of the origin and its major axis is along $x$-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area of the quadrilateral inscribed in the ellipse with diagonals as major and minor axis of the ellipse is

(a) 8 (b) 32 (c) 80 (d) 40

8. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(a) $2ab$ (b) $ab$ (c) $\sqrt{ab}$ (d) $\frac{a}{b}$

9. An ellipse has $OB$ as semi minor axes, $F$ and $F'$ its foci and the angle $FBF'$ is a right angle. Then the eccentricity of the ellipse is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

10. The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

11. If the two tangents drawn from a point $P$ to the parabola $y^2=4x$ are at right angles then the locus of $P$ is

(a) $2x+1=0$ (b) $x=-1$ (c) $2x-1=0$ (d) $x=1$

12. The circle passing through (1,-2) and touching the axis of $x$ at (3,0) passing through the point

(a) (-5, 2) (b) (2,-5) (c) (5, -2) (d) (-2,5)

13. The locus of a point whose distance from (-2,0) is $\frac{2}{3}$ times its distance from the line $x=\frac{-9}{2}$ is

(a) a parabola (b) a hyperbola (c) an ellipse (d) a circle

14. The values of $m$ for which the line $y=mx+2\sqrt{5}$ touches the hyperbola $16x^2-9y^2=144$ are the roots of $x^2-(a+b)x-4=0$, then the value of (a+b) is

(a) 2 (b) 4 (c) 0 (d) -2

15. If the coordinates at one end of a diameter of the circle $x^2+y^2-8x-4y+c=0$ are (11,2) the coordinates of the other end are

(a) (-5, 2) (b) (2,-5) (c) (5, -2) (d) (-2,5)

16.The equation of the circle passing through (1,5) and (4,1) and touching $y$ -axis is $x^2+y^2-5x-6y+9+\lambda(4x+3y-19)=0$ where $\lambda$ is equal to

(a) $0, \frac{-40}{9}$ (b) 0 (c) $\frac{40}{9}$ (d) $\frac{-40}{9}$

17. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$

18.The circle $x^2+y^2=4x+8y+5$ intersects the line $3x-4y=m$ at two distinct points if

(a) $15<m<65$ (b) $35<m<85$ (c) $-85<m<-35$ (d) $-35<m<15$

19. The length of the diameter of the circle which touches the $x$ -axis at the point (1,0) and passes through the point (2,3).

(a) $\frac{6}{5}$ (b) $\frac{5}{3}$ (c) $\frac{10}{3}$ (d) $\frac{3}{5}$

20. The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

(a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$

21. The centre of the circle inscribed in a square formed by the lines $x^2 - 8x - 12 = 0$ and $y^2 - 14y + 45 = 0$ is

(a) (4,7) (b) (7,4) (c) (9,4) (d) (4,9)

22. The eqn of the normal to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ which is parallel to the line $2x + 4y = 3$ is

(a) $x + 2y = 3$ (b) $x + 2y + 3 = 0$ (c) $2x + 4y + 3 = 0$ (d) $x - 2y + 3 = 0$

23. If $P(x, y)$be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then $PF_1 + PF_2$ is

(a) 8 (b) 6 (c) 10 (d) 12

24. The radius of the circle passing through the point (6,2) two of whose diameter are $x + y = 6$ and $x + 2y = 4$ is

(a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4

25. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is

(a)$4(a^2 + b^2)$ (b) $2(a^2 + b^2)$ (c) $a^2 + b^2$ (d) $\frac{1}{2}(a^2 + b^2)$

## 6.APPLICTIONS OF VECTOR ALGEBRA

1. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + \hat{\jmath}$ , $\vec{c} = \hat{\imath}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$ then the value of $\lambda + \mu$ is

(a) 0 (b) 1 (c) 6 (d) 3

2. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$ then $\{[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]\}^2$ is equal to

(a) 81 (b) 9 (c) 27 (d)18

3. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) π

4. If the planes $\vec{r}.(2\hat{\imath} - \lambda\hat{\jmath} + \hat{k}) = 3$ and $\vec{r}.(4\hat{\imath} + \hat{\jmath} - \mu\hat{k}) = 5$ are parallel, then the value of λ and μ are

(a) $\frac{1}{2}, -2$ (b) $\frac{-1}{2}, 2$ (c) $\frac{-1}{2}, -2$ (d) $\frac{1}{2}, 2$

5.If length of perpendicular from the origin to the plane $2x + 3y + \lambda z = 1$ , λ >0 is $\frac{1}{5}$, then the value of λ is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1

6. If the volume of the parallelepiped with $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ as coterminous edges is 8 cubic units, then the volume of the parallelepiped with $(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c}), (\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})$ and $(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})$ as coterminous edges is,

(a) 8 cu. units (b) 512 cu. units (c) 64 cu. units (d) 24 cu. Units

7. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to

(a) 2 (b) -1 (c) 1 (d) 0

8. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$and $\vec{\gamma}$ , then

(a) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 1$ (b) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = -1$ (c) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ (d) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 2$

9. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$ then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

(a) $|\vec{a}||\vec{b}||\vec{c}|$ (b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (c) 1 (d) -1

10. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that is $\vec{a}$ perpendicular to $\vec{b}$ , and is parallel to $\vec{c}$ then $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to

(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$

11. If $[\vec{a}, \vec{b}, \vec{c}]$=1 then the value of $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}} + \frac{\vec{b}.(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b}).\vec{c}} + \frac{\vec{c}.(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b}).\vec{a}}$ is

(a) 1 (b) -1 (c) 2 (d) 3

12. The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath}+\hat{\jmath},\ \hat{\imath}+2\hat{\jmath},\ \hat{\imath}+\hat{\jmath}+\pi\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\pi$ (d) $\frac{\pi}{4}$

13. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a},\vec{b},\vec{a}\times\vec{b}]=\frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

14. Consider the vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$. Let $P_1$ and $P_2$ be the planes determined by the pairs of vectors $\vec{a},\vec{b}$ and $\vec{c},\vec{d}$ respectively. Then the angle between $P_1$ and $P_2$ is

(a) 0° (b) 45° (c) 60° (d) 90°

15. If $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$ where $\vec{a},\vec{b},\vec{c}$ are any three vectors such that $\vec{b}.\vec{c}\neq 0$ and $\vec{a}.\vec{b}\neq 0$ then $\vec{a}$ and $\vec{c}$ are

(a) perpendicular (b) parallel (c) inclined at $\frac{\pi}{3}$ (d) inclined at $\frac{\pi}{6}$

16. If $\vec{a}=2\hat{\imath}+3\hat{\jmath}-\hat{k}$, $\vec{b}=\hat{\imath}+2\hat{\jmath}-5\hat{k}$ and $\vec{c}=3\hat{\imath}+5\hat{\jmath}-\hat{k}$ then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is

(a) $-17\hat{\imath}+21\hat{\jmath}-97\hat{k}$ (b) $17\hat{\imath}+21\hat{\jmath}-123\hat{k}$ (c) $-17\hat{\imath}-21\hat{\jmath}+97\hat{k}$ (d) $-17\hat{\imath}-21\hat{\jmath}-97\hat{k}$

17. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

18. If the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta=0$ then $(\alpha,\beta)$ is

(a) (-5,5) (b) (-6, 7) (c) (5, -5) (d) (6, -7)

19. The angle between the line $\vec{r}=(\hat{\imath}+2\hat{\jmath}-3\hat{k})+t(2\hat{\imath}+\hat{\jmath}-2\hat{k})$ and the plane $\vec{r}.(\hat{\imath}+\hat{\jmath})+4=0$ is

(a) 0° (b) 30° (c) 45° (d) 90°

20. The coordinates of the point where the line $\vec{r}=(6\hat{\imath}-\hat{\jmath}-3\hat{k})+t(-\hat{\imath}+4\hat{k})$ meets the plane $\vec{r}.(\hat{\imath}+\hat{\jmath}-\hat{k})=3$ are

(a) (2, 1, 0) (b) (7, -1, -7) (c) (1, 2, -6) (d) (5, -1, 1)

21. Distance from the origin to the plane $3x-6y+2z+7=0$ is

(a) 0 (b) 1 (c) 2 (d) 3

22.The distance between the planes $x+2y+3z+7=0$ and $2x+4y+6z+7=0$ is

(a) $\frac{\sqrt{7}}{2\sqrt{2}}$ (b) $\frac{7}{2}$ (c) $\frac{\sqrt{7}}{2}$ (d) $\frac{7}{2\sqrt{2}}$

23. If the direction cosines of a line are $\frac{1}{c},\frac{1}{c},\frac{1}{c}$, then

(a) $c=\pm 3$ (b) $c=\pm\sqrt{3}$ (c) c > 0 (d) 0 < c < 1

24. The vector equation $\vec{r}=(\hat{\imath}-2\hat{\jmath}-\hat{k})+t(6\hat{\imath}-\hat{k})$ represents a straight line passing through the points

(a)(0,6,-1) and (1,-2,-1) (b) (0,6,-1) and (1,-4,-2) (c) (1,-2,-1) and (1,4,-2) (d) (1,-2,-1) and (0,-6,1)

25.If the distance of the point (1,1,1) from the origin is half of its distance from the plane $x+y+z+k=0$, then the values of $k$ are

(a) $\pm 3$ (b) $\pm 6$ (c) -3, 9 (d) 3, -9

## 7.APPLICATIONS OF DIFFERENTIAL CALCULUS

1.The minimum value of the function $|3-x|+9$ is

(a) 0 (b) 3 (c) 6 (d)9

2.The max slope of the tangent to the curve $y = e^x sinnx, x \in [0,2\pi]$ is at

(a)$x = \frac{\pi}{4}$ (b) $x = \frac{\pi}{2}$ (c) $x = \pi$ (d) $x = \frac{3\pi}{2}$

3.The maximum value of the function $x^2 e^{-2x}, x > 0$ is

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) $\frac{1}{e^2}$ (d) $\frac{4}{e^4}$

4.One of the closest point on the curve $x^2 - y^2 = 4$ to the point (6,0) is

(a) (2,0) (b) $(\sqrt{5}, 1)$ (c) $(3, \sqrt{5})$ (d)$(\sqrt{13}, -\sqrt{3})$

5.The maximum value of the product of two positive numbers, when the sum of squares is 200, is

(a) 100 (b) $25\sqrt{7}$ (c) 28 (d) $24\sqrt{14}$

6.The curve $y = ax^4 + bx^2$ with ab>0

(a)has no horizontal tangent (b) is concave up (c)is concave down (d)has no point of inflection

7.The point of inflection of the curve $y = (x-1)^3$ is

(a) (0,0) (b) (0,1) (c) (1,0) (d) (1,1)

8.The volume of sphere is increasing in volume at the rate of $3\pi\ cm^3/sec$. the rate of change of its radius is $\frac{1}{2}\ cm$

(a) $3cm/s$ (b) 2 $cm/s$ (c) 1 $cm/s$ (d) $\frac{1}{2} cm/s$

9.A balloon rises straight up 10 m/s. an observer is 40m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30metres above the ground.

(a) $\frac{3}{25} radians/sec$ (b) $\frac{4}{25} radians/sec$ (c) $\frac{1}{5} radians/sec$ (d) $\frac{1}{3} radians/sec$

10.The position of the particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$ .the time at which the particle is at rest is

(a) $t = 0$ (b) $t = \frac{1}{3}$ (c) $t = 1$ (d) $t = 3$

11.A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$ the stone reaches the max height in t seconds is given by

(a) 2 (b) 2.5 (c) 3 (d)3.5

12.Find the point on the curve $6y = x^3 + 2$ at which y-coordinate change 8 times as fast as x- coordinate is

(a) (4,11) (b) (4,-11) (c) (-4,11) (d) (-4,-11)

13.The value of the limit $\lim_{x\to 0}\left(cotx - \frac{1}{x}\right)$ is

(a)0 (b)1 (c) 2 (d)$\infty$

14.The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

(a) $y = 0$ (b) $y = \pm\sqrt{3}$ (c) $y = \frac{1}{2}$ (d) $y = \pm 3$

15.Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(a)$tan^{-1}\frac{3}{4}$ (b) $tan^{-1}\left(\frac{4}{3}\right)$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

16.The function $sin^4x + cos^4x$ is increasing in the interval

(a)$\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left[0, \frac{\pi}{4}\right]$

17.The no. given by Rolle's theorem for the function $x^3 - 3x^2, x \in [0,3]$is

(a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (d)2

18.The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25?

(a) -8 (b) -4 (c) -2 (d)0

19.The slope of the normal line of the curve $f(x) = 2cos4x$ at $x = \frac{\pi}{12}$ is

(a) $-4\sqrt{3}$ (b) -4 (c) $\frac{\sqrt{3}}{12}$ (d) $4\sqrt{3}$

20. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1,9]$ is

(a) 2 (b) 2.5 (c) 3 (d)3.5

## 8.DIFFERENTIALS AND PARTIAL DERIVATIVES

1.If $g(x,y) = 3x^2 - 5y + 2y^2, x(t) = e^t$ and $y(t) = cost$,then $\frac{\partial g}{\partial t} =$

(a) $6e^{2t} + 5sint - 4costsint$ (b) $6e^{2t} - 5sint + 4costsint$

(c) $3e^{2t} + 5sint + 4costsint$ (d) $3e^{2t} - 5sint + 4costsint$

2.If $f(x) = \frac{x}{x+1}$ ,then its differential is given by

(a) $\frac{-1}{(x+1)^2}dx$ (b) $\frac{1}{(x+1)^2}dx$ (c) $\frac{1}{x+1}dx$ (d) $\frac{-1}{x+1}dx$

3.If $u(x,y) = x^2 + 3xy + y - 2019$ ,then $\left.\frac{\partial u}{\partial x}\right|_{(4,-5)}$ is equal to

(a) -4 (b) -3 (c)-7 (d) 13

4. A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. then the percentage error in calculating area of this template is

(a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

5. The percentage error of fifth root of 31is approximately how many times the percentage error in 31?

(a) $\frac{1}{31}$ (b) $\frac{1}{5}$ (c) 5 (d)31

6. If $(x,y) = e^{x^2+y^2}$ , then $\frac{\partial u}{\partial x}$ is equal to

(a) $e^{x^2+y^2}$ (b) $2xu$ (c) $x^2u$ (d) $y^2u$

7. If $v(x,y) = log(e^x + e^y)$ ,then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

(a) $e^x + e^y$ (b) $\frac{1}{e^x+e^y}$ (c) 2 (d)1

8.The change in the surface area $S = 6x^2$ of a cube when the edge length varies from $x_0$ to $x_0 + dx$ is

(a) $12x_0 + dx$ (b) $12x_0dx$ (c) $6x_0dx$ (d) $6x_0 + dx$

9.If $f(x,y,z) = xy + yz + zx$, then $f_x - f_z$ is equal to

(a) $z - x$ (b) $y - z$ (c) $x - z$ (d) $y - x$

10.The approximate change in the volume $V$ of a cube of side $x$ metres caused by increasing the side by 1% is

(a) $0.3xdxm^3$ (b) $0.03xm^3$ (c) $0.03x^2m^3$ (d) $0.03x^3m^3$

11.Linear approximation for $g(x) = cosx$ at $x = \frac{\pi}{2}$ is

(a) $x + \frac{\pi}{2}$ (b) $-x + \frac{\pi}{2}$ (c) $x - \frac{\pi}{2}$ (d) $-x - \frac{\pi}{2}$

12.If $w(x,y,z) = x^2(y-z) + y^2(z-x) + z^2(x-y)$, then $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$ is

(a)$xy + yz + zx$ (b) $x(y+z)$ (c) $y(z+x)$ (d) 0

13. If $w(x,y) = x^y, x > 0$ , then $\frac{\partial w}{\partial x}$ is equal to

(a) $x^y \log x$ (b) $y\log x$ (c) $yx^{y-1}$ (d) $x \log y$

14. If $f(x,y) = e^{xy}$ ,then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

(a) $xye^{xy}$ (b) $(1+xy)e^{xy}$ (c) $(1+y)e^{xy}$ (d) $(1+x)e^{xy}$

15. If we measure the side of the cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(a) 0.4 $cu.cm$ (b) 0.45 $cu.cm$ (c) 2 $cu.cm$ (d) 4.8 $cu.cm$

## 9.APPLICATIONS OF INTEGRATION

1.The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^2xcosxdx$ is

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 0 (d) $\frac{2}{3}$

2.The value of $\int_{-4}^{4}\left[tan^{-1}\left(\frac{x^2}{x^4+1}\right) + tan^{-1}\left(\frac{x^4+1}{x^2}\right)\right]dx$

(a) $\pi$ (b) $2\pi$ (c) $3\pi$ (d) $4\pi$

3.The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\frac{2x^7-3x^5+7x^3-x+1}{cos^2x}\right)dx$ is

(a) 4 (b) 3 (c) 2 (d) 0

4.The value of $\int_0^{\frac{2}{3}}\frac{dx}{\sqrt{4-9x^2}}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\pi$

5.For any value of $n \in \mathbb{Z}$, $\int_0^{\pi} e^{cos^2x}cos^3x[(2n+1)x]dx$ is

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) 0 (d) 2

6.The value of $\int_{-1}^{2}|x|dx$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

7.If $f(x) = \int_0^x tcostdt$, then $\frac{df}{dx} =$

(a) $cosx - xsinx$ (b) $sinx + xcosx$ (c) $xcosx$ (d) $xsinx$

8. The area between $y^2 = 4x$ and its latus rectum is

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{5}{3}$

9.The value of $\int_0^1 x(1-x)^{99}dx$ is

(a) $\frac{1}{11000}$ (b) $\frac{1}{10100}$ (c) $\frac{1}{10010}$ (d) $\frac{1}{10001}$

10. The value of $\int_0^{\pi}\frac{dx}{1+5^{cosx}}$ is

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) $\frac{3\pi}{2}$ (d) $2\pi$

11.If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$ , then the value of $f(1)$ is

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{3}{4}$

12.If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

(a) 10 (b) 5 (c) 8 (d) 9

13.the value of $\int_0^{\frac{\pi}{6}} cos^3 3x dx$ is

(a) $\frac{2}{3}$ (b) $\frac{2}{9}$ (c) $\frac{1}{9}$ (d) $\frac{1}{3}$

14. The value of $\int_0^1 (sin^{-1}x)^2 dx$ is

(a) $\frac{\pi^2}{4} - 1$ (b) $\frac{\pi^2}{4} + 2$ (c) $\frac{\pi^2}{4} + 1$ (d) $\frac{\pi^2}{4} - 2$

Z5. The value of $\int_0^{\infty} e^{-3x} x^2 dx$ is

(a) $\frac{7}{27}$ (b) $\frac{5}{27}$ (c) $\frac{4}{27}$ (d) $\frac{2}{27}$

16. the value of $\int_0^a \left(\sqrt{a^2 - x^2}\right)^3 dx$ is

(a) $\frac{\pi a^3}{16}$ (b) $\frac{3\pi a^4}{16}$ (c) $\frac{3\pi a^2}{8}$ (d) $\frac{3\pi a^4}{8}$

17. The value of $\int_0^{\pi} sin^4 x dx$ is

(a) $\frac{3\pi}{10}$ (b) $\frac{3\pi}{8}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

18.If $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$ then a is

(a)4 (b) 1 (c) 3 (d)2

19.the volume of solid of revolution of the region bounded by $y^2 = x(a - x)$ about x-axis is

(a) $\pi a^3$ (b) $\frac{\pi a^3}{4}$ (c) $\frac{\pi a^3}{5}$ (d) $\frac{\pi a^3}{6}$

20.If $f(x) = \int_1^x \frac{e^{sinu}}{u} du, x > 1$ and $\int_1^3 \frac{e^{sinx^2}}{x} dx = \frac{1}{2}[f(a) - f(1)]$ , then one of the value of a is

(a) 3 (b) 6 (c) 9 (d)5

## 10.ORIDNARY DIFFERENTIAL EQUATIONS

1.Integrating factor of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+1}$

(a) $\frac{1}{x+1}$ (b) $x + 1$ (c) $\frac{1}{\sqrt{x+1}}$ (d) $\sqrt{x + 1}$

2.The population $P$ in any year t is such that the rate of increase in the population is proportional to the population. Then

(a) $P = ce^{kt}$ (b) $P = ce^{-kt}$ (c) $P = ckt$ (d) $P = c$

3. $P$ is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount of remaining, then

(a) $P = ce^{kt}$ (b) $P = ce^{-kt}$ (c) $P = ckt$ (d) $Pt = c$

4.If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle ,then the value of a is

(a) 2 (b) -2 (c) 1 (d)-1

5.The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through (-1,1). Then the equation of the curve is

(a) $y = x^3 + 2$ (b) $y = 3x^2 + 4$ (c) $y = 3x^3 + 4$ (d) $y = x^3 + 5$

6.The order and degree of the differential eqn $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are respectively

(a) 2,3 (b) 3,3 (c) 2,6 (d) 2,4

7.The differential eqn representing the family of curves $y = Acos(x+B)$ where A and B are parameters, is

(a) $\frac{d^2y}{dx^2} - y = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$ (c) $\frac{d^2y}{dx^2}$=0 (d) $\frac{d^2x}{dy^2} = 0$

8. The order and degree of the D.E $\sqrt{sinx}(dx+dy) = \sqrt{cosx}(dx-dy)$ is

(a) 1,2 (b) 2,2 (c) 1,1 (d)2,1

9. The order of the differential equation of all circles with centre at $(h,k)$ and radius 'a'is

(a) 2 (b)3 (c) 4 (d)1

10. The differential equation of the family of curves $y = Ae^x + Be^{-x}$ where A and B are arbitrary constants is

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$ (c) $\frac{dy}{dx} + y = 0$ (d) $\frac{dy}{dx} - y = 0$

11.Tthe general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(a) $xy = k$ (b) $y = klogx$ (c) $y = kx$ (d) $logy = kx$

12.The solution of the differential equation $\frac{dy}{dx} = 2xy$

(a)$y = ce^{x^2}$ (b)$y = 2x^2 + c$ (c) $y = ce^{-x^2} + c$ (d) $y = x^2 + c$

13.The general solution of the differential equation $log\left(\frac{dy}{dx}\right) = (x+y)$

(a) $e^x + e^y = c$ (b) $e^x + e^{-y} = c$ (c) $e^{-x} + e^y = c$ (d) $e^{-x} + e^{-y} = c$

14.The solution of $\frac{dy}{dx} = 2^{y-x}$

(a) $2^x + 2^y = c$ (b) $2^x - 2^y = c$ (c) $\frac{1}{2^x} - \frac{1}{2^y} = c$ (d)$x = y = c$

15.The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\emptyset\left(\frac{y}{x}\right)}{\emptyset\prime\left(\frac{y}{x}\right)}$ is

(a) $x\emptyset\left(\frac{y}{x}\right) = k$ (b) $\emptyset\left(\frac{y}{x}\right) = kx$ (c) $y\emptyset\left(\frac{y}{x}\right) = k$ (d) $\emptyset\left(\frac{y}{x}\right) = ky$

16.If $sinx$ is the integrating factor of linear differential eqn $\frac{dy}{dx} + Py = Q$

(a) $logsinx$ (b) $cosx$ (c) $tanx$ (d) $cotx$

17.The number of arbitrary constants in the general solution of order $n$ and $n+1$ respectively

(a) $n-1, n$ (b) $n, n+1$ (c) $n+1, n+2$ (d) $n+1, n$

18.The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents

(a) straight line (b) circles (c)parabola (d) ellipse

19.The solution of $\frac{dy}{dx} + p(x)y = 0$ is

(a) $y = ce^{\int pdx}$ (b) $y = ce^{-\int pdx}$ (c) $x = ce^{-\int pdy}$ (d) $x = ce^{\int pdy}$

20.The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

(a)$\frac{x}{e^{\lambda}}$ (b) $\frac{e^{\lambda}}{x}$ (c) $\lambda e^x$ (d) $e^x$

21.The integrating factor of the differential eqn $\frac{dy}{dx} + p(x)y = Q(x)$ is x, then $p(x)$ is

(a) $x$ (b) $\frac{x^2}{2}$ (c) $\frac{1}{x}$ (d)$\frac{1}{x^2}$

22.The deg of the differential eqn $y(x) = 1 + \frac{dy}{dx} + \frac{1}{1.2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1.2.3}\left(\frac{dy}{dx}\right)^3 + \cdots$ is

(a)2 (b) 3 (c) 1 (d) 4

23.If p and q are the order and degree of the differential equation $y\frac{dy}{dx}+x^3\left(\frac{d^2y}{dx^2}\right)+xy=cosx$, when

(a) p < q (b)p = q (c) p > q (d) p exist and q does not exist

24.The solution of the differential equation $\frac{dy}{dx}+\frac{1}{\sqrt{1-x^2}}=0$ is

(a) $y+sin^{-1}x=c$ (b) $x+sin^{-1}y=c$ (c) $y^2+2sin^{-1}x=c$ (d) $x^2+2sin^{-1}y=c$

25.The number of arbitrary constants in the particular solution of the differential equation of third order is

(a) 3 (b) 2 (c) 1 (d)0

## 11.PROBABILITY DISTRIBUTION

1. Let x have a Bernoulli distribution with mean 0.4 then variance of (2X-3) is

(a)0.24 (b)0.48 (c) 0.6 (d) 0.96

2. If in 6 trials, X is a binomial variable which follows the relation 9P(X=4)=P(X=2)then the probability of success is

(a)0.125 (b)0.25 (c)0.375 (d)0.75

3. A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?

(a) $\frac{57}{20^3}$ (b) $\frac{57}{20^2}$ (c) $\frac{19^3}{20^3}$ (d) $\frac{57}{20}$

4.Let X be a random variable with probability density function $f(x)=\begin{cases}\frac{2}{x^3}, & x\geq 1\\ 0, & x<1\end{cases}$ which of the following statement is correct?

(a) both mean and variance exist (b) mean exist but variance does not exist

(c) both mean and variance do not exist (d)variance exist but mean does not exist

5.A rod of length $2l$ is broken into two pieces at random. The probability density function of the shorter of the two pieces is $f(x)=\begin{cases}\frac{1}{l}, & 0<x<l\\ 0, & l\leq x<2l\end{cases}$ The mean and variance of the shorter of the two pieces are respectively

(a)$\frac{l}{2},\frac{l^2}{3}$ (b) $\frac{l}{2},\frac{l^2}{6}$ (c) $l,\frac{l^2}{12}$ (d) $\frac{l}{2},\frac{l^2}{12}$

6.Consider a game where the player tosses a six sided fair die. If the face that comes up is 6, the player wins Rs.36, otherwise he loses Rs. $k^2$,where $k$ is the face that comes up $k=\{1,2,3,4,5\}$.

The expected amount to win at this game in Rs. is

(a) $\frac{19}{6}$ (b) $\frac{-19}{6}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

7. Four buses carrying 160 students from the same school arrive at a football stadium. The buses carry,respectively,42,36,34 and 48 students.one of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student.one of the 4 bus drivers is also randomly selected. Let Y denote the number of students on the bus. Then E[X] and E[Y] respectively are

(a) 50 ,40 (b) 40,50 (c) 40.75,40 (d) 41,41

8.If $P(X=0)=1-P(X=1)$. If$E[X]=3Var(X)$, then $P(X=0)$ is

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

9.A pair of dice numbered 1,2,3,4,5,6 of a six-sided die and 1,2,3,4 of a four sided die is rolled and the sum is determined. Let the random variable X denote this sum. Then the number if elements in the inverse image of 7 is

(a) 1 (b) 2 (c) 3 (d)4

10.A random variable X has binomial distribution with n=25 and p=0.8 then standard deviation of X is

(a) 6 (b) 4 (c) 3 (d)2

11.Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible value of X are

(a) $i + 2n, i = 0,1,2, \ldots n$ (b) $2i - n, i = 0,1,2, \ldots n$ (c) $n - i, i = 0,1,2, \ldots n$ (d) $2i + 2n, i = 0,1,2, \ldots n$

12.If the function $f(x) = \frac{1}{12} for\ a < x < b$ represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

(a) 0 and 12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24

13. If X is a bionomial random variable with expected value 6 and variance 2.4 then $P(X = 5)$ is

(a)$\binom{10}{5}\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$ (b) $\binom{10}{5}\left(\frac{3}{5}\right)^{10}$ (c) $\binom{10}{5}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^6$ (d) $\binom{10}{5}\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$

14.Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.5. assume that the results of the flips are independent, And let X equal the total number of heads that result. The value of E[X] is

(a) 0.11 (b) 1.1 (c) 11 (d)1

15. On a multiple choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

(a) $\frac{11}{243}$ (b) $\frac{3}{8}$ (c) $\frac{1}{243}$ (d) $\frac{5}{243}$

16. The random variable X has the probability density function $f(x) = \begin{cases} ax + b, & 0 < x < 1 \\ 0, & otherwise \end{cases}$ and E[X]=$\frac{7}{12}$, then a and b respectively

(a) 1 and $\frac{1}{2}$ (b) $\frac{1}{2}$ and 1 (c) 2 and 1 (d)1 and 2

17. Suppose that X takes on one of the values 0, 1 and 2.If for some constant k ,$P(X = i) = kP(X = i - 1)$ for $i = 1,2$ and $P(X = 0) = \frac{1}{7}$ then the value of k is

(a) 1 (b) 2 (c) 3 (d)4

18. Which of the following is a discrete random variable?

I. The number of cars crossing a particular signal in a day.

II. The number of customers in a queue to buy train tickets at a moment .

III. The time taken to complete a telephone call.

(a) I and II (b) II only (c)III only (d) II and III

19. If $f(x) = \begin{cases} 2x, & 0 \le x \le a \\ 0, & otherwise \end{cases}$ is a probability density function of a random variable , then the value of a is

(a) 1 (b) 2 (c) 3 (d)4

20. The probability mass function of a random variable is defined as

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | k | 2k | 3k | 4k | 5k |

Then E[x] is equal to

(a) $\frac{1}{15}$ (b) $\frac{1}{10}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

## 12.DISCRETE MATHEMATICS

1. The operation $*$ defined by $a * b = \frac{ab}{7}$ is not a binary operation on

(a) $Q^+$ (b) z (c) R (d) C

2. In the set Q defined $a \odot b = a + b + ab$ for what value of y, $3 \odot (y \odot 5) = 7$?

(a) $y = \frac{2}{3}$ (b) $y = \frac{-2}{3}$ (c) $y = \frac{-3}{2}$ (d) $y = 4$

3. If $a * b = \sqrt{a^2 + b^2}$ on a real numbers then $*$ is

(a) commutative but not associative (b) associative but not commutative

(c) both commutative and associative(d) neither commutative nor associative

4. Which one of the following statements has the truth value T?

(a) $sinx$ is an even function (b) Every square matrix is non-singular

(c) The product of complex number and its conjugate is purly imaginary (d)$\sqrt{5}$ is an irrational number

5. Which one of the following statements has the truth value F?

(a) Chennai is in india or $\sqrt{2}$ is an integer (b) Chennai is in india or $\sqrt{2}$ is an irrational number

(c) Chennai is in china or $\sqrt{2}$ is an integer (d) Chennai is in china or $\sqrt{2}$ is an irrational number

6. A binary operation on a set S is a function from

(a) $S \to S$ (b) $(S \times S) \to S$ (c) $S \to (S \times S)$ (d) $(S \times S) \to (S \times S)$

7. Subtraction is not a binary operation in

(a) R (b) Z (c) N (d) Q

8. Which one is the contrapositive of the statement $(p \vee q) \to r$ ?

(a) $\neg r \to (\neg p \wedge \neg q)$ (b) $\neg r \to (p \vee q)$ (c) $r \to (p \wedge q)$ (d) $p \to (q \vee r)$

9. The truth table for $(p \wedge q) \vee \neg q$ is given,

which one of the following is true?

| p | q | $(p \wedge q) \vee (\neg q)$ |
|---|---|---|
| **T** | **T** | **(a)** |
| **T** | **F** | **(b)** |
| **F** | **T** | **(c)** |
| **F** | **F** | **(d)** |

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| (a) | T | T | T | T |
| (b) | T | F | T | T |
| (c) | T | T | F | T |
| (d) | T | F | F | F |

10.In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value 'F' are

(a) 1 (b) 2 (c) 3 (d) 4

11.Which one of the following is incorrect? For any two propositions p and q , we have

(a) $\neg (p \vee q) \equiv \neg p \wedge \neg q$ (b) $\neg (p \wedge q) \equiv \neg p \vee \neg q$ (c) $\neg (p \vee q) \equiv \neg p \vee \neg q$ (d) $\neg(\neg p) \equiv p$

12. which one of the following is correct for the truth value of $(p \wedge q) \rightarrow \neg p$

| P | q | $(p \wedge q) \rightarrow \neg p$ |
|---|---|---|
| T | T | (a) |
| T | F | (b) |
| F | T | (c) |
| F | F | (d) |

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| (a) | T | T | T | T |
| (b) | F | T | T | T |
| (c) | F | F | T | T |
| (d) | T | T | T | F |

13. Which one of the following is a binary operation on N ?

(a) subtraction (b) Multiplication (c) division (d) all the above

14. In the set R of real numbers '$*$' is defined as follows. Which one of the following is not a binary operation on R?

(a) $a * b = \min(a, b)$ (b) $a * b = \max(a, b)$ (c) $a * b = a$ (d) $a * b = a^b$

15. If a compound statement involves 3 simple statements then the number of rows in the truth table is

(a) 9 (b) 8 (c) 6 (d) 3

16. Which one is the inverse of the statement $(p \vee q) \rightarrow (p \wedge q)$?

(a) $(p \wedge q) \rightarrow (p \vee q)$ (b) $\neg(p \vee q) \rightarrow (p \wedge q)$ (c) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$ (d) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$

17.Determine the truth value of each of the following statement?

(a) 4+2=5 and 6+3=9 (b) 3+2=5 and 6+1=7 (c) 4+5=9 and 1+2=4 (d) 3+2=5 and 4+7=11

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| (1) | F | T | F | T |
| (2) | T | F | T | F |
| (3) | T | T | F | F |
| (4) | F | F | T | T |

18.Which one of the following is not true?

(a) Negation of a negation of a statement is the statement itself

(b) If the last column of the truth table contains only T then it is a tautology

(c) If the last column of the truth table contains only F then it is a contradiction

(d) If p and q are any two statements then $p \leftrightarrow q$ is a tautology

19.The dual of $\neg(p \vee q) \vee [p \vee (p \wedge \neg r)]$ is

(a) $\neg(p \wedge q) \wedge [p \vee (p \wedge \neg r)]$ (b) $(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$ (c) $\neg(p \wedge q) \wedge [p \wedge (p \wedge r)]$ (d) $\neg(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

20.Tthe proposition $p \wedge (\neg p \vee q)$ is

(a)a tautology (b) a contradiction (c) logically equivalent to $p \wedge q$ (d) logically equivalent to $p \vee q$

# VECTOR ALGEBRA

FORMULA:

1. $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\,\theta$
2. $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\,\theta\,\hat{n}$
3. Work done $W = \vec{F}\cdot\vec{d}$
4. Torque $\vec{t} = \vec{r}\times\vec{F}$
5. $\vec{a},\vec{b},\vec{c}$ are coplanar vectors $\Leftrightarrow [\vec{a},\vec{b},\vec{c}] = 0$
6. $\vec{a},\vec{b}$ are perpendicular vectors $\Leftrightarrow \vec{a}\cdot\vec{b} = 0$
7. $\vec{a},\vec{b}$ are parallel vectors $\Leftrightarrow \vec{a}\times\vec{b} = \vec{0}$
8. Volume of parallelepiped with coterminous vectors $V = |[\vec{a},\vec{b},\vec{c}]|$ Cubic units
   Equation of straight lines

If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ are two vector thenthe scalar product (or dot product) is denoted by $\vec{a}\cdot\vec{b}$ and is calculated by $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

and the vector product (or cross product) is denoted by $\vec{a}\times\vec{b}$, and is calculated by $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$

If $\theta$ is the acute angle between two straight lines $\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{d}$, then $\theta = \cos^{-1}\left(\frac{|\vec{b}\cdot\vec{d}|}{|\vec{b}||\vec{d}|}\right)$

The acute angle $\theta$ between the two planes $\vec{r}\cdot\vec{n}_1 = p_1$ and $\vec{r}\cdot\vec{n}_2 = p_2$ is $\theta = \cos^{-1}\left(\frac{|\vec{n}_1\cdot\vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}\right)$

If $\theta$ is the acute angle between the line $\vec{r} = \vec{a} + t\vec{b}$ and the plane $\vec{r}\cdot\vec{n} = p$, then $\theta = \sin^{-1}\left(\frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}\right)$

Let $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}, \vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$. Then,

we have $\vec{a},\vec{b},\vec{c}$ are coplanar $\Leftrightarrow [\vec{a},\vec{b},\vec{c}] = 0 \Leftrightarrow \begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix} = 0$

If two lines $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$ and $\frac{x-x_2}{d_1} = \frac{y-y_2}{d_2} = \frac{z-z_2}{d_3}$ intersect, then $\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ b_1 & b_2 & b_3\\ d_1 & d_2 & d_3\end{vmatrix} = 0$

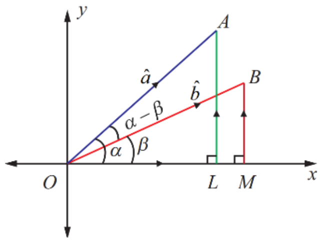
| **Vector Equation/ Parametric** | **Non Parametric** | **Cartiesan Equation** |
|---|---|---|
| $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$ | $(\vec{r} - \vec{a})\cdot(\vec{b}\times\vec{c}) = 0$ | $\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix} = 0$ |
| $\vec{r} = (1-s)\vec{a} + s\vec{b} + t\vec{c}$ | $(\vec{r} - \vec{a})\cdot[(\vec{b} - \vec{a})\times\vec{c}] = 0$ | $\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ c_1 & c_2 & c_3\end{vmatrix} = 0$ |
| $\vec{r} = (1-s-t)\vec{a} + s\vec{b} + t\vec{c}$ | $(\vec{r} - \vec{a})\cdot[(\vec{b} - \vec{a})\times(\vec{c} - \vec{a})] = 0$ | $\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{vmatrix} = 0$ |

# 5 MARKS

**1. By vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$**

Solution:

Let $\hat{a}$ and $\hat{b}$ are two unit vectors




$$\hat{a}=\cos\alpha\hat{\imath}+\sin\alpha\hat{\jmath}$$

$$\hat{b}=\cos\beta\hat{\imath}+\sin\beta\hat{\jmath}$$

$$\hat{b}\cdot\hat{a}=|\vec{b}||\vec{a}|\cos(\alpha-\beta)=\cos(\alpha-\beta)\text{-----}\rightarrow(1)$$

$$\hat{b}\cdot\hat{a}=(\cos\beta\hat{\imath}+\sin\beta\hat{\jmath})\cdot(\cos\alpha\hat{\imath}+\sin\alpha\hat{\jmath})$$

$$=\cos\alpha\cos\beta+\sin\alpha\sin\beta\text{-----}\rightarrow(2)$$

From (1)&(2) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

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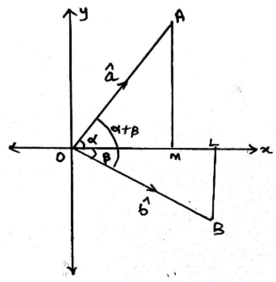
**2. By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$**

Solution:

Let $\hat{a}$ and $\hat{b}$ are two unit vectors




$$\hat{a}=\cos\alpha\hat{\imath}+\sin\alpha\hat{\jmath}$$

$$\hat{b}=\cos\beta-\sin\beta\hat{\jmath}$$

$$\hat{b}\cdot\hat{a}=|\vec{b}||\vec{a}|\cos(\alpha+\beta)=\cos(\alpha+\beta)\text{-----}\rightarrow(1)$$

$$\hat{b}\cdot\hat{a}=(\cos\beta\hat{\imath}-\sin\beta\hat{\jmath})\cdot(\cos\alpha\hat{\imath}+\sin\alpha\hat{\jmath})$$

$$=\cos\alpha\cos\beta-\sin\alpha\sin\beta\text{-----}\rightarrow(2)$$

From (1)&(2) $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

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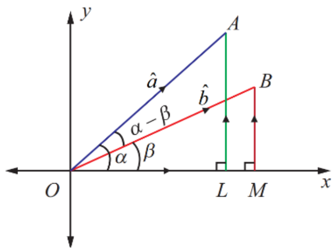
**3. By vector method, prove that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$**

Solution:

Let $\hat{a}$ and $\hat{b}$ are two unit vectors




$$\hat{a}=\cos\alpha\hat{\imath}+\sin\alpha\hat{\jmath}$$

$$\hat{b}=\cos\beta\hat{\imath}+\sin\beta\hat{\jmath}$$

$$\hat{b}\times\hat{a}=|\vec{b}||\vec{a}|\sin(\alpha-\beta)(\hat{k})=\sin(\alpha-\beta)(\hat{k})\text{-----}\rightarrow(1)$$

$$\hat{b}\times\hat{a}=\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\\cos\beta&\sin\beta&0\\\cos\alpha&\sin\alpha&0\end{vmatrix}=(\sin\alpha\cos\beta-\cos\alpha\sin\beta)(\hat{k})\text{-----}\rightarrow(2)$$

From (1) & (2) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

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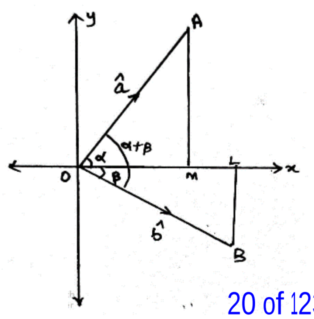
**4. By vector method, prove that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$**

Solution:

Let $\hat{a}$ and $\hat{b}$ are two unit vectors

$$\hat{a}=\cos\alpha\hat{\imath}+\sin\alpha\hat{\jmath}$$




$$\hat{b}=\cos\beta\hat{\imath}-\sin\beta\hat{\jmath}$$

$$\hat{b}\times\hat{a}=|\vec{b}||\vec{a}|\sin(\alpha+\beta)\hat{k}=\sin(\alpha+\beta)\hat{k}\text{-----}\rightarrow(1)$$

$$\hat{b}\times\hat{a}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ cos\beta & -sin\beta & 0\\ cos\alpha & sin\alpha & 0\end{vmatrix}=(sin\alpha cos\beta+cos\alpha sin\beta)\hat{k}\;----\rightarrow(2)$$

From (1)&(2) $sin(\alpha+\beta)=sin\alpha cos\beta+cos\alpha sin\beta$

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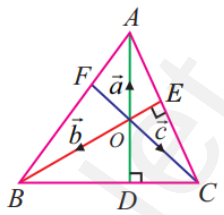
**5. Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.**

Solution:

$\overrightarrow{OA}=\vec{a}$ , $\overrightarrow{OB}=\vec{b}$, $\overrightarrow{OC}=\vec{c}$

AD⊥BC ; BE ⊥CA To prove CF⊥BA




Case:1 AD⊥BC

$$\overrightarrow{AD}\cdot\overrightarrow{BC}=0$$
$$\overrightarrow{OA}\cdot\overrightarrow{BC}=0$$
$$\overrightarrow{OA}\cdot(\overrightarrow{OC}-\overrightarrow{OB})=0$$
$$\vec{a}\cdot(\vec{c}-\vec{b})=0$$
$$\vec{a}\cdot\vec{c}-\vec{a}\cdot\vec{b}=0\longrightarrow(1)$$

Case:2 BE ⊥ CA

$$\overrightarrow{BE}\cdot\overrightarrow{CA}=0$$
$$\overrightarrow{OB}\cdot\overrightarrow{CA}=0$$
$$\overrightarrow{OB}\cdot(\overrightarrow{OA}-\overrightarrow{OC})=0$$
$$\vec{b}\cdot(\vec{a}-\vec{c})=0$$
$$\vec{b}\cdot\vec{a}-\vec{b}\cdot\vec{c}=0--\longrightarrow(2)$$

From (1)+(2)⟹ $\vec{a}\cdot\vec{c}-\vec{b}\cdot\vec{c}=0$

$$(\vec{a}-\vec{b})\cdot\vec{c}=0$$
$$(\overrightarrow{OA}-\overrightarrow{OB})\cdot\overrightarrow{OC}=0$$
$$\overrightarrow{BA}\cdot\overrightarrow{OC}=0\Longrightarrow\overrightarrow{BA}\cdot\overrightarrow{CF}=0\Longrightarrow \text{CF}\perp\text{BA}$$

Hence, the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

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**6. If $\vec{a}=\hat{\imath}-\hat{\jmath}$, $\vec{b}=\hat{\imath}-\hat{\jmath}-4\hat{k}$, $\vec{c}=3\hat{\jmath}-\hat{k}$, and $\vec{d}=2\hat{\imath}+5\hat{\jmath}+\hat{k}$ verify that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$**

Solution:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -1 & 0\\ 1 & -1 & -4\end{vmatrix}=\hat{\imath}(4-0)-\hat{\jmath}(-4-0)+\hat{k}(-1+1)=4\hat{\imath}+4\hat{\jmath}$$

$$\vec{c}\times\vec{d}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 0 & 3 & -1\\ 2 & 5 & 1\end{vmatrix}=\hat{\imath}(3+5)-\hat{\jmath}(0+2)+\hat{k}(0-6)=8\hat{\imath}-2\hat{\jmath}-6\hat{k}$$

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 4 & 4 & 0\\ 8 & -2 & -6\end{vmatrix}=\hat{\imath}(-24-0)-\hat{\jmath}(-24-0)+\hat{k}(-8-32)$$
$$=-24\hat{\imath}+24\hat{\jmath}-40\hat{k}$$

$$[\vec{a},\vec{b},\vec{d}]=\begin{vmatrix}1 & -1 & 0\\ 1 & -1 & -4\\ 2 & 5 & 1\end{vmatrix}=1(-1+20)+1(1+8)+0(5+2)=28$$

$$[\vec{a},\vec{b},\vec{c}]=\begin{vmatrix}1 & -1 & 0\\ 1 & -1 & -4\\ 0 & 3 & -1\end{vmatrix}=1(1+12)+1(-1+0)+0(3+0)=12$$

$$[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}=28(3\hat{\jmath}-\hat{k})-12(2\hat{\imath}+5\hat{\jmath}+\hat{k})$$
$$=-24\hat{\imath}+24\hat{\jmath}-40\hat{k}$$

Hence $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$

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**7. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, $\vec{b} = 3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}$, and $\vec{c} = -\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$**

Solution:

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 5 & 2 \\ -1 & -2 & 3 \end{vmatrix} = \hat{\imath}(15+4) - \hat{\jmath}(9+2) + \hat{k}(-6+5) = 19\hat{\imath} - 11\hat{\jmath} - \hat{k}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & -1 \\ 19 & -11 & -1 \end{vmatrix} = \hat{\imath}(-3-11) - \hat{\jmath}(-2+19) + \hat{k}(-22-57) = -14\hat{\imath} - 17\hat{\jmath} - 79\hat{k}$$

$$\vec{a} \cdot \vec{c} = (2\hat{\imath} + 3\hat{\jmath} - \hat{k}) \cdot (-\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) = 2(-1) + 3(-2) - 1(3) = -11$$

$$\vec{a} \cdot \vec{b} = (2\hat{\imath} + 3\hat{\jmath} - \hat{k}) \cdot (3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}) = 2(3) + 3(5) - 1(2) = 19$$

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = -11(3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}) - 19(-\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) = -14\hat{\imath} - 17\hat{\jmath} - 79\hat{k}$$

Hence $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

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**8. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (0,1,-5) and parallel to the straight lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + s(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and $\vec{r} = (\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) + t(\hat{\imath} + \hat{\jmath} - \hat{k})$.**

Solution:

$\vec{a} = 0\hat{\imath} + \hat{\jmath} - 5\hat{k}$  $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$  $\vec{c} = \hat{\imath} + \hat{\jmath} - \hat{k}$

V.E : $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (0\hat{\imath} + \hat{\jmath} - 5\hat{k}) + s(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}) + t(\hat{\imath} + \hat{\jmath} - \hat{k})$$

C.E: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x - 0 & y - 1 & z + 5 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$$(x-0)(-3-6) - (y-1)(-2-6) + (z+5)(2-3) = 0$$
$$-9x + 8y - z - 13 = 0$$

N.P.V.E: $(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c}) = 0$

$$\Rightarrow \vec{r} \cdot (-9\hat{\imath} + 8\hat{\jmath} - \hat{k}) = 13$$

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**9. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$**

Solution:

$\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$  $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$  $\vec{c} = 2\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$

V.E : $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}) + s(2\hat{\imath} + 3\hat{\jmath} + \hat{k}) + t(2\hat{\imath} - 5\hat{\jmath} - 3\hat{k})$$

C.E: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x-2 & y-3 & z-6 \\ 2 & 3 & 1 \\ 2 & -5 & -3 \end{vmatrix} = 0$$

$(x-2)(-9+5)-(y-3)(-6-2)+(z-6)(-10-6)=0$

$-4x+8y-16z+80=0$ (or) $x-2y+4z-20=0$

N.P.V.E: $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$

$$\Rightarrow \vec{r}\cdot(\hat{\imath}-2\hat{\jmath}+4\hat{k})-20=0$$

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**10. Find the non-parametric form of vector equation and Cartesian equation of the plane passing through the point (1,-2,4) and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$ .**

Solution:

$\vec{a}=\hat{\imath}-2\hat{\jmath}+4\hat{k}$ $\quad \vec{b}=\hat{\imath}+2\hat{\jmath}-3\hat{k}$ $\quad \vec{c}=3\hat{\imath}-\hat{\jmath}+\hat{k}$

V.E : $\vec{r}=\vec{a}+s\vec{b}+t\vec{c}$

$\vec{r}=(\hat{\imath}-2\hat{\jmath}+4\hat{k})+s(\hat{\imath}+2\hat{\jmath}-3\hat{k})+t(3\hat{\imath}-\hat{\jmath}+\hat{k})$

C.E: $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x-1 & y+2 & z-4 \\ 1 & 2 & -3 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$(x-1)(2-3)-(y+2)(1+9)+(z-4)(-1-6)=0$

$-x-10y-7z+9=0$ $(or)$ $x+10y+7z-9=0$

N.P.V.E: $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$

$$\Rightarrow \vec{r}\cdot(\hat{\imath}+10\hat{\jmath}+7\hat{k})-9=0$$

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**11. Find the parametric form of vector equation, & Cartesian equations of the plane containing the line $\vec{r}=(\hat{\imath}-\hat{\jmath}+3\hat{k})+t(2\hat{\imath}-\hat{\jmath}+4\hat{k})$and perpendicular to plane $\vec{r}.(\hat{\imath}+2\hat{\jmath}+\hat{k})=8$.**

Solution:

$\vec{a}=\hat{\imath}-\hat{\jmath}+3\hat{k}$ $\quad \vec{b}=2\hat{\imath}-\hat{\jmath}+4\hat{k}$ $\quad \vec{c}=\hat{\imath}+2\hat{\jmath}+\hat{k}$

V.E : $\vec{r}=\vec{a}+s\vec{b}+t\vec{c}$

$\vec{r}=(\hat{\imath}-\hat{\jmath}+3\hat{k})+s(2\hat{\imath}-\hat{\jmath}+3\hat{k})+t(\hat{\imath}+2\hat{\jmath}+\hat{k})$

C.E: $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x-1 & y+1 & z-3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$(x-1)(-1-8)-(y+1)(2-4)+(z-3)(4+1)=0$

$-9x+2y+5z-4=0$ $(or)$ $9x-2y-5z+4=0$

N.P.V.E: $(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$

$$\Rightarrow \vec{r}\cdot(9\hat{\imath}-2\hat{\jmath}-5\hat{k})+4=0$$

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**12. Find the non-parametric form of vector eqn, and Cartesian eqns of the plane $\vec{r} = (6\hat{\imath} - \hat{\jmath} + \hat{k}) + s(-\hat{\imath} + 2\hat{\jmath} + \hat{k}) + t(-5\hat{\imath} - 4\hat{\jmath} - 5\hat{k})$.**

Solution:

$\vec{a} = 6\hat{\imath} - \hat{\jmath} + \hat{k}$ $\quad \vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ $\quad \vec{c} = -5\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$

V.E : $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (6\hat{\imath} - \hat{\jmath} + \hat{k}) + s(-\hat{\imath} + 2\hat{\jmath} + \hat{k}) + t(-5\hat{\imath} - 4\hat{\jmath} - 5\hat{k})$$

C.E: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x - 6 & y + 1 & z - 1 \\ -1 & 2 & 1 \\ -5 & -4 & -5 \end{vmatrix} = 0$

$$(x - 6)(-10 + 4) - (y + 1)(5 + 5) + (z - 1)(4 + 10) = 0$$

$$-6x - 10y + 14z + 12 = 0 \ \ (or) \qquad 3x + 5y - 7z - 6 = 0$$

N.P.V.E: $(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c}) = 0$

$$\Rightarrow \ \vec{r} \cdot (3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}) - 6 = 0$$

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**13. Find the non-parametric and Cartesian form of the eqn of the plane passing through the points (-1,2,0), (2,2,-1) and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$**

Solution: $\vec{a} = -\hat{\imath} + 2\hat{\jmath} + 0\hat{k}$ $\quad \vec{b} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$ $\quad \vec{c} = \hat{\imath} + \hat{\jmath} - \hat{k}$

V.E: $\vec{r} = (1 - s)\vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (1 - s)(-\hat{\imath} + 2\hat{\jmath}) + s(2\hat{\imath} + 2\hat{\jmath} - \hat{k}) + t(\hat{\imath} + \hat{\jmath} - \hat{k})$$

C.E: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x + 1 & y - 2 & z - 0 \\ 2 + 1 & 2 - 2 & -1 - 0 \\ 1 & 1 & -1 \end{vmatrix} = 0$

$\begin{vmatrix} x + 1 & y - 2 & z - 0 \\ 3 & 0 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$

$$(x + 1)(0 + 1) - (y - 2)(-3 + 1) + (z - 0)(3 - 0) = 0$$

$$x + 2y + 3z - 3 = 0$$

N.P.V.E: $(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times \vec{c}] = 0$

$$\Rightarrow \vec{r} \cdot (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = 3$$

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**14.Find the non-parametric form of vector eqn, Cartesian eqns of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$.**

Solution: $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ $\quad \vec{b} = 9\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ $\quad \vec{c} = 2\hat{\imath} + 6\hat{\jmath} + 6\hat{k}$

V.E: $\vec{r} = (1 - s)\vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (1 - s)(2\hat{\imath} + 2\hat{\jmath} + \hat{k}) + s(9\hat{\imath} + 3\hat{\jmath} + 6\hat{k}) + t(2\hat{\imath} + 6\hat{\jmath} + 6\hat{k})$$

C.E: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ 9-2 & 3-2 & 6-1 \\ 2 & 6 & 6 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix} = 0$$

$(x-2)(6-30)-(y-2)(42-10)+(z-1)(42-2)=0$

$-24x-32y+40z+72=0$ (or) $3x+4y-5z-9=0$

N.P.V.E: $(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times\vec{c}]=0$

$\Rightarrow \vec{r}\cdot(3\hat{\imath}+4\hat{\jmath}-5\hat{k})-9=0$

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**15. Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2,1), (1,-2,3) and parallel to the straight line passing through the points (2, 1, −3) and (−1,5, −8).**

Solution:

$\vec{a}=2\hat{\imath}+2\hat{\jmath}+\hat{k}$ $\vec{b}=\hat{\imath}-2\hat{\jmath}+3\hat{k}$ $\vec{c}=-3\hat{\imath}+4\hat{\jmath}-5\hat{k}$

V.E: $\vec{r}=(1-s)\vec{a}+s\vec{b}+t\vec{c}$

$\vec{r}=(1-s)(2\hat{\imath}+2\hat{\jmath}+\hat{k})+s(\hat{\imath}-2\hat{\jmath}+3\hat{k})+t(-3\hat{\imath}+4\hat{\jmath}-5\hat{k})$

$$\text{C.E: } \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ 1-2 & -2-2 & 3-1 \\ -3 & 4 & -5 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ -1 & -4 & 2 \\ -3 & 4 & -5 \end{vmatrix} = 0$$

$(x-2)(20-8)-(y-2)(5+6)+(z-1)(-4-12)=0$

$12x-11y-16z+14=0$

N.P.V.E: $(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times\vec{c}]=0$

$\Rightarrow \vec{r}\cdot(12\hat{\imath}-11\hat{\jmath}-16\hat{k})+14=0$

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**16. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-Collinear points (3,6, −2), (−1, −2,6) , and (6, 4, –2).**

Solution: $\vec{a}=3\hat{\imath}+6\hat{\jmath}-2\hat{k}$ $\vec{b}=-\hat{\imath}-2\hat{\jmath}+6\hat{k}$ $\vec{c}=6\hat{\imath}+4\hat{\jmath}-2\hat{k}$

V.E: $\vec{r}=(1-s-t)\vec{a}+s\vec{b}+t\vec{c}$

$\vec{r}=(1-s-t)(3\hat{\imath}+6\hat{\jmath}-2\hat{k})+s(-\hat{\imath}-2\hat{\jmath}+6\hat{k})+t(6\hat{\imath}-4\hat{\jmath}-2\hat{k})$

$$\text{C.E: } \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-3 & y-6 & z+2 \\ -1-3 & -2-6 & 6+2 \\ 6-3 & 4-6 & -2+2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-3 & y-6 & z+2 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$

$$(x-3)(0+16)-(y-6)(0-24)+(z+2)(8+24)=0$$
$$16x-48+24y-144+32z+64=0 \quad (or) \quad 16x+24y+32z+128=0$$
$$2x+3y+4z-16=0$$

N.P.V.E: $(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$

$$\Rightarrow \vec{r}\cdot(2\hat{\imath}+3\hat{\jmath}+8\hat{k})-16=0$$

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## DISCRETE MATHEMATICS

Important Points:

Let * be a binary operation on S

i) Closure property : $\forall a,b\in S \Rightarrow a*b\in S$

ii) Commutative property : $\forall a,b\in S$ [a * b = b * a]

iii) Associative property : $a*(b*c)=(a*b)*c \ \forall a,b,c\in S$

iv) Existence of identity : $a*e=e*a=a$, e is the identity element

v) Existence of inverse : $a^{-1}$ is the inverse of a $a*a^{-1}=a^{-1}*a=e$

**1.Verify closure, commutative, associative, existence of identity, and existence of inverse for $m*n=m+n-mn, \quad m,n\epsilon Z$**

Soln:

**Closure property**: $m,n\epsilon Z$, clearly $m+n-mn\epsilon Z$

$\therefore$ closure property true

**Associative property:** $(l*m)*n=l*(m*n)$

$(l*m)*n=(l+m-lm)*n=(l+m-lm)+n-(l+m-lm)n$

$=l+m+n-lm-mn-nl+lmn$

Similarly, $l*(m*n)=l+m+n-lm-mn-nl+lmn$

$\therefore$ associative property true

**Identity property**: $m*e=e*m=m$

$m*e=m \quad \Rightarrow m+e-me=m \quad \Rightarrow \quad e-me=0$

$\Rightarrow e(1-m)=0 \Rightarrow e=0$

$\therefore$ identity property true

**Inverse property**: $m*m^{-1}=m^{-1}*m=e=0$

$m*m^{-1}=0 \Rightarrow \ m+m^{-1}-mm^{-1}=0$

$\Rightarrow m^{-1}-mm^{-1}=-m$

$\Rightarrow \ m^{-1}(1-m)=-m$

$m^{-1}=\frac{-m}{1-m}\notin z$

$\therefore$ inverse property not true

**Commutative property:** $m*n=m+n-mn=n+m-nm=n*m$

$\therefore$ commutative property true

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**2.Verify closure, commutative, associative, existence of identity, and existence of inverse for $x*y=x+y-xy, \ \forall \ x,y\epsilon Q\setminus\{1\}$.**

Soln:

**Closure property:** $x,y\epsilon Q\setminus\{1\}, \quad x\neq 1, \ y\neq 1 \implies x-1\neq 0, \ y-1\neq 0$

$\Rightarrow (x-1)(y-1)\neq 0$

$\Rightarrow x\,y-x-y+1\neq 0$

$\Rightarrow \ xy-x-y\neq -1$

$\Rightarrow \ x+y-xy\neq 1$

$x*y\neq 1 \ \therefore$ closure property true

**Associative property:** $(x*y)*z=x*(y*z)$

$\Rightarrow (x*y)*z=(x+y-xy)*z=(x+y-xy)+z-(x+y-xy)z$

$=x+y+z-xy-yz-zx+xyz$

$\Rightarrow x * (y * z) = x * (y + z - yz) = x + (y + z - yz) - x(y + z - yz)$
$= x + y + z - xy - yz - zx + xyz$

$\therefore$ associative property true

**Identity property:** $x * e = e * x = x$

$\Rightarrow \quad x * e = x$

$\Rightarrow x + e - xe = x \quad \Rightarrow \quad e - xe = 0$

$\Rightarrow e(1 - x) = 0 \quad \Rightarrow \quad e = 0$

$\therefore$ identity property true

**Inverse property:** $x * x^{-1} = x^{-1} * x = e = 0$

$\Rightarrow \quad x * x^{-1} = 0$

$\Rightarrow \quad x + x^{-1} - xx^{-1} = 0 \quad \Rightarrow \quad x^{-1} - xx^{-1} = -x$

$\Rightarrow \quad x^{-1}(1 - x) = -x \quad \Rightarrow \quad x^{-1} = \frac{-x}{1-x} \epsilon Q \setminus \{1\}$

$\therefore$ inverse property true

**Commutative property:** $m * n = m + n - mn$
$= n + m - nm = n * m$

$\therefore$ commutative property true

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**3. Verify closure, commutative , associative, existence of identity, and inverse for $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} \in R - \{0\}\right\}$. Let $*$be the matrix multiplication.**

Soln:

**Closure property:** Let, $A = \begin{pmatrix} x & x \\ x & x \end{pmatrix}, B = \begin{pmatrix} y & y \\ y & y \end{pmatrix} \quad \because x, y \neq 0 \Longrightarrow 2xy \neq 0$

$AB = \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} y & y \\ y & y \end{pmatrix} = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}$

$\therefore$ closure property true

**Commutative property :** $AB = \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} y & y \\ y & y \end{pmatrix} = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \quad \because xy = yx$

$BA = \begin{pmatrix} y & y \\ y & y \end{pmatrix}\begin{pmatrix} x & x \\ x & x \end{pmatrix} = \begin{pmatrix} 2yx & 2yx \\ 2yx & 2yx \end{pmatrix}$

$\therefore$ commutative property true

**Associative property:**

Matrix multiplication always satisfies associative property

**Existence of identity property:** $A * E = E * A = A$

$\begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} e & e \\ e & e \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$

$\begin{pmatrix} 2ex & 2ex \\ 2ex & 2ex \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$

$2ex = x$

$2e = 1 \Longrightarrow e = \frac{1}{2} \qquad 2e = 1 \Longrightarrow e = \frac{1}{2} \therefore \; E = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$\therefore$ identity property true

**Existence of inverse property:** $A * A^{-1} = A^{-1} * A = E = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$\begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} a & a \\ a & a \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$\begin{pmatrix} 2ax & 2ax \\ 2ax & 2ax \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$2ax = \frac{1}{2} \quad \Longrightarrow a = \frac{1}{4x} \qquad 2ax = \frac{1}{2} \Longrightarrow \quad a = \frac{1}{4x}$

$\therefore \; A^{-1} = \begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix}$

$\therefore$ inverse property true

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**4. Verify closure property, commutative property, associative property ,existence of identity, and existence of inverse for the operation $\times_{11}$ on a subset $A$ = {1,3,4,5,9}of the set of remainders {0,1,2,3,4,5,6,7,8,9,10}.**

**Soln:**

| $\times_{11}$ | 1 | 3 | 4 | 5 | 9 |
|---|---|---|---|---|---|
| 1 | 1 | 3 | 4 | 5 | 9 |
| 3 | 3 | 9 | 1 | 4 | 5 |
| 4 | 4 | 1 | 5 | 9 | 3 |
| 5 | 5 | 4 | 9 | 3 | 1 |
| 9 | 9 | 5 | 3 | 1 | 4 |

**Closure property:**
All elements in the table are in set A
∴ closure property true
**Commutative property:**
Symmetry about diagonal
∴ commutative property true
**Associative property:**
$\times_{11}$ always associative property true
**Identity property:**
Identity element $1 \epsilon A$
∴ identity property true
**Inverse property:**
Each row and column contain 1
∴ inverse property true.
Inverse element of 1,3,4,5 and 9 are 1,4,3,9 and 5 respectively.
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**5. Verify closure property, commutative property, associative property,existence of identity, and existence of inverse for the operation $+_5$ on $Z_5$ using table corresponding to addition modulo 5.**

Soln:$Z_5 = \{0,1,2,3,4\}$

| $+_5$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 1 | 2 | 3 | 4 | 0 |
| 2 | 2 | 3 | 4 | 0 | 1 |
| 3 | 3 | 4 | 0 | 1 | 2 |
| 4 | 4 | 0 | 1 | 2 | 3 |

**Closure** : All elements in the table are in set $Z_5$
∴ closure property true
**Commutative property**:
Symmetry about diagonal
∴ commutative property true
**Associative property**:
$+_5$ always associative property true
**Identity property**:
identity element $0 \epsilon Z_5$
∴ identity property true.
**Inverse property**:
Each row and column contain 0
∴ inverse property true
Inverse element of 0,1,2,3 and 4 are 0,4,3,2 and 1 respectively.
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**6. Verify closure, commutative, associative, identity, and inverse property for $a * b = \frac{a+b}{2}$ $\forall$ $a, b \epsilon Q$**

Soln:

**Closure property:**

Clearly $a, b \epsilon Q \implies \frac{a+b}{2} \epsilon Q$

∴ closure property true

**Associative property:**

$(a * b) * c = \left(\frac{a+b}{2}\right) * c = \frac{\frac{a+b}{2}+c}{2} = \frac{a+b+2c}{4}$

$a * (b * c) = a * \left(\frac{b+c}{2}\right) = \frac{a+\frac{b+c}{2}}{2} = \frac{2a+b+c}{4}$

$(a * b) * c \neq a * (b * c)$

∴ Associative property is not true

**Identity property**: $a * e = e * a = a$

$a * e = a$

$\frac{a+e}{2} = a$

$\Rightarrow a + e = 2a \quad \Rightarrow e = a$

Uniqueness of identity is not preserved
Because we substitute ‘b’ instead of ‘a’ we get identity is ‘b’.

$\therefore$ identity property is not true

**Inverse property:** $x * x^{-1} = x^{-1} * x = e$

identity property is not true

$\therefore$ inverse property is not true

**Commutative property:** $a * b = \frac{a+b}{2} = \frac{b+a}{2} = b * a$

$\therefore$ commutative property true

## COMPLEX NUMBERS

**Important Hints**:

$i^2 = -1,\ i^3 = -i,\ i^4 = 1,\ i^{4n} = 1$

Rectangular form of a complex number is x + iy real part is x, Imaginary part is y.

The conjugate of the complex number z = x + iy is x −iy and is denoted by $\bar{z}$

If $z = x +$ iy then modulus of $z$ is $|z| = \sqrt{x^2 + y^2}$

Triangle inequality:

For any two complex number $z_1$ and $z_2$, $|z_1 + z_2| \le |z_1| + |z_1|$

$$\sqrt{a+ib} = \pm\left[\sqrt{\frac{|z|+a}{2}} + \frac{ib}{|b|}\sqrt{\frac{|z|-a}{2}}\right]$$

Additive inverse of $z$ is $-z$ multiplicative inverse of $z$ is $1/z$

z is real if any only if z = $\bar{z}$ and z is purely imaginary if and only if z = $-\bar{z}$

Distance between two complex numbers, $z_1$ and $z_2$ is $|z_1 - z_2|$

$|z - z_0| = r$ is the complex form of the equation of a circle. Centre is $z_0$ and radius is r.

# 5 Marks

**1. If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of $z$ is $2x^2 + 2y^2 + x - 2y = 0$.**

Soln: Given $Im\left(\frac{2z+1}{iz+1}\right) = 0$

put $z = x + iy$

$$Im\left(\frac{2(x+iy)+1}{i(x+iy)+1}\right) = 0$$

$$Im\left(\frac{2x+i2y+1}{ix+i^2y+1}\right) = 0$$

$$Im\left(\frac{(2x+1)+i2y}{(1-y)+ix}\right) = 0$$

2x + 1 ↖ ↗ 2y

1 − y (1) ✕ (2) $x$

$$\left(\frac{2y(1-y)-x(2x+1)}{(1-y)^2+x^2}\right) = 0$$

$2y - 2x^2 - 2y^2 - x = 0 \quad (or)\quad 2x^2 + 2y^2 + x - 2y = 0$

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**2. If $z = x + iy$ is a complex number such that $Re\left(\frac{z-1}{z+1}\right) = 0$, show that the locus of $z$ is $x^2 + y^2 = 1$.**

Soln: Given $Re\left(\frac{z-1}{z+1}\right) = 0$

put $z = x + iy$

$$Re\left(\frac{x+iy-1}{x+iy+1}\right) = 0$$

$$Re\left(\frac{(x-1)+iy}{(x+1)+iy}\right) = 0$$

↓ $x-1$ ↓ y
$x+1$ y

$$\left(\frac{(x-1)(x+1)+y^2}{(x+1)^2+y^2}\right) = 0$$

$$x^2 - 1 + y^2 = 0 \implies x^2 + y^2 = 1$$

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**3.If $z = x + iy$ is a complex number such that $arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that the locus of $z$ is $x^2 + y^2 = 1$.**

Soln: Given $arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$

put $z = x + iy$

$$arg\left(\frac{x+iy-1}{x+iy+1}\right) = \frac{\pi}{2}$$

$$arg\left(\frac{(x-1)+iy}{(x+1)+iy}\right) = \frac{\pi}{2}$$

(3) $x-1$ (1) y (4)
$x+1$ (2) y

$$tan^{-1}\left(\frac{y(x+1)-y(x-1)}{(x-1)(x+1)+y^2}\right) = \frac{\pi}{2}$$

$$\left(\frac{y(x+1)-y(x-1)}{(x-1)(x+1)+y^2}\right) = tan\frac{\pi}{2} = \infty$$

$$\left(\frac{(x-1)(x+1)+y^2}{y(x+1)-y(x-1)}\right) = 0$$

$$x^2 - 1 + y^2 = 0 \implies x^2 + y^2 = 1$$

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**4. If $z = x + iy$ is a complex number such that $arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that the locus of $z$ is $x^2 + y^2 + 3x - 3y + 2 = 0$.**

Soln: Given $arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$

put $z = x + iy$

$$arg\left(\frac{x+iy-i}{x+iy+2}\right) = \frac{\pi}{4}$$

$$arg\left(\frac{x+i(y-1)}{(x+2)+iy}\right) = \frac{\pi}{4}$$

(3) $x$ (1) $y-1$ (4)
$x+2$ (2) y

$$tan^{-1}\left(\frac{(x+2)(y-1)-xy}{x(x+2)+y(y-1)}\right) = \frac{\pi}{4}$$

$$\left(\frac{(x+2)(y-1)-xy}{x(x+2)+y(y-1)}\right) = tan\frac{\pi}{4} = 1$$

$$(x+2)(y-1) - xy = x(x+2) + y(y-1)$$

$$xy + 2y - x - 2 - xy = x^2 + 2x + y^2 - y$$

$$2y - x - 2 = x^2 + 2x + y^2 - y$$

$$x^2 + y^2 + 3x - 3y + 2 = 0$$

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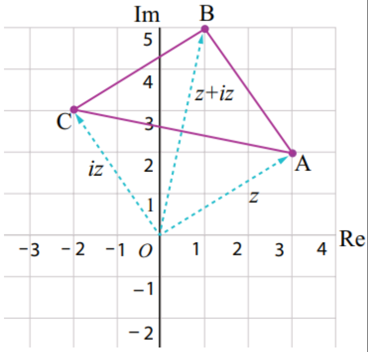
## Try yourself

**If $z = x + iy$ is a complex number such that $arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$, show that the locus of $z$ is $\sqrt{3}x^2 + \sqrt{3}y^2 - 2y - 3 = 0$.**

**5.If $z = 3 + 2i$, represent the complex numbers $z, iz,$ and $z + iz$ in one Argand plane. S.t. these complex numbers form the vertices of an isosceles right triangle.**




**Soln:** Given, $z = 3 + 2i$

Then $iz = i(3 + 2i) = 3i - 2 = -2 + 3i$ ;

$z + iz = 1 + 5i$

Let $z_1 = z = 3 + 2i \quad z_2 = iz = -2 + 3i \quad z_3 = z + iz = 1 + 5i$

$AB = |z_1 - z_2| = |(3 + 2i) - (-2 + 3i)| = |5 - i| = \sqrt{(5)^2 + (-1)^2} = \sqrt{26}$

$BC = |z_2 - z_3| = |(-2 + 3i) - (1 + 5i)| = |-3 - 2i| = \sqrt{(-3)^2 + (-2)^2} = \sqrt{13}$

$CA = |z_3 - z_1| = |(1 + 5i) - (3 + 2i)| = |-2 + 3i| = \sqrt{(-2)^2 + (3)^2} = \sqrt{13}$

$BC^2 + CA^2 = AB^2 \implies (\sqrt{13})^2 + (\sqrt{13})^2 = (\sqrt{26})^2 \implies 26 = 26$

∴ Given complex numbers form the vertices of an isosceles right triangle.

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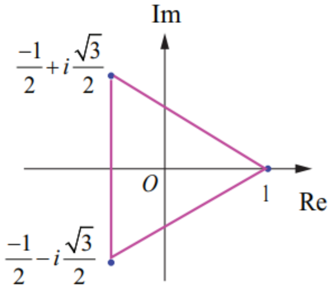
**6. Show that the points 1, $\frac{-1}{2} + i\frac{\sqrt{3}}{2}$ and $\frac{1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.**




**Soln:** Let $z_1 = 1 \quad z_2 = \frac{-1}{2} + i\frac{\sqrt{3}}{2} \quad z_3 = \frac{1}{2} - i\frac{\sqrt{3}}{2}$

$AB = |z_1 - z_2| = \left|1 - \left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right)\right| = \left|\frac{3}{2} + i\frac{\sqrt{3}}{2}\right| = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}$

$BC = |z_2 - z_3| = \left|\left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right) - \left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right)\right| = |0 + i\sqrt{3}| = \sqrt{3}$

$CA = |z_3 - z_1| = \left|\left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right) - 1\right| = \left|\frac{-3}{2} - i\frac{\sqrt{3}}{2}\right| = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}$

AB=BC=CA ∴ Given points are the vertices of an equilateral triangle.

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**7. If $z_1, z_2$ and $z_3$ are three complex numbers S.T $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_3 + z_yz_3| = 6$.**

**Soln:**

Given $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|z_1 + z_2 + z_3| = 1 \qquad \because |z|^2 = z\bar{z}$

$z_1\overline{z_1} = 1, z_2\overline{z_2} = 4 \quad , z_3\overline{z_3} = 9$

$|9z_1z_2 + 4z_1z_3 + z_2z_3| = |z_3\overline{z_3}z_1z_2 + z_2\overline{z_2}z_1z_3 + z_1\overline{z_1}z_2z_3|$

$= |z_1z_2z_3(\overline{z_3} + \overline{z_2} + \overline{z_1})|$

$= |z_1z_2z_3(\overline{z_1 + z_2 + z_3})|$

$= |z_1||z_2||z_3|\,|z_1 + z_2 + z_3|$

$= 1 \times 2 \times 3 \times 1$

$= 6$

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**8. If $z_1, z_2$ and $z_3$ are three complex number S.T. $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$. Prove that $\left|\frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3}\right| = r$.**

**Soln:** Given $|z_1| = |z_2| = |z_3| = r \qquad \because |z|^2 = z\bar{z}$

$z_1\overline{z_1} = z_2\overline{z_2} = z_3\overline{z_3} = r^2$

$z_1 = \frac{r^2}{\overline{z_1}}, z_2 = \frac{r^2}{\overline{z_2}}, z_3 = \frac{r^2}{\overline{z_3}}$

$|z_1 + z_2 + z_3| = \left|\frac{r^2}{\overline{z_1}} + \frac{r^2}{\overline{z_2}} + \frac{r^2}{\overline{z_3}}\right|$

$$= r^2 \left| \frac{\overline{z_1}\,\overline{z_2}+\overline{z_2}\,\overline{z_3}+\overline{z_3}\,\overline{z_1}}{\overline{z_1}\,\overline{z_2}\,\overline{z_3}} \right|$$

$$= r^2 \left| \frac{\overline{z_1 z_2+z_2 z_3+z_1 z_3}}{\overline{z_1\, z_2\, z_3}} \right|$$

$$= r^2 \frac{|z_1 z_2+z_2 z_3+z_1 z_3|}{|z_1||z_2||z_3|}$$

$$|z_1+z_2+z_3| = r^2 \frac{|z_1 z_2+z_2 z_3+z_1 z_3|}{r^3}$$

$$= \frac{|z_1 z_2+z_2 z_3+z_1 z_3|}{r}$$

$$\therefore \quad \left| \frac{z_1 z_2+z_2 z_3+z_3 z_1}{z_1+z_2+z_3} \right| = r$$

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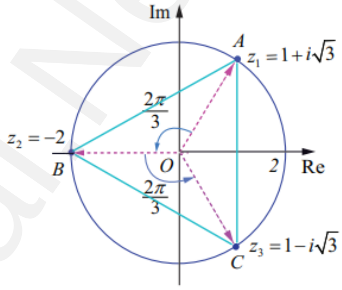
**9. Suppose $z_1, z_2$ and $z_3$ are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find $z_2$ and $z_3$.**




**Soln:** Given, $|z| = r = 2$ and $z_1 = 1 + i\sqrt{3}$;

$$\theta = \alpha = tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

$\therefore$ Euler's form of $z_1 = re^{i\theta} = 2e^{i\frac{\pi}{3}}$

Clearly, $z_2$ is rotation of $z_1$ anti-clockwise by $\frac{2\pi}{3}$

$$z_2 = z_1 e^{i\frac{2\pi}{3}} = 2e^{i\frac{\pi}{3}} e^{i\frac{2\pi}{3}} = 2e^{i\pi} = -2$$

Clearly, $z_3$ is rotation of $z_1$ clockwise by $\frac{2\pi}{3}$

$$z_3 = z_1 e^{-i\frac{2\pi}{3}} = 2e^{i\frac{\pi}{3}} e^{-i\frac{2\pi}{3}} = 2e^{-i\frac{\pi}{3}} = 2\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 1 - i\sqrt{3}$$

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**10. Triangle inequality.**

**Statement:**

For any two complex numbers $z_1, z_2$, we have $|z_1 + z_2| \le |z_1| + |z_2|$

**Proof:**

$$|z_1+z_2|^2 = (z_1+z_2)\overline{(z_1+z_2)}$$

$$= (z_1+z_2)(\overline{z_1}+\overline{z_2})$$

$$= z_1\overline{z_1} + z_1\overline{z_2} + z_2\overline{z_1} + z_2\overline{z_2}$$

$$= |z_1|^2 + |z_2|^2 + z_1\overline{z_2} + \overline{z_1\overline{z_2}}$$

$$= |z_1|^2 + |z_2|^2 + 2Re(z_1\overline{z_2})$$

$$|z_1+z_2|^2 \le |z_1|^2 + |z_2|^2 + 2|z_1\overline{z_2}| = |z_1|^2 + |z_2|^2 + 2|z_1||z_2| = (|z_1| + |z_2|)^2$$

Taking square root on both sides, we get $|z_1 + z_2| \le |z_1| + |z_2|$

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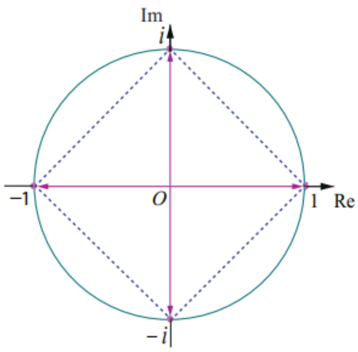
**11.Find the fourth roots of unity.**



Fourth roots of unity

**Soln:** Given $z^4 = 1$

$$z = (1)^{\frac{1}{4}}\, z = \left(e^{i0}\right)^{\frac{1}{4}}$$

$$z = \left(e^{i(0+2k\pi)}\right)^{\frac{1}{4}} \quad k = 0, 1, 2, 3$$

$$z = e^{i\frac{2k\pi}{4}} \quad k = 0, 1, 2, 3$$

For $k = 0$, $\quad z = e^{i0} = 1$

For $k = 1$, $\quad z = e^{i\frac{2\pi}{4}} = e^{i\frac{\pi}{2}} = i$

For $k=2$, $z=e^{i\frac{4\pi}{4}}=e^{i\pi}=-1$

For $k=3$, $z=e^{i\frac{6\pi}{4}}=e^{i\frac{3\pi}{2}}=-i$

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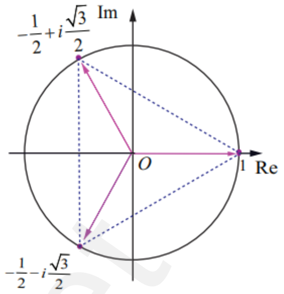
**12. Find the cube roots of unity.**



Cube roots of unity

Soln: Given $z^3=1 \Rightarrow z=(1)^{\frac{1}{3}}$

$$z=\left(e^{i0}\right)^{\frac{1}{3}}$$

$$z=\left(e^{i(0+2k\pi)}\right)^{\frac{1}{3}} \quad k=0,1,2$$

$$z=e^{i\frac{2k\pi}{3}} \quad k=0,1,2$$

For $k=0$, $z=e^{i0}=1$

For $k=1$, $z=e^{i\frac{2\pi}{3}}=\frac{-1}{2}+i\frac{\sqrt{3}}{2}=\frac{-1+i\sqrt{3}}{2}$

For $k=2$, $z=e^{i\frac{4\pi}{3}}=\frac{-1}{2}-i\frac{\sqrt{3}}{2}=\frac{-1-i\sqrt{3}}{2}$

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**13. Find all the cube roots of $\sqrt{3}+i$**

Soln: Let $z^3=re^{i\theta} \Rightarrow z=\left(re^{i\theta}\right)^{\frac{1}{3}}$

We have to find $z=\left(\sqrt{3}+i\right)^{\frac{1}{3}}$

$r=\sqrt{\left(\sqrt{3}\right)^2+(1)^2}=2$, $\theta=\alpha=tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$

$$z=\left(2e^{i\frac{\pi}{6}}\right)^{\frac{1}{3}}$$

$$z=\left(e^{i\left(\frac{\pi}{6}+2k\pi\right)}\right)^{\frac{1}{3}} \quad k=0,1,2$$

$$z=\left(e^{i\frac{(1+12k)\pi}{6}}\right)^{\frac{1}{3}} \quad k=0,1,2$$

$$z=e^{i\frac{(1+12k)\pi}{18}} \quad k=0,1,2$$

For $k=0$, $z=e^{i\frac{\pi}{18}}$

For $k=1$, $z=e^{i\frac{13\pi}{18}}$

For $k=2$, $z=e^{i\frac{25\pi}{18}}$

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**14. Solve the equation $z^3+8i=0$, where $z \in C$.**

Soln:

Given $z^3+8i=0$

$$z^3=-8i$$

$$z^3=8i^3=(2i)^3$$

$$z=\left[(2i)^3\right]^{\frac{1}{3}} \Rightarrow z=2i[1]^{\frac{1}{3}}$$

$$z=2i\left(e^{i0}\right)^{\frac{1}{3}}$$

$$z = 2i\left(e^{i(0+2k\pi)}\right)^{\frac{1}{3}} \ ; \ k = 0,1,2$$

$$z = 2ie^{i\frac{2k\pi}{3}} \ ; \ k = 0,1,2$$

**For $k = 0$,** $z = 2ie^{i0} = 2i$

**For $k = 1$,** $z = 2ie^{i\frac{2\pi}{3}} = 2i\left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right) = -i - \sqrt{3}$

**For $k = 2$,** $z = 2ie^{i\frac{4\pi}{3}} = 2i\left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right) = -i + \sqrt{3}$

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# TWO DIMENSIONAL ANALYTICAL GEOMETRY

**Important Points:**

Circle:

- Equation of circle having centre (0,0) & radius $r$
  $$x^2 + y^2 = r^2$$
- Equation of circle having centre $(h,k)$& radius $r$
  $$(x-h)^2 + (y-k)^2 = r^2$$
- Equation of circle having $(x_1, y_1)$ and $(x_2, y_2)$ as end points of diameter
  $$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$
- Equation of circle in general form
  $$x^2 + y^2 + 2gx + 2fy + c = 0$$
  Centre $(-g,-f)$ radius $= \sqrt{g^2 + f^2 - c}$

# 5 Marks

**1. Find the equation of the circle passing through the points $(1,1)$, $(2,-1)$, and $(3,2)$**

$$A(1,1), B(2,-1), C(3,2)$$
$$M_1 = \text{Slope of } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-1}{2-1} = -2$$
$$M_2 = \text{Slope of } AC = \frac{2-1}{3-1} = \frac{1}{2}$$
$$m_1 \times m_2 = -1 \ \therefore \angle A = 90^\circ$$

End points of diameter B, C

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$
$$(x-2)(x-3) + (y+1)(y-2) = 0$$
$$x^2 + y^2 - 5x - y + 4 = 0$$

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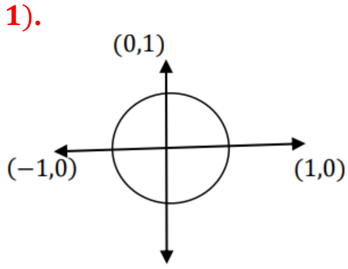
**2.Find the equation of the circle through the points $(1,0)$, $(-1,0)$, and $(0,1)$.**




End point of diameter of (1,0), (−1,0)

Centre(0,0), radius=1

Equation of circle $x^2 + y^2 = 1$

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**3.Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$.Also find the coordinates of the point of contact.**

Solution: $x - y = 4 = 0$

$y = x + 4$

$y = mx + 4$

$m = 1 \quad c = 4$

$x^2 + 3y^2 = 12$

$\frac{x^2}{12} + \frac{y^2}{4} = 1$

$a^2 = 12 \quad b^2 = 4$

Condition: $c^2 = a^2m^2 + b^2$

$a^2m^2 + b^2 = 12 \times 1 + 4 = 16 = c^2$

$x - y + 4 = 0$ is a tangent to $x^2 + 3y^2 = 12$

Point of contact: $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right) = \left(\frac{-12 \times 1}{4}, \frac{4}{4}\right) = (-3, 1)$

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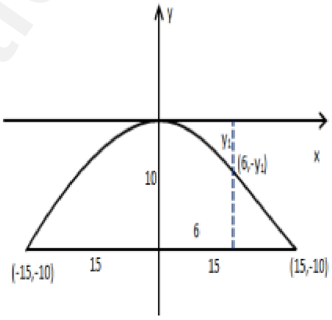
**4.A bridge has a parabolic arch that is 10$m$ high in the centre and 30$m$ wide at the bottom. Find the height of the arch 6$m$ from the centre, on either sides.**

Soln: Equation of downward open parabola $x^2 = -4ay ----\rightarrow (1)$

At $(15, -10)$

$(1) \Rightarrow (15)^2 = -4a(-10) \Rightarrow a = \frac{225}{40}$

$(1) \Rightarrow x^2 = -4\left(\frac{225}{40}\right)y ----\rightarrow (2)$




At $(6, -y_1)$

$(2) \Rightarrow (6)^2 = -4 \times \frac{225}{40}(-y_1)$

$\frac{36 \times 40}{4 \times 225} = y_1 \Rightarrow y_1 = 1.6$

Required height is $10 - y_1 = 10 - 1.6 = 8.4$ m

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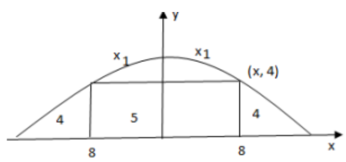
**5. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16$m$, and the height at the edge of the road must be sufficient for a truck 4$m$ high to clear if the highest point of the opening is to be 5$m$ approximately . How wide must the opening be?**

Soln: Equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 ----\rightarrow (1)$

Given $b = 5$ $(1) \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{5^2} = 1 ----\rightarrow (2)$




At (8,4)

$(2) \Rightarrow \frac{8^2}{a^2} + \frac{4^2}{5^2} = 1 \Rightarrow \frac{8^2}{a^2} = 1 - \frac{16}{25} = \frac{25 - 16}{25} = \frac{9}{25} = \left(\frac{3}{5}\right)^2$

$$\frac{8^2}{a^2} = \left(\frac{3}{5}\right)^2 \Rightarrow \frac{8}{a} = \frac{3}{5} \Rightarrow a = \frac{40}{3}$$

Required opening is $2a = \frac{80}{3} = 26.66$ m

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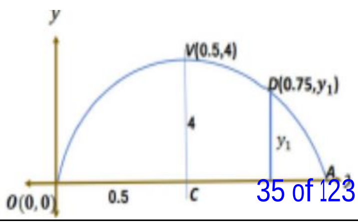
**6. At a water fountain, water attains a maximum height of 4$m$ at horizontal distance of 0.5 $m$ from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 $m$ from the point of origin.**

Soln: Equation of downward open parabola $x^2 = -4ay ----\rightarrow (1)$

At $(-0.5, -4)$



(1) $\Rightarrow (-\frac{1}{2})^2 = -4a(-4) \Rightarrow a = \frac{1}{64}$

$$(1) \Rightarrow x^2 = -4\left(\frac{1}{64}\right)y = -\left(\frac{1}{16}\right)y ----\rightarrow (2)$$

At $(0.25, -y_1)$

(2) $\Rightarrow \left(\frac{1}{4}\right)^2 = -4 \times \frac{1}{64}(-y_1) \Rightarrow \frac{64}{4\times16} = y_1 \Rightarrow y_1 = 1$

Required distance is $4 - y_1 = 4 - 1 = 3$ m

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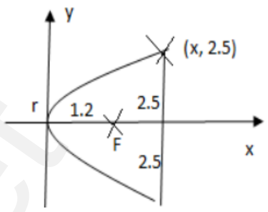
**7. An engineer designs a satellite dish with a parabolic cross section. The dish is 5*m* wide at the opening, and the focus is placed 1.2 *m* from the vertex**

**(a) Position a coordinate system with the origin at the vertex and the *x* -axis on the parabola's axis of symmetry and find an equation of the parabola.**

**(b) Find the depth of the satellite dish at the vertex.**




Soln: Equation of rightward open parabola $y^2 = 4ax ----\rightarrow (1)$

Given $a = 1.2$

(i)equation of parabola $y^2 = 4 \times 1.2 \times x = 4.8x$

$$y^2 = 4.8x ----\rightarrow (2)$$

(ii)At $(x_1, 2.5)$

(2) $\Rightarrow (2.5)^2 = 4.8x_1 \Rightarrow \frac{6.25}{4.8} = x_1 \therefore x_1 = 1.3$ m

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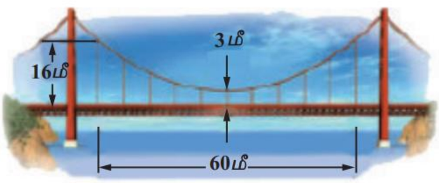
**8. Parabolic cable of a 60*m* portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6*m* along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.**




Soln: Equation of upward open parabola $x^2 = 4ay ----\rightarrow (1)$

At $(30,13) \Rightarrow 30^2 = 4a(13) \Rightarrow a = \frac{900}{52}$

Equation of parabola $x^2 = 4 \times \frac{900}{52}y \Rightarrow x^2 = \frac{900}{13}y ----\rightarrow (2)$

(i)At $(6, y_1)$

(2) $\Rightarrow 6^2 = \frac{900}{13}y_1 \Rightarrow \frac{36\times13}{900} = y_1 \Rightarrow y_1 = 0.52$

Height of the first cable is $3 + y_1 = 3 + 0.52 = 3.52$

(i)At $(12, y_2)$

(2) $\Rightarrow 12^2 = \frac{900}{13}y_2 \Rightarrow \frac{144\times13}{900} = y_2 \Rightarrow y_2 = 2.08$

Height of the second cable is $3 + y_2 = 3 + 2.08 = 5.08$ m

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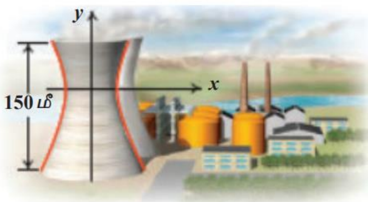
**9. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$ The tower is 150*m* tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.**




Soln: Given Equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1 ----\rightarrow (1)$

At $(x_1, 50)$

(1) $\Rightarrow \frac{(x_1)^2}{30^2} - \frac{(50)^2}{44^2} = 1 \Rightarrow \frac{(x_1)^2}{30^2} = 1 + \frac{(50)^2}{44^2}$

$$x_1{}^2 = 30^2\left[\frac{44^2+50^2}{44^2}\right]$$

$$x_1 = \frac{30}{44}\sqrt{44^2+50^2} = 45.41$$

$\therefore$ the diameter of the top is $2x_1 = 2(45.41) = 90.82$

At $(x_2, 100)$

(1) $\Rightarrow \frac{(x_2)^2}{30^2} - \frac{(100)^2}{44^2} = 1 \Rightarrow \frac{(x_2)^2}{30^2} = 1 + \frac{(100)^2}{44^2}$

$$x_2{}^2 = 30^2\left[\frac{44^2+100^2}{44^2}\right]$$
$$x_2 = \frac{30}{44}\sqrt{44^2+100^2} = 74.49$$

∴ the diameter of the top is $2x_2 = 2(74.49) = 148.98$ m

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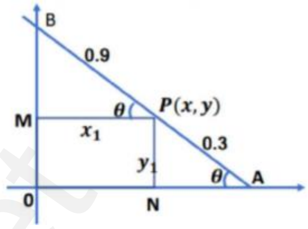
**10.A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of the point P on the rod 0.3 m from the end in contact with x-axis is an ellipse. Find the eccentricity.**

Soln: From the triangle ACP




$sin\theta = \frac{y_1}{0.3} \Longrightarrow sin^2\theta = \frac{y_1{}^2}{0.09} ----\longrightarrow (1)$

From triangle BPD

$cos\theta = \frac{x}{0.9} \Longrightarrow cos^2\theta = \frac{x_1{}^2}{0.81} ----\longrightarrow (2)$

$$(1)^2 + (2)^2 \Rightarrow \frac{x_1{}^2}{0.81} + \frac{y_1{}^2}{0.09} = cos^2\theta + sin^2\theta = 1$$

Eccentricity $e = \sqrt{\frac{a^2-b^2}{a^2}}$ $e = \sqrt{\frac{0.81-0.09}{0.81}} = \sqrt{\frac{0.72}{0.81}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$ m

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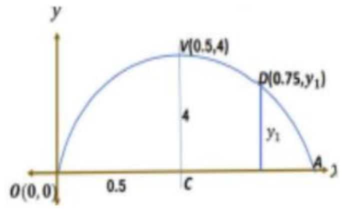
**11. Assume that water issuing from the end of a horizontal pipe, 7.5 *m* above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 *m* below the line of the pipe, the flow of water has curved outward 3*m* beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?**

Soln: Equation of downward open parabola $x^2 = -4ay ----\longrightarrow (1)$




At $(3, -2.5)$

$(1) \Longrightarrow (3)^2 = -4a(-2.5) \Longrightarrow a = \frac{9}{10}$

$x^2 = -4\left(\frac{9}{10}\right)y ----\longrightarrow (2)$

At $(x_1, -7.5)$

$(2) \Longrightarrow (x_1)^2 = -4 \times \frac{9}{10}(-7.5)$

$\Longrightarrow (x_1)^2 = 9 \times 3 \Longrightarrow x_1 = 3\sqrt{3}$ m

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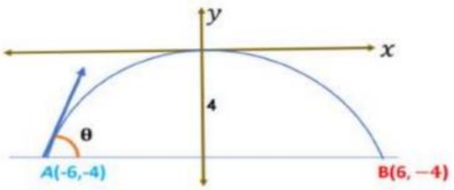
**12. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4*m* when it is 6*m* away from the point of projection. Finally it reaches the ground 12*m* away from the starting point. Find the angle of projection.**

Soln: Equation of downward open parabola $x^2 = -4ay ----\longrightarrow (1)$

At $(6, -4)$

$(1) \Longrightarrow (6)^2 = -4a(-4) \Longrightarrow a = \frac{36}{16} = \frac{9}{4}$

$(1) \Rightarrow x^2 = -4ay \Longrightarrow x^2 = -4\left(\frac{9}{4}\right)y \Longrightarrow x^2 = -9y ----\longrightarrow (2)$




$(2)$ $d.w.r.t.'x' \Longrightarrow 2x = -9\frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \frac{2x}{-9}$

At $(-6, -4) \Longrightarrow \frac{dy}{dx} = \frac{2(-6)}{-9}$

$\frac{dy}{dx} = tan\theta = \frac{4}{3}$

$\therefore \theta = tan^{-1}\left(\frac{4}{3}\right)$

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**13.Points $A$ and $B$ are 10$km$ apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 $km$ closer to $A$ than $B$ . Show that the location of the explosion is restricted to a particular curve and find an equation of it.**

Soln: $2ae = 10 \quad \Rightarrow ae = 5\,; \quad 2a = 6 \quad \Rightarrow a = 3$

1,2 $\quad 3e = 5 \quad \Rightarrow \quad e = \frac{5}{3} > 1$, ∴ The curve is an hyperbola.

$b^2 = a^2(e^2 - 1) \Rightarrow b^2 = 9\left(\frac{25}{9} - 1\right) \Rightarrow b^2 = 9\left(\frac{25-9}{9}\right) \Rightarrow b^2 = 16$

Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

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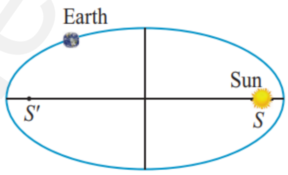
**14.The maximum and minimum distances of the Earth from the Sun respectively are $152 \times 10^6$ km and $94.5 \times 10^6$ km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.**

$$\begin{aligned} AS &= 94.5 \times 10^6 \text{ km}, SA' = 152 \times 10^6 \text{ km} \\ a + c &= 152 \times 10^6 \\ a - c &= 94.5 \times 10^6 \end{aligned}$$




Subtracting $2c = 57.5 \times 10^6 = 575 \times 10^5$ km

Distance of the Sun from the other focus is $SS' = 575 \times 10^5$ km.

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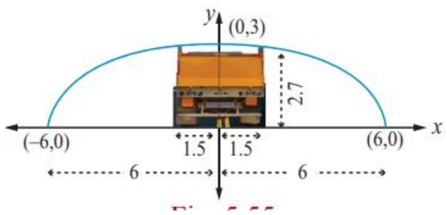
**15.A semielliptical archway over a one-way road has a height of 3 m and a width of 12 m. The truck has a width of 3 m and a height of 2.7 m. Will the truck clear the opening of the archway? (Fig.)**

Soln.

From the diagram $a = 6$ and $b = 3$

Equation of ellipse as $\frac{x^2}{6^2} + \frac{y^2}{3^2} = 1.$




Substituting $x = 1.5 = \frac{3}{2}$ and solving for $y$

$$\begin{aligned} \frac{\left(\frac{3}{2}\right)^2}{36} + \frac{y^2}{9} &= 1 \\ y^2 &= 9\left(1 - \frac{9}{144}\right) \\ &= \frac{135}{16} \\ y &= \frac{\sqrt{135}}{4} = 2.90m \end{aligned}$$

Thus the height of arch way 1.5 m from the centre is approximately 2.90 m. Since the truck's height is 2.7 m, the truck will clear the archway.

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# 8. *Differentials and Partial Derivatives*

*linear approximation* : $L(x) = f(x_0) + f'(x_0)(x - x_0)$

**1. *Find the linear approximation for $f(x) = \sqrt{1 + x}, x \geq -1$, at $x_0 = 3$. Use the linear approximation to estimate $f(3.2)$.***

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

$f(x) = \sqrt{1 + x}$ , $x_0 = 3, \Delta x = 0.2$ and hence $f(3) = \sqrt{1 + 3} = 2.$

$$f'(x) = \frac{1}{2\sqrt{1+x}} \Rightarrow f'(3) = \frac{1}{2\sqrt{1+3}} = \frac{1}{4}$$

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$= 2 + \frac{1}{4}(x-3) = \frac{x}{4} + \frac{5}{4}$$

$$f(3.2) = \sqrt{4.2} \cong L(3.2) = \frac{3.2}{4} + \frac{5}{4} = 2.050$$

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**2. Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator**

$$f(x) = \sqrt{x}, x_0 = 9, \Delta x = 0.2$$

$$f(9) = 3,$$

$$f'(x) = \frac{1}{2\sqrt{x}} \quad , \quad f'(x) = \frac{1}{2\sqrt{9}} = \frac{1}{(2\times 3)} = \frac{1}{6}$$

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$\sqrt{9.2} = f(9) + f'(9)(x-9)$$

$$= 3 + \frac{1}{6}(9.2 - 9) = 3 + \frac{0.2}{6} = 3.0333$$

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**Euler theorem :** $\frac{x\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$

**3. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$**

$$u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

$$f = \sin u = \left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

$$Degree = n = N.degree - D.degree$$

$$n = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore u(x,y) is\ a\ homgeneous\ function\ of\ degree\ is\ n = \frac{1}{2}$$

*By Euler theorem,*

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \text{n}f$$

$$x\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) = \frac{1}{2}\sin u$$
$$x\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y} = \frac{1}{2}\sin u$$
$$\cos u\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) = \frac{1}{2}\sin u$$
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\frac{\sin u}{\cos u}$$
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$

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**4. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$.**

$$u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$$
$$Degree = n = N.degree - D.degree$$
$$n = 2 - \frac{1}{2} = \frac{3}{2}$$
$$\therefore u(x,y) \text{ is a homgeneous function of degree is } n = \frac{3}{2}$$

*By Euler theorem,*

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$$

**5. If $v(x,y) = \log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$**

$$v(x,y) = \log\left(\frac{x^2+y^2}{x+y}\right)$$
$$f = e^v = \frac{x^2+y^2}{x+y}$$

$$Degree = n = N.degree - D.degree$$
$$n = 2 - 1 = 1$$
$$\therefore v(x,y) \text{ is a homgeneous function of degree is } n = 1$$

*By Euler theorem,*

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$

$$x\frac{\partial e^v}{\partial x} + y\frac{\partial e^v}{\partial y} = (1)e^v$$
$$xe^v\frac{\partial v}{\partial x} + ye^v\frac{\partial v}{\partial y} = e^v$$

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$$

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**6.** $\boldsymbol{If\ w(x,y,z) = log\left(\frac{5x^3y^4 + 7y^2xz^4 - 75y^3z^4}{x^2+y^2}\right), find\ x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}}$

$$w(x,y,z) = \log\left(\frac{5x^3y^4 + 7y^2xz^4 - 75y^3z^4}{x^2+y^2}\right)$$

$$f = e^w = \left(\frac{5x^3y^4 + 7y^2xz^4 - 75y^3z^4}{x^2+y^2}\right)$$

$$Degree = n = N.degree - D.degree$$
$$n = 7 - 2 = 5$$
$$\therefore w(x,y,z) is\ a\ homgeneous\ function\ of\ degree\ is\ \ n = 5$$

*By Euler theorem,*

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = nf$$

$$x\frac{\partial e^w}{\partial x} + y\frac{\partial e^w}{\partial y} + z\frac{\partial e^w}{\partial z} = (5)e^w$$
$$xe^w\frac{\partial v}{\partial x} + ye^w\frac{\partial v}{\partial y} + ze^w\frac{\partial v}{\partial z} = 5e^w$$

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + z\frac{\partial v}{\partial z} = 5$$

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7. *Prove that* $g(x,y) = xlog\left(\frac{y}{x}\right)$ *is homogeneous, what is the degree? verify Euler's theorem for g*

$$g(x,y) = xlog\left(\frac{y}{x}\right)$$

$$Degree = n = N.degree - D.degree$$
$$n = 2 - 1 = 1$$
$$\therefore v(x,y) is\ a\ homgeneous\ function\ of\ degree\ is\ \ n = 1$$

*By Euler theorem,*

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$
$$LHS = x\frac{\partial g}{\partial x} + y\frac{\partial g}{\partial y} = x\frac{\partial}{\partial x}\left(x\log\frac{y}{x}\right) + y\frac{\partial}{\partial y}\left(x\log\frac{y}{x}\right)$$
$$= x\left[(1)\log\frac{y}{x} + x\cdot\frac{1}{(y/x)}\cdot\frac{y}{x^2}\right] + y\left[x\cdot\frac{1}{(y/x)}\cdot\frac{1}{x}\right]$$

$$= x\left[\log\frac{y}{x} + x\cdot\frac{x}{y}\cdot\frac{-y}{x^2}\right] + y\left[x\cdot\frac{x}{y}\cdot\frac{1}{x}\right]$$

$$= x\left(\log\frac{y}{x} - 1\right) + y\left(\frac{x}{y}\right)$$

$$= x\log\frac{y}{x} - x + x$$

$$= x\log\frac{y}{x} = g$$

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# DIFFERENTIAL EQUATIONS

**1. The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?**

**Soln: Let A be the no.of bacteria at present**

| t | A |
|---|---|
| 0 | $A_0$ |
| 5 | $3A_0$ |
| 10 | ? |

$\frac{dA}{dt} \propto A$

$\Rightarrow \frac{dA}{dt} = kA$

$\Rightarrow \quad A = Ce^{kt} ---\rightarrow (1)$

Let $A_0$ be the initial no. of bacteria

i.e, $t = 0, A = A_0$

$(1) \Rightarrow A_0 = Ce^0 \Rightarrow A_0 = C$

$\therefore A = A_0 e^{kt} ----\rightarrow (2)$

Put $A = 3A_0$ at $t = 5$

$(2) \Rightarrow 3A_0 = A_0 e^{5k} \quad \Rightarrow 3 = e^{5k}$

A=? at $t = 10$ $A = A_0 e^{10k} \Rightarrow A = A_0(e^{5k})^2 \Rightarrow A = A_0(3)^2 = 9A_0$

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**2. Find the population of a city at any time *t*, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.**

Soln: Let A be the population of a city at present

| t | A |
|---|---|
| 0 | 300000 |
| 40 | 400000 |
| t | ? |

$\frac{dA}{dt} \propto A$

$\Rightarrow \frac{dA}{dt} = kA$

$\Rightarrow \quad A = Ce^{kt} ----\rightarrow (1)$

Let 3,00,000 be the initial population of a city

i.e, $t = 0, A = 30000$

$(1) \Rightarrow 3{,}00{,}000 = Ce^0 \Rightarrow 3{,}00{,}000 = C$

$\therefore A = 3{,}00{,}000e^{kt} ----\rightarrow (2)$

Put $A = 4{,}00{,}000$ at $t = 40$

$(2) \Rightarrow 4{,}00{,}000 = 3{,}00{,}000e^{40k}$

$\Rightarrow \frac{4}{3} = e^{40k} \Rightarrow log\left(\frac{4}{3}\right) = 40k \Rightarrow k = \frac{1}{40} log\left(\frac{4}{3}\right)$

$A = 3{,}00{,}000e^{\frac{1}{40}log\left(\frac{4}{3}\right)t} \Rightarrow A = 3{,}00{,}000(\frac{4}{3})^{\frac{t}{40}}$

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**3.The engine of a motor boat moving at 10 *m / s* is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine.**

**Soln: Let A be the no.of bacteria at present**

| t | A |
|---|---|
| 0 | 10 |
| 2 | ? |

$\frac{dA}{dt} \propto A$

$\Rightarrow \frac{dA}{dt} = -A$

$\Rightarrow \quad A = Ce^{-t} ----\rightarrow (1)$

Let 10 be the initial velocity

i.e, $t = 0, A = 10$

$(1) \Rightarrow 10 = Ce^0 \Rightarrow 10 = C$

$\therefore\ A = 10e^{-t} ----\rightarrow (2)$

A=? at $t = 2$

$(2) \Rightarrow A = 10e^{-2}\ (or)\ \ \Rightarrow A = {}^{10}/_{e^2}$

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**4.Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?**

Soln: Let A be the no.of bacteria at present

$\frac{dA}{dt} \propto A$

$\Longrightarrow \frac{dA}{dt} = 0.05A$

$\Longrightarrow\ \ A = Ce^{0.05t} ----\longrightarrow (1)$

K=0.05

| t | x |
|---|---|
| 0 | 10,000 |
| 1.5 | ? |

Let A=10,000 be the initial deposit

i.e, $t = 0, A = 10000$

$(1) \Rightarrow 10{,}000 = Ce^{0}\ \Longrightarrow\ 10{,}000 = C$

$\therefore\ A = 10{,}000e^{0.05t} ----\longrightarrow (2)$

put $t = \frac{3}{2}$=1.5 then A=?

$(2) \Rightarrow A = 10{,}000e^{0.05\left(\frac{3}{2}\right)}\ \Longrightarrow A = 10{,}000e^{0.075}$

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5.Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years?

Soln: Let A be the no.of bacteria at present

$\frac{dA}{dt} \propto A$

$\Longrightarrow \frac{dA}{dt} = -kA$

$\Longrightarrow\ \ A = Ce^{-kt} ----\longrightarrow (1)$

| t | A |
|---|---|
| 0 | 100 |
| 100 | 90 |
| 1000 | ? |

Let $A_0$ be the initial no. of bacteria

i.e, $t = 0, A = A_0$

$(1) \Rightarrow A_0 = Ce^{0}\ \Longrightarrow\ A_0 = C$

$\therefore\ A = A_0e^{-kt} ----\longrightarrow (2)$

Put $A = \frac{9}{10}A_0$ at $t = 100$

$(2) \Longrightarrow\ \frac{9}{10}A_0 = A_0e^{-100k}$

$\Longrightarrow\ \frac{9}{10} = e^{-100k} \Longrightarrow k = \frac{-1}{100}log\left(\frac{9}{10}\right)$

$t = 1000$, then A=?

$(2) \Rightarrow A = A_0e\ -^{1000k} \Longrightarrow A = A_0e^{-1000\left[\frac{-1}{100}log\left(\frac{9}{10}\right)\right]}$

$\Longrightarrow\ \frac{A}{A_0} = e^{10log\left(\frac{9}{10}\right)} = \left(\frac{9}{10}\right)^{10}$

Percentage of radioactive nuclei $\frac{A}{A_0} \times 100 = \left(\frac{9}{10}\right)^{10} \times 100\ (or)\ \frac{9^{10}}{10^8}\%$

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6.Water at temperature 100°C cools in 10 minutes to 80°C in a room temperature of 25°C .Find(i) The temperature of water after 20 minutes (ii) The time when the temperature is 40°C. $\left[log\frac{11}{15} = -0.3101; log5 = 1.6094\right]$.

Soln:

$\frac{dT}{dt} \propto T - T_m$

$\Longrightarrow \frac{dT}{dt} = k(T - T_m)$

$\Longrightarrow T - T_m = Ce^{kt}$

$\Longrightarrow T - 25 = Ce^{kt} ----\longrightarrow (1)$

| t | T | S |
|---|---|---|
| 0 | 100 | 25 |
| 10 | 80 | |
| 20 | ? | |
| ? | 40 | |

Put T=100 at t=0

(1) $\Longrightarrow C = 75$ ,

(1) $\Longrightarrow T - 25 = 75e^{kt} ----\longrightarrow (2)$

Put T=80 at t=10,

(2) $\Longrightarrow$ 80-25 =$75e^{10k}$ $\Longrightarrow 55 = 75e^{10k}$

$\frac{55}{75} = e^{10k} \Longrightarrow e^{10k} = \frac{11}{15} \Longrightarrow k = \frac{1}{10} log\left(\frac{11}{15}\right)$

Put t=20, T=?

(2) $\Rightarrow T - 25 = 75e^{20k} \Rightarrow$T=25+75$(e^{10k})^2$ $\Rightarrow$T=25+75$\left(\frac{11}{15}\right)^2$ $\Rightarrow T = 65.33$

Put T=40, t=?

(2) $\Rightarrow 40 - 25 = 75e^{kt} \Rightarrow \frac{15}{75} = e^{kt} \Rightarrow log\left(\frac{15}{75}\right) = kt \Rightarrow log\left(\frac{15}{75}\right)$

$$= \frac{1}{10} log\left(\frac{11}{15}\right) t$$

$t = \frac{10log\left(\frac{15}{75}\right)}{log\left(\frac{11}{15}\right)} \Rightarrow t = 53.46\, min$

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7. At 10.00 A.M. a woman took a cup of hot instant coffee from her microwave oven and placed it on a nearby Kitchen counter to cool. At this instant the temperature of the coffee was 180°F, and 10 minutes later it was 160°F . Assume that constant temperature of the kitchen was 70°F . (i) What was the temperature of the coffee at 10.15A.M.? (ii) The woman likes to drink coffee when its temperature is between 130°F and 140°F.between what times should she have drunk the coffee?

Soln:

| t | T | S |
|---|---|---|
| 0 | 180 | 70 |
| 10 | 160 | |
| 15 | ? | |
| ? | 130 | |
| ? | 140 | |

$\frac{dT}{dt} \propto T - T_m ----\longrightarrow (1)$

$\Longrightarrow \frac{dT}{dt} = k(T - T_m)$

$\Longrightarrow T - T_m = Ce^{kt}$

$\Longrightarrow T - 70 = Ce^{kt} ----\longrightarrow (2)$

At T=180 at t=0

(2) $\Longrightarrow C = 110$

(2) $\Longrightarrow T - 70 = 110e^{kt} ----\longrightarrow (3)$

At T=1600 at t=10

(3) $\Longrightarrow 160 - 70 = 110e^{10k}$

$\Longrightarrow e^{10k} = \frac{90}{110}$

$\Longrightarrow e^k = \left(\frac{9}{11}\right)^{\frac{1}{10}}$

At t=15 , T=?

(3) $\Rightarrow T - 70 = 110e^{kt}$

$\Longrightarrow T=70+110e^{15k}$

$\Longrightarrow T=70+110\left(\frac{9}{11}\right)^{\frac{15}{10}}$

$\Longrightarrow T = 151.33$

At T=130, t=?

(3) $\Rightarrow 130 - 70 = 110e^{kt}$

$\Longrightarrow \frac{60}{110} = e^{kt}$

$\Longrightarrow log\left(\frac{6}{11}\right) = kt$

$\Longrightarrow log\left(\frac{6}{11}\right) = \frac{1}{10}log\left(\frac{9}{11}\right)t$

$t = \frac{10log\left(\frac{6}{11}\right)}{log\left(\frac{9}{11}\right)} \quad \Longrightarrow t = 30.20\ min$

At T=140, t=?

$140 - 70 = 110e^{kt} \Longrightarrow \frac{70}{110} = e^{kt} \Longrightarrow log\left(\frac{7}{11}\right) = kt \Longrightarrow log\left(\frac{7}{11}\right) = \frac{1}{10}log\left(\frac{9}{11}\right)t$

$t = \frac{10log\left(\frac{7}{11}\right)}{log\left(\frac{9}{11}\right)} \quad \Longrightarrow t = 22.52\ min$

She drunk coffee between 10.22 min to 10.30 min

8.A pot of boiling water at 100°C is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C , and another 5 minutes later it has dropped to 65°C. Determine the temperature of the kitchen.

Soln:

$\frac{dT}{dt} \propto T - T_m$

$\Longrightarrow \frac{dT}{dt} = k(T - T_m)$

$\Longrightarrow T - T_m = Ce^{kt} ----\longrightarrow (1)$

| t | T | s |
|---|---|---|
| 0 | 100 | |
| 5 | 80 | ? |
| 10 | 65 | |

At T=100 at t=0

(1) $\Longrightarrow 100 - T_m = C$

(1) $\Longrightarrow \quad T - T_m = (100 - T_m)e^{kt} ----\longrightarrow (2)$

At T=80 at t=5

(2) $\Rightarrow 80 - T_m = (100 - T_m)e^{5k}$

$\Longrightarrow \quad e^{5k} = \frac{80-T_m}{100-T_m}$

At T=65 at, t=10

(2) $\Rightarrow 65 - T_m = (100 - T_m)e^{10k}$

$\Longrightarrow e^{10k} = \frac{65 - T_m}{100 - T_m}$

$\frac{65-T_m}{100-T_m} = \left(e^{5k}\right)^2 = \left(\frac{80-T_m}{100-T_m}\right)^2$

$(65 - T_m)(100 - T_m) = (80 - T_m)^2$

$T_m = 20$

……………………………………………………………………………………………

9.A tank initially contains 50 litres of pure water. Starting at time $t = 0$ a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time $t > 0$ .

| t | A |
|---|---|
| 0 | 0 |

Soln: $\frac{dA}{dt} = IN - OUT$

$\frac{dA}{dt} = 50 - 0.01A$

$\frac{dA}{dt} = -0.01(A - 50000)$

$\Rightarrow \quad A - 5000 = Ce^{-0.01t} ----\rightarrow (1)$

Put A = 100, t = 0

$(1) \Rightarrow \quad 100 - 5000 = C \quad \Rightarrow C = -4900$

$\therefore A - 5000 = -4900e^{-0.01t}$

……………………………………………………………………………………

10.The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple.

Soln: Let A be the no.of bacteria at present

$\frac{dA}{dt} \propto A$

$\Rightarrow \frac{dA}{dt} = kA$

$\Rightarrow \quad A = Ce^{kt} ----\rightarrow (1)$

| t | A |
|---|---|
| 0 | $A_0$ |
| 50 | $2A_0$ |
| ? | $3A_0$ |

Let $A_0$ be the initial no. of bacteria

i.e, $t = 0, A = A_0$

$(1) \Rightarrow A_0 = Ce^0 \Rightarrow A_0 = C$

$\therefore A = A_0 e^{kt} ----\rightarrow (2)$

Put $A = 2A_0$ at $t = 50$

$(2) \Rightarrow 2A_0 = A_0 e^{50k} \Rightarrow 2 = e^{50k}$

$\Rightarrow k = \frac{1}{50} log2$

Put A=$3A_0$ at $t$ =?

$(2) \Rightarrow 3A_0 = A_0 e^{kt}$

$\Rightarrow 3 = e^{kt} \Rightarrow log3 = kt$

$\Rightarrow log3 = (\frac{1}{50} log2)t$

$t = 50\frac{log3}{log2}.$

11. A radioactive isotope has an initial mass 200mg, which two years later is 50mg . Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life means the time taken for the radioactivity of a specified isotope to fall to half its original value).

Soln: Let A mass at present

$\frac{dA}{dt} \propto A$

$\Rightarrow \frac{dA}{dt} = -kA$

$\Rightarrow \quad A = Ce^{-kt} ----\rightarrow (1)$

| t | A |
|---|---|
| 0 | 200 |
| 2 | 150 |
| ? | 100 |

Let 200 be the initial mass

i.e, $t = 0, A = 200$

$(1) \Rightarrow 200 = Ce^0 \Rightarrow 200 = C$

$\therefore\ A = 200e^{-kt} ----\longrightarrow (2)$

Put $A = 150$ at $t = 2$

$(2) \Longrightarrow 150 = 200e^{-2k}$

$\Longrightarrow\ \frac{3}{4} = e^{-2k} \Longrightarrow log\left(\frac{3}{4}\right) = -2k$

Put A=100 at $t$ =?

$(2) \Rightarrow 100 = 200e^{kt}$

$\Longrightarrow\ \frac{1}{2} = e^{kt}\ \ \Longrightarrow\ log\left(\frac{1}{2}\right) = kt$

From 1,2 $\frac{-kt}{-2k} = \frac{log\left(\frac{1}{2}\right)}{log\left(\frac{3}{4}\right)}$

$\Longrightarrow t = 2\,\frac{log\left(\frac{1}{2}\right)}{log\left(\frac{3}{4}\right)}$

……………………………………………………………………………………………

12. In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur? [log(2.43) = 0.88789; log(0.5) = −0.69315].

Soln

$\frac{dT}{dt} \propto T - T_m$

$\Longrightarrow \frac{dT}{dt} = k(T - T_m)$

$\Longrightarrow T - T_m = Ce^{kt}$

$\Longrightarrow T - 50 = Ce^{kt} ----\longrightarrow (1)$

| t | T | s |
|---|---|---|
| 0 | 70 | 50 |
| 2 | 60 | |
| $t_1$ | 98.6 | |

Put T=70 at t=0

$(1) \Longrightarrow C\ = 20$

$(1) \Longrightarrow\ T - 50 = 20e^{kt} ----\longrightarrow (2)$

Put T=60 at t=2

(2) $\Longrightarrow$ 60-50 =20$e^{2k}$

$\Longrightarrow\ e^{2k} = \frac{1}{2}\ \ \Longrightarrow k = \frac{1}{2}log\left(\frac{1}{2}\right)$

Put T=98.6, t=?

$(2) \Rightarrow 98.6 - 50 = 20e^{kt}$

$\Longrightarrow \frac{48.6}{20} = e^{kt} \Longrightarrow log(2.43) = kt \Longrightarrow log(2.43) = \frac{1}{2}log\left(\frac{1}{2}\right)t$

$t = \frac{2log(2.43)}{log\left(\frac{1}{2}\right)}\ \ \ \Longrightarrow t \cong -2.56$

$\therefore$ the murder time is $8 - 2.56 \cong 5{:}30\ PM$

13. A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (Brine is a high-concentration solution of salt usually sodium chloride) in water runs in a rate of 10 litres per minute, and each litre contains 5grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time $t$ .

Soln: $\frac{dA}{dt} = IN - OUT$

| t | A |
|---|---|
| 0 | 100 |

$\frac{dA}{dt} = 6 - \frac{3}{50}A = \frac{300-3A}{50}$

$\frac{dA}{dt} = \frac{-3}{50}(A - 100)$

$\Rightarrow \quad A - 100 = Ce^{\frac{-3}{50}t} - - - \longrightarrow (1)$

Put A = 0, t = 0

$(1) \Rightarrow \quad -100 = C$

$\therefore \quad A - 100 = -100e^{\frac{-3}{50}t}$

……………………………………………………………………………………

14.The equation of electromotive force for an electric circuit containing resistance and self-inductance is $E = Ri + L\frac{di}{dt}$, where $E$ is the electromotive force is given to the circuit, $R$ the resistance and $L$, the coefficient of induction. Find the current $i$ at time $t$ when $E = 0$.

Soln: Given $E = Ri + L\frac{di}{dt}$

$\Rightarrow \quad \frac{di}{dt} + \frac{Ri}{L} = \frac{E}{L}$ P= $\frac{R}{L}$, Q= $\frac{E}{L}$

Integrating factor, IF= $e^{\int P dt} = e^{\int \frac{R}{L}dt} = e^{\frac{Rt}{L}}$

Complete solution i(IF)= $\int Q(\text{IF})dt + c$

$i\,(e^{\frac{Rt}{L}}) = \int \frac{E}{L}\left(e^{\frac{Rt}{L}}\right)dt + c \quad \Rightarrow \quad ie^{\frac{Rt}{L}} = \frac{E}{L}\left(\frac{e^{\frac{Rt}{L}}}{\frac{R}{L}}\right) + c \Rightarrow i = \frac{E}{R} + ce^{-\frac{Rt}{L}}$

put E=0 $\Rightarrow \quad i = ce^{-\frac{Rt}{L}}$

……………………………………………………………………………………

## PROBABILITY DISTRIBUTIONS

1.A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If $X$ denotes the total score in two throws. Find (i) The probability mass function. (ii) The cumulative distribution function.(iii) $P(3 \le X < 6)$ (iv) $P(X \ge 4)$.

Soln: The random variable X takes the value 2,3,4,5 and 6.

| + | 1 | 2 | 2 | 3 | 3 | 3 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 3 | 4 | 4 | 4 |
| 2 | 3 | 4 | 4 | 5 | 5 | 5 |
| 2 | 3 | 4 | 4 | 5 | 5 | 5 |
| 3 | 4 | 5 | 5 | 6 | 6 | 6 |
| 3 | 4 | 5 | 5 | 6 | 6 | 6 |
| 3 | 4 | 5 | 5 | 6 | 6 | 6 |

| X | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| PMF | $\frac{1}{36}$ | $\frac{4}{36}$ | $\frac{10}{36}$ | $\frac{12}{36}$ | $\frac{9}{36}$ |
| CDF | $\frac{1}{36}$ | $\frac{5}{36}$ | $\frac{15}{36}$ | $\frac{27}{36}$ | $\frac{36}{36}$ |

(iii) $P(3 \le X < 6) = P(x = 3) + P(x = 4) + P(x = 5)$

$= \frac{4}{36} + \frac{10}{36} + \frac{12}{36} = \frac{26}{36}$

(iv) $P(X \ge 4) = P(x = 4) + P(x = 5) + P(x = 6)$

$= \frac{10}{36} + \frac{12}{36} + \frac{9}{36} = \frac{31}{36}$

……………………………………………………………………………………

2. A six sided die is marked '1' on one face, '3' on two of its faces ,and '5' on remaining three faces. The die is rolled twice. If $X$ denotes the total score in two throws. Find (i) The probability mass function (ii) The cumulative distribution function.(iii) $P(4 \le X < 10)$ (iv) $P(X \ge 6)$.

Soln: The random variable X takes the value 2,4,6,8 and 10.

| + | 1 | 3 | 3 | 5 | 5 | 5 |
|---|---|---|---|---|---|---|
| 1 | 2 | 4 | 4 | 6 | 6 | 6 |
| 3 | 4 | 6 | 6 | 8 | 8 | 8 |
| 3 | 4 | 6 | 6 | 8 | 8 | 8 |
| 5 | 6 | 8 | 8 | 10 | 10 | 10 |
| 5 | 6 | 8 | 8 | 10 | 10 | 10 |
| 5 | 6 | 8 | 8 | 10 | 10 | 10 |

| X | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| PMF | $\frac{1}{36}$ | $\frac{4}{36}$ | $\frac{10}{36}$ | $\frac{12}{36}$ | $\frac{9}{36}$ |
| CDF | $\frac{1}{36}$ | $\frac{5}{36}$ | $\frac{15}{36}$ | $\frac{27}{36}$ | $\frac{36}{36}$ |

(iii) $P(4 \le X < 10)$

$= P(x = 4) + P(x = 6) + P(x = 8)$

$= \frac{4}{36} + \frac{10}{36} + \frac{12}{36} = \frac{26}{36}$

(iv) $P(X \ge 6) = P(x = 6) + P(x = 8) + P(x = 10)$

$= \frac{10}{36} + \frac{12}{36} + \frac{9}{36} = \frac{31}{36}$

……………………………………………………………………………………

3.A random variable $X$ has the following probability mass function.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$ | $k$ | $2k$ | $6k$ | $5k$ | $6k$ | $10k$ |

Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X < 5)$ (iii) $P(X \le 4)$ (iv) $P(3 < X)$

Soln: Given $f$ is P.M.F

$$\therefore \sum f(x) = 1$$

$$k + 2k + 6k + 5k + 6k + 10k = 1$$

$$30k = 1 \implies k = \frac{1}{30}$$

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$ | $\frac{1}{30}$ | $\frac{2}{30}$ | $\frac{6}{30}$ | $\frac{5}{30}$ | $\frac{6}{30}$ | $\frac{10}{30}$ |

(i) $P(2 < X < 6) = P(x=3) + P(x=4) + P(x=5) = \frac{6}{30} + \frac{5}{30} + \frac{6}{30} = \frac{17}{30}$

(ii) $P(2 \le X < 5) = P(x=2) + P(x=3) + P(x=4) = \frac{2}{30} + \frac{6}{30} + \frac{5}{30} = \frac{13}{30}$

(iii) $P(X \le 4) = P(x=1) + P(x=2) + P(x=3) + P(x=4) = \frac{1}{30} + \frac{2}{30} + \frac{6}{30} + \frac{5}{30} = \frac{14}{30}$

(iv) $P(3 < X) = P(x=4) + P(x=5) + P(x=6) = \frac{5}{30} + \frac{6}{30} + \frac{10}{30} = \frac{21}{30}$

……………………………………………………………………………………

4. A random variable $X$ has the following probability mass function.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | $k^2$ | $2k^2$ | $3k^2$ | $2k$ | $3k$ |

Find (i) $P(2 \le X < 5)$ (ii) $P(3 < X)$

Soln: Given $f$ is P.M.F $\therefore \sum f(x) = 1$

$$k^2 + 2k^2 + 3k^2 + 2k + 3k = 1$$

$$6k^2 + 5k - 1 = 0$$

$$k = -1 \ , \ k = \frac{1}{6}$$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{2}{6} = \frac{12}{36}$ | $\frac{3}{6} = \frac{18}{36}$ |

(i) $P(2 \le X < 5) = P(x=2) + P(x=3) + P(x=4) = \frac{2}{36} + \frac{3}{36} + \frac{12}{36} = \frac{17}{36}$

(ii) $P(3 < X) = P(x=4) + P(x=5) = \frac{12}{36} + \frac{18}{36} = \frac{30}{36}$

……………………………………………………………………………………

5.The cumulative distribution function of a discrete random variable is given by
find

$$F(x) = \begin{cases} 0 & ; -\infty < x < 0 \\ \frac{1}{2} & ; 0 \le x < 1 \\ \frac{3}{5} & ; 1 \le x < 2 \\ \frac{4}{5} & ; 2 \le x < 3 \\ \frac{9}{10} & ; 3 \le x < 4 \\ 1 & ; 4 \le x < \infty \end{cases}.$$

(i) The Probability mass function $f(x)$
(ii) $P(X < 3)$
(iii) (iii) $P(X \ge 2)$

Soln.

The values of the discrete random variable $X$

are 0,1,2,3,4.

(i) The Probability mass function $f(x)$ :

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $F(x)$ | $\frac{1}{2}$ | $\frac{3}{5}$ | $\frac{4}{5}$ | $\frac{9}{10}$ | 1 |
| $f(x)$ | $\frac{1}{2}$ or $\frac{5}{10}$ | $\frac{1}{5}$ or $\frac{2}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ |

(i) $P(X<3)=P(X=0)+P(X=1)+P(X=2)=\frac{5}{10}+\frac{1}{10}+\frac{2}{10}=\frac{8}{10}=\frac{4}{5}$

(ii) $P(X\geq 2)=P(X=2)+P(X=3)+P(X=4)=\frac{2}{10}+\frac{1}{10}+\frac{1}{10}=\frac{4}{10}=\frac{2}{5}$

………………………………………………………………………………………………..

6.The cumulative distribution function of a discrete random variable is given by $F(x)=\begin{cases}0 & ;-\infty<x<-1\\ 0.15 & ;-1\leq x<0\\ 0.35 & ;0\leq x<1\\ 0.60 & ;1\leq x<2\\ 0.85 & ;2\leq x<3\\ 1 & ;3\leq x<\infty\end{cases}$

Find (i)the probability mass function ( ii )$p(X<1)$ and(iii) $P(X\geq 2)$

The values of the discrete random variable $X$ are $-1,0,1,2,3$.

(i)The Probability mass function $f(x)$ :

| $x$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $F(x)$ | 0.15 | 0.35 | 0.60 | 0.85 | 1 |
| $f(x)$ | 0.15 | 0.20 | 0.25 | 0.25 | 0.15 |

(i) $P(X<1)=P(X=-1)+P(X=0)=0.15+0.20=0.35$

(ii) $P(X\geq 2)=P(X=2)+P(X=3)=0.25+0.15=0.40$

# Applications of Matrices and Determinants

**2, 3 Mark Questions**:

**1. Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$**

Solution:

$|A| = \begin{vmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{vmatrix}$ = 2 (9 – 2) + 1 (-15 + 3)+3 (-10 +9)

= 2(7)+1(-12)+3(-1) = 14-12-3 = -1≠0

$$\begin{bmatrix} \cancel{2} & \cancel{-1} & \cancel{3} & \cancel{2} & \cancel{-1} & \cancel{3} \\ \cancel{-5} & 3 & 1 & -5 & 3 & \cancel{1} \\ \cancel{-3} & 2 & 3 & -3 & 2 & \cancel{3} \\ \cancel{2} & -1 & 3 & 2 & -1 & \cancel{3} \\ \cancel{-5} & 3 & 1 & -5 & 3 & \cancel{1} \\ \cancel{-3} & \cancel{2} & \cancel{3} & \cancel{-3} & \cancel{2} & \cancel{3} \end{bmatrix}$$

$$\text{adj A} = \begin{bmatrix} 9-2 & -3+15 & -10+9 \\ 6+3 & 6+9 & 3-4 \\ -1-9 & -15-2 & 6-5 \end{bmatrix}^T$$

$$\text{adjA} = \begin{bmatrix} 7 & 12 & -1 \\ 9 & 15 & -1 \\ -10 & -17 & 1 \end{bmatrix}^T = \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj A} = \frac{1}{-1}\begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$$

**2. If A= $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that A (adj A) = (adj A) A= $|A|I_2$**

Solution :

adj A= $\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$|A| = \begin{vmatrix} 8 & -4 \\ -5 & 3 \end{vmatrix}$ = 24 – 20 = 4

$|A|I_2 = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ ……….. (1)

A (adj A) = $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ ……………… (2)

(adj A) A= $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ ………………..(3)

From (1), (2) and (3)

A (adj A) = (adj A) A = $|A|I_2$

**3.** **If A= $\begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$,verify that $(AB)^{-1} = B^{-1}A^{-1}$**

Solution:

$$A\,B = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+3 \\ -2+0 & -3-4 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -2 & -7 \end{bmatrix}$$

$|AB|$ = 0 + 6 = 6; adj (AB) = $\begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{|AB|}$ adj (AB) = $\frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix}$ ……… (1)

$B^{-1}A^{-1} = \frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix}$ ………………… (2)

From (1) and (2)

$(AB)^{-1} = B^{-1}A^{-1}$

**4.** **If A= $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and B= $\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$**

Solution:

$$A\,B = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} -3+10 & -9+4 \\ -7+25 & -21+10 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 18 & -11 \end{bmatrix}$$

$|AB|$ = -77 + 90 = 13; adj (AB) = $\begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{|AB|}$ adj (AB) = $\frac{1}{13}\begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix}$ ……… (1)

$B^{-1}A^{-1} = \frac{1}{13}\begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix}$ ………………… (2)

From (1) and (2)

$(AB)^{-1} = B^{-1}A^{-1}$

**5.** **Find a matrix A if adj A = $\begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$**

Solution:

$A = \pm \frac{1}{\sqrt{|adjA|}}$ adj (adj A)

$|adjA|$ = 2 (24 -0) +4 (-6 -14) +2 ( 0 + 24)

= 48 – 80 + 48 = 16

$\sqrt{|adjA|} = \sqrt{16}$ = 4

$$\text{adj (adj A)} = \begin{bmatrix} 24-0 & 14+6 & 0+24 \\ 0+8 & 4+4 & 8-0 \\ 28-24 & -6+14 & 24-12 \end{bmatrix}^T \left| \begin{matrix} \cancel{2} & -\cancel{4} & \cancel{2} & \cancel{2} & -\cancel{4} & \cancel{2} \\ -\cancel{3} & 12 & -7 & -3 & 12 & -\cancel{7} \\ -\cancel{2} & 2 & 2 & -2 & 0 & \cancel{2} \\ \cancel{2} & 0 & 2 & 2 & -4 & \cancel{2} \\ -\cancel{3} & 12 & -7 & -3 & 12 & -\cancel{7} \\ -\cancel{2} & \cancel{0} & \cancel{2} & -\cancel{2} & \cancel{0} & \cancel{2} \end{matrix} \right.$$

$$= \begin{bmatrix} 24 & 20 & 24 \\ 8 & 8 & 8 \\ 4 & 8 & 12 \end{bmatrix}^T$$

$$= \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix}$$

$$A = \pm \frac{1}{\sqrt{|adj A|}} \text{adj (adj A)} = \pm \frac{1}{4} \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix} = \pm \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

6. **If** $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ **Prove that** $A^{-1} = A^T$

Solution:

$AA^T = I \Rightarrow A^T = A^{-1}$

$$A^T = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$$

$$AA^T = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix} \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix} = \frac{1}{81}\begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$A^{-1} = A^T$$

7. **If** $\mathbf{F(\alpha)} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$ **show that** $(F(\alpha))^{-1} = F(-\alpha)$

Solution:

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & 0 & \sin(-\alpha) \\ 0 & 1 & 0 \\ -\sin(-\alpha) & 0 & \cos(-\alpha) \end{bmatrix}$$

$$F(-\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad \text{........ (1)}$$

$$(F(\alpha))^{-1} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad \text{........ (2)}$$

From (1) and (2), $(F(\alpha))^{-1} = F(-\alpha)$

**8.** **If adj (A) =** $\begin{bmatrix} 0 & -2 & 0 \\ 6 & -2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$**, find** $A^{-1}$

Solution:

$|adjA| = 0\ (12 - 0) + 2\ (36 - 18) + 0\ (0 + 6) = 36$

$\sqrt{|adjA|} = \sqrt{36} = 6$

$A^{-1} = \pm \frac{1}{\sqrt{|adjA|}} adjA$

$= \pm \frac{1}{6} \begin{bmatrix} 0 & -2 & 0 \\ 6 & -2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

**9.** **If adj A =** $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, $\boldsymbol{find} \quad \boldsymbol{A^{-1}}$

Solution:

$|adjA| = -1(1-4) - 2(1-4) + 2(2-2) = 3 + 6 + 0 = 9$

$\sqrt{|adjA|} = \sqrt{9} = 3$

$A^{-1} = \pm \frac{1}{\sqrt{|adj\ A|}} adjA = \pm \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

**10.** **Verify the property** $(A^T)^{-1} = (A^{-1})^T$ **with** $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$

Solution:

$|A| = 14 - 9 = 5$

$adjA = \begin{bmatrix} 7 & -9 \\ -1 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{5} \begin{bmatrix} 7 & -9 \\ -1 & 2 \end{bmatrix}$

$(A^{-1})^T = \frac{1}{5} \begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$ ……………………. (1)

$(A^T)^{-1} = \frac{1}{5} \begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$ ……………………. (2)

From (1) and (2) $(A^{-1})^T = (A^T)^{-1}$

**11. Prove that $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is orthogonal**

Solution

If $AA^T = A^TA = I$ then A is orthogonal

$A = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

$AA^T = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

$= \begin{bmatrix} cos^2\theta + sin^2\theta & cos\theta\, sin\theta - sin\theta \cos\theta \\ sin\theta cos\theta - cos\theta sin\theta & sin^2\theta + cos^2\theta \end{bmatrix}$

$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad similarly \quad A^TA = I$

∴ A is orthogonal.

**12. Find the matrix A for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$**

Solution:

$AB = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix} B^{-1}$

$|B| = \begin{vmatrix} 5 & 3 \\ -1 & -2 \end{vmatrix} = -10 + 3 = -7 \,;\, adjB = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|}$ adj B $= -\frac{1}{7}\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$

$A = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}(-\frac{1}{7})\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -28+7 & -42+35 \\ -14+7 & -21+35 \end{bmatrix}$

$A = -\frac{1}{7}\begin{bmatrix} -21 & -7 \\ -7 & 14 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}$

**13. Find the rank of the matrix $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$ by minor method**

Solution:

Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$ Then A is a matrix of order 3 x 3

$\rho(A) \leq \min(3, 3) = 3$

$|A| = \begin{vmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{vmatrix} = 3(6-6) - 2(6-6) + 5(3-3) = 0$

$\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 - 2 = 1 \neq 0$

$\therefore \rho(A) = 2$

**14. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form:**

Solution:

The rank of a matrix is equal to the number of non-zero rows in a row –echelon form of the matrix.

$$A = \begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix} \sim \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 2 & 8 & 7 \\ 0 & 8 & -13 & -2 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to 2R_2 + 3R_1}$$

$$\sim \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 2 & 8 & 7 \\ 0 & 0 & -45 & -30 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 4R_2} \sim \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 2 & 8 & 7 \\ 0 & 0 & 3 & 2 \end{bmatrix} R_3 \to R_3 \div (-15)$$

$\therefore \rho(A) = 3$

**15. Find the rank of the matrix = $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ by minor method**

Solution:

Let A = $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ p(A) ≤ min (3, 3) = 3

$|A| = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{vmatrix}$ = 1 (-4 + 6) +2 (-2 + 30) +3 (2 – 20)

= 2 + 56 – 54 = 4

$\therefore \rho(A) = 3$

**16. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ by reducing it to an echelon form:**

Solution:

The rank of a matrix is equal to the number of non-zero rows in a row-echelon form of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -4 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 - 2R_1}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_2}$$

ρ(A) = 2

**17.** **A = $\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ show that $A^2$ -3A – 7$I_2$ = 0 Hence find $A^{-1}$.**

Solution:

$$A^2 = AA = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 25-3 & 15-6 \\ -5+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$A^2 - 3A - 7I_2 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22-15-7 & 9-9-0 \\ -3+\ 3-0 & 1+6-7 \end{bmatrix}$$

$$|A| = -10+3 = -7$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0_2$$

$$\text{adj } A = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

**18.** **Solve 2x – y = 8 ; 3x + 2y = -2 Using matrix inversion method**

Solution:

$$\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$$

AX = B

$$|A| = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 4+3 = 7 \neq 0; \text{ adj } A = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}adjA = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ -2 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 16-2 \\ -24-4 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 \\ -28 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$\therefore$ x = 2, y = -4

**19.** **Solve $\frac{3}{x} + 2y = 12$, $\frac{2}{x} + 3y = 13$ by Cramer's rule**

Solution:

$$\Delta = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 9 - 4 = 5$$

$$\Delta_1 = \begin{vmatrix} 12 & 2 \\ 13 & 3 \end{vmatrix} = 36 - 26 = 10$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 \\ 2 & 13 \end{vmatrix} = 39 - 24 = 15$$

$$\frac{1}{x} = \frac{\Delta_1}{\Delta} = \frac{10}{5} = 2 \Rightarrow x = \frac{1}{2}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{15}{5} = 3$$

$$\therefore x = \frac{1}{2}, \ y = 3$$

**20. Solve : 5x – 2y + 16 = 0, x + 3y – 7 = 0 by Cramer's rule**

Solution:

5x – 2y = -16, x + 3y = 7

$\Delta = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix} = 15 + 2 = 17$

$\Delta_1 = \begin{vmatrix} -16 & -2 \\ 7 & 3 \end{vmatrix} = -48 + 14 = -34$

$\Delta_2 = \begin{vmatrix} 5 & -16 \\ 1 & 7 \end{vmatrix} = 35 + 16 = 51$

$x = \frac{\Delta_1}{\Delta} = \frac{-34}{17} = -2$

$y = \frac{\Delta_2}{\Delta} = \frac{51}{17} = 3$

$\therefore x = -2$ , y = 3

**Important points:**

1. $A^{-1} = \frac{1}{|A|} adjA$
2. $A^{-1} = \pm \frac{1}{\sqrt{|adjA|}} adjA$
3. $A = \pm \frac{1}{\sqrt{|adjA|}} adj\,(adjA)$
4. The rank of a matrix is equal to the number of non-zero rows in row-echelon form of the matrix
5. Matrix inversion method: A X = B ==> $X = A^{-1}B$
6. Cramer's method:

   $x = \frac{\Delta_1}{\Delta}, \; y = \frac{\Delta_2}{\Delta}, \; z = \frac{\Delta_3}{\Delta}$
7. If $AA^T = A^TA = I$ then A is orthogonal

# COMPLEX NUMBERS

## Important Hints:

- $i^2 = -1,$ $i^3 = -i\,,$ $i^4 = 1\,,$ $i^{4n} = 1$
- *Re*ctangular form of a complex number is x + iy real part is x, Imaginary part is y.
- The conjugate of the complex number z = x + iy is x –iy and is denoted by $\bar{z}$
- If z = x + iy then modulus of z is $|z| = \sqrt{x^2 + y^2}$
- Triangle inequality:
  For any two complex number $z_1$ *and* $z_2$, $|z_1 + z_2| \leq |z_1| + |z_1|$
- $\sqrt{a+ib} = \pm\left[\sqrt{\frac{|z|+a}{2}} + \frac{ib}{|b|}\sqrt{\frac{|z|-a}{2}}\right]$
- Additive inverse of z is –z multiplicative inverse of z is 1/z
- z is real if any only if z = $\bar{z}$ and z is purely imaginary if and only if z = - $\bar{z}$
- Distance between two complex numbers, $z_1$ and $z_2$ is $|z_1 - z_2|$
- $|z - z_0| = r$ is the complex form of the equation of a circle. Centre is $z_0$ and radius is r.

## Two Marks Questions:

1. **a. Simplify:** $\boldsymbol{i^{1948} - i^{-1869}}$

$= i^{1948} - i^{-1868}\, i^{-1}$

$= 1 - (1)\frac{1}{i}$

$= 1 - \frac{i}{i^2} = 1 + i$

**b. Simplify:** $\boldsymbol{i^{59} + \frac{1}{i^{59}} = i^{56}.i^3 + \frac{1}{i^{56}.i^3} = i^3 + \frac{1}{i^3}}$

$= -i - \frac{1 \times i}{i \times i}$

$= -i - \frac{i}{i^2}$

$\boldsymbol{= -i + i = 0}$

**c. Simplify:** $\sum_{n=1}^{10} i^{n+50} = \boldsymbol{i^{51} + i^{52} + i^{53} + \cdots \ldots + i^{60}}$ $\left[\because \boldsymbol{i^{51} + i^{52} + i^{53} + i^{54} = 0}\right.$

$= i^{59} + i^{60} = i^{56}.i^3 + i^{60}$ $i^{55} + i^{56} + i^{57} + i^{58} = 0$

$= -i + 1 = 1 - i$

Do it yourself:

Simplify: a) $i^{-1924} + i^{2018}$ b) $\sum_{n=1}^{10} i^n$ c) $i\ \ i^2\, i^3 \ldots\ldots i^{40}$

**2.** **If z= 5- 2i, w = -1+3i find** **a) z –iw** **b) $z^2 + 2zw + w^2$**

a) z – iw = 5 – 2i –i(-1 + 3i)

= 5 – 2i + i - $3i^2 = 8 - i$

b) $z^2 + 2zw + w^2 = (z + w)^2 = [(5 - 2i) + (-1 + 3i)] = (4 + i)^2$

= $4^2 + i^2 + 8i = 15 + 8i$

Do it yourself:

(i) zw (ii) 2z + 3w

**3.** **If $z_1 = 1 - 3i$, $z_2 = -4i$, $z_3 = 5$ *prove*: $(z_1 z_2)z_3 = z_1(z_2 z_3)$**

LHS = $(z_1 z_2)z_3$ = {(1 – 3i) (-4i)} 5= (-4i+12i$^2$)5

= -20i+60i$^2$ = -60 – 20i

RHS = $z_1(z_2 z_3) = (1 - 3i)\{(-4i)5\}$= (1 – 3i) (-20i)

= -20i + $60i^2 = -60 - 20i$

Do it yourself:

$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

**4.** **Write in rectangular form:**

$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$

$\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{1^2-i^2} = \frac{1+i^2+2i}{2} = \frac{2i}{2} = i$

$\frac{1-i}{1+i} \times \frac{1-i}{1-i} = \frac{(1-i)^2}{1^2-i^2} = \frac{1+i^2-2i}{2} = \frac{-2i}{2} = -i$

$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = (i)^3 - (-i)^3 = i^3 + i^3 = 2i^3 = -2i$

**5.** **If z = (2 + 3i) (1 – i) find $z^{-1}$**

z = (2 + 3i) (1 – i) = 2 – 2i + 3i - $3i^2 = 5 + i$

$z^{-1} = \frac{1}{z} = \frac{1}{5+i} = \frac{1}{5+i} \times \frac{5-i}{5-i} = \frac{5-i}{5^2-i^2} = \frac{5-i}{26}$

Do it yourself:

1) Write $\frac{3+4i}{5-12i}$ in the x + iy form, hence find its real and imaginary parts:

2) If $z_1$ = 3 – 2i, $z_2 = 6 + 4i$ find $\frac{z_1}{z_2}$ in the rectangular form

**6.** **Write $\overline{3\iota} + \frac{1}{2-i}$ in rectangular form:**

$$\overline{3\iota} + \frac{1}{2-i} = -3i + \frac{1\times(2+i)}{(2-i)(2+i)} = -3i + \frac{2+i}{2^2-i^2}$$

$$= -3i + \frac{2+i}{5} = \frac{-15i+2+i}{5} = \frac{2-14i}{5} = \frac{2}{5} - \frac{14}{5}i$$

**7.** **If z = x + iy, write $Re(i\bar{z})$ in rectangular form**

$$Re(i\bar{z}) = Re(i(x-iy)) = Re(ix+y) = y$$

Do it yourself:

1) (5 + 9i) + (2 -4i) 2) $Re(1/z)$

**8.** **Find $\left|\frac{i\,(2+i)^3}{(1+i)^2}\right|$**

$$\left|\frac{i\,(2+i)^3}{(1+i)^2}\right| = \frac{|i||2+i|^3}{|1+i|^2} = \frac{1\left(\sqrt{4+1}\right)^3}{\left(\sqrt{1+1}\right)^2} = \frac{\sqrt{5}^3}{\sqrt{2}^2} = \frac{5\sqrt{5}}{2}$$

Do it yourself:

Find the modulus of the following:

1. $\frac{2\,i}{3+4i}$ 2. $(1-i)^{10}$

**9.** **If $|z| = 3$, prove that $7 \le |z+6-8i| \le 13$**

$* \; ||z_1| - |z_2|| \le |z_1 + z_2| \le |z_1| + |z_2|$

Let $z_1 = z$, $z_2 = 6 - 8i$ $|z_2| = \sqrt{36+64} = 10$

$||z| - |6-8i|| \le |z+6-8i| \le |z| + 10$

$|3-10| \le |z+6-8i| \le 3 + 10$

$7 \le |z+6-8i| \le 13$

Do it yourself:

If $|z| = 2$, $P.T.$ $3 \le |z+3+4i| \le 7$

**10. Find the square root of the following:**

**i) -6 + 8i ii) -5 – 12i**

$$\sqrt{a+ib} = \pm\left[\sqrt{\frac{|z|+a}{2}} + \frac{ib}{|b|}\sqrt{\frac{|z|-a}{2}}\right]$$

Let z = -6+8i a = -6, b = 8 , $|z| = \sqrt{36+64} = 10$

$$\sqrt{-6+8i} = \pm\left[\sqrt{\frac{10+(-6)}{2}} + \frac{i8}{|8|}\sqrt{\frac{10-(-6)}{2}}\right] = \pm\left[\sqrt{2}+i\sqrt{8}\right]$$

**ii) z = -5-12i**

a = -5, b = -12

$|z| = \sqrt{25+144} = 13$

$$\sqrt{-5-12i} = \pm\left[\sqrt{\frac{13+(-5)}{2}} + \frac{i\,(-12)}{|-12|}\sqrt{\frac{13-(-5)}{2}}\right] = \pm[2-3i]$$

Do it yourself:

$\sqrt{6+8i}$, $\sqrt{4+3i}$ - find

**11. Obtain the Cartesian form of the locus of z = x + iy in each of the following:**

**1. $[Re(iz)]^2 = 3$ 2. $\overline{z} = z^{-1}$**

z = x + iy

$iz = i(x+iy) = ix - y = -y + ix \Rightarrow Re[iz] = -y$

$[Re(iz)]^2 = (-y)^2 = 3$

$y^2 - 3 = 0$

2. $\overline{z} = z^{-1}$

$x - iy = \frac{1}{x+iy}$

$(x+iy)(x-iy) = 1$

$x^2 + y^2 = 1$

Do it yourself:

i) $|z| = |z-i|$ ii) $|z+i| = |z-1|$

**12. Show that the following equations represent a circle and find its centre and radius**

1. $|2z+2-4i| = 2$

**(÷2)** $|z+1-2i| = 1$

$|z-(-1+2i)| = 1$ It is of the form $|z-z_0| = r$ and so it represents a circle

Centre = -1+2i = (-1, 2) radius = 1

2. $|3z-6+12i| = 8$

**(÷3)** $|z-2+4i| = 8/3$

$|z-(2-4i| = 8/3$

Centre = 2 – 4i = (2, -4) radius = 8/3

Do it yourself:

1. $|z-2-i| = 3$ 2. $|3z-5+i| = 4$

13. **If $z = 3 + 2i$ then show that $z, iz$ and $z + iz$ form the vertices of an isosceles right triangle**

A ⇒ z = 3 + 2i

B ⇒ iz = i (3 + 2i) = -2+3i

C ⇒ z + iz = 1 + 5i

AB $= |z - iz|$

$= |3 + 2i + 2 - 3i|$

$= |5 - i| = \sqrt{25 + 1} = \sqrt{26}$

BC $= |iz - z - iz|$

$= |-z| = |-3 - 2i|$

$= \sqrt{9 + 4} = \sqrt{13}$

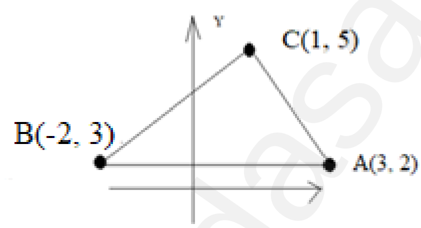
AC $= |z - z - iz|$

$= |- iz|$

$= |2 - 3i| = \sqrt{4 + 9} = \sqrt{13}$

$AB^2 = AC^2 + BC^2$

∴ ΔABC is an isosceles right triangle.




## THREE MARKS QUESTION

1. Find the values of the real numbers x and y, if the complex numbers

(3-i)x - (2-i)y + 2i + 5 and 2x + (-1 + 2i) y + 3 + 2i Equating real and imaginary parts

(3-i)x – (2-i)y + 2i+5 = 2x + (-1 + 2i)y + 3 + 2i

3x –ix – 2y + iy + 2i + 5 = 2x – y +2iy + 3 + 2i

3x – 2y + 5 = 2x - y + 3 -x + y + 2 = 2y + 2

x – y + 2 = 0 …(1) -x – y = 0 ………… (2)

(1) + (2) ⇒ -2y + 2 = 0

y = 1 from equ. (2)–x -1 = 0

x = -1

**Do it yourself:**

Find the value of the real numbers and y if the complex number

(2+i)x + (1-i)y + 2i -3 and x+(-1+2i)y + 1 + i are equal

2. **Find the additive and multiplicative inverse of -3-4i**

**Solution:** **let z = -3 -4i**

Additive inverse of z => -z = 3 + 4i

Multiplicative inverse of z => $\frac{1}{z} = \frac{1}{-3-4i}$

$= \frac{1 \times (-3+4i)}{(-3-4i)(-3+4i)} = \frac{-3+4i}{9+16} = \frac{-3+4i}{25}$

**Do it yourself:**

1). Find the additive and multiplicative inverse of 2+5i

2) If $z_1 = 3$, $z_2 = -7i$, $z_3 = 5+4i$ then prove that $z_1(z_2+z_3) = z_1 z_2 + z_1 z_3$

3. **The complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If v = 3 – 4i and w = 4+3i find u – in rectangular form.**

$\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$

$= \frac{1}{3-4i} + \frac{1}{4+3i}$

$\frac{1}{u} = \frac{3+4i}{9+16} + \frac{4-3i}{16+9} = \frac{7+i}{25}$

$\therefore u = \frac{25}{7+i} = \frac{75(7-i)}{(7+i)(7-i)} = \frac{25(7-i)}{50} = \frac{7-i}{2}$

4. **Which one of the points 10-8i, 11+6i is closest to 1 + i**

Let z = 1 + i, $z_1 = 10 - 8i$, $z_2 = 11 + 6i$

$|z - z_1| = |(1+i) - (10-8i)| = |-9+9i| = \sqrt{81+81} = \sqrt{162}$

$|z - z_2| = |(1+i) - (11-6i)| = |-10-5i| = \sqrt{100+25} = \sqrt{125}$

$\sqrt{125} < \sqrt{162}$

$z_2 = 11 + 6i$ is closest to 1 +i

5. **If $z_1$, $z_2$ and $z_3$ are complex numbers such that $|z_1| = |z_2| = |z_3|$**

$|z_1 + z_2 + z_3| = 1$ find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$

$|z_1| = |z_2| = |z_3| = 1$

$|z_1|^2 = 1 \quad \Rightarrow \quad z_1\bar{z_1} = 1$

$\bar{z_1} = 1/z_1$, $\Rightarrow \bar{z_2} = 1/z_2$, $\bar{z_3} = 1/z_3$

$\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = |\bar{z_1} + \bar{z_2} + \bar{z_3}| = |\overline{z_1 + z_2 + z_3}| = |z_1 + z_2 + z_3| = 1$

6. **Show that the equation $z^2 = \bar{z}$ has four solutions:.**

$z^2 = \bar{z}$

$|z^2| = |\bar{z}| = |z|$

$|z^2| = |z|$ $\qquad$ $|z| = 1$

$|z|(|z| - 1) = 0$ $\qquad$ $|z^2| = 1$

$|z| = 0$ $\qquad$ $z\bar{z} = 1$

$z = 0$ is a solution $\qquad$ ஃ $\bar{z} = 1/z$

$z^2 = \bar{z} = \frac{1}{z}$

$z^3 = 1 \quad z^3 - 1 = 0$ It has 3 non zero solution including zero solution, there are four solutions.

**Do it yourself:**

1. Show that the equation $z^3 + 2\bar{z} = 0$ has five solution

# Theory of Equation

**Important Points:**

1. Quadratic equation is $ax^2 + bx + c = 0$

The roots are $\alpha, \beta$

$\Sigma_1 = \alpha + \beta = -\frac{b}{a}$

$\Sigma_2 = \alpha\beta = \frac{c}{a}$

If roots are given then the equation is

$x^2 - \Sigma_1 x + \Sigma_2 = 0$

2. Cubic equation is $ax^3 + bx^2 + cx + d = 0$

The roots are $\alpha, \beta, \gamma$

$\Sigma_1 = \alpha + \beta + \gamma = -\frac{b}{a}$

$\Sigma_2 = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\Sigma_3 = \alpha\beta\gamma = -\frac{d}{a}$

If roots are given then the equation is

$x^3 - \Sigma_1 x^2 + \Sigma_2 x - \Sigma_3 = 0$

3. Fourth degree equation is

$ax^4 + bx^3 + cx^2 + dx + e = 0$

The roots are $\alpha, \beta, \gamma, \delta$

$\Sigma_1 = \alpha + \beta + \gamma + \delta = -\frac{b}{a}$

$\Sigma_2 = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$

$\Sigma_3 = \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a}$

$\Sigma_4 = \alpha\beta\gamma\delta = \frac{e}{a}$

If roots are given then the equation is

$x^4 - \Sigma_1 x^3 + \Sigma_2 x^2 - \Sigma_3 x + \Sigma_4 = 0$

**1. If α, β are the roots of the equation $17x^2 + 43x - 73 = 0$ construct a quadratic equation whose roots are α + 2, β +2**

**Solution:**

$17x^2 + 43x - 73 = 0$

a = 17, b = 43, c = -73

$\Sigma_1 = \alpha + \beta = -\frac{b}{a} = -\frac{43}{17}$

$\Sigma_2 = \alpha\beta = \frac{c}{a} = -\frac{73}{17}$

Given roots are α + 2, β +2

$\Sigma_1 = \alpha + 2 + \beta + 2 = \alpha + \beta + 4 = -\frac{43}{17} + 4 = -\frac{43+68}{17}$

$= \frac{25}{17}$

$\Sigma_2 = (\alpha + 2)(\beta + 2) = \alpha\beta + 2\alpha + 2\beta + 4$

$= \alpha\beta + 2(\alpha + \beta) + 4$

$= -\frac{73}{17} + 2\left(\frac{-43}{17}\right) + 4$

$= \frac{-73-86+68}{17} = \frac{-91}{17}$

∴ equation $x^2 - \Sigma_1 x + \Sigma_2 = 0$

$x^2 - \frac{25}{17} \times \frac{-91}{17} = 0$

x17 $17x^2 - 25x - 91 = 0$

**2. If α, β are roots of $2x^2 - 7x + 13 = 0$ construct a quadratic equation whose roots are $\alpha^2, \beta^2$** **(Eg: 3.2)**

**Solution:**

$2x^2 - 7x + 13 = 0$

a = 2, b = -7, c = 13

$\Sigma_1 = \alpha + \beta = \frac{-b}{a} = -\left(\frac{-7}{2}\right) = \frac{7}{2}$

$\Sigma_2 = \alpha\beta = \frac{c}{a} = \frac{13}{2}$

To form the equation whose roots are $\alpha^2, \beta^2$

$\Sigma_1 = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{7}{2}\right)^2 - 2\left(\frac{13}{2}\right) = \frac{49}{4} - 13$

$= \frac{49-52}{4} = \frac{-3}{4}$

$\Sigma_2 = \alpha^2\beta^2 = (\alpha\beta)^2 = \left(\frac{13}{2}\right)^2 = \frac{169}{4}$

∴ equation $x^2 - \Sigma_1 x + \Sigma_2 = 0$

$x^2 - \left(\frac{-3}{4}\right)x + \frac{169}{4} = 0$

$x^2 + \frac{3}{4}x + \frac{169}{4} = 0$

x 4 $4x^2 + 3x + 169 = 0$

**3. Find the polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root .**

(Eg 3.2 (4))

**Solution:** $\sqrt{5} - \sqrt{3}$ is a root

$\sqrt{5} + \sqrt{3}, -\sqrt{5} + \sqrt{3}, -\sqrt{5} - \sqrt{3}$ are also roots

$x = \sqrt{5} - \sqrt{3}$

squaring both sides: $[(a-b)^2 = a^2 + b^2 - 2ab]$

$x^2 = \left(\sqrt{5} - \sqrt{3}\right)^2 = \left(\sqrt{5}\right)^2 + \left(\sqrt{3}\right)^2 - 2\sqrt{5}\sqrt{3}$

$x^2 = 5 + 3 - 2\sqrt{5}\sqrt{3}$

$x^2 = 8 - 2\sqrt{5}\sqrt{3}$

$x^2 - 8 = -2\sqrt{5}\sqrt{3}$

Again squaring on both sides

$(x^2 - 8)^2 = \left(-2\sqrt{5}\sqrt{3}\right)^2$

$x^4 + 64 - 2x^2(8) = 4(5)(3)$

$x^4 + 64 - 16x^2 = 60$

$\therefore x^4 - 16x^2 + 64 - 60 = 0$

$x^4 - 16x^2 + 4 = 0$

**4. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root**

(Eg: 3.10)

$x = \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$

Squaring on both sides $x^2 = \frac{\sqrt{2}}{\sqrt{3}}$

Again squaring on both sides

$x^4 = \frac{2}{3}$

$3x^4 - 2 = 0$

**5.** **Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x** (Eg 3.11)

Solution::

Discriminant $\Delta = b^2 - 4ac$

Here a = 2, b = -6, c = 7

$\Delta = (-6)^2 - 4(2)(7) = 36 - 56$

$\therefore \Delta = -20 < 0$

$\because \Delta < 0$ the roots are imaginary

**6.** **If $x^2 + 2(k+2)x + 9k = 0$ has equal roots, find k** (Eg 3.12)

**Solution:** Equal roots $\Delta = b^2 - 4ac$

Here a = 1 b = 2(K+2), c = 9k

Equal roots $\Delta = 0$

$\therefore [2(k+2)]^2 - 4(1)(9k) = 0$

$4(k+2)^2 - 4(9k) = 0$

÷4 $(k+2)^2 - 9k = 0$

$K^2 + 4 + 4k - 9k = 0$

$K^2 - 5k + 4 = 0$

$(k-4)(k-1) = 0$

k = 4, k = 1

| 4 | | |
|---|---|---|
| -4 | -1 | -5 |

**7.** **Solve the cubic equation $2x^3 - 9x^2 + 10x = 3$** **(Eq.3.3 – 6(i))**

**Solution:**

$2x^3 - 9x^2 + 10x = 3$ First rewrite the given equation as below

$2x^3 - 9x^2 + 10x - 3 = 0$

The Coefficients are 2, –9, 10, -3

Sum of the coefficients = 2 – 9 + 10 – 3 = 12-12 = 0

$\therefore x = 1$ is root

If sum of the coefficients

| x = 1 | 2 | -9 | 10 | -3 |
|---|---|---|---|---|
| | 0 | 2 | -7 | 3 |
| | 2 | -7 | 3 | 0 |

$2x^2 - 7x + 3 = 0$

$(x - 3)(x - ½) = 0 \quad \therefore x = 3, x = ½$

| 6 | | |
|---|---|---|
| -6 | -1 | -7 |
| -6/2 | -1/2 | |
| -3 | -1/2 | |

Solution: x = 1, x = 3, x = 1/2

**8. Solve the equation: $x^3 - 3x^2 - 33x + 35 = 0$ (Eg: 3.17)**

Solution:

The co-efficients 1 -3 -33 35

Sum of the roots 1- 3-33+35 = 36 – 36 = 0

| 36 | | |
|---|---|---|
| 12 | -3 | 9 |

∴ x = 1 is a real

| x = 1 | 1 | -3 | -33 | 35 |
|---|---|---|---|---|
| | 0 | 1 | -2 | -35 |
| | 1 | -2 | -35 | 0 |

$x^2 - 2x - 35 = 0$

| -35 | | |
|---|---|---|
| -7 | 5 | -2 |

$(x - 7)(x + 5) = 0$

$x = 7, \quad x = -5$

∴ Solution: x = 1, x = 7, x = -5

**9. Solve the cubic equation $8x^3 - 2x^2 - 7x + 3 = 0$ (Eg3.3 6(ii))**

Solution:

The co-efficient are 8 -2 -7 3

$S_1$ $S_2$

$S_1 = 8 - 7 = 1$, $S_2 = -2 + 3 = 1$

$S_1 = S_2$ ∴ x = -1 is a root

| x = -1 | 8 | -2 | -7 | 3 |
|---|---|---|---|---|
| | 0 | -8 | 10 | -3 |
| | 8 | -10 | 3 | 0 |

| 4 | | |
|---|---|---|
| -6 | -4 | -10 |
| -6/8 | -4 / 8 | |
| -3/4 | -1/2 | |

$8x^2 - 10x + 3 = 0$

$\left(x - \frac{3}{4}\right)\left(x - \frac{1}{2}\right) = 0$

$x = \frac{3}{4}, \quad x = \frac{1}{2}$

Solution: $x = -1, \quad x = \frac{3}{4}, \quad x = \frac{1}{2}$

**10. Solve the cubic equation $2x^3 + 11x^2 - 9x - 18 = 0$ (Eg: 3.18)**

The co-efficient are 2 11 -9 -18

$S_1$ $S_2$

$S_1 = 2 - 9 = -7$, $S_2 = 11 - 18 = -7$

$S_1 = S_2$ ∴ $x = -1$ is a root

x = -1

| | | | |
|---|---|---|---|
| 2 | 11 | -9 | -18 |
| 0 | -2 | -9 | 18 |
| 2 | 9 | -18 | 0 |

| | 36 | |
|---|---|---|
| 12 | -3 | 9 |
| 12/2 | -3/2 | |
| 6 | -3/2 | |

$2x^2 + 9x - 18 = 0$

$(x + 6)\left(x - \frac{3}{2}\right) = 0$

$x = -6$, $x = \frac{3}{2}$

∴ Solution: $x = -6$, $x = \frac{3}{2}$, $x = -1$

**11. Solve the equation: $7x^3 - 43x^2 = 43x - 7$ (Eg: 3.27)**

Solution: $7x^3 - 43x^2 = 43x - 7$

Rewriting the equation in correct order $7x^3 - 43x^2 - 43x + 7 = 0$

The coefficients are 7 -43 -43 7

$S_1$ $S_2$

$S_1 = 7 - 43 = -36$, $S_2 = -43 + 7 = -36$

$S_1 = S_2$ ∴ $x = -1$ is a root

x = -1

| | | | |
|---|---|---|---|
| 7 | -43 | -43 | 7 |
| 0 | -7 | 50 | -7 |
| 7 | -50 | 7 | 0 |

| | 49 | |
|---|---|---|
| -49 | -1 | -50 |
| -49/7 | -1/7 | |
| -7 | -1/7 | |

$7x^2 - 50x + 7 = 0$

$(x - 7)\left(x - \frac{1}{7}\right) = 0$

$x = 7$, $x = \frac{1}{7}$

∴ Solution: $x = -1$, $x = 7$, $x = \frac{1}{7}$

**12. Solve : $x^4 + 3x^3 - 3x - 1 = 0$ (Eg3.5 5(ii))**

Solution:

The coefficients are 1 3 0 -3 -1 [Note $x^2$ term is missing]

Sum of the coefficients $1 + 3 + 0 - 3 - 1 = 4 - 4 = 0$

∴ $x = 1$ is a root

$S_2$

1 3 0 -3 -1

$S_1$

$S_1 = 1+0-1 = 0$ , $S_2 = 3 - 3 = 0$

$S_1 = S_2$ ∴ $x = -1$ is a root

| | | | | | |
|---|---|---|---|---|---|
| x = 1 | 1 | 3 | 0 | -3 | -1 |
| | 0 | 1 | 4 | 4 | 1 |
| x = -1 | 1 | 4 | 4 | 1 | 0 |
| | 0 | -1 | -3 | -1 | |
| | 1 | 3 | 1 | 0 | |

$x^2 + 3x + 1 = 0$

$a = 1$, $b = 3$, $c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2}$$

∴ Solution: $x = 1$, $x = -1$, $x = \frac{-3+\sqrt{5}}{2}$, $x = \frac{-3-\sqrt{5}}{2}$

**13.** Solve the cubic equation $2x^3 - x^2 - 18x + 9 = 0$ if sum of its two roots vanishes: (Eg3.3(1))

Solution: Clearly sum $= 2 - 1 - 18 + 9 \neq 0$

$x = 1$ is not a root

$S_1 \neq S_2$ ∴ $x = -1$ is not a root

$x = 2$, $x = -2$ are also not roots

By trail put $x = 3$

| | | | | |
|---|---|---|---|---|
| x = 3 | 2 | -1 | -18 | 9 |
| | 0 | 6 | 15 | -9 |
| | 2 | 5 | -3 | 0 |

Remainder is 0 $x = 3$ is a root

Put $x = -3$

| | | | |
|---|---|---|---|
| x = -3 | 2 | 5 | -3 |
| | 0 | -6 | 3 |
| | 2 | -1 | 0 |

Remainder is 0 x = -3is also a root

Note Clearly sum of two of its roots 3 -3 = 0 vanishes

2x -1 = 0

2x = 1

x = ½

∴solution x = 3, x = -3, x = ½

**14. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$ if it is known that $1+2i$ and $\sqrt{3}$ are two of its roots** (Eg 3.3 (5))

**Solution :**

The root are $1+2i$, $1-2i$, $\sqrt{3}$, $-\sqrt{3}$, $\alpha$, $\beta$

$$\Sigma_1 = 1 + 2i + 1 - 2i + \sqrt{3} - \sqrt{3} + \alpha + \beta = -\frac{Coefficient\ of\ x^5}{Coefficient\ of\ x^6}$$

$$2 + \alpha + \beta = \frac{-(-3)}{1} = 3$$

$$\alpha + \beta = 1 \quad \ldots\ldots\ldots\ldots\ldots (1)$$

$$\Sigma_6 = (1 + 2i)(1 - 2i)(\sqrt{3})(-\sqrt{3})\,\alpha\beta = \frac{Constant}{Coefficient\ of\ x^6}$$

$(1^2 + 2^2)(-3)\,\alpha\beta = 135 / 1 = 135$

$(1 + 4)(-3)\,\alpha\beta = 135$

$5(-3)\,\alpha\beta = 135$

$\alpha\beta = \frac{135}{-15} = -9$

$\alpha\beta = -9$

$x^2 - (\alpha\beta)\,x + \alpha\beta = 0$

$x^2 - x - 9 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-9)}}{2} = \frac{1 \pm \sqrt{1+36}}{2} = \frac{1 \pm \sqrt{37}}{2}$$

Solution : $1 + 2i,\ 1 - 2i,\ \sqrt{3}, -\sqrt{3}, \frac{1+\sqrt{37}}{2}, \frac{1-\sqrt{37}}{2}$

**15. Solve the equation: $x^4 - 9x^2 + 20 = 0$** **(Eg: 3.16)**

Put $x^2 = y$

$y^2 - 9y + 20 = 0$

| | 20 | |
|---|---|---|
| -5 | -4 | -9 |

$(y - 5)(y - 4) = 0$

$y = 5$, $y = 4$

$y = 5$ If $x^2 = 5 \Rightarrow x = \pm\sqrt{5}$

$y = 4$ If $x^2 = 4 \Rightarrow x = \pm 2$

Solution $x = 2, -2, \sqrt{5}, -\sqrt{5}$

Solve the equation: $x^4 - 14x^2 + 45 = 0$ Eg: 3.3(7)

Put $x^2 = y$

$y^2 - 14y + 45 = 0$

| | 45 | |
|---|---|---|
| -9 | -5 | -14 |

$(y - 9)(y - 5) = 0$

$y = 9, \quad y = 5$

$y = 9$ If $x^2 = 9 \Rightarrow x = \pm 3$

$y = 5$ If $x^2 = 5 \Rightarrow x = \pm\sqrt{5}$

Solution $x = 3, -3, \sqrt{5}, -\sqrt{5}$

16. **Solve: $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$** **(Eg3.5.5 (i)**

Solution: This is a reciprocal equation

x = 2put

| x = 2 | 6 | -35 | 62 | -35 | 6 |
|---|---|---|---|---|---|
| | 0 | 12 | -46 | 32 | -6 |
| | 6 | -23 | 16 | -3 | 0 |

Remainder is 0 , x = 2 is a root

x = 3 is a root

| x = 3 | 6 | -23 | 16 | -3 |
|---|---|---|---|---|
| | 0 | 18 | -15 | 3 |
| | 6 | -5 | 1 | 0 |

$6x^2 - 5x + 1 = 0$

$\left(x - \frac{1}{3}\right)\left(x - \frac{1}{2}\right) = 0$

Since the given equation is a reciprocal equation

x = 2, x = 3 are roots x = ½, x =1/3 are also root

∴ Solution x = 2, x =3, x = ½, x = 1/3

17. **Solve: $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$, if it is known that x = 1/3 is a solution**

Solution: This is a reciprocal equation

Given x = 1/3 is a root x = 3 is a root

| x = 3 | 6 | -5 | -38 | -5 | 6 |
|---|---|---|---|---|---|
| | 0 | 18 | 39 | 3 | -6 |
| x = -2 | 6 | 13 | 1 | -2 | 0 |
| | 0 | -12 | -2 | 2 | |
| | 6 | 1 | -1 | 0 | |

x = -2

Remainder x = -2 is a solution

The given equation is a reciprocal equation x = -1/2 is also a solution

∴Solution x = 3, x =1/3, x = -2, x = -½

**18.** **Solve :** $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ **(Eg: 3.28)**

Solution:

Put : $x + \frac{1}{x} = y$

$y^2 - 10y + 24 = 0$

$(y - 4)(y - 6) = 0$

| | 24 | |
|---|---|---|
| -4 | -6 | -10 |

y = 4 , y = 6

(i) If y $= 4$ $x + \frac{1}{x} = 4$ $\rightarrow$ $\frac{x^2+1}{x} = 4 \rightarrow$ $x^2 + 1 = 4x$

$x^2 - 4x + 1 = 0$ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ a = 1, b = -4, c = 1

$x = \frac{4 \pm \sqrt{(-4)^2-4(1)(1)}}{2} = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2}$

$= \frac{4 \pm \sqrt{4 \times 3}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = \frac{2\,(2 \pm \sqrt{3})}{2} = (2 \pm \sqrt{3})$

(ii) If y = 6 $\rightarrow x + \frac{1}{x} = 6 \rightarrow$ $\frac{x^2+1}{x} = 6 \rightarrow$ $x^2 + 1 = 6x$

$x^2 - 6x + 1 = 0$ $x = \frac{6 \pm \sqrt{(-6)^2-4(1)(1)}}{2}$

$x = \frac{6 \pm \sqrt{36-4}}{2} = \frac{6 \pm \sqrt{32}}{2} = \frac{6 \pm \sqrt{16 \times 2}}{2}$

$= \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$

**Solution:** $2 + \sqrt{3},\ 2 - \sqrt{3},\ 3 + 2\sqrt{2},\ 3 - 2\sqrt{2}$

**19.** **Discuss the maximum possible number of positive and negative roots of the polynomial equation** $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$ **(Ex: 3.6(1))**

**Solution :** $P(x) = 9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$

P(x) + - + - + + + + +

1 2 3 4

Number of sign changes in P(x) is 4

Maximum number of positive roots is 4

P(-x): - - - - - - + - +

1 2 3

Number of sign changes in P(-x) is 3

Maximum number of negative roots is = Total roots - positive roots

=9-7=2

**20. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7$** **(Eg: 3.6(4))**

Solution : $P(x) = 9x^9 - 4x^8 + 4x^7$

P(x) = + - -

1

Number of sign changes in P(x) is 1

Maximum number of positive roots is 1

P(-x) = - - +

1

Number of sign changes in P(-x) is 1

Maximum number of negative roots is 1

**21. Find the exact number of real roots and imaginary of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$** **(Eg: 3.6(5))**

Solution : P(x) = + + + + +

No sign change

No positive real root

P(-x) = - - - - -

No sign change

No negative real root

Since there is no constant term x = 0 is a root

∴ There are 8 imaginary roots

**22. Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2 = 0$ has atleast six imaginary roots.** **(Eg: 3.30)**

Solution: P(x) = + + - - +

1 2

Number of sign changes in P(x) is 2

Maximum number of positive roots is 2

P(-x) = - - - - +

1

Number of sign changes in P(-x) is 1

Maximum number of negative roots is 1

There is a constant term (2) in P(x)

Zero is not a root

Minimum number of imaginary roots is

$$9 - (2 + 1) = 9 - 3 = 6$$

# Inverse Trigonometric Functions

**Important Points:**

| Function | Domain | Range |
|---|---|---|
| $sin^{-1}x$ | $[-1, 1]$ | $[-{}^{\pi}/_{2}\ ,\ {}^{\pi}/_{2}]$ |
| $cos^{-1}x$ | $[-1, 1]$ | $[0\ ,\ \pi]$ |
| $tan^{-1}x$ | $R$ | $(-{}^{\pi}/_{2}\ ,\ {}^{\pi}/_{2})$ |
| $cosec^{-1}x$ | $R \setminus (-1, 1)$ | $[-{}^{\pi}/_{2}\ {}^{\pi}/_{2}] \setminus \{0\}$ |
| $sec^{-1}x$ | $R \setminus (-1, 1)$ | $[0\ ,\ \pi] \setminus \{{}^{\pi}/_{2}\}$ |
| $cot^{-1}x$ | $R$ | $(0\ ,\ \pi)$ |

If y = A sinαx then the period = $\frac{2\pi}{|\alpha|}$ and amplitude = $|A|$

## 2 & 3 Mark Questions

1. **Find the value of $sin^{-1}(\sin {}^{2\pi}/_{3})$**

Solution:

$$= sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$$
$$= sin^{-1}(\sin({}^{\pi}/_{3}))$$
$$= {}^{\pi}/_{3} \in [-{}^{\pi}/_{2}, {}^{\pi}/_{2}]$$

2. **Find the value of $sin^{-1}\left(\sin\frac{5\pi}{4}\right)$**

Solution:

$$= sin^{-1}\left(\sin\left(\pi + \frac{\pi}{4}\right)\right)$$
$$= sin^{-1}(-\sin({}^{\pi}/_{4}))$$
$$= sin^{-1}(\sin(-{}^{\pi}/_{4}))$$
$$= -\frac{\pi}{4} \in [-{}^{\pi}/_{2}\ , {}^{\pi}/_{2}]$$

3. **Find the value of $cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$**

Solution:

$$= cos^{-1}\left(\cos\left(\pi + \frac{\pi}{6}\right)\right)$$
$$= cos^{-1}\left(-\cos\frac{\pi}{6}\right)$$
$$= cos^{-1}\left(\cos\left(\pi - \frac{\pi}{6}\right)\right)$$
$$= cos^{-1}\left(\cos\frac{5\pi}{6}\right)$$
$$= \frac{5\pi}{6} \in [0\ , \pi]$$

4. **Find the value: $tan^{-1}\left(\tan \frac{5\pi}{4}\right)$**

Solution:

$$tan^{-1}\left(\tan \frac{5\pi}{4}\right) = tan^{-1}\left(\tan\left(\pi + \frac{\pi}{4}\right)\right)$$
$$= tan^{-1}\left(\tan {}^{\pi}/_{4}\right)$$
$$= \frac{\pi}{4} \in \left(-{}^{\pi}/_{2}, {}^{\pi}/_{2}\right)$$

5. **Find the principal value of $sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$**

Solution:

$$sec^{-1}\left({}^{2}/_{\sqrt{3}}\right) = cos^{-1}\left({}^{\sqrt{3}}/_{2}\right) = {}^{\pi}/_{6}$$
$$sec^{-1}x = cos^{-1}\left(\frac{1}{x}\right)$$

6. **$cot^{-1}({}^{1}/_{7}) = \theta$ Find the value of $\cos\theta$**

Solution:

$$\theta = cot^{-1}({}^{1}/_{7})$$
$$\cot\theta = {}^{1}/_{7} \Rightarrow \tan\theta = 7$$
$$\sec\theta = \sqrt{1 + tan^2\theta} = \sqrt{1 + 49} = \sqrt{50} = 5\sqrt{2}$$
$$\cos\theta = \frac{1}{5\sqrt{2}}$$

7. **Find the principal value: $cosec^{-1}(-\sqrt{2})$**

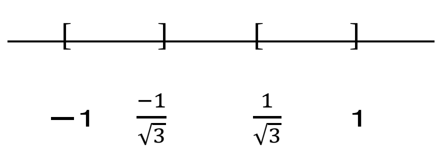
Solution:

$$cosec^{-1}(-\sqrt{2}) = sin^{-1}\left({}^{-1}/_{\sqrt{2}}\right)$$
$$= -sin^{-1}\left({}^{1}/_{\sqrt{2}}\right) \qquad sin^{-1}(-x) = -sin^{-1}x$$
$$= -{}^{\pi}/_{4} \qquad x \;\varepsilon\; [-1, 1]$$

8. **$sin^{-1}(2 - 3x^2)$ in Domain**

Solution:

$$-1 \le 2 - 3x^2 \le 1$$
$(-2)$ $$-3 \le -3x^2 \le -1$$
$(\div 3)$ $$-1 \le -x^2 \le -{}^{1}/_{3}$$
$(-1)$ $$1 \ge x^2 \ge {}^{1}/_{3}$$
$$1 \ge |x| \ge {}^{1}/_{\sqrt{3}}$$
$$\frac{1}{\sqrt{3}} \le |x| \le 1$$




$$x\varepsilon\left[-1, -{}^{1}/_{\sqrt{3}}\right] \cup \left[{}^{1}/_{\sqrt{3}}, 1\right]$$

**9.** $cos^{-1}[\cos(-\pi/6)] \neq -\pi/6$ **True? Justify your answer**

Sol:

$cos^{-1}(\cos(-\pi/6)) = cos^{-1}(\cos\pi/6) = \pi/6$

$cos^{-1}(\cos(-\pi/6)) \neq -\pi/6$

**10. Find the Domain F(x) =** $sin^{-1}\left(\frac{x^2+1}{2x}\right)$

Sol:

$\left|\frac{x^2+1}{2x}\right| \leq 1$

$x^2 + 1 \leq 2|x|$

$x^2 + 1 - 2|x| \leq 0$

$(|x|-1)^2 \leq 0$

$|x| - 1 \leq 0$

$x \in [-1, 1]$

**11.** $cos^{-1}\left(\frac{2+\sin x}{3}\right)$ **Find the Domain**

Sol: $-1 \leq \frac{2+\sin x}{3} \leq 1$

(x 3) $\rightarrow$ $-3 \leq 2+\sin x \leq 3$

(-2) $\rightarrow$ $-5 \leq sin\, x \leq 1$

$-5 \leq \sin x \leq 1$

$- \quad \pi/2 \leq x \leq \pi/2$

$x \in [-\pi/2, \pi/2]$

**12. Find the value** $sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + cos\frac{5\pi}{9}\,.sin\frac{\pi}{9}\right)$

Sol:

$= sin^{-1}\left(\sin\left(\frac{5\pi}{9}+\frac{\pi}{9}\right)\right)$

$= sin^{-1}\left(\sin\left(\frac{6\pi}{9}\right)\right)$

$= sin^{-1}(\sin(2\pi/3))$

$= sin^{-1}(\sin(\pi - \pi/3))$

$= sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$= \frac{\pi}{3} \in [-\pi/2, \pi/2]$

# Applications of vector Algebra

**Important Points:**

1. $\vec{a}\,.\,\vec{b} = |\vec{a}||\vec{b}|cos\theta$
2. $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|sin\theta \ \hat{n}$
3. Work done $W = \vec{F}\,.\,\vec{d}$
4. Torque $\vec{t} = \vec{r} \times \vec{F}$
5. $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors ⇔ $[\vec{a}, \vec{b}, \vec{c}] = 0$
6. $\vec{a}, \vec{b}$ *are perpendicular vectors* ⇔ $\vec{a}\,.\,\vec{b} = 0$
7. $\vec{a}, \vec{b}$ are parallel vectors ⇔ $\vec{a} \times \vec{b} = \vec{0}$
8. Volume of parallelepiped with coterminous vectors $V = |[\vec{a}, \vec{b}, \vec{c}]|$ Cubic units

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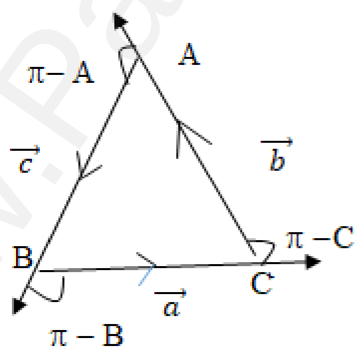
**2 marks/ 3 marks question**

1. **Cosine formulae:**

With usual notations in ΔABC, prove $a^2 = b^2 + c^2 - 2bc \cos A$

Solution:

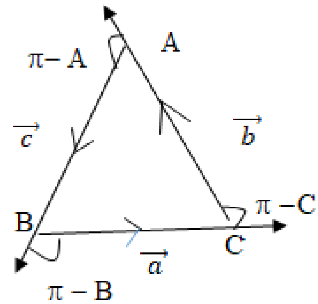



In$\Delta ABC$

$$\vec{a} + \vec{b} + \vec{c} = 0$$
$$\vec{a} = -(\vec{b} + \vec{c})$$
$$(\vec{a})^2 = (\vec{b} + \vec{c})^2$$
$$a^2 = b^2 + c^2 + 2\,\vec{b}\,.\,\vec{c}$$
$$a^2 = b^2 + c^2 + 2\,bc\cos(\pi - A)$$
$$a^2 = b^2 + c^2 - 2\,bc\cos A$$

2. With usual notations prove that in $\Delta ABC$, prove $a = b\cos C + c\cos B$ by vector method .

Solution:




In $\Delta ABC$

$\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} = -\vec{b} - \vec{c}$

$\vec{a}.\vec{a} = -\vec{b}.\vec{a} - \vec{c}.\vec{a}$

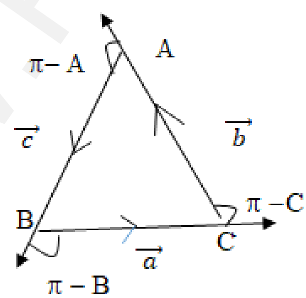
$a^2 = -ba\, cos(\pi - C) - ca\cos(\pi - B)$

$a^2 = ab\cos C + ac\cos B$

$a = b\cos C + c\cos B$

3. With usual notations in $\Delta ABC$ prove that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ by vector method

Solution:




Area of $\Delta ABC$

$\frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}|\vec{b} \times \vec{c}| = \frac{1}{2}|\vec{c} \times \vec{a}|$

$|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{c}| = |\vec{c} \times \vec{a}|$

$ab\sin(\pi - C) = bc\sin(\pi - A) = ca\sin(\pi - B)$

$ab\sin C = bc\sin A = ca\sin B$

$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

4. **A particle acted upon by constant forces $2\,\hat{\imath}+5\,\hat{\jmath}+6\,\hat{k}$ and $-\hat{\imath}-2\,\hat{\jmath}-\hat{k}$ is displaced from the point (4, -3, -2) to the point (6, 1, -3) find the total workdone by the forces:**

**Solution:**

$$\vec{F}=(2\,\hat{\imath}+5\,\hat{\jmath}+6\,\hat{k})+(-\hat{\imath}-2\,\hat{\jmath}-\hat{k})$$
$$\vec{F}=\hat{\imath}+3\,\hat{\jmath}+5\,\hat{k}$$
$$\vec{d}=\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$$
$$=(6\,\hat{\imath}+\hat{\jmath}-3\,\hat{k})-(4\hat{\imath}-3\,\hat{\jmath}-2\,\hat{k})$$
$$=6\,\hat{\imath}+\hat{\jmath}-3\,\hat{k}-4\hat{\imath}+3\,\hat{\jmath}+2\,\hat{k}$$
$$\vec{d}=2\,\hat{\imath}+4\,\hat{\jmath}-\hat{k}$$

The work done $W=\vec{F}\,.\,\vec{d}$

$$=(\hat{\imath}+3\,\hat{\jmath}+5\,\hat{k}).(2\,\hat{\imath}+4\,\hat{\jmath}-\hat{k})$$
$$=2+12-5=9 \text{ Units}$$

5. **A particle is acted upon by the forces $3\,\hat{\imath}-2\,\hat{\jmath}+2\,\hat{k}$ and $2\,\hat{\imath}+\hat{\jmath}-\hat{k}$ is displaced from the point $(1,3,-1)$ to the point $(4,-1,\lambda)$. If the work done by the forces is 16 units, find the value of λ**

Solution:

$$\vec{F}=(3\,\hat{\imath}-2\,\hat{\jmath}+2\,\hat{k})+(2\,\hat{\imath}+\hat{\jmath}-\hat{k})$$
$$\vec{F}=5\hat{\imath}-\hat{\jmath}+\hat{k}$$
$$\vec{d}=(4\,\hat{\imath}-\hat{\jmath}-\lambda\hat{k})-(\hat{\imath}+3\,\hat{\jmath}-\hat{k})$$
$$\vec{d}=4\,\hat{\imath}-\hat{\jmath}+\lambda\hat{k}-\hat{\imath}-3\,\hat{\jmath}+\hat{k}$$
$$\vec{d}=3\,\hat{\imath}-4\,\hat{\jmath}+(\lambda+1)\,\hat{k}$$

Given W = 16

$$\vec{F}\,.\,\vec{d}=16$$
$$(5\,\hat{\imath}-\hat{\jmath}+\hat{k}).(3\,\hat{\imath}-4\,\hat{\jmath}+(\lambda+1)\hat{k})=16$$
$$15+4+\lambda+1=16$$
$$\lambda+20=16$$
$$\lambda=16-20$$
$$\lambda=-4$$

6. A particle is acted upon by the forces $8\hat{\imath}-2\,\hat{\jmath}+6\,\hat{k}$ and $6\hat{\imath}+2\hat{\jmath}-2\,\hat{k}$ is displaced from the point$(1,2,3)$ to the point $(5,4,-1)$. Find the total work done by the forces.

Solution:

$$\vec{F}=(8\,\hat{\imath}+2\,\hat{\jmath}-6\,\hat{k})+(6\,\hat{\imath}+2\hat{\jmath}-2\,\hat{k})$$
$$\vec{F}=14\hat{\imath}+4\,\hat{\jmath}-8\,\hat{k}$$
$$\vec{d}=(5\,\hat{\imath}+4\,\hat{\jmath}+\hat{k})-(\hat{\imath}+2\,\hat{\jmath}+3\,\hat{k})$$
$$\vec{d}=5\,\hat{\imath}+4\,\hat{\jmath}+\hat{k}-\hat{\imath}-2\,\hat{\jmath}-3\,\hat{k}$$
$$\vec{d}=4\,\hat{\imath}+2\,\hat{\jmath}-2\,\hat{k}$$

The work done $W=\vec{F}\,.\vec{d}$

$$=(14\,\hat{\imath}+4\,\hat{\jmath}-8\,\hat{k}).(4\,\hat{\imath}+2\,\hat{\jmath}-2\hat{k})$$
$$=56+8+16$$
$$=80 \text{ Units}$$

7. **Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\,\hat{\imath} + 4\,\hat{\jmath} + 5\,\hat{k}$ and $10\,\hat{\imath} + 6\,\hat{\jmath} - 8\,\hat{k}$ respectively, act on a particle which is displaced from the point with position vectors $4\,\hat{\imath} - 3\,\hat{\jmath} - 2\,\hat{k}$ to the point with position vector $6\,\hat{\imath} + \hat{\jmath} - 3\,\hat{k}$ Find the work done by the forces.**

Solution: $\vec{F} = \frac{5\sqrt{2}(3\,\hat{\imath}+4\,\hat{\jmath}+5\,\hat{k})}{5\sqrt{2}} + \frac{10\sqrt{2}(10\,\hat{\imath}+16\,\hat{\jmath}-8\,\hat{k})}{10\sqrt{2}}$

$\vec{F} = 13\hat{\imath} + 10\,\hat{\jmath} - 3\,\hat{k}$

$\vec{d} = (6\,\hat{\imath} + \hat{\jmath} - 3\,\hat{k}) - (4\,\hat{\imath} - 3\,\hat{\jmath} - 2\,\hat{k})$

$\vec{d} = 6\,\hat{\imath} + \hat{\jmath} - 3\,\hat{k} - 4\hat{\imath} + 3\,\hat{\jmath} + 2\,\hat{k}$

$\vec{d} = 2\,\hat{\imath} + 4\,\hat{\jmath} - \hat{k}$

Work done $W = \vec{F}\,.\vec{d}$

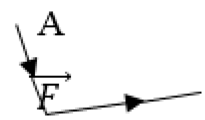
$= (13\,\hat{\imath} + 10\,\hat{\jmath} - 3\,\hat{k}).(2\,\hat{\imath} + 4\,\hat{\jmath} - \hat{k})$

W $= 26 + 40 + 3 = 69$ Units

## Torque($\vec{t}$) :

If a force $\vec{F}$ is applied on the particle at a point with position vector $\vec{r}$ then the torque or moment on the particle is given by $\vec{t} = \vec{r} \times \vec{F}$

8. **Find the magnitude and the direction cosines of the torque about the point (2, 0, -1) of the force $2\,\hat{\imath} + \hat{\jmath} - \hat{k}$, Whose line of action passes through the origin.**




Solution:-

$F = 2\,\hat{\imath} + \hat{\jmath} - \hat{k}$

$\vec{r} = \vec{AO} \quad = -2\,\hat{\imath} + \hat{k}$

The torque $\vec{t} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 0 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$= \hat{\imath}(0 - 1) - \hat{\jmath}(2 - 2) + \hat{k}\,(-2 - 0)$

$= -\hat{\imath} - 2\,\hat{k}$

Magnitude $= \sqrt{1+4} = \sqrt{5}$

Direction cosine $\Longrightarrow \left(\frac{-1}{\sqrt{5}}\,,\ 0\,,\ \frac{-2}{\sqrt{5}}\right)$

9. **Find the magnitude and direction cosines of the torque of a force represented by $3\,\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ about the point with the position vector $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ acting through a point whose position vector is $4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$**

Solution :

$\vec{F} = 3\,\hat{\imath} + 4\,\hat{\jmath} - 5\hat{k}$

$\vec{r} = (4\,\hat{\imath} + 2\,\hat{\jmath} - 3\hat{k}) - (2\,\hat{\imath} - 3\,\hat{\jmath} + 4\hat{k})$

$\vec{r} = 4\,\hat{\imath} + 2\,\hat{\jmath} - 3\hat{k} - 2\,\hat{\imath} + 3\,\hat{\jmath} - 4\hat{k}$

$\vec{r} = 2\,\hat{\imath} + 5\,\hat{\jmath} - 7\hat{k}$

Torque $\vec{t} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 5 & -7 \\ 3 & 4 & -5 \end{vmatrix} = \hat{\imath}(-25+28) - \hat{\jmath}(-10+21) + \hat{k}\,(8-15)$$

$= 3\,\hat{\imath} - 11\,\hat{\jmath} - 7\hat{k}$

Magnitude $= \sqrt{9+121+49} = \sqrt{179}$

Direction cosine $= \left(\frac{3}{\sqrt{179}}, \frac{-11}{\sqrt{179}}, \frac{-7}{\sqrt{179}}\right)$

10. **Find the torque of the resultant of the three forces represented by $-3\,\hat{\imath} + 6\,\hat{\jmath} - 3\hat{k}$, $4\,\hat{\imath} - 10\,\hat{\jmath} + 12\hat{k}$ and $4\,\hat{\imath} + 7\hat{\jmath}$ *acting at the point with position vector* $8\hat{\imath} - 6\,\hat{\jmath} - 4\hat{k}$**

Hint: $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} = 5\,\hat{\imath} + 3\,\hat{\jmath} + 9\hat{k}$

$\vec{r} = -10\,\hat{\imath} - 9\,\hat{\jmath} + 5\hat{k}$

Torque $\vec{t} = \vec{r} \times \vec{F}$

$\vec{t} = -96\,\hat{\imath} + 115\,\hat{\jmath} + 15\hat{k}$

11. **Find the volume of the parallelepiped whose coterminous edges are given by the vectors $2\,\hat{\imath} - 3\,\hat{\jmath} + 4\hat{k}$, $\hat{\imath} + 2\,\hat{\jmath} - \hat{k}$ and $3\,\hat{\imath} - \hat{\jmath} + 2\hat{k}$**

Solution: $\vec{a} = 2\,\hat{\imath} - 3\,\hat{\jmath} + 4\hat{k}$

$\vec{b} = \hat{\imath} + 2\,\hat{\jmath} - \hat{k}$

$\vec{c} = 3\,\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$= 2(4-1) + 3(2+3) + 4(-1-6)$

$= 6 + 15 - 28 = -7$

∴ Volume of *parallelepiped* = 7 Cubic units

**12.** **Show that the vectors $\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} + \hat{\jmath} - \hat{k}$ are coplanar;**

Solution: $[\vec{a}\ ,\ \vec{b}\ ,\ \vec{c}] = \begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix}$

$= 1(1-2) - 2(-2-3) - 3(2+3)$

$= 1(-1) - 2(-8) - 3(5)$

$= -1 + 16 - 15$

$[\vec{a}\ ,\ \vec{b}\ ,\ \vec{c}] = 0$

$\Rightarrow \vec{a}\ ,\ \vec{b}\ ,\ \vec{c}$ are coplanar vectors

**13.** **The volume of parallelepiped whose coterminous edges are $\hat{\imath} + \lambda\hat{\jmath} - 3\hat{k}$, $\hat{\imath} + 2\hat{\jmath} - \hat{k}$, $-3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$ is 90 cubic units. Find the value of $\lambda$**

Solution: $[\vec{a}\ ,\ \vec{b}\ ,\ \vec{c}] = 90$

$$\begin{vmatrix} 7 & \lambda & -3 \\ 1 & 2 & -1 \\ -3 & 7 & 5 \end{vmatrix} = 90$$

$7(10+7) - \lambda(5-3) - 3(7+6) = 90$

$7(17) - \lambda(2) - 3(13) = 90$

$119 - 2\lambda - 39 = 90$

$-2\lambda + 80 = 90$

$-2\lambda = 10$

$\lambda = -5$

**14.** **Let $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath}$, and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ If $c_1 = 1$ and $c_2 = 2$ such that $\vec{a}\ ,\vec{b}\ ,\ \vec{c}$ are coplanar.**

Solution: $[\vec{a}\ ,\ \vec{b}\ ,\ \vec{c}] = 0$

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & x \end{vmatrix} = 0$

1(0) – 1(x) + 1(2-0) = 0

-x + 2 = 0

-x = -2 ⇒ x = 2

**15.** **If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$, $\vec{c} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$ Find $\vec{a}\ .(\vec{b}\ \times \vec{c}\ )$**

Solution: $\vec{a}\ .(\vec{b}\ \times \vec{c}) = [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{vmatrix}$

= 1(1 + 4) + 2(2 + 6) + 3(4-3)

= 1(5) + 2(8) + 3(1)

= 5 + 16 + 3

= 24

16. **If the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar, then prove that the vectors $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ are also coplanar**

Solution: $\vec{a}, \vec{b}, \vec{c}$ are coplanar $\Rightarrow [\vec{a}, \vec{b}, \vec{c}] = 0$

$[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2\,[\vec{a}, \vec{b}, \vec{c}] = 0$

$\Rightarrow \vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are coplanar vectors.

17. **If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors, prove that**

$$[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$$

Solution :

$$[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} [\vec{a}, \vec{b}, \vec{c}]$$

$$= [\vec{a}, \vec{b}, \vec{c}]$$

18. **Prove that $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$**

Solution :

$$[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = (\vec{a}\times\vec{b}).[(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})]$$

$$= (\vec{a}\times\vec{b}).\{[\vec{b}, \vec{c}, \vec{a}]\vec{c} - [\vec{b}, \vec{c}, \vec{c}]\vec{a}\}$$

$$= [\vec{b}, \vec{c}, \vec{a}](\vec{a}\times\vec{b}).\vec{c}$$

$$= [\vec{a}, \vec{b}, \vec{c}][\vec{a}, \vec{b}, \vec{c}]$$

$$= [\vec{a}, \vec{b}, \vec{c}]^2$$

**Note:**

1. $\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$
2. $\vec{a}\times(\vec{b}\times\vec{c}) \neq (\vec{a}\times\vec{b})\times\vec{c}$

19. **If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$ Find**

**(i) $(\vec{a}\times\vec{b})\times\vec{c}$ (ii) $\vec{a}\times(\vec{b}\times\vec{c})$**

**Solution :**

i) $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 1 & -2 \end{vmatrix}$

$= \hat{i}(4-3) - \hat{j}(-2-6) + \hat{k}(1+4)$

$= \hat{i} + 8\hat{j} + 5\hat{k}$

$(\vec{a}\times\vec{b})\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 8 & 5 \\ 3 & 2 & 1 \end{vmatrix}$

$= \hat{i}(8-10) - \hat{j}(1-15) + \hat{k}(2-24)$

$= -2\hat{i} + 14\hat{j} - 22\hat{k}$

ii) $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{vmatrix}$

$= \hat{\imath}(1+4) - \hat{\jmath}(2+6) + \hat{k}(4-3)$

$= 5\hat{\imath} - 8\hat{\jmath} + \hat{k}$

$\vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 3 \\ 5 & -8 & 1 \end{vmatrix}$

$= \hat{\imath}(-2+24) - \hat{\jmath}(1-15) + \hat{k}(-8+10)$

$= 22\hat{\imath} + 14\hat{\jmath} + 2\hat{k}$

**20.** **For any vectors $\vec{a}$ Prove that**
$\hat{\imath} \times (\vec{a} \times \hat{\imath}) + \hat{\jmath} \times (\vec{a} \times \hat{\jmath}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$

Solution :

Let $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\hat{\imath} \times (\vec{a} \times \hat{\imath}) = (\hat{\imath}.\hat{\imath})\vec{a} - (\hat{\imath}.\vec{a})\hat{\imath} \quad = \vec{a} - x\hat{\imath}$

$\hat{\imath} \times (\vec{a} \times \hat{\imath}) = \vec{a} - x\,\hat{\imath}$ ................ (1)

$\hat{\jmath} \times (\vec{a} \times \hat{\jmath}) = \vec{a} - y\,\hat{\jmath}$ ................ (2)

$\hat{k} \times (\vec{a} \times \hat{k}) = \vec{a} - z\,\hat{k}$ ................ (3)

(1) + (2) + (3)

$\Rightarrow \hat{\imath} \times (\vec{a} \times \hat{\imath}) + \hat{\jmath} \times (\vec{a} \times \hat{\jmath}) + \hat{k} \times (\vec{a} \times \hat{k})$

$= 3\vec{a} - \vec{a} = 2\vec{a}$

**21.** **Prove that $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}] = 0$**

Solution :

$[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}]$

$= \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} [\vec{a}, \vec{b}, \vec{c}]$

$= [1(1-0) + 1(0-1) + 0][\vec{a}, \vec{b}, \vec{c}]$

$= (1-1)[\vec{a}, \vec{b}, \vec{c}]$

$= 0$

**22.** **Find the parametric form of vector equation and Cartesian equations of the straight line passing through the point $(-2, 3, 4)$ $parallel\ to$ $\frac{x-1}{-4} = \frac{y+3}{5} = \frac{8-z}{6}$**

Solution : $\vec{a} = -2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ ; $\vec{b} = -4\hat{\imath} + 5\hat{\jmath} - 6\hat{k}$

Parametric vector equation:

$$\vec{r} = \vec{a} + t\vec{b}$$

$$\vec{r} = (-2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) + t(-4\hat{\imath} + 5\hat{\jmath} - 6\hat{k})$$

Cartesian equation:

$$\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$$

$$\frac{x+2}{-4} = \frac{y-3}{5} = \frac{z-4}{-6}$$

**23.** **Find the direction cosines of the straight line passing through the points (5, 6, 7) and (7, 9, 13) also, find the parametric form of vector equation and Cartesian equations of the straight line through two given points**

Solution: $\vec{a} = 5\hat{\imath} + 6\hat{\jmath} + 7\hat{k}$

$\vec{b} = 7\hat{\imath} + 9\hat{\jmath} + 13\hat{k}$

$\vec{b} - \vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$

Direction cosines: $\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$

**Parametric vector equation:**

$$\vec{r} = \vec{a} + t(\vec{b} - \vec{a})$$

$$\therefore \vec{r} = (5\hat{\imath} + 6\hat{\jmath} + 7\hat{k}) + t(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$$

**Cartesian equation:**

$$\frac{x-5}{2} = \frac{y-6}{3} = \frac{z-7}{6}$$

**24.** **Find the acute angle between $\vec{r} = (4\hat{\imath} - \hat{\jmath}) + t(\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$, and $\vec{r} = (\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) + S(-\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$**

Solution: $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ , $\vec{d} = -\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$$\cos\theta = \frac{|\vec{b}\cdot\vec{d}|}{|\vec{b}||\vec{d}|} = \frac{|-1-4-4|}{\sqrt{1+4+4}\sqrt{1+4+4}} = \frac{|-9|}{\sqrt{9}\sqrt{9}} = \frac{9}{9} = 1$$

$$cos\theta = 1$$

$$\theta = cos^{-1}(1)$$

$$\theta = 0^o$$

# Differentials and partial Derivatives

## Important Points:

1. $L(x) = f(x_0) + f'(x_0)(x - x_0), \quad \forall\, x \in (a, b)$

(or)

$f(x + \Delta x) \approx (x) + f'(x)\Delta x$

2. Absolute error = Actual error – Approximate error
3. Relative error $= \frac{Absoluteerror}{Actualerror}$
4. Percentage error = Relative error x 100
5. Differential of $f$; $df = f'(x)\Delta x$
6. Recall, Differentiation formulae

## 2, 3 Mark questions

**1. Evaluate $df$ for $f(x) = x^2 + 3x$ , $if\ x = 3$ and $dx = 0.02$**

Solution: $df = f'(x)\, dx$

$df = (2x + 3)\, dx$

$= (2(3) + 3)\,(0.02)$

$df = 0.18$

**2. Find an approximate value at given point:**

$$f(x) = x^3 - 5x + 12, x_0 = 2$$

Solution:

$L(x) = f(x_0) + f'(x_0)(x - x_0)$

$f(x) = x^3 - 5x + 12$ $\qquad f(x_0) = f(2) = 8 - 10 + 12 = 10$

$f'(x) = 3x^2 - 5$ $\qquad f'(x_0) = f'(2) = 12 - 5 = 7$

$L(x) = 10 + 7(x - 2)$

$= 10 + 7x - 14$

$= 7x - 4$

**3. Find the differential dy, $y = (3 + \sin 2x)^{2/3}$**

Solution: $dy = \frac{2}{3}(3 + \sin 2x)^{-1/3}\,(2\cos 2x)\, dx$

$dy = \frac{4\cos 2x}{3\,(3+\sin 2x)^{1/3}}$ $\qquad [\because dy = y'\, dx]$

4. **Assume that the cross section of the artery of human is circular. A drug is given to patient to dilate his arteries of the radius of an artery is increased from 2mm to 2.1mm, how much is cross sectional area increased approximately?**

Solution:

$df = f'(x)\Delta x$

r = 2mm, dr = 2.1—2 = 0.1mm

Cross sectional area = $A = \pi r^2$

$dA = 2\pi r dr$

$= 2 \times \pi \times 2 \times 0.1$

$dA = 0.4\pi mm^2$

5. **A circular plate expands uniformly under the influence of heat. If the radius increased from 10.5cm to 10.75cm, there find an approximate change in the area and the approximate percentage changes in the area**

$r = 10.5,$ $dr = 10.75 - 10.50$

$= 0.25$

Solution

Area of circle = $A = \pi r^2$

$dA = \pi(2rdr)$

$= 2\pi(10.5)(0.25)$

$dA = 5.25\pi cm^2$

Percentage error = $\frac{dA}{A} \times 100 = \frac{5.25\pi}{\pi(10.5)^2} \times 100 = 4.762\%$

## 5 marks Questions

1. **Show that the percentage error in the nth root of a number is approximately $\frac{1}{n}$ times the percentage error in the numbers**

Proof:: $y = x^{\frac{1}{n}}$

$\log y = {}^{1}/_{n} \log x$

$\frac{dy}{y} \times 100 = \frac{1}{n}\left(\frac{dx}{x} \times 100\right)$

$\frac{dy}{y} \times 100 = \frac{1}{n}$ (percentage error in the number hence proved)

2. **The trunk of a tree has diameter 30cm. During the following year, the circumference grew 6cm. (1) Approximately, how much did the tree's diameter grow? II) What is the percentage increase in area of the tree's cross section?**

Solution: Diameter d = 30cm  Radius r = 15cm

$i.$Increase in circumference = 6

$$2\pi r_2 - 2\pi r_1 = 6$$

Approximate change in diameter $= 2(r_2 - r_1) = \frac{6}{\pi}$

$ii. 2(r_2 - r_1) = \frac{6}{\pi} \Rightarrow r_2 - r_1 = \frac{3}{\pi} \Rightarrow$ dr $= \frac{3}{\pi}$

A = $\pi r^2$ $\Rightarrow$ dA = $2\pi r dr$

$= 2\pi(15)\frac{3}{\pi}$

= 90cm$^2$

Percentage increase $= \frac{dA}{A} \times 100 = \frac{90}{\pi(15)^2} \times 100 = \frac{40}{\pi}\%$

3. **The time T taken for a complete oscillation of a simple pendulum with length $l$ is given by the equation T $=2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculate value of T corresponding to an error of 2 percent in the value of $l$.**

Solution:

d $l$ = (2%) $l = \frac{2l}{100}$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\log T = \log 2\pi + {}^1/_2 \log l - {}^1/_2 \log g$$

$$\frac{dT}{T} \times 100 = \frac{dl}{2l} \times 100$$

$$= \frac{2l}{100} \times \frac{100}{2l}$$

$$\Rightarrow \frac{dT}{T} \times 100 = 1\%$$

4. A sphere is made of ice having radius 10cm its radius decreases from 10cm to 9.8cm. Find approximately for the following:

i) Change in the volume  ii) Change in surface area:

Solution: r = 10,  dr = 9.8 − 10 = −0.2

i) Volume of the sphere : V $= \frac{4}{3}\pi r^3$

dV $= {}^4/_3\,\pi(3r^2 dr)$

$= 4\,\pi(10)^2(-0.2)$

dV = $-80\pi$ cm$^3$

ii. Surface area of the sphere:

$$S = 4\,\pi r^2$$

$$dS = 4\pi\,(2rdr)$$

$$= 8\pi(10)\,(-0.2) \Rightarrow -16\,\pi cm^2$$

**5. A right circular cylinder has radius r = 10cm and height h = 20cm. Suppose that the radius of the cylinder is increased from 10cm to 10.1cm and the height does not change. Estimate the change in the volume of the cylinder. Also calculate the relative error and percentage error.**

Solution :

i. Absolute error = Actual error – Approximate error

ii. Relative error $= \frac{Absolute\ error}{Actual\ error}$

iii. Percentage error = Relative error x 100

Given

r = 10 cm, h = 20 cm ; dr = 10.1 – 10.0 = 0.1cm

Volume of the cylinder $V = \pi r^2 h = 20\,\pi r^2$

$dV = 40\,\pi r dr$

$= 40\,\pi(10)(0.1)$

Approximation error $= 40\,\pi c^3$

Actual error $\Rightarrow V(10.1) - V(10)$

$\Rightarrow 2040.2\pi - 2000\pi$

$\Rightarrow 40.2\pi cm^3$

Absolute error = Actual error – Approximate error

$= 40.2\,\pi - 40\pi$

$= 0.2\pi$

Relative error $= \frac{0.2\pi}{40.2\,\pi} = 0.00497$

Percentage error = Relative error x 100 = 0.00497 x 100 = 0.497

**6. Find an approximate value $(123)^{2/3}$ using $L(x)$**

Solution: $Let\ x_0 = 125, \quad \Delta x = -2$

$f(x) = x^{2/3}\ , \quad f(x_0) = 25$

W.K.T. $f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\,\Delta x$

$(123)^{2/3} = 25 + \frac{2}{3x^{1/3}}(-2)$ $\qquad |\because x^{1/3} = (125)^{1/3}$

$= 25 + \frac{2}{3 \times 5} \times (-2)$ $\qquad =(5^3)^{1/3}$

$= 25 - \frac{4}{15}$ $\qquad = 5$

$= 25 - 0.2666$

∴ $(123)^{2/3} = 24.7334$

**7. $f(x) = \sqrt[3]{x}$ , $x = 27$ find an approximate value of $\sqrt[3]{27.2}$**

Solution: $x = 27$

$f(x) = \sqrt[3]{27} = 3$

$W.K.T.$ $f(x_0 + \Delta x) \approx f(x_0) + f(x_0)\,\Delta x$

$f(27.2) = 3 + \frac{1}{3x^{2/3}} \times 0.2$

$= 3 + \frac{1}{3 \times 9} \times 0.2$

$= 3 + \frac{2}{270}$

∴ $\sqrt[3]{27.3}$ $= 3 + 0.0074$

$= 3.0074$

# Applications of Integral Calculus

**Important points:**

- The curve y = f(x), Area lies above the x - axis A = $\int_a^b y\,dx$
- The curve y = f(x) Area lies below the x - axis A = $\int_a^b -y\,dx$
- The curve x = g(y), Area lies right of y - axis A= $\int_c^d x\,dy$
- The curve x = g(y), Area lies Left of y – axis A = $\int_c^d -x\,dy$
- Common area of the region bounded by the curves $y_u = f(x), y_L = g(x)$ about x-axis

  A= $\int_a^b (y_u - y_L)dx$
- Common area of the region bounded by the curves $x_u = f(y)$, $x_L = g(y)$ about y-axis

  A = $\int_c^d (x_u - x_L)dy$
- If $f(-x) = -f(x)$ then f is an odd function
- If $f(-x) = f(x)$ then f is an even function
- Gamma Integral : $\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$
- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
- $\int_0^1 x^m (1-x)^n dx = \frac{m! \times n!}{(m+n+1)!}$

## 2 and 3 marks question

❖ **Reduction formulae:**

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx = \int_0^{\frac{\pi}{2}} \cos^n x dx = \begin{cases} \frac{n-1}{n} \cdot \frac{n-3}{n-2} \ldots\ldots \frac{2}{3}.1\,; & n \text{ is odd} \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \ldots\ldots \frac{1}{2}.\frac{\pi}{2}\,; & n \text{ is even} \end{cases}$$

1. **Evaluate:** $\int_0^{\frac{\pi}{2}} \sin^{10} x dx$

Solution:

n = 10, $I_{10} = \frac{9}{10}.\frac{7}{8}.\frac{5}{6}.\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2} = \frac{63\pi}{512}$

2. **Evaluate:** $\int_0^{\frac{\pi}{2}} \cos^7 x dx$

Solution:

n = 7, $I_7 = \frac{6}{7}.\frac{4}{5}.\frac{2}{3}.1 = \frac{16}{35}$

3. **Evaluate::** $\int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^4 x\right)dx$

Solution:

$$I = \int_0^{\frac{\pi}{2}} \sin^2 x dx + \int_0^{\frac{\pi}{2}} \cos^4 x dx$$

$$= I_2 + I_4$$

$$= \frac{1}{2}.\frac{\pi}{2} + \frac{3}{4}.\frac{1}{2}.\frac{\pi}{2} = \frac{\pi}{4} + \frac{3\pi}{16} = \frac{7\pi}{16}$$

4. **Evaluate** $\int_0^{\frac{\pi}{2}} \left(\sin^2 x \times \cos^4 x\right)dx$

Solution:

$$I = \int_0^{\frac{\pi}{2}} \left(1 - \cos^2 x\right)\cos^4 x dx$$

$$= \int_0^{\frac{\pi}{2}} \cos^4 x dx - \int_0^{\frac{\pi}{2}} \cos^6 x dx$$

$$= I_4 - I_6$$

$$= \frac{3}{4}.\frac{1}{2}.\frac{\pi}{2} - \frac{5}{6}.\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2} = \frac{3\pi}{16}\left(1 - \frac{5}{6}\right) = \frac{\pi}{32}$$

**5.** **Evaluate::** $\int_0^{\frac{\pi}{2}} \begin{vmatrix} \cos^4 x & 7 \\ \sin^5 x & 3 \end{vmatrix} dx$

Solution:

$$I = \int_0^{\frac{\pi}{2}} (3\cos^4 x - 7\sin^5 x)dx$$

$$= 3\int_0^{\frac{\pi}{2}} \cos^4 x dx - 7\int_0^{\frac{\pi}{2}} \sin^5 x dx$$

$$= 3I_4 - 7I_5$$

$$= 3\left(\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}\right) - 7\left(\frac{4}{5}.\frac{2}{3}.1\right) = \frac{9\pi}{16} - \frac{56}{15}$$

**6.** **Evaluate:** $\int_0^{\frac{\pi}{4}} \sin^6(2x)dx$

| x | 0 | $\pi/4$ |
|---|---|---|
| t | 0 | $\pi/2$ |

Solution:

$$Let \ \ t = 2x, \ \frac{dt}{dx} = 2, \frac{dt}{2} = dx$$

$$I = \int_0^{\frac{\pi}{2}} \sin^6 t.\frac{dt}{2} = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin^6 t.dt = \frac{1}{2}I_6$$

$$I = \frac{1}{2}\left(\frac{5}{6} \ . \ \frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}\right) = \frac{5\pi}{64}$$

**7.** **Evaluate:** $\int_0^{\frac{\pi}{6}} \sin^5(3x)dx$

| x | 0 | $\pi/6$ |
|---|---|---|
| t | 0 | $\pi/2$ |

Solution:

$$Let \ \ t = 3x, \ \frac{dt}{dx} = 3, \frac{dt}{3} = dx$$

$$I = \int_0^{\frac{\pi}{2}} \sin^5 t.\frac{dt}{3} = \frac{1}{3}\int_0^{\frac{\pi}{2}} \sin^5 t.dt = \frac{1}{3}I_5$$

$$I = \frac{1}{3}\left(\frac{4}{5} \ . \ \frac{2}{3}.1\right) = \frac{8}{45}$$

| x | 0 | $2\pi$ |
|---|---|---|
| t | 0 | $\pi/2$ |

**8. Evaluate:** $\int_0^{2\pi} \sin^7 \frac{x}{4} dx$

Solution:

$$Let \;\; t = \frac{x}{4}, \; \frac{dt}{dx} = \frac{1}{4}, 4dt = dx$$

$$\text{I} = \int_0^{\frac{\pi}{2}} \sin^7 t(4dt) = 4\int_0^{\frac{\pi}{2}} \sin^7 t.dt = 4\, I_7$$

$$I = 4 \left[\frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3} .1\right] = \frac{64}{35}$$

❖ Gamma Integration: $\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}, \; a > 0$

**9. Evaluate:** $\int_0^{\infty} x^5 e^{-3x} dx$

Solution: n = 5, a = 3

$$\text{I} = \frac{5\,!}{3^{5+1}} = \frac{5\,!}{3^6}$$

**10. Evaluate** $\int_0^{\infty} x^3 e^{-\alpha x^2} dx =$ **32, α > 0 Find α**

| x | 0 | ∞ |
|---|---|---|
| t | 0 | ∞ |

Solution:: $\text{I} = \int_0^{\infty} x^2 e^{-\alpha x^2} (xdx) = 32$

$$t = x^2 , \frac{dt}{dx} = 2x, \frac{dt}{2} = x\,dx$$

$$\text{I} = \int_0^{\infty} t.e^{-\alpha t}.\frac{dt}{2} = 32 \Rightarrow \frac{1}{2}\int_0^{\infty} t^1 e^{-\alpha t}\, dt = 32$$

$$\int_0^{\infty} t^1 e^{-\alpha t}\, dt = 64 \;\; \text{(By Gamma Integral)}$$

$$\frac{1\,!}{\alpha^{1+1}} = 64 \Rightarrow \alpha^2 = \frac{1}{64} \Rightarrow \alpha = \frac{1}{8} \;\; (\because \alpha > 0)$$

❖ $\int_0^{1} x^m (1-x)^n dx = \frac{m\,! \times n\,!}{(m+n+1)!}$ (Where m and n – are positive integers)

**11. Evaluate:** $\int_0^{1} x^3 (1-x)^4 dx$

Solution: m= 3 , n = 4 m + n + 1 = 8

$$\text{I} = \frac{3\,! \times 4\,!}{8\,!} = \frac{6 \times 4\,!}{(8.7.6.5)\times 4\,!} = \frac{1}{8.7.5} = \frac{1}{280}$$

**12.** **Evaluate:** $\int_0^1 x^2(1-x)^3\,dx$

Solution: m = 2, n = 3, m + n + 1 = 6

$I = \frac{2!\times 3!}{6!} = \frac{2\times 6}{720} = \frac{12}{720} = \frac{1}{60}$

❖ Properties of Integral:

i. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ ii. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

**13.** **Evaluate:** $\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx$

Solution $I = \int_0^a \frac{f(x)}{f(x)+f(a-x)}dx$ ……………. (1)

By property, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^a \frac{f(a-x)}{f(a-x)+f(x)}dx$ ……………………. (2)

$x \leftrightarrow a-x$

$(1)+(2) \Rightarrow 2I = \int_0^a \frac{f(x)+f(a-x)}{f(x)+f(a-x)}dx$

$2I = \int_0^a dx = [x]_0^a = a$

$I = \frac{a}{2}$

**14.** **Evaluate:** $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$

Solution $I = \int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ ………………. (1)

By property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_2^3 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}dx$ ……………………. (2) $x \leftrightarrow 5-x$

$$(1) + (2) \Rightarrow 2I = \int_2^3 \frac{\sqrt{x}+\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\, dx$$

$$2I = \int_2^3 dx = (x)_2^{\,3} = 3 - 2 = 1$$

$$I = \frac{1}{2}$$

**15.** **Evaluate:** $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}\, dx$

Solution $I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}$

$$I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

By property, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

sin x $\leftrightarrow$ cos x

$$(1) + (2) \Rightarrow 2I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} dx$$

$$= (x)_{\pi/8}^{3\pi/8}$$

$$2I = \frac{3\pi}{8} - \frac{\pi}{8} = \frac{2\pi}{8}$$

$$2I = \frac{2\pi}{8}$$

$$I = \frac{\pi}{8}$$

❖ $\int_{-a}^{a} f(x)dx = \begin{cases} 0 & ; f \text{ is an odd function} \\ 2\int_{0}^{a} f(x)dx & ; f \text{ is an even function} \end{cases}$

**16.** **Evaluate** $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x dx$

Solution: $f(x) = sin^2 x = (\sin x)^2$

$f(-x) = [\sin(-x)]^2 = (-\sin x)^2 = sin^2 x = f(x)$

f(-x) = f(x) ∴ fis an even function

$I = 2\int_{0}^{\frac{\pi}{4}} \sin^2 x dx = 2\int_{0}^{\frac{\pi}{4}} \frac{(1-\cos 2x)}{2} dx$

$I = \left[x - \frac{\sin 2x}{2}\right]_0^{\pi/4} = \left(\frac{\pi}{4} - \frac{1}{2}\right) - 0 = \frac{\pi - 2}{4}$

**17.** **Evaluate:** $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x dx$

Solution: $f(x) = x \, cosx$ $f(-x) = (-x) \, cos\,(-x) = -xcos\, x$

$\therefore f(-x) = -f(x)$

$f(x)$ is an odd function

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x dx = 0$

**18.** **Evaluate:** $\int_{3}^{4} \frac{dx}{x^2 - 4}$

Solution: $I = \int_3^4 \frac{dx}{x^2 - 2^2}$ $\because \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right)$

$I = \left[\frac{1}{2(2)} \log\left(\frac{x-2}{x+2}\right)\right]_3^4$ a = 2

$= \frac{1}{4}\left[\log\frac{2}{6} - \log\frac{1}{5}\right]$

$= \frac{1}{4}\left[\log\left(\frac{\frac{1}{3}}{\frac{1}{5}}\right)\right]$

$= \frac{1}{4}\log\left(\frac{5}{3}\right)$

**19. Evaluate:** $\int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1+sec^2 x}\, dx$

| x | 0 | $\pi/3$ |
|---|---|---|
| t | 1 | 2 |

Solution: $t = \sec x$ , $\frac{dt}{dx} = \sec x\, tanx,$ dt = secx tan dx

$I = \int_1^2 \frac{dt}{1+t^2}$ $\because \int \frac{dx}{1+x^2} = tan^{-1}x$

$=[tan^{-1}(t)]_1^{\ 2} = tan^{-1}(2) - tan^{-1}(1)$

$= tan^{-1}(2) - \frac{\pi}{4}$

**20. Evaluate:** $\int_0^9 \frac{1}{x+\sqrt{x}}\, dx$

| x | 0 | 9 |
|---|---|---|
| t | 0 | 3 |

Solution: $\sqrt{x} = t$, $x = t^2$, $\frac{dt}{dx} = 2t, dx = 2tdt$

$I = \int_0^3 \frac{1}{t^2+t}(2t\, dt) = \int_0^3 \frac{dt}{1+t} = 2\,[\log(1+t)]_0^{\ 3}$

I = 2[log 4 – log 1] = 2 log 4 – 0 = log $4^2$ = log 16

**21. Evaluate:** $\int_1^2 \frac{x}{(x+1)(x+2)}\, dx$

Solution: $I = \int_1^2 \frac{x}{(x+1)(x+2)}\, dx$

$By\ partial\ Fractions,\ \frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} = \frac{-1}{x+1} + \frac{2}{x+2}$

$I = \int_1^2 \left(\frac{-1}{x+1} + \frac{2}{x+2}\right) dx$

$=[-\log(x+1) + 2\log(x+2)]_1^2$

$=[\log(x+2)^2 - \log(x+1)]_1^2$

$=\left[\log\frac{(x+2)^2}{(x+1)}\right]_1^2$

$=\log\left(\frac{16}{3}\right) - \log\left(\frac{9}{2}\right)$

$=\log\left(\frac{16/3}{9/2}\right)$ $=log\left(\frac{32}{27}\right)$

**22. Evaluate:** $\int_{-4}^{4} |x+3|dx$

Solution: $|x+3| = \begin{cases} -(x+3) & ; \quad x < -3 \\ x+3 & ; \quad x \geq -3 \end{cases}$

$$I = \int_{-4}^{-3} -(x+3)dx + \int_{-3}^{4} (x+3)dx$$

$$= -\left[\frac{x^2}{2} + 3x\right]_{-4}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{4}$$

$$= -\left[\left(\frac{9}{2} - 9\right) - (8 - 12)\right] + \left[(8 + 12) - \left(\frac{9}{2} - 9\right)\right]$$

$$= -\left[-\frac{9}{2} + 4\right] + \left[20 + \frac{9}{2}\right]$$

$$= \frac{9}{2} - 4 + 20 + \frac{9}{2} = 16 + 9 = 25$$

- ❖ If n is odd and m is any positive integer (even or odd)
- ❖ $\int_{0}^{\frac{\pi}{2}} \sin^m x \cos^n x \, dx = \frac{n-1}{m+n} \cdot \frac{n-3}{m+n-2} \cdot \frac{n-5}{m+n-4} \dots \dots \frac{2}{m+3} \cdot \frac{1}{m+1}$
- ❖ $\int_{0}^{\frac{\pi}{2}} \sin^m x \cos^n x \, dx = \int_{0}^{\frac{\pi}{2}} \sin^n x \cos^m x \, dx$

$$I = \frac{m-1}{n+m} \cdot \frac{m-3}{n+m-2} \cdot \frac{m-5}{n+m-2} \dots \dots \frac{2}{n+3} \cdot \frac{1}{n+1}$$

- ❖ If n or m any one odd

**23. Evaluate:** $\int_{0}^{\frac{\pi}{2}} \sin^3 x \cos^5 x \, dx$

Solution: Here m = 3, n = 5 , m + n = 8

$$I = \frac{4}{8} \cdot \frac{2}{6} \cdot \frac{1}{4} = \frac{1}{24}$$

**24. Evaluate:** $\int_{0}^{\frac{\pi}{2}} \sin^5 x \cos^4 x \, dx = \int_{0}^{\frac{\pi}{2}} \sin^4 x \cos^5 x \, dx$

Solution: $m = 5, n = 4, m + n = 9$

$$I = \frac{4}{9} \cdot \frac{2}{7} \cdot \frac{1}{5} = \frac{8}{315}$$

# PRABABILITY DISTRIBUTION

Random Variable

$X : S \to R$

Discrete Random variable X has finite (or) countably infinite values

Probability mass function

$f(x)$- P.M.F.

$f(x)$is P.M.F. then

$i. f(x_u) \geq 0, \quad k = 1, 2, \dots . m$

$ii. \sum_{k} f(x_u) = 1$

Cumulative distribution $F(X)$Function

$F(X) = p(X \leq x) = \sum_{x_i < x} f(x_i)$

$x_i < x \quad x \in R$

Continuous Random variable X has values in the Interval I is a set of R

Probability density Function

$f(x) - p.d.F.$

$f(x)$ $is\ a\ p.d.F.$

$i. f(x) \geq 0 \quad \forall x$

$ii. \int_{-\infty}^{\infty} f(x)dx = 1$

Cumulative distribution Function$F(X)$

$F(X) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt$

$-\infty < x < \infty$

# 2 Marks Questions

**1. Suppose two coins are tossed once. If X denote the number of tails**
**i) Write the sample space** **ii) Find the Inverse image of 1**

Sample space S= $\{HH,\ HT,\ TH,\ TT\}$

X – denote no of tails in 2 tosses

X = 0, 1, 2

X (HT) = 1 ⇒ H.T = $x^{-1}(1)$

X (T H) = 1 ⇒ T.H = $x^{-1}(1)$

∴ $Inverse$ image of 1 is {HT, TH}

**2. Suppose a pair of unbiased dice is rolled once. If X denote the total score of two dice write down**
**i) Sample space** **ii) Inverse image of 10**

$$S = \begin{cases} (1,1)(1,2) \ldots \ldots \ldots \ldots\ (1,6) \\ (2,1)(2,2) \ldots \ldots \ldots \ldots .. (2,6) \\ \quad .. \\ \quad .. \\ (6,1),(1,2) \ldots \ldots \ldots \ldots (6,6)\} \end{cases}$$

Let the random variable X- denote sum of the numbers shown in both the dice

X= 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

X (4, 6) = X (5, 5) = X (6, 4) = 10

∴ $X^{-1}(10) = (4,6),\quad x^{-1}(10) = (5,5)$

$X^{-1}(10) = (6,4)$

$Inverse$ image of 10 = { (4, 6), (5, 5), (6, 4)}

**3. Suppose a discrete random variable can only take the values 0, 1, 2. The probability mass function is**

$$\boldsymbol{f(x) = \begin{cases} \frac{x^2+1}{k} & , \quad x = 0,1,2 \\ 0 & , \quad Otherwise \end{cases}}$$

Find the out probability mass function

$f(0) = 0 + \frac{1}{k} = \frac{1}{k}$ , $f(1) = \frac{1+1}{K} = \frac{2}{K}$

$f(2) = \frac{2^2+1}{k} = \frac{5}{k}$

$\sum f(x_k) = 1$

$\frac{1}{k} + \frac{2}{k} + \frac{5}{k} = 1 \Rightarrow \frac{8}{k} = 1$

k = 8

P.M.F.

| x | 0 | 1 | 2 |
|---|---|---|---|
| F(x) | $\frac{1}{8}$ | $\frac{2}{8}$ | $5/8$ |

Each $p_i > 0$ $\sum p_i = 1$

**4. If X is a random variable with distribution function F(x) given by**

**Solution**:

$$F(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < 1 \\ 1, & x \ge 1 \end{cases}$$ Find the probability density function

Density function

$$f(x) = F'(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 \le x < 1 \\ 0, & x \ge 1 \end{cases}$$

# 3 Marks Questions

**5. An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of apples taken is a random variable then. Find the values of the random variable and number of points in the inverse images.**

| | |
|---|---|
| Mango | – M |
| Apple | – A |

Solution:

S = {4 apples, 5 mangoes}

Let the random variable X-denote getting no of apples in 3 draws, X = 0, 1, 2, 3

X(MMM) = 0

X (AMM) = X (MAM) = X (MMA) = 1

X (AAM) = X (AMA) = X (MAA) = 2

X (AAA) = 3

| Values of X | 0 | 1 | 2 | 3 | Total |
|---|---|---|---|---|---|
| Number of points in inverse images | 1 | 3 | 3 | 1 | 8 |

**6. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win Rs.15 for each red ball selected and we lose Rs.10. For each black ball selected X-denote the winning amount, then find the values of and number of points in its inverse images.**

S= {6 red, 8 black}

Let the Random variable x- denote the winning amount

X = -20, 05, 30

X (BB) = (-10 -10 = -20) $[8c_2 = \frac{\overset{4}{\cancel{8}} \times 7}{1 \times \cancel{2}} = 28$

X (RB) = $(15 - 10 = 5)$ $[6c_1 \text{ x } 8c_1 = 6 \text{ x } 8 = 48]$

X(RB) = X(BR)

X (RR) = 15 + 15 = 30 $[6c_2 = \frac{\overset{3}{\cancel{6}} \times 5}{1 \times \cancel{2}} = 15$

| Values of X | -20 | 5 | 30 | Total |
|---|---|---|---|---|
| Number of elements in inverse image | 28 | 48 | 15 | 91 |

**7. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred**

S= {HHH, HHT HTH, HTT, THH, THT, TTH, TTT}

(Let the random variable X denotes number of heads occurred)

X = 0, 1, 2, 3

P(s) = 1/2

P(F) = 1/2

| 4 | 2 | 1 |
|---|---|---|
| H | H | H |
| H | H | T |
| H | T | H |
| H | T | T |
| T | H | H |
| T | H | T |
| T | T | H |
| T | T | T |

P(x = 0) = P(F) x P(F) x P(F)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

P(X =1) = P(s) x P(F) x P(F) x 3

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 3 = \frac{3}{8}$

P (X = 2) = P(s) x P(s) x P(F) x 3

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 3 = \frac{3}{8}$

P(X = 3) = P(s) x P(s) x P(s) = $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

P.M.F.:

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| f(x) | 1/8 | 3/8 | 3/8 | 1/8 |

All $Pi > 0$

$\Sigma Pi = 1$

**8. The cumulative distribution function of a discrete random variable is given by**

$$F(x) = \begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \le x < 0 \\ 0.35 & 0 \le x < 1 \\ 0.60 & 1 \le x < 2 \\ 0.85 & 2 \le x < 3 \\ 1 & 3 \le x < \infty \end{cases}$$

Find the probability mass function of P(X< 1)

The value of X = -1, 0, 1, 2, 3

f(-1) = F (-1) – F $(-1) = 0.15 - 0 = 0.15$

f(0) = F(0) – F(-1) = 0.35 – 0.15 = 0.20

f(1) = F(1) – F(0) = 0.60 – 0.35 = 0.25

f(2) = F(2) – F(1) = 0.85 – 0.60 = 0.25

f(3) = F(3) – F(2) = 1- 0.85 = 0.15

**P.M.F.**

| X | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| f(x) | 0.15 | 0.20 | 0.25 | 0.25 | 0.15 |

P(X < 1) = P(X = -1) + P (x = 0)

= 0.15 + 0.20 = 0.35

**9. A random variable X has the following probability mass function**

| x | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| f(x) | $k^2$ | $2k^2$ | $3k^2$ | 2k | 3k |

Find 1) k ii) P (2≤ $x < 5$)

since f(x) is a P.M.F

**Solution:**

$$\sum_{x=1}^{n} P(X=x) = \sum f(x) = 1$$

$k^2 + 2k^2 + 3k^2 + 2k + 3k = 1$

$6k^2 + 5k - 1 = 0$ P= -6

(k + 1) $\left(k - {}^{1}/_{6}\right) = 0$ s= 5 $[\frac{6}{6}, \frac{-1}{6}]$

k + 1 = 0 | $k - {}^{1}/_{6} = 0$

k = -1 | k = ${}^{1}/_{6}$

no + possible

P (2 ≤ $X < 5$ ) $= P(x=2) + P(x=3) + P(x=4)$

$= 2k^2 + 3k^2 + 2k$

$= 5k^2$ + 2k = 5 x $\frac{1}{36} + 2 \times \frac{1}{6} = \frac{17}{36}$

**10. The probability during function of X is given by f(x) = $\begin{cases} kxe^{-2x} & , \quad x > 0 \\ 0 & , \quad x \le 0 \end{cases}$ find k**

**Since f(x) is a p.d.f.**

Solution:

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

k $\int_0^\infty x^1 e^{-2x} dx = 1$

by Gamma Integral $\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$

n = 1, a = 2

k x $\frac{1!}{2^{1+1}} = 1$

$\frac{k}{4} = 1$

$k = 4$

**11. If X is the random variable with distribution F(x) given by**

$$F(x) = \begin{cases} 0 & , \quad x < 0 \\ \frac{1}{2}(x^2 + x) & , \quad 0 \le x < 1 \\ 1 & , \quad x \ge 1 \end{cases}$$

Find i) probability density function

ii) P(0.3 ≤ $x$ ≤ 0.6)

**Solution:**

density function f(x) = $\begin{cases} \frac{1}{2}(2x+1); 0 \le x < 1 \\ 0 \text{ ; otherwise} \end{cases}$

$$f(x) = \begin{cases} 0 & , \quad x < 0 \\ \frac{1}{2}(x^2 + x) & , \quad 0 \le x < 1 \\ 1 & , \quad x \ge 1 \end{cases}$$

$P(0.3 \le x \le 0.6) = F(0.6) - F(0.3)$

$= \frac{1}{2}[((0.6)^2 + 0.6) - (0.3)^2 \times (0.3)]$

$= \frac{1}{2}$ x 0.57 = 0.285

# DISCRETE MATHEMATICS

**Important Points:**

Let * be a binary operation on S

i) Closure property : $\forall\, a, b \in S \implies a * b \in S$

ii) Commutative property : $\forall\, a, b \in S$ [a * b = b * a]

iii) Associative property : $a * (b * c) = (a * b) * c \quad \forall\, a, b, c \in S$

iv) Existence of identity : $a * e = e * a = a$ , e is the identity element

v) Existence of inverse : $a^{-1}$is the inverse of a $a * a^{-1} = a^{-1} * a = e$

**Truth Table**

| | | **V(big) ∧ (small) write the big one** | **Write the small one** | **T, F → *F*** | **V (+) , ∧(-) multiplication rule** | **Not p** | **Not q** |
|---|---|---|---|---|---|---|---|
| p | q | p v q | p ∧ q | p → q | p ↔ q | ¬ p | ¬q |
| T | T | T | T | T | T | F | F |
| T | F | T | F | F | F | F | T |
| F | T | T | F | T | F | T | F |
| F | F | F | F | T | T | T | T |

# 2, 3 Mark Questions

**1.** $p \to q \equiv (\sim p) \vee q$ **using truth table**

| p | q | p → q | ~ p | (~p)v q |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T ↳ (1) | T | T ↳ (2) |

From (1), (2) $p \to q \equiv (\sim p) \vee q$

**2.** $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ **using truth table**

| p | q | p ↔ q | p → q | q → p | (p → q)Λ(q → p) |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | F | T | F |
| F | T | F | T | F | F |
| F | F | T ↳ (1) | T | T | T ↳ (2) |

From (1) and (2) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

**3.** $p \wedge q \to p \vee q$ ***P.T. tautology***

| p | q | p Λ q | p v q | p Λ q → p vq |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | F | F | T |

$p \wedge q \to p \vee q$ is tautology

**4. Verify whether the compound statement (p Λ q) Λ ( ¬ ( p v q)) is a tautology or** contradiction

| p | q | (p Λ q) | pvq | ¬ (p v q) | (p Λ q) Λ ( ¬ ( p v q)) |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | F | F | T | F | F |
| F | T | F | T | F | F |
| F | F | F | F | T | F |

(p Λ q) ( ¬ ( p v q)) is a contradiction

**5. Using truth table check whether the statement ¬(pvq) v (¬p Λ q) and ¬ *p* or logically equivalent.**

| p | q | $\neg p$ | (pvq) | ¬ (p v q) | (¬p Λ q) | ¬(pvq) v (¬p Λ q) |
|---|---|---|---|---|---|---|
| T | T | F | T | F | F | F |
| T | F | F | T | F | F | F |
| F | T | T | T | F | T | T |
| F | F | T ↳ (1) | F | T | F | T ↳ (2) |

**From (1) & (2)** **¬(pvq) v (¬p Λ q)** ≡ $\neg p$

**6. P.T. p → (¬ *q*vr) ≡ ¬*p*v (¬qvr)**

| p | q | r | $\neg q$ | $(\neg qvr)$ | p→$(\neg qvr)$ | $\neg p$ | $\neg pv(\neg qvr)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | T | T | F | T |
| T | T | F | F | F | F | F | F |
| T | F | T | T | T | T | F | T |
| T | F | F | T | T | T | F | T |
| F | T | T | F | T | T | T | T |
| F | T | F | F | F | T | T | T |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T ↳ (1) | T | T ↳ (2) |

$p \rightarrow (\neg q\text{v r}) \equiv \neg pv(\neg qvr)$

Do it yourself : Ex: 12.2 6, 7, 9, 9, 10, 11, 13, 15

**7. Let A = $\{a+\sqrt{5}\,b\,; a, b\ \in z\}$ check whether the usual multiplication is a binary operation on A.**

Solution:

Let $a+\sqrt{5}\,b$, $c+\sqrt{5}\,d$, ∈A

$(a+\sqrt{5}\,b)(c+\sqrt{5}\,d) = ac + 5bd + \sqrt{5}\,(bc+ad)$ ∈A

Hence usual multiplication is binary on A.

**8. On Z define ⊗ by (m ⊗ n) = $m^n + n^m$ $\forall m, n \in z$ Is ⊗ binary on z?**

Solution: when n = -p where p > 0

$m^n + m^{-p} = \frac{1}{m^p}$ Now $m^n + n^m$ need not be in z

Hence ⊗ is not binary on Z.

**9. Let * be defined on R by (a * b) = a + b + ab -7.**

**Is * binary on R? If so find 3* $\left(-7/_{15}\right)$**

Solution: a + b, ab, -7 are real numbers that their sum is also a real number. Therefore * is binary

$3 * \left(-7/_{15}\right) = 3 + \left(-7/_{15}\right) + 3\left(-7/_{15}\right) - 7$

$= 3\ -7/_{15} - 21/_{15}\ -7 = -88/_{15}$

**10) A = $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ B = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ find A $\vee B$, $A \wedge B$**

Solution:

$A \vee B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A \wedge B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

**11. Let p: Jupiter is a planet and**

**q : India is an island be any two simple statements**

Give verbal sentence describing i) $\neg\, pvq$ ii) $p\wedge\neg q$ iii) $p \rightarrow \neg q$

i) $\neg\, pvq$ Jupiter is not a planet or India is an island

ii) $p\wedge\neg q$ Jupiter is a planet and India is not an island

iii) $p \rightarrow \neg q$ If jupiter is a planet then India is not an island

Do it yourself: Ex 12.2 2, 3, 4, 5

12.2 6.Construct the truth table for the following statements.

(i) $\neg p \wedge \neg q$

Solution:

No of simple statements = 2; No. of rows = $2^2 = 4$

| p | q | ¬p | ¬q | ¬p ∧ ¬q |
|---|---|---|---|---|
| T | T | F | F | F |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

Last column corresponding to $\neg p \wedge \neg q$

(ii) $\neg(p \wedge \neg q)$

Solution:

No of simple statements = 2; No. of rows = $2^2 = 4$

| p | q | ¬q | p ∧ ¬q | ¬(p ∧ ¬q) |
|---|---|---|---|---|
| T | T | F | F | T |
| T | F | T | T | F |
| F | T | F | F | T |
| F | F | T | F | T |

Last column corresponding to $\neg(p \wedge \neg q)$

(iii) $(p \vee q) \vee \neg q$

Solution: No of simple statements = 2; No. of rows = $2^2 = 4$

| p | q | ¬q | p ∨ q | (p ∨ q) ∨ ¬q |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | T | T | T |
| F | T | F | T | T |
| F | F | T | F | T |

Last column corresponding to $(\mathbf{p} \vee \mathbf{q}) \vee \neg q$

(iv) $(\neg \mathbf{p} \to \mathbf{r}) \wedge (\mathbf{p} \leftrightarrow \mathbf{q})$

Solution:

No of simple statements = 3; No. of rows $= 2^3 = 8$

| p | q | r | ¬ p | ¬p → r | p ↔ q | ¬p → r) ∧ (p ↔ q) |
|---|---|---|---|---|---|---|
| T | T | T | F | T | T | T |
| T | T | F | F | T | T | T |
| T | F | T | F | T | F | F |
| T | F | F | F | T | F | F |
| F | T | T | T | T | F | F |
| F | T | F | T | F | F | F |
| F | F | T | T | T | T | T |
| F | F | F | T | F | T | F |

Last column corresponding $(\neg p \to r) \wedge (p \leftrightarrow q)$

Exercise 12.2: (7)

Verify whether the following compound propositions are tautologies or contradictions or contingency:

(i) $(p \wedge q) \wedge \neg(p \vee q)$

Solution:

No of simple statements = 2; No. of rows $= 2^2 = 4$

| p | q | p ∧ q | p ∨ q | ¬(p ∨ q) | (p ∧ q) ∧ ¬(p ∨ q) |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | F | F | T | F | F |
| F | T | F | T | F | F |
| F | F | F | F | T | F |

Since last column contains ONLY F So it is contradiction

(ii) $((p \vee q) \wedge \neg p) \rightarrow q$

Solution:

No of simple statements $= 2$; No. of rows $= 2^2 = 4$

| p | q | p ∨ q | ¬p | (p ∨ q) ∧ ¬p | ((p ∨ q) ∧ ¬p) → q |
|---|---|---|---|---|---|
| T | T | T | F | F | T |
| T | F | T | F | F | T |
| F | T | T | T | T | T |
| F | F | F | T | F | T |

Since last column contains ONLY T so it is tautology

:(iii) $(p \rightarrow q) \leftrightarrow (\neg p \rightarrow q)$

Solution:

No of simple statements $= 2$; No. of rows $= 2^2 = 4$

| p | q | p → q | ¬p | ¬p → q | ( p→ q) ↔ (¬p→ q) |
|---|---|---|---|---|---|
| T | T | T | F | T | T |
| T | F | F | F | T | F |
| F | T | T | T | T | T |
| F | F | T | T | F | F |

Since last column contains both T and F it is contigencey

(iv) $((\mathbf{p} \rightarrow \mathbf{q}) \wedge (\mathbf{q} \rightarrow \mathbf{r})) \rightarrow (\mathbf{p} \rightarrow \mathbf{r})$

Solution:

No of simple statements = 3; No. of rows = $2^3 = 8$

| p | q | r | $p \rightarrow q$ | $q \rightarrow r$ | $(p \rightarrow q) \wedge (q \rightarrow r)$ | $p \rightarrow r$ | $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | T | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | F | T | T |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T |

Since last column contains ONLY T so it is tautology

Exercise 12.2: (8): Show that (i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Solution:No of simple statements = 2; No. of rows = $2^2 = 4$

| p | q | $p \wedge q$ | $\neg(p \wedge q)$ | $\neg p$ | $\neg q$ | $\neg p \vee \neg q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | F | T | F | T | T |
| F | T | F | T | T | F | T |
| F | F | F | T | T | T | T |

Since column corresponding to L.H.S and R.H.S are identical, Hence $\neg(p \wedge q) \equiv \neg p \vee \neg q$

(ii) $\neg(p \rightarrow q) \equiv p \wedge \neg q$

Solution:No of simple statements = 2; No. of rows = $2^2 = 4$

| p | q | p → q | ¬(p → q) | ¬q | p ∧ ¬q |
|---|---|---|---|---|---|
| T | T | T | F | F | F |
| T | F | F | T | T | T |
| F | T | T | F | F | F |
| F | F | T | F | T | F |

Since column corresponding to L.H.S and R.H.S are identical, Hence $\neg(p \rightarrow q) \equiv p \wedge \neg q$

Exercise 12.2: (9): Prove that $\mathbf{q} \rightarrow \mathbf{p} \equiv \neg p \rightarrow \neg q$
Solution:No of simple statements = 2; No. of rows = $2^2 = 4$

| p | q | q → p | ¬p | ¬q | ¬p → ¬q |
|---|---|---|---|---|---|
| T | T | T | F | F | T |
| T | F | T | F | T | T |
| F | T | F | T | F | F |
| F | F | T | T | T | T |

Since column corresponding to L.H.S and R.H.S are identical, Hence $q \rightarrow p \equiv \neg p \rightarrow \neg q$

Exercise 12.2: (10):
Show that $\mathbf{p} \rightarrow \mathbf{q}$ and $\mathbf{q} \rightarrow \mathbf{p}$ are not equivalent
Solution:
No of simple statements = 2; No. of rows = $2^2 = 4$

| p | q | p → q | q → p |
|---|---|---|---|
| T | T | T | T |
| T | F | F | T |
| F | T | T | F |
| F | F | T | T |

Since column corresponding p → q AND q → p are NOT identical. p → q and q → p are not equivalent

Exercise 12.2: (11):
Show that $\neg(\mathbf{p} \leftrightarrow \mathbf{q}) \equiv \mathbf{p} \leftrightarrow \neg\mathbf{q}$
Solution:
No of simple statements = 2; No. of rows = $2^2 = 4$

| p | q | p ↔ q | ¬(p ↔ q) | ¬q | p ↔ ¬q |
|---|---|---|---|---|---|
| T | T | T | F | F | F |
| T | F | F | T | T | T |
| F | T | F | T | F | T |
| F | F | T | F | T | F |

Since column corresponding p → q AND q → p are NOT identical
p → q and q → p are not equivalent
Exercise 12.2: (13):
Using truth table check whether the statements ¬(pVq) ∨ (¬p ∧ q) and ¬p are logically equivalent.
Solution:
No of simple statements = 2; No. of rows = $2^2 = 4$

| p | q | p ∨ q | ¬(p ∨ q | ¬p | ¬p ∧ q | ¬(p ∨ q) ∨ (¬ p ∧ q) |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | T | F | F | F | F |
| F | T | T | F | T | T | T |
| F | F | F | T | T | F | T |

Since column corresponding $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are identical
Hence $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent

*ANSWERS*

*1. Applications of Matrices and Determinants*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.b** | **2.a** | **3.d** | **4.b** | **5.c** | **6.b** | **7.c** | **8.d** | **9.b** | **10.d** |
| **11.b** | **12.a** | **13.b** | **14.d** | **15.a** | **16.d** | **17.b** | **18.d** | **19.b** | **20.d** |
| **21.d** | **22.c** | **23.a** | **24.d** | **25.d** | | | | | |

*2.Complex numbers*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.c** | **2.a** | **3.d** | **4.a** | **5.a** | **6.a** | **7.a** | **8.b** | **9.d** | **10.b** |
| **11.c** | **12.a** | **13.a** | **14.b** | **15.d** | **16.b** | **17.a** | **18.a** | **19.b** | **20.c** |
| **21.a** | **22.c** | **23.d** | **24.d** | **25.b** | | | | | |

*3. Theory of Equations*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.a** | **2.b** | **3.a** | **4.c** | **5.a** | **6.d** | **7.d** | **8.a** | **9.c** | **10.c** |

*4. Inverse Trigonometric Functions*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.a** | **2.b** | **3.c** | **4.b** | **5.a** | **6.c** | **7.a** | **8.c** | **9.c** | **10.b** |
| **11.b** | **12.d** | **13.d** | **14.d** | **15.d** | **16.c** | **17.b** | **18.b** | **19.a** | **20.c** |

*5. Two-Dimensional Analytical Geometry 2*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.a** | **2.d** | **3.c.** | **4.c** | **5.a** | **6.d** | **7.d** | **8.a** | **9.a** | **10.b** |
| **11.b** | **12.c** | **13.c** | **14.c** | **15.b** | **16.a** | **17.c** | **18.d** | **19.c** | **20.c** |
| **21.a** | **22.a** | **23.c** | **24.b** | **25.b** | | | | | |

*6.Applications of Vector Algebra*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.a** | **2.a** | **3.b** | **4.c** | **5.a** | **6.c** | **7.d** | **8.c** | **9.a** | **10.b** |
| **11.a** | **12.c** | **13.a** | **14.a** | **15.b** | **16.d** | **17.d** | **18.b** | **19.c** | **20.d** |
| **21.b** | **22.a** | **23.b** | **24.c** | **25.d** | | | | | |

*7.Applications of Differential Calculus*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.d | 2.b | 3.c | 4.c | 5.a | 6.d | 7.c | 8.a | 9.b | 10.b |
| 11.b | 12.a | 13.a | 14.d | 15.c | 16.c | 17.d | 18.b | 19.c | 20.c |

*8.Differentials and Partial Derivatives*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.a | 2.b | 3.c | 4.b | 5.b | 6.b | 7.d | 8.b | 9.a | 10.c |
| 11.b | 12.d | 13.c | 14.b | 15.d | | | | | |

*9. Applications of Integration*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.d | 2.d | 3.c | 4.a | 5.c | 6.c | 7.c | 8.c | 9.b | 10.a |
| 11.a | 12.d | 13.b | 14.d | 15.d | 16.b | 17.b | 18.d | 19.d | 20.c |

*10.Ordinary Differential Equations*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.a | 2.a | 3.b | 4.b | 5.a | 6.a | 7.b | 8.c | 9.b | 10.b |
| 11.c | 12.a | 13.b | 14.c | 15.b | 16.d | 17.b | 18.c | 19.b | 20.b |
| 21.c | 22.c | 23.c | 24.a | 25.d | | | | | |

*11. Probability Distributions*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.d | 2.b | 3.a | 4.b | 5.d | 6.b | 7.c | 8.d | 9.d | 10.d |
| 11.b | 12.d | 13.d | 14.b | 15.a | 16.a | 17.b | 18.a | 19.a | 20.d |

*12.Discrete Mathematics*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.b | 2.b | 3.c | 4.d | 5.c | 6.b | 7.c | 8.a | 9.c | 10.c |
| 11.c | 12.b | 13.b | 14.d | 15.b | 16.d | 17.a | 18.d | 19.d | 20.c |

All the best