2nd VOLUME TEST

12th Standard

Date : 30-Dec-22

Reg.No. :

Maths

Time : 00:03:00 Hrs

Total Marks : 90

20 x 1 = 20

1) (a) 2 < t < 5

2) (a) -2

3) (a) $\frac{1}{2}$

4) (d) $\frac{\log\left(\frac{a}{b}\right)}{\log\left(\frac{c}{d}\right)}$

5) (b) 0

6) (a) xy^{x-1}

7) (a) $\frac{2}{(x+1)^2}dx$

8) (a) $\frac{1}{28}$

9) (c) $\frac{\pi}{12}$

10) (c) 0

11) (a) xdy + ydx = 0

12) (b) $y = 2x\left(\frac{dy}{dx}\right)$

13) (b) log y

14) (d) $\frac{dp}{dt} = -kp$

15) (a) 1

16) (b) $F(-\infty) = -1$

17) (a) $\sigma = \sqrt{npq}$

18) (c) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

19) (d) Q- {0}

20) (c) -1

answer the seven question 10th compulsory 7x 2 = 14

21) Rolle's theorem is not valid.

22) Area of the circle

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

When r = 3cm,

$\frac{dA}{dr} = 2\pi \times 3 = 6\pi\text{cm}$

23) Area of a circular template A = πr^2

dA = 2πrdr

$\text{dA} = 2 \times \pi \times 10 \times 0.02$

$= 0.4\pi\text{cm}^2$

The possible error in calculating the template area is approximately 0.4π cm^2.

24) $\frac{x^3}{3}+c$

25) $\frac{\pi}{60}$

26) $\mathrm{I}=\mathrm{I}=\int_0^1 \frac{|x|}{x}\,\mathrm{d}x$

$=\int_0^1 1\,\mathrm{d}x$

$=[x]_0^1$

$\mathrm{I}=1$

$$f(x)=\begin{cases}\frac{x}{x}, x>0\\ \frac{-x}{x}, x\leq 0\end{cases}$$

$$=\begin{cases}+1, x>0\\ -1, x\leq 0\end{cases}$$

27) order – 2 : degree 2

28) Given n = 5 and

P(X = 2) = P(X = 3)

P(X = x) = $n\mathrm{C}_x p^x q^{n-x}$

x = 0, 1, 2,...n

Therefore (1) becomes,

$5\mathrm{C}_2 p^2 q^3=5\mathrm{C}_3 p^3 q^2$

$\Rightarrow \frac{5\times 4}{1\times 2}q=\frac{5\times 4\times 3}{3\times 2\times 1}p$

$\Rightarrow$ Since $p+q=p$

$\Rightarrow p+p=1$

$\Rightarrow 2p=1$

$\Rightarrow p=\frac{1}{2}$

29) Let (S, $*$) be an algebraic structure. Assume that the identity element of S exists in S .

It is to be proved that the identity element is unique. Suppose that e_1 and e_2 be any two identity elements of S.

First treat e_1 as the identity and e_2 as an arbitrary element of S

Then by the existence of identity property $e_2 * e_1=e_1 * e_2=e_2$ ... (1)

Interchanging the role of e1 and e2 $e_1 * e_2=e_2 * e_1=e_1$ ... (2)

From (1) and (2), $e_1 = e_2$. Hence the identity element is unique which completes the proof

30) Let e be the identity element in Q.

Then for every a € Q such that

A*e = e * a = a

Given a^* b $=\frac{ab}{8}$

Now, $a^*\mathrm{e}=a$

$\frac{ae}{8}=a$

$\mathrm{e}=8\in \mathrm{Q}$

Hence 8 is the identity element for given operation.

answer the seven question 10th compulsory 7 x 3 = 21

31) $C=\frac{3\pi}{4}$

32) $\mathrm{V}=\frac{4}{3}\pi r^3$ $\mathrm{S}=4\pi r^2$

$\frac{dV}{dr}=4\pi r^2$ $\frac{dS}{dr}=8\pi r$

$\frac{dV}{dS}=\frac{dV/dr}{dS/dr}=\frac{4\pi r^2}{8\pi r}=\frac{r}{2}$

Hence $\left(\frac{dV}{dS}\right)_{\mathrm{r}=2}=\frac{2}{2}=1$

33) Let f(x) = tan x, x_o= 45, dx = 1

f(x_o) = f(45) = tan 45 = 1

f'(x) = $\sec^2$ x dx

f'(x_o) = f'(45) = $\sec^2$ 45 (1)

$(\sqrt{2})^2$ =2 (0.01745)

= 0.03490

$\therefore \tan 46° = f(x_o) + f'(x_o)\,dx$
$= 1 + 0.03490 = 1.03490$

34) Let $y = (x)^{\frac{1}{5}}$

$$dy = \frac{1}{5}(x)^{\frac{1}{5}-1}\,dx$$
$$dy = \frac{1}{5}(x)^{\frac{-4}{5}}\,dx$$
$$x_0 = 32, \Delta x = -1$$
$$f'(x_0)\,\Delta x = dy = \frac{1}{5}(32)^{\frac{-4}{5}}(-1)$$
$$= \frac{-1}{5}\left(2^5\right)^{\frac{-4}{5}}$$
$$f'(x_0)\,\Delta_x = 0.0125$$
$$f(x+\Delta x) \approx f(x_0) + f'(x_0)\,\Delta x$$
$$\approx 2 + (-0.0125)$$
$$\sqrt[5]{31} \approx 1.9875$$

35) $\frac{3}{2}$

36) $I = \int_0^1 \log\left(\frac{1}{x} - 1\right)dx$

$$= \int_0^1 \log\left(\frac{1-x}{x}\right)dx \text{ ....(1)}$$
$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$
$$\therefore I = \int_0^1 \log\left(\frac{1-1+x}{1-x}\right)dx$$
$$= \int_0^1 \log\left(\frac{x}{1-x}\right)dx \text{ ....(2)}$$

Add (1) & (2),

$$2I = \int_0^1 \left[\log\left(\frac{1-x}{x}\right) + \log\left(\frac{x}{1-x}\right)\right]dx$$
$$= \int_0^1 \log\left(\frac{1-x}{x}\cdot\frac{x}{1-x}\right)dx = \int_0^1 \log 1\,dx$$

2I = 0

I = 0

$$\therefore \int_0^1 \log\left(\frac{1}{x} - 1\right)dx = 0$$

37) $(x-y)^2\left[1 + \left(\frac{dy}{dx}\right)^2\right] = \left(x + y\frac{dy}{dx}\right)^2$

38) Given y = A cos 2x- B sin 2x ....(1)

Here two arbitrary constant, so differentiating equation (1) two times, we get

$$\frac{dy}{dx} = A(-\sin 2x)2 - B\cos 2x(2)$$
$$= -2\,A\sin 2x - 2\,B\cos 2x$$
$$\frac{dy}{dx} = -2[\,A\sin 2x + B\cos 2x] \text{ ....(2)}$$

Again differentiating equation (2), we get

$$\frac{d^2y}{dx^2} = -2[\,A\cos 2x(2) - B\sin 2x(2)]$$
$$= -4[\,A\cos 2x - B\sin 2x]$$
$$\frac{d^2y}{dx^2} = -4y\ [\because y = A\cos 2x - B\sin 2x]$$

$\frac{d^2y}{dx^2} + 4y = 0$ is the e required differential equation.

y = A cos 2x - B sin 2x is the general solution of the differential equation $\frac{d^2y}{dx^2} + 4y = 0$

39) Given n = 10

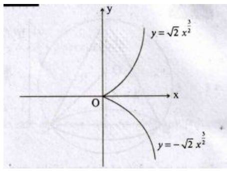
$$p = \frac{20}{100} = \frac{1}{5}$$
$$\therefore q = 1 - \frac{1}{5} = \frac{4}{5}$$
$$\therefore P(X = x) = nC_x p^r q^{n-x}$$
$$x = 0, 1, 2, \ldots n$$
$$\therefore P(X = 2) = 10C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^n$$
$$= \frac{10\times 9}{2\times 1}\left(\frac{48}{5^{10}}\right)$$
$$P(X = 2) = 45\left(\frac{48}{5^{10}}\right)$$

40) gf

answer the six question 11th compulsory 7x 5 = 35

41) $\frac{ds}{dt} = 60cm^2/sec\frac{dt}{dr} = \frac{1}{4\pi}cm/sec$

42)



1. When x > 0, y is well defined. As $x \to \pm\infty$ we find that $y \to \pm\infty$
2. For x < 0, y is imaginary and so the curve exists in the first and fourth quadrants only
3. When x = 0, y = 0 and when y = 0, x = 0 clearly the curve passes through the origin
4. The curve is symmetrical about x-axis only
5. As $x \to \infty, y \to \pm\infty$ and vice versa. Hence the curve does not admit any asymptote.

$\frac{dy}{dx} = \pm\frac{3}{2}\sqrt{x}$

For x $> 0, \frac{dy}{dx} > 0$ and $\frac{dy}{dx} < 0$ due to symmetry.

Hence the branch $y = \sqrt{2}x^{\frac{3}{2}}$ of the curve is increasing and the branch $y = \sqrt{2}x^{\frac{3}{2}}$ decreasing and they do not intersect the axes at points other than the origin.

The point (0, 0) is not a point of inflection.

43) 2.0116 approx.

44) $\mathrm{u} = \sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$

u is not a homogeneous

$f = \sec u = \frac{x^3-y^3}{x+y}$

$f(x,y) = \frac{x^3-y^3}{x+y}$

$f(tx,ty) = \frac{t^3x^3-t^3y^3}{tx+ty}$

$= \frac{t^3(x^3-y^3)}{t(x+y)}$

$f(tx,ty) = t^2 f(x,y)$

f is homogeneous of degree 2

$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = pf$

$x\frac{\partial(\sec u)}{\partial x} + y\frac{\partial(\sec u)}{\partial y} = 2\sec u$

$\sec u \tan \mathrm{u}\left(x\frac{\partial u}{\partial x}\right) + \sec u \tan x\left(y\frac{\partial u}{\partial x}\right) = 2\sec u$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{2\sec u}{\sec u\tan u}$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$

Hence proved

45) Given x = t^2, y = t - $\frac{t^3}{3}$

Point of intersection of the curve with x-axis is obtained by putting y = 0.

∴ y = 0

⇒ $t - \frac{t^3}{3} = 0$

⇒ t = 0 or ± $\sqrt{3}$

∴ The limit is from t = 0 to t = $\sqrt{3}$

∴ Volume = $\pi\int_0^{\sqrt{3}} y^2dx = \pi\int_0^{\sqrt{3}}\left(t-\frac{t^3}{3}\right)^2(2t\,dt)$

[∵ x = t^2 ⇒ dx = 2t dt]

$= 2\pi\int_0^{\sqrt{3}}\left(\frac{3t-t^3}{3}\right)^2 t\,dt$

$= \frac{2\pi}{9}\int_0^{\sqrt{3}}(9t^2-6t^4+t^6)t\,dt$

$= \frac{2\pi}{9}\int_0^{\sqrt{3}}(9t^3-6t^5+t^7)dt$

$= \frac{2\pi}{9}\left[\frac{9t^4}{4} - \frac{6t^6}{6} + \frac{t^8}{8}\right]_0^{\sqrt{3}}$

$= \frac{2\pi}{9}\left[\frac{81}{4} - 27 + \frac{81}{8} - 0\right]$

Volume $= \frac{2\pi}{9}\left(\frac{27}{8}\right) = \frac{3\pi}{4}$ cubic units

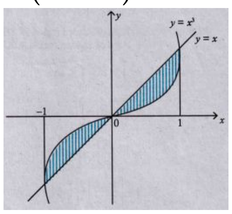
46) The line y = x lies above the curve $y = x^3$ in the first quadrant $y = x^3$ lies above the line y = x in the third quadrant.

To get the point of intersection solve the curves $y = x^3$, y = x

$\mathbf{x}^3 = \mathrm{x}$

$\mathrm{x}^3 - \mathrm{x} = 0$

$\mathbf{x}\left(\mathrm{x}^2 - \mathrm{x}\right) = 0 \quad \Rightarrow \quad \mathrm{x} = 0, \mathrm{x} = \pm 1$



The required Area

$= \int_{-1}^{0}[g(x) - f(x)]dx + \int_0^1[f(x) - g(x)]dx$

$= \int_{-1}^{0}\left(x^3 - x\right)dx + \int_0^1\left(x - x^3\right)dx$

$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$

$= 0 - \frac{1}{4} - \frac{1}{2} + \frac{1}{2} - 0 - \frac{1}{4}$

$= -\frac{1}{4} + \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{1}{2}$

47) Let P_o be the initial population and the population after t year be P.

Given $\frac{dp}{dt} = \frac{2p}{100} \Rightarrow \frac{dp}{dt} = \frac{p}{50}$

$\Rightarrow \frac{dp}{p} = \frac{dt}{50} \Rightarrow \int \frac{dp}{p} = \int \frac{dt}{50}$

$\Rightarrow$ log p = $\frac{t}{50}$ + c ...(1)

when t = 0, p = 0

$\Rightarrow$ log p_0 = 0+c $\Rightarrow$ log p_0 ....(1)

$\therefore$ (1) becomes, log p = $\frac{t}{50}$+log P_0.

$\Rightarrow \log\left(\frac{P}{p_0}\right) = \frac{t}{50}$

$\Rightarrow$ t = 50 $\log\left(\frac{P}{p_0}\right)$

when p = $2p_0$, t = 50 $\log\left(\frac{2P_0}{p_0}\right)$ = 50 log 2

= 50(0.3) = 15 years.

Hence the population doubles in 15 years.

48) Let x denote the number of bacteria at time t hours

Given $\frac{dx}{dt} = \mathrm{k}x$

($\because$ x varies with respect to time t, $\frac{dx}{dt}$)

The equation can be written as,

$\frac{dx}{x} = \mathrm{k} \cdot \mathrm{dt}$ ....(1)

Integrating of (1) on both sides, we get

$\int \frac{dx}{x} = \int kdt$

$\log x = kt + \log c$

$\log x - \log \mathrm{c} = \mathrm{kt}$

$\log \frac{x}{c} = \mathrm{kt}$

$\frac{x}{c} = \mathrm{e}^{\mathrm{kt}}$

$x = \mathrm{ce}^{\mathrm{kt}}$

Initial condition:

At time t = 4, x = $2x_o$

$\therefore (2) \Rightarrow 2x_0 = 0e^{kt}$

$2 = e^{4k}$

$e^{4k} = 2$

when t = 12,

$x = x_0 \mathrm{e}^{12\mathrm{k}}$

$= x_0 e^{(4k)^3}$

$= x_0 2^3$

$x = x_0(8) = 8x_0$

$\therefore$ At the end of 12 hours, there are 8 times the original number.

49) Since f(x) is a p.d.f $\int_{-\infty}^{\infty} f(x)dx = 1$

$\int_{-\infty}^{\infty} ke^{-|x|}dx = 1$

$k\int_{-\infty}^{\infty} e^{-|x|}dx = 1$

$2k\int_{0}^{\infty} e^{-|x|}dx = 1$

[$\because e^{-|x|}$ is an even function]

$2k\int_{0}^{\infty} e^{-x}dx = 1$

$2k\left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1$

$-2\mathrm{k}\left(\mathrm{e}^{-\infty} - \mathrm{e}^0\right) = 1\mathrm{k} = \frac{1}{2}$

Mean :

$\mathrm{E(X)} = \int_{-\infty}^{\infty} xf(x)dx$

$= \int_{-\infty}^{\infty} xke^{-|x|}dx$

$= \frac{1}{2}\int_{-\infty}^{\infty} xe^{-|x|}dx$

= 0

[$\because xe^{-|x}$ is an odd function]

Variance :

$E\left(X^2\right) = \int_{-\infty}^{\infty} x^2 f(x)dx$

$= \int_{-\infty}^{\infty} x^2 ke^{-|x|}dx$

$= k\int_{-\infty}^{\infty} x^2 e^{-|x|}dx$

$= \frac{1}{2}\times 2\int_0^{\infty} x^2 e^{-x}dx$

[$\because xe^{-|x}$ is an even function]

$= \int_0^{\infty} x^2 e^{-x}dx$

$\because \int_0^{\infty} x^n e^{-\alpha x}dx = \frac{n!}{\alpha^{n+1}}$

$\mathrm{V}(\mathbf{X}) = E\left(X^2\right) - [E(X)]^2$

= 2 - 0

= 2

50) 51) (i) We have $(a, b) * (c, d) = (a + c, b + d)$

Let (a, b), (c, d), (e, f) $\in$ A. Then

[(a, b) * (c. d)] * (e, f) = (a + c, b + d) * (e, f)

= ((a + c) + e, (b + d) + f)

= (a +(c +e), b+ (d+f)

= (a, b) * (c+e, d+f)

= (a, b) * [(c, d) * (e, f)]

(A, *) is associative.

(ii) Let (a, b), (c, d) $\in$ A. Then

(a, b) * (c, d)

= (a + c, b+d)

=(c, d) * (a, b)

(A, *) is commutative.

(iii) Let (x, y) be the identity element in A. Then, for every (a, b) $\in$ A.

(a, b) * (x, y) = (a, b)

(a + x, b + y) = (a, b)

a + x = a, b + y = b

which gives x = 0, y = 0

But (0, 0) € N$\times$ N

Hence, identity element does not exist.