20. $i + i^{22} + i^{23} + i^{24} + i^{25} = i + i^2 + i^3 + i^4 + i^5$ $\quad \because [i^4 = 1$

$= i - 1 - i + 1 + i$ $\boxed{= i}$

21. $\overline{i^{13} + i^{14} + i^{15} + i^{16}} = + \overline{i - 1 - i + 1} = 0$

23. $x^2 -$ (sum of the roots) $x +$ (product of the roots) $= 0$

$x^2 - 0x + 7 = 0$

$\boxed{x^2 + 7 = 0}$

$\because \left[\begin{array}{c|c} +i\sqrt{7} & (i\sqrt{7}).(-i\sqrt{7}) = -i^2.7 \\ -i\sqrt{7} & = +7 \\ 0 & \end{array}\right.$

24. $x^2 - (8)x + (25) = 0$

$\boxed{x^2 - 8x + 25 = 0}$

$\left[\begin{array}{l} 4 - 3i \\ 4 + 3i \\ 8 \end{array}\right.$ $\quad (4 - 3i)(4 + 3i) = 16 + 9 = 25$

25. $\dfrac{1-i}{1+i} = \dfrac{(1-i)(1-i)}{(1+i)(1-i)} = \dfrac{-2i}{2} = -i$ (root)

$= i$ (another root)

$(a, b) = (1, 0)$

$\therefore$ equation

$x^2 - 0x + 1 = 0$

$ax^2 + bx + 1 = 0$

$\boxed{a = 1}$ $\boxed{b = 0}$

26. $(3 - i).(3 + i) = 9 + 1$

$= 10 = k$

27. $(1 - \omega + \omega^2)^4 + (1 + \omega - \omega^2)^4 = (-\omega - \omega)^4 + (-\omega^2 - \omega^2)^4$

$= (-2\omega)^4 + (-2\omega^2)^4$

$= 16\omega^4 + 16\omega^8$

$= 16(\omega + \omega^2)$

$\boxed{= -16}$

$\because \left[\begin{array}{l|l} 1 + \omega + \omega^2 = 0 & \omega^3 = 1 \\ 1 + \omega = -\omega^2 & \\ 1 + \omega^2 = -\omega & \\ \omega + \omega^2 = -1 & \end{array}\right.$

29. $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8) = (1 - \omega)(1 - \omega^2)(1 - \omega)(1 - \omega^2)$

$= (1 - \omega)^2 (1 - \omega^2)^2 = [(1 - \omega)(1 - \omega^2)]^2$

$= [2 - \omega - \omega^2]^2 = [2 - (\omega + \omega^2)]^2$

$= [2 - (-1)]^2 = \boxed{9}$

30. $\sqrt{-35} = i\sqrt{35}$ $\quad \because [\sqrt{-1} = i$

31. $3 \;\; \sqrt{-7} = 3 - i\sqrt{7}$

33. $0 + \dfrac{3}{2}i$ $\quad \left(0, \dfrac{3}{2}\right)$

36. $Z = \sqrt{5} + 0i$

$\overline{Z} = \overline{\sqrt{5} + 0i} = \sqrt{5} - 0i = \sqrt{5}$

37. $3 + 2i - 7 - i = -4 + i$

38. $a + ib = 8 - 6i - 2i + 7$

$a + ib = 15 - 8i$

$\boxed{a = 15}$; $\boxed{b = -8}$

39. $p + iq = (2 - 3i)(4 + 2i)$

$= (8 + 4i - 12i + 6)$

$p + iq = 14 - 8i$

$\boxed{q = -8}$

40. $Z = (2 + i)(3 - 2i)$

$= 6 - 4i + 3i + 2$

$Z = 8 - i$

$\overline{Z} = \overline{8 - i}$

$\overline{Z} = 8 + i$

41. $(2 + i)(3 - 2i) = 6 - 4i + 3i + 2$

$= 8 - i \quad (8, -1)$

42. $|-2 + 2i| = \sqrt{4 + 4} = 2\sqrt{2}$

$|2 - 3i| = \sqrt{4 + 9} = \sqrt{13}$

43. $|-3 + 2i| = \sqrt{9 + 4} = \sqrt{13}$

$|4 + 3i| = \sqrt{16 + 9} = 5$

44. $1, \omega, \omega^2$ They are in G.P.

45. $\left(\omega = \cos\dfrac{2\pi}{n} + i\sin\dfrac{2\pi}{n}\right)$

48. $\overline{\overline{Z}} + \overline{Z} = Z + \overline{Z} = 2\,\text{Re}(Z)$ $\quad \because [\overline{\overline{Z}} = Z$

51. AP = PB

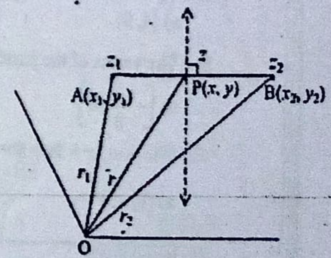
$\sqrt{(x - x_1)^2 + (y - y_1)^2} = \sqrt{(x - x_2)^2 + (y - y_2)^2}$

$(x - x_1)^2 + (y - y_1)^2 = (x - x_2)^2 + (y - y_2)^2$

$|z - z_1| = |z - z_2|$

$\therefore$ If AP = PB then the locus of z is a perpendicular bisector of the line joining z_1 and z_2.

where $\left[\begin{array}{l} z_1 = x_1 + iy_1 \\ z_2 = x_2 + iy_2 \\ z = x + iy \end{array}\right.$



53. Note : i) For an arbitrary $z \neq 0$ the principal value of arg z is $-\pi < \arg z \leq \pi$. $(-\pi, \pi]$

ii) For $z = 0$ the arg z is indeterminate.

59. $e^{i\theta} + e^{-i\theta} = \cos\theta + i\sin\theta + \cos\theta - i\sin\theta = 2\cos\theta$

60. $e^{i\theta} - e^{-i\theta} = (\cos\theta + i\sin\theta) - (\cos\theta - i\sin\theta) = 2i\sin\theta$

62. $z_1 - z_2 = a + ib + a - ib = 2a$ (in real axis)

4. Analytical Geometry

Blue Print	Part - I 1 Mark		Part - II 6Marks		Part - III 10 Marks		Total Marks
	No. of Qns.	Marks	No. of Qns.	Marks	No. of Qns.	Marks	
	4	4	1	6	3	30	40

Choose the correct answer (Multiple Choice Questions)

I. Book Questions

1. The axis of the parabola $y^2 - 2y + 8x - 23 = 0$ is *(Sep '08, Mar '11, Sep '12 & Jun '13)*
   a) $y = -1$ b) $x = -3$ c) $x = 3$ d) $y = 1$
2. $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ represents *(Jun '08 & Sep '10)*
   a) an ellipse b) a circle c) a parabola d) a hyperbola
3. The line $4x + 2y = c$ is a tangent to the parabola $y^2 = 16x$ then c is *(Jun '09, Mar '10, Mar '11, Sep '13 & Mar '15)*
   a) -1 b) -2 c) 4 d) -4
4. The point of intersection of the tangents at $t_1 = t$ and $t_2 = 3t$ to the parabola $y^2 = 8x$ is *(Mar '08, Sep '09 & June '14)*
   a) $(6t^2, 8t)$ b) $(8t, 6t^2)$ c) $(t^2, 4t)$ d) $(4t, t^2)$
5. The length of the latus rectum of the parabola $y^2 - 4x + 4y + 8 = 0$ is *(Mar '07, Mar '09, Mar '14 & Jun '15)*
   a) 8 b) 6 c) 4 d) 2
6. The directrix of the parabola $y^2 = x + 4$ is *(Jun '10)*
   a) $x = \frac{15}{4}$ b) $x = -\frac{15}{4}$ c) $x = -\frac{17}{4}$ d) $x = \frac{17}{4}$
7. The length of the latus rectum of the parabola whose vertex is $(2, -3)$ and the directrix $x = 4$ is *(Sep '07 & Mar '16)*
   a) 2 b) 4 c) 6 d) 8
8. The focus of the parabola $x^2 = 16y$ is *(Sep '15)*
   a) $(4, 0)$ b) $(0, 4)$ c) $(-4, 0)$ d) $(0, -4)$
9. The vertex of the parabola $x^2 = 8y - 1$ is *(March '12)*
   a) $\left(-\frac{1}{8}, 0\right)$ b) $\left(\frac{1}{8}, 0\right)$ c) $\left(0, \frac{1}{8}\right)$ d) $\left(0, -\frac{1}{8}\right)$
10. The line $2x + 3y + 9 = 0$ touches the parabola $y^2 = 8x$ at the point *(March '06 & March '13)*
    a) $(0, -3)$ b) $(2, 4)$ c) $\left(-6, \frac{9}{2}\right)$ d) $\left(\frac{9}{2}, -6\right)$

11. The tangents at the end of any focal chord to the parabola $y^2 = 12x$ intersect on the line *(Jun '07 & Sep '11)*
    a) $x - 3 = 0$ b) $x + 3 = 0$ c) $y + 3 = 0$ d) $y - 3 = 0$
12. The angle between the two tangents drawn from the point $(-4, 4)$ to $y^2 = 16x$ is *(June '08)*
    a) 45° b) 30° c) 60° d) 90°
13. The eccentricity of the conic $9x^2 + 5y^2 - 54x - 40y + 116 = 0$ is *(Mar '07 & Sep '11)*
    a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{4}{9}$ d) $\frac{2}{\sqrt{5}}$
14. The length of the semi-major and the length of the semi-minor axis of the ellipse $\frac{x^2}{144} + \frac{y^2}{169} = 1$ are *(Jun '10 & Mar '15)*
    a) 26, 12 b) 13, 24 c) 12, 26 d) 13, 12
15. The distance between the foci of the ellipse $9x^2 + 5y^2 = 180$ is *(Jun '09, Jun '12, Mar '13, Jun '13, Sep '13 & Mar '14)*
    a) 4 b) 6 c) 8 d) 2
16. If the length of major and semi-minor axes of an ellipse are 8, 2 and their corresponding equations are $y - 6 = 0$ and $x + 4 = 0$ then the equations of the ellipse is
    a) $\frac{(x+4)^2}{4} + \frac{(y-6)^2}{16} = 1$ b) $\frac{(x+4)^2}{16} + \frac{(y-6)^2}{4} = 1$
    c) $\frac{(x+4)^2}{16} - \frac{(y-6)^2}{4} = 1$ d) $\frac{(x+4)^2}{4} - \frac{(y-6)^2}{16} = 1$
17. The straight line $2x - y + c = 0$ is a tangent to the ellipse $4x^2 + 8y^2 = 32$ if c is *(Sep '08, Sep '10, Mar '12 & Sep '12)*
    a) $\pm 2\sqrt{3}$ b) ± 6 c) 36 d) ± 4
18. The sum of the distance of any point on the ellipse $4x^2 + 9y^2 = 36$ from $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$ is *(Mar '06 & Sep '09)*
    a) 4 b) 8 c) 6 d) 18
19. The radius of the director circle of the conic $9x^2 + 16y^2 = 144$ is *(Sep '06, Jun '08, Mar '11, Sep '14 & Mar '16)*
    a) $\sqrt{7}$ b) 4 c) 3 d) 5
20. The locus of foot of perpendicular from the focus to a tangent of the curve $16x^2 + 25y^2 = 400$ is *(March '09)*
    a) $x^2 + y^2 = 4$ b) $x^2 + y^2 = 25$ c) $x^2 + y^2 = 16$ d) $x^2 + y^2 = 9$
21. The eccentricity of the hyperbola $12y^2 - 4x^2 - 24x + 48y - 127 = 0$ is *(Sep '09, Mar '10 & Sep '15)*
    a) 4 b) 3 c) 2 d) 6
22. The eccentricity of the hyperbola whose latus rectum is equal to half of its conjugate axis is *(Jun '06, Jun '07 & Mar '15)*
    a) $\frac{\sqrt{3}}{2}$ b) $\frac{5}{3}$ c) $\frac{3}{2}$ d) $\frac{\sqrt{5}}{2}$

23. The difference between the focal distances of any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is 24 and the eccentricity is 2. Then the equation of the hyperbola is *(Mar '07 & Jun '11)*

a) $\frac{x^2}{144}-\frac{y^2}{432}=1$ b) $\frac{x^2}{432}-\frac{y^2}{144}=1$ c) $\frac{x^2}{12}-\frac{y^2}{12\sqrt{3}}=1$ d) $\frac{x^2}{12\sqrt{3}}-\frac{y^2}{12}=1$

24. The directrix of the hyperbola $x^2-4(y-3)^2=16$ is *(Mar '06, Jun '09, Sep '14 & Mar '16)*

a) $y=\pm\frac{8}{\sqrt{5}}$ b) $x=\pm\frac{8}{\sqrt{5}}$ c) $y=\pm\frac{\sqrt{5}}{8}$ d) $x=\pm\frac{\sqrt{5}}{8}$

25. The line $5x-2y+4k=0$ is a tangent to $4x^2-y^2=36$ then k is *(Sep '06 & Jun '15)*

a) $\frac{4}{9}$ b) $\frac{2}{3}$ c) $\frac{9}{4}$ d) $\frac{81}{16}$

26. The equation of the chord of contact of tangents from (2, 1) to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is *(Jun '13)*

a) $9x-8y-72=0$ b) $9x+8y+72=0$ c) $8x-9y-72=0$ d) $8x+9y+72=0$

27. The angle between the asymptotes to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is *(March '08, Mar '12 & Sep '13)*

a) $\pi-2\tan^{-1}\left(\frac{3}{4}\right)$ b) $\pi-2\tan^{-1}\left(\frac{4}{3}\right)$ c) $2\tan^{-1}\left(\frac{3}{4}\right)$ d) $2\tan^{-1}\left(\frac{4}{3}\right)$

28. The asymptotes of the hyperbola $36y^2-25x^2+900=0$ are

a) $y=\pm\frac{6}{5}x$ b) $y=\pm\frac{5}{6}x$ c) $y=\pm\frac{36}{25}x$ d) $y=\pm\frac{25}{36}x$

29. The product of the perpendiculars drawn from the point (8, 0) on the hyperbola to its asymptotes is $\frac{x^2}{64}-\frac{y^2}{36}=1$ is *(Jun '10 & June '14)*

a) $\frac{25}{576}$ b) $\frac{576}{25}$ c) $\frac{6}{25}$ d) $\frac{25}{6}$

30. The locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is *(Sep '08 & Jun '12)*

a) $x^2+y^2=25$ b) $x^2+y^2=4$ c) $x^2+y^2=3$ d) $x^2+y^2=7$

31. The eccentricity of the hyperbola with asymptotes $x+2y-5=0$, $2x-y+5=0$ is *(Sep '06 & Sep '07)*

a) 3 b) $\sqrt{2}$ c) $\sqrt{3}$ d) 2

32. Length of the semi-transverse axis of the rectangular hyperbola $xy=8$ is *(Sep '10, Sep '12 & Mar '14)*

a) 2 b) 4 c) 16 d) 8

33. The asymptotes of the rectangular hyperbola $xy=c^2$ are *(March '13)*

a) $x=c, y=c$ b) $x=0, y=c$ c) $x=c, y=0$ d) $x=0, y=0$

34. The co-ordinate of the vertices of the rectangular hyperbola $xy=16$ are *(Mar '07 & Sep '11)*

a) (4, 4), (–4, –4) b) (2, 8), (–2, –8) c) (4, 0), (–4, 0) d) (8, 0), (–8, 0)

35. One of the foci of the rectangular hyperbola $xy=18$ is *(March '09)*

a) (6, 6) b) (3, 3) c) (4, 4) d) (5, 5)

36. The length of the latus rectum of the rectangular hyperbola $xy=32$ is *(Mar '08, Mar '10, Jun '11, Sep '14 & Sep '15)*

a) $8\sqrt{2}$ b) 32 c) 8 d) 16

37. The area of the triangle formed by the tangent at any point on the rectangular hyperbola $xy=72$ and its asymptotes is *(Mar '11 & Jun '15)*

a) 36 b) 18 c) 72 d) 144

38. The normal to the rectangular hyperbola $xy=9$ at $\left(6,\frac{3}{2}\right)$ meets the curve again at *(Jun '12 & June '14)*

a) $\left(\frac{3}{8},24\right)$ b) $\left(-24,\frac{-3}{8}\right)$ c) $\left(\frac{-3}{8},-24\right)$ d) $\left(24,\frac{3}{8}\right)$

II. COME Book Questions (PTA Questions)

39. The axis of the parabola $y^2=4x$ is

a) $x=0$ b) $y=0$ c) $x=1$ d) $y=1$

40. The vertex of the parabola $y^2=4x$ is

a) (1, 0) b) (0, 1) c) (0, 0) d) (0, –1)

41. The focus of the parabola $y^2=4x$ is

a) (0, 1) b) (1, 1) c) (0, 0) d) (1, 0)

42. The directrix of the parabola $y^2=4x$ is

a) $y=-1$ b) $x=-1$ c) $y=1$ d) $x=1$

43. The equation of the latus rectum of $y^2=4x$ is

a) $x=1$ b) $y=1$ c) $x=4$ d) $y=-1$

44. The length of the L.R. of $y^2=4x$ is

a) 2 b) 3 c) 1 d) 4

45. The axis of the parabola $x^2=-4y$ is

a) $y=1$ b) $x=0$ c) $y=0$ d) $x=1$

46. The vertex of the parabola $x^2=-4y$ is

a) (0,1) b) (0, –1) c) (1, 0) d. (0, 0)

47. The focus of the parabola $x^2=-4y$ is

a) (0, 0) b) (0, –1) c) (0,1) d) (1, 0)

48. The directrix of the parabola $x^2 = -4y$ is

a) $x = 1$ b) $x = 0$ c) $y = 1$ d) $y = 0$

49. The equation of the L.R. of $x^2 = -4y$ is

a) $x = -1$ b) $y = -1$ c) $x = 1$ d) $y = 1$

50. The length L.R. of $x^2 = -4y$ is

a) 1 b) 2 c) 3 d) 4

51. The axis of the parabola $y^2 = -8x$ is

a) $x = 0$ b) $x = 2$ c) $y = 2$ d) $y = 0$

52. The vertex of the parabola $y^2 = -8x$ is

a) (0, 0) b) (2, 0) c) (0, –2) d) (2, –2)

53. The focus of the parabola $y^2 = -8x$ is

a) (0, –2) b) (0,2) c) (–2, 0) d) (2, 0)

54. The equation of the directrix of the parabola $y^2 = -8x$ is

a) $y + 2 = 0$ b) $x - 2 = 0$ c) $y - 2 = 0$ d) $x + 2 = 0$

55. The equation of the latus rectum of $y^2 = -8x$ is

a) $y - 2 = 0$ b) $y + 2 = 0$ c) $x - 2 = 0$ d) $x + 2 = 0$

56. The length of the latus rectum of $y^2 = -8x$ is

a) 8 b) 6 c) 4 d) –8

57. The axis of the parabola $x^2 = 20y$ is

a) $y = 5$ b) $x = 5$ c) $x = 0$ d) $y = 0$

58. The vertex of the parabola $x^2 = 20y$ is

a) (0, 5) b) (0, 0) c) (5, 0) d) (0, –5)

59. The focus of the parabola $x^2 = 20y$ is

a) (0, 0) b) (5, 0) c) (0, 5) d) (–5, 0)

60. The equation of the directrix of the parabola $x^2 = 20y$ is

a) $y - 5 = 0$ b) $x + 5 = 0$ c) $x - 5 = 0$ d) $y + 5 = 0$

61. The equation of the latus rectum of the parabola $x^2 = 20y$ is

a) $x - 5 = 0$ b) $y - 5 = 0$ c) $y + 5 = 0$ d) $x + 5 = 0$

62. The length of the latus rectum of the parabola $x^2 = 20y$ is

a) 20 b) 10 c) 5 d) 4

63. If the centre of the ellipse is (2,3) one of the foci is (3, 3) then the other focus is

a) (1,3) b) (–1, 3) c) (1, –3) d) (–1, –3)

64. The equation of the major and minor axes of $\frac{x^2}{9} + \frac{y^2}{4} = 1$ respectively are (June '06)

a) $x = 3, y = 2$ b) $x = -3, y = -2$ c) $x = 0, y = 0$ d) $y = 0, x = 0$

65. The equation of the major and minor axes of $4x^2 + 3y^2 = 12$ are (Sep '08)

a) $x = \sqrt{3}, y = 2$ b) $x = 0, y = 0$ c) $x = -\sqrt{3}, y = -2$ d) $y = 0, x = 0$

66. The lengths of minor and major axes of $\frac{x^2}{9} + \frac{y^2}{4} = 1$ are

a) 6, 4 b) 3, 2 c) 4, 6 d) 2, 3

67. The lengths of major and minor axes of $4x^2 + 3y^2 = 12$ are (June '07)

a) $4, 2\sqrt{3}$ b) $2, \sqrt{3}$ c) $2\sqrt{3}, 4$ d) $\sqrt{3}, 2$

68. The equation of the directrices of $\frac{x^2}{16} + \frac{y^2}{9} = 1$ are (Sep '15)

a) $y = \pm\frac{4}{\sqrt{7}}$ b) $x = \pm\frac{16}{\sqrt{7}}$ c) $x = \pm\frac{16}{7}$ d) $y = \pm\frac{16}{\sqrt{7}}$

69. The equation of the directrices of $25x^2 + 9y^2 = 225$ are.

a) $x = \pm\frac{4}{25}$ b) $x = \pm\frac{25}{4}$ c) $y = \pm\frac{4}{25}$ d) $y = \pm\frac{25}{4}$

70. The equation of the latus rectum of $\frac{x^2}{16} + \frac{y^2}{9} = 1$ are (Mar '10)

a) $y = \pm\sqrt{7}$ b) $x = \pm\sqrt{7}$ c) $x = \pm 7$ d) $y = \pm 7$

71. The equation of the L.Rs of $25x^2 + 9y^2 = 225$ are

a) $y = \pm 5$ b) $x = \pm 5$ c) $y = \pm 4$ d) $x = \pm 4$

72. The length of the L.R.of $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is

a) $\frac{9}{2}$ b) $\frac{2}{9}$ c) $\frac{9}{16}$ d) $\frac{16}{9}$

73. The length of the L.R. of $25x^2 + 9y^2 = 225$ is

a) $\frac{9}{5}$ b) $\frac{18}{5}$ c) $\frac{25}{9}$ d) $\frac{5}{18}$

74. The eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is

a) $\frac{1}{5}$ b) $\frac{3}{5}$ c) $\frac{2}{5}$ d) $\frac{4}{5}$

75. The eccentricity of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is

a) $\frac{\sqrt{5}}{3}$ b) $\frac{\sqrt{3}}{5}$ c) $\frac{3}{5}$ d) $\frac{2}{3}$

76. The eccentricity of the ellipse $16x^2 + 25y^2 = 400$ is

a) $\frac{4}{5}$ b) $\frac{3}{5}$ c) $\frac{3}{4}$ d) $\frac{2}{5}$

77. Centre of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ is

a) (0, 0) b) (5, 0) c) (3, 5) d) (0, 5)

78. The centre of the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is

a) (0, 3) b) (2, 3) c) (0, 0) d) (3, 0)

79. The foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ are

a) $(0, \pm 5)$ b) $(0, \pm 4)$ c) $(\pm 5, 0)$ d) $(\pm 4, 0)$

80. The foci of the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ are (March '08)

a) $(\pm 5, 0)$ b) $(0, \pm\sqrt{5})$ c) $(0, \pm 5)$ d) $(\pm\sqrt{5}, 0)$

81. The foci of the ellipse $16x^2+25y^2=400$ are

a) $(\pm 3, 0)$ b) $(0, \pm 3)$ c) $(0, \pm 5)$ d) $(\pm 5, 0)$

82. The vertices of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ are

a) $(0, \pm 5)$ b) $(0, \pm 3)$ c) $(\pm 5, 0)$ d) $(\pm 3, 0)$

83. The vertices of the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ are

a) $(0, \pm 3)$ b) $(\pm 2, 0)$ c) $(\pm 3, 0)$ d) $(0, \pm 2)$

84. The vertices of the ellipse $16x^2+25y^2=400$ are

a) $(0, \pm 4)$ b) $(\pm 5, 0)$ c) $(\pm 4, 0)$ d) $(0, \pm 5)$

85. If the centre of the ellipse is (4, –2) and one of the foci is (4, 2), then the other focus is

a) (4, 6) b) (6, –4) c) (4, –6) d) (6, 4)

86. The equations of transverse and conjugate axes of the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ are

a) $x=2; y=3$ b) $y=0; x=0$ c) $x=3; y=2$ d) $x=0; y=0$

87. The equations of transverse and conjugate axes of the hyperbola $16y^2-9x^2=144$ are

a) $y=0; x=0$ b) $x=3; y=4$ c) $x=0; y=0$ d) $y=3; x=4$

88. The equations of transverse and conjugate axes of the hyperbola $144x^2-25y^2=3600$ are

a) $y=0; x=0$ b) $x=12; y=5$ c) $x=0; y=0$ d) $x=5; y=12$ (March '06)

89. The equations of transverse and conjugate axes of the hyperbola $8y^2-2x^2=16$ are

a) $x=2\sqrt{2}; y=\sqrt{2}$ b) $x=\sqrt{2}; y=2\sqrt{2}$ c) $x=0; y=0$ d) $y=0; x=0$

90. The equations of the directrices of the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ are

a) $y=\pm\frac{9}{\sqrt{13}}$ b) $x=\pm\frac{13}{9}$ c) $y=\pm\frac{\sqrt{13}}{9}$ d) $x=\pm\frac{9}{\sqrt{13}}$

91. The equations of the directrices of the hyperbola $16y^2-9x^2=144$ are

a) $x=\pm\frac{5}{9}$ b) $y=\pm\frac{9}{5}$ c) $x=\pm\frac{9}{5}$ d) $y=\pm\frac{5}{9}$

92. The equations of the L.R's of the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ are

a) $y=\pm 13$ b) $y=\pm\sqrt{13}$ c) $x=\pm 13$ d) $x=\pm\sqrt{13}$

93. The equations of the L.R's of the hyperbola $16y^2-9x^2=144$ are

a) $y=\pm 5$ b) $x=\pm 5$ c) $y=\pm\sqrt{5}$ d) $x=\pm\sqrt{5}$

94. The length of the L.R of $\frac{x^2}{9}-\frac{y^2}{4}=1$ is

a) $\frac{4}{3}$ b) $\frac{8}{3}$ c) $\frac{3}{2}$ d) $\frac{9}{4}$

95. The eccentricity of the hyperbola $\frac{y^2}{9}-\frac{x^2}{25}=1$ is

a) $\frac{34}{3}$ b) $\frac{5}{3}$ c) $\frac{\sqrt{34}}{3}$ d) $\frac{\sqrt{34}}{5}$

96. The centre of the hyperbola $25x^2-16y^2=400$ is

a) (0, 4) b) (0, 5) c) (4, 5) d) (0, 0)

97. The foci of the hyperbola $\frac{y^2}{9}-\frac{x^2}{25}=1$ are

a) $(0, \pm\sqrt{34})$ b) $(\pm 34, 0)$ c) $(0, \pm 34)$ d) $(\pm\sqrt{34}, 0)$

98. The vertices of the hyperbola $25x^2-16y^2=400$ are

a) $(0, \pm 4)$ b) $(\pm 4, 0)$ c) $(0, \pm 5)$ d) $(\pm 5, 0)$

99. The equation of the tangent at (3, – 6) to the parabola $y^2=12x$ is

a) $x-y-3=0$ b) $x+y-3=0$ c) $x-y+3=0$ d) $x+y+3=0$

100. The equation of the tangent at (–3, 1) to the parabola $x^2=9y$ is

a) $3x-2y-3=0$ b) $2x-3y+3=0$ c) $2x+3y+3=0$ d) $3x+2y+3=0$

101. The equation of chord of contact of tangents from the point (– 3, 1) to the parabola $y^2=8x$ is (Sep '97)

a) $4x-y-12=0$ b) $4x+y+12=0$ c) $4y-x-12=0$ d) $4y-x+12=0$

102. The equation of chord of contact of tangents from (2, 4) to the ellipse $2x^2+5y^2=20$ is

a) $x-5y+5=0$ b) $5x-y+5=0$ c) $x+5y-5=0$ d) $5x-y-5=0$

103. The equation of chord of contact of tangents from (5, 3) to the hyperbola $4x^2-6y^2=24$ is (Sep '09)

a) $9x+10y+12=0$ b) $10x+9y-12=0$ c) $9x-10y+12=0$ d) $10x-9y-12=0$

104. The combined equation of the asymptotes to the hyperbola $36x^2-25y^2=900$ is

a) $25x^2+36y^2=0$ b) $36x^2-25y^2=0$ c) $36x^2+25y^2=0$ d) $25x^2-36y^2=0$

105. The angle between the asymptotes of the hyperbola $24x^2 - 8y^2 = 27$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$ c) $\frac{2\pi}{3}$ d) $\frac{-2\pi}{3}$

106. The point of contact of the tangent $y = mx + c$ and the parabola $y^2 = 4ax$ is *(Sep '06)*

a) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ b) $\left(\frac{2a}{m^2}, \frac{a}{m}\right)$ c) $\left(\frac{a}{m}, \frac{2a}{m^2}\right)$ d) $\left(\frac{-a}{m^2}, \frac{-2a}{m}\right)$

107. The point of contact of the tangent $y = mx + c$ and the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is *(Sep '10 & Mar '11)*

a) $\left(\frac{b^2}{c}, \frac{a^2m}{c}\right)$ b) $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right)$ c) $\left(\frac{a^2m}{c}, \frac{-b^2}{c}\right)$ d) $\left(\frac{-a^2m}{c}, \frac{-b^2}{c}\right)$

108. The point of contact of the tangent $y = mx + c$ and the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

a) $\left(\frac{am^2}{c}, \frac{b^2}{c}\right)$ b) $\left(\frac{a^2m}{c}, \frac{b^2}{c}\right)$ c) $\left(\frac{-a^2m}{c}, \frac{-b^2}{c}\right)$ d) $\left(\frac{-am^2}{c}, \frac{-b^2}{c}\right)$

109. The true statements of the following are

(i) Two tangents and 3 normals can be drawn to a parabola from a point.

(ii) Two tangents and 4 normals can be drawn to an ellipse from a point.

(iii) Two tangents and 4 normals can be drawn to a hyperbola from a point.

(iv) Two tangents and 4 normals can be drawn to a R.H. from a point.

a) (i), (ii), (iii) and (iv) b) (i), (ii) only c) (iii), (iv) only d) (i), (ii) and (iii)

110. If 't_1' 't_2' are the extremities of any focal chord of a parabola $y^2 = 4ax$ then $t_1 t_2$ is

a) -1 b) 0 c) ± 1 d) $\frac{1}{2}$ *(March '12)*

111. The normal at 't_1' on the parabola $y^2 = 4ax$ meets the parabola at 't_2' then $\left(t_1 + \frac{2}{t_1}\right)$ is

a) $-t_2$ b) t_2 c) $t_1 + t_2$ d) $\frac{1}{t_2}$ *(Jun '10 & Mar '11)*

112. The condition that the line $lx + my + n = 0$ may be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is *(Sep '12)*

a) $al^3 + 2alm^2 + m^2n = 0$ b) $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

c) $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2-b^2)^2}{n^2}$ d) $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

113. The condition that the line $lx + my + n = 0$ may be a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

a) $al^3 + 2alm^2 + m^2n = 0$ b) $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

c) $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2-b^2)^2}{n^2}$ d) $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

114. The condition that the line $lx + my + n = 0$ may be a normal to the parabola $y^2 = 4ax$ is *(March '13)*

a) $al^3 + 2alm^2 + m^2n = 0$ b) $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

c) $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2-b^2)^2}{n^2}$ d) $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

115. The chord of contact of tangents from any point on the directrix of the parabola $y^2 = 4ax$ passes through its

a) vertex b) focus c) directrix d) latus rectum

116. The chord of contact of tangents from any point on the directrix of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through its

a) vertex b) focus c) directrix d) latus rectum

117. The chord of contact of tangents from any point on the directrix of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through its *(March '09)*

a) vertex b) focus c) directrix d) latus rectum

118. The point of intersection of tangents at 't_1' and 't_2' to the parabola $y^2 = 4ax$ is *(Jun '09, Jun '13 & Mar '14)*

a) $(a(t_1 + t_2), at_1t_2)$ b) $(at_1t_2, a(t_1 + t_2))$ c) $(at^2, 2at)$ d) $(at_1t_2, a(t_1 - t_2))$

119. If the normal to the R.H. $xy = c^2$ at 't_1' meets the curve again at 't_2' then $t_1^3 t_2 =$ *(June '08)*

a) 1 b) 0 c) -1 d) -2

120. The locus of the point of intersection of perpendicular tangents to the parabola $y^2 = 4ax$ is *(Sep '13 & Mar '16)*

a) latus rectum b) directrix c) tangent at the vertex d) axis of the parabola

121. The locus of the foot of perpendicular from the focus on any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

a) $x^2 + y^2 = a^2 - b^2$ b) $x^2 + y^2 = a^2$ c) $x^2 + y^2 = a^2 + b^2$ d) $x = 0$

122. The locus of the foot of perpendicular from the focus on any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

a) $x^2 + y^2 = a^2 - b^2$ b) $x^2 + y^2 = a^2$ c) $x^2 + y^2 = a^2 + b^2$ d) $x = 0$

123. The locus of the foot of perpendicular from the focus on any tangent to the parabola $y^2 = 4ax$ is *(June '12, June '14 & Sep '14)*

a) $x^2 + y^2 = a^2 - b^2$ b) $x^2 + y^2 = a^2$ c) $x^2 + y^2 = a^2 + b^2$ d) $x = 0$

124. The locus of the point of intersection of perpendicular tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is *(Sep '11)*

a) $x^2 + y^2 = a^2 - b^2$ b) $x^2 + y^2 = a^2$ c) $x^2 + y^2 = a^2 + b^2$ d) $x = 0$

125. The locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

a) $x^2 + y^2 = a^2 - b^2$ b) $x^2 + y^2 = a^2$ c) $x^2 + y^2 = a^2 + b^2$ d) $x = 0$

126. The condition that the line $lx + my + n = 0$ may be a tangent to the parabola $y^2 = 4ax$ is

a) $a^2l^2 + b^2m^2 = n^2$ b) $am^2 = ln$ c) $a^2l^2 - b^2m^2 = n^2$ d) $4c^2lm = n^2$

127. The condition that the line $lx + my + n = 0$ may be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is *(June '15)* *(June '15)*

a) $a^2l^2 + b^2m^2 = n^2$ b) $am^2 = ln$ c) $a^2l^2 - b^2m^2 = n^2$ d) $4c^2lm = n^2$

128. The condition that the line $lx + my + n = 0$ may be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

a) $a^2l^2 + b^2m^2 = n^2$ b) $am^2 = ln$ c) $a^2l^2 - b^2m^2 = n^2$ d) $4c^2lm = n^2$

129. The condition that the line $lx + my + n = 0$ may be a tangent to the rectangular hyperbola $xy = c^2$ is *(March '15)*

a) $a^2l^2 + b^2m^2 = n^2$ b) $am^2 = ln$ c) $a^2l^2 - b^2m^2 = n^2$ d) $4c^2lm = n^2$

130. The foot of a perpendicular from a focus of the hyperbola on an asymptote lies on the

a) centre b) corresponding directrix c) vertex d) L.R

III. Additional Questions

131. The normal at the point 't_1' on the parabola $y^2 = 4ax$ again at 't_2' then $-t_2 =$

a) t_1^2 b) $\frac{1}{t_1}$ c) t_1 d) $t_1 + \frac{2}{t_1}$

132. The tangent at any point P on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ whose centre C meets the major axis at T and PN is the perpendicular to the major axis then CN . CT = *Deletion (Sep '07)*

a) $\sqrt{6}$ b) 3 c) $\sqrt{3}$ d) 6

133. If the normal at the end of the latus rectum of the ellipse $x^2 + 3y^2 = 12$ intersect the major axis at G, then CG is

a) $\frac{16}{3}$ b) $\frac{8\sqrt{2}}{3}$ c) $\frac{4\sqrt{2}}{3}$ d) $\frac{32}{3}$

134. If B and B' are the ends of the minor axis, F_1 and F_2 are the foci of the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ then the area and F_1 B F_2 B' is *Deletion* *(Jun '06 & Jun '07)*

a) 16 b) 8 c) $16\sqrt{2}$ d) $32\sqrt{2}$

135. P is any point on the hyperbola $\frac{x^2}{36} - \frac{y^2}{4} = 1$. The ordinate at P meets the asymptotes in Q and Q' then QP. Q'P is *Deletion* *(June '06)*

a) 36 b) 6 c) 4 d) 2

IV. Answers

Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.
1	d	24	b	47	b	70	b	93	a	116	b
2	d	25	c	48	c	71	c	94	b	117	b
3	d	26	a	49	b	72	a	95	c	118	b
4	a	27	c	50	d	73	b	96	d	119	c
5	c	28	b	51	d	74	d	97	a	120	b
6	c	29	b	52	a	75	a	98	b	121	b
7	d	30	d	53	c	76	b	99	d	122	b
8	b	31	b	54	b	77	a	100	c	123	d
9	c	32	b	55	d	78	c	101	a	124	c
10	d	33	d	56	a	79	d	102	c	125	a
11	b	34	a	57	c	80	b	103	d	126	b
12	d	35	a	58	b	81	a	104	b	127	a
13	b	36	d	59	c	82	c	105	c	128	c
14	d	37	d	60	d	83	a	106	a	129	d
15	c	38	c	61	b	84	b	107	b	130	b
16	b	39	b	62	a	85	c	108	c	131	d
17	b	40	c	63	a	86	b	109	d	132	d
18	c	41	d	64	c	87	c	110	a	133	c
19	d	42	b	65	b	88	a	111	a	134	b
20	b	43	a	66	c	89	c	112	c	135	c
21	c	44	d	67	a	90	d	113	d		
22	d	45	b	68	b	91	b	114	a		
23	a	46	d	69	d	92	d	115	b		

V. Solutions

1. $y^2 - 2y + 8x - 23 = 0$

$\underbrace{y^2 - 2y + 1^2} - 1^2 + 8x - 23 = 0$

$(y-1)^2 = -8x + 24$

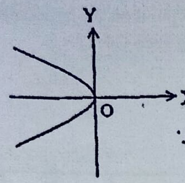
$(y-1)^2 = -8(x-3)$

$Y^2 = -4aX$

axis- X axis $\Rightarrow Y = 0$

$y - 1 = 0$

$\boxed{y = 1}$



$Y^2 = -4aX$

$\because \begin{bmatrix} X = x - 3 \\ Y = y - 1 \\ a = 2 \end{bmatrix}$

2. $16x^2 + 0xy - 3y^2 - 32x - 12y - 44 = 0$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$a = 16$ | $2h = 0$, $h = 0$ | $b = -3$

$h^2 = 0$: $ab = -48$

$h^2 > ab$

$0 > -48$ (is a hyperbola)

Note:

(i) If $h^2 = ab$ It is a parabola

(ii) If $h^2 > ab$ It is an ellipse

(iii) If $h^2 > ab$ It is a hyperbola

(iv) If $h^2 > ab$ & $a + b = 0$ It is a R .H

3. $4x + 2y = c$

$y = -2x + \frac{c}{2}$

$y = mx + c$

Where $\begin{bmatrix} m = -2 \\ c = \frac{c}{2} \end{bmatrix}$

$y^2 = 16x$

$y^2 = 4ax$

$\begin{bmatrix} 4a = 16 \\ a = 4 \end{bmatrix}$

condition for the tangency

$c = \frac{a}{m}$

$\frac{c}{2} = \frac{-4}{2}$

$\boxed{c = -4}$

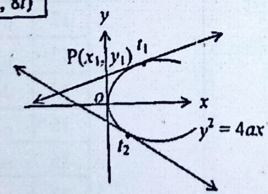
4. $y^2 = 8x$

$y^2 = 4ax$

$a = 2$

$\boxed{t_1 = t}$ $\boxed{t_2 = 3t}$

$\boxed{P(x_1, y_1) = (6t^2, 8t)}$



The point of intersection of the tangents

$\because [P(x_1, y_1) = [at_1 t_2, a(t_1 + t_2)]$

5. $y^2 + 4y - 4x + 8 = 0$

$\underbrace{y^2 + 4y + 2^2} - 2^2 - 4x + 8 = 0$

$(y+2)^2 = 4x - 4$

$(y+2)^2 = 4(x-1)$

$Y^2 = 4aX$

$\because \begin{bmatrix} X = x - 1 \\ Y = y + 2 \\ \boxed{4a = 4} \text{ Latus rectum} \end{bmatrix}$

6. $y^2 = (x + 4)$

$Y^2 = 4aX$

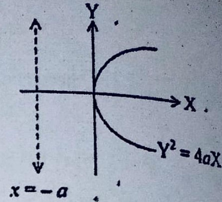
$\begin{bmatrix} Y = y \\ X = x + 4 \\ 4a = 1 \end{bmatrix}$

$\Rightarrow a = \frac{1}{4}$

directrix $X = -a$

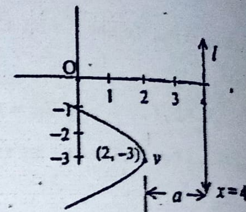
$x + 4 = -\frac{1}{4}$

$x = -\frac{17}{4}$



7. From this diagram $a = 2$ units

Latus rectum = $\boxed{4a = 8}$



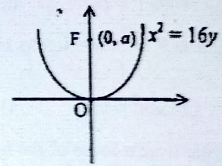
8. $x^2 = 16y$

$x^2 = 4ay$

$4a = 16$

$a = 4$

$\therefore$ F(0, a) = (0, 4)



9. $x^2 = 8y - 1$

$x^2 = 8\left(y - \frac{1}{8}\right)$

$X^2 = 4aY$

Here $\begin{cases} X = x \\ Y = y - \frac{1}{8} \\ 4a = 8 \end{cases}$

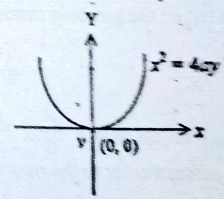
Vertex v (0, 0)

$X = 0; Y = 0$

$x = 0;\ y - \frac{1}{8} = 0$

$x = 0;\ y = \frac{1}{8}$

$\boxed{v(x, y) = \left(0, \frac{1}{8}\right)}$



10. $2x+3y+9=0$
$3y=-2x-9$
$y=\left(\frac{-2}{3}\right)x-3$
$y=mx+c$
Where $m=\frac{-2}{3}$
$c=-3$

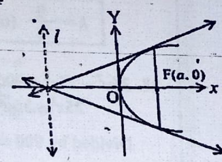
$y^2=8x$
$y^2=4ax$
$4a=8$
$a=2$

[∵ Touching point
$P(x_1, y_1)=\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
$=\left(\frac{c}{m}, \frac{2a}{m}\right)$

$P(x_1, y_1)=\left(\frac{2}{\frac{4}{9}}, \frac{4}{\frac{-2}{3}}\right)$

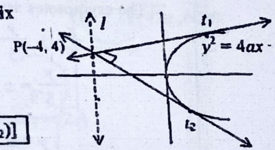
$\boxed{P(x_1, y_1)=\left(\frac{9}{2}, -6\right)}$

11. $y^2=12x$
$a=3$ [Intersect on the line be the directrix
$x=-a$
$x=-3 \Rightarrow \boxed{x+3=0}$



$x=-a$

12. Note:
The locus of the point of intersection of perpendicular tangents to $y^2=4ax$ is its directrix
$\boxed{\theta=90°}$
(i) perpendicular ⇒ $\boxed{\theta=90°}$
(ii) point of intersection $\boxed{P(x_1, y_1)=[at_1t_2, a(t_1+t_2)]}$
(iii) directrix $\boxed{x=-a}$

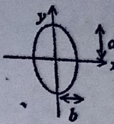


$x=-a$

13. $9x^2-54x+5y^2-40y+116=0$
$9(x^2-6x)+5(y^2-8y)+116=0$
$9[x^2-6x+3^2-3^2]+5[y^2-8y+4^2-4^2]+116=0$
$9(x-3)^2-81+5(y-4)^2-80+116=0$
$9(x-3)^2+5(y-4)^2=45$
$\frac{(x-3)^2}{5}+\frac{(y-4)^2}{9}=1$
$\frac{X^2}{b^2}+\frac{Y^2}{a^2}=1$ $\left[a^2=9 \quad b^2=5\right.$

$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{5}{9}}$
$\boxed{e=\frac{2}{3}}$

14. $a^2=169$; $b^2=144$
$a=13$; $b=12$
$\boxed{13, 12}$



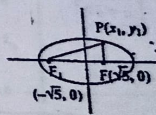
Length of semi-major axis $=a$
Length of semi-minor axis $=b$

15. Divided by 180 ⇒ $\frac{x^2}{20}+\frac{y^2}{36}=1$ $\left(\frac{X^2}{b^2}+\frac{Y^2}{a^2}=1\right)$ $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{20}{36}}=\frac{4}{6}=\frac{2}{3}$
$b^2=20$; $a^2=36$
$b=2\sqrt{5}$; $a=6$
$2ae=2\times 6\times\frac{2}{3}$
$\boxed{2ae=8}$

16. $2a=8$; $b=2$
$a=4$; $b^2=4$ $(a>b)$
$a^2=16$
The equation is $\frac{(x+4)^2}{16}+\frac{(y-6)^2}{4}=1$

17. $y=2x+c$
$y=mx+c$
Where $\left[\begin{array}{l}m=2\\c=c\end{array}\right.$
$4x^2+8y^2=32$
$\frac{x^2}{8}+\frac{y^2}{4}=1$
$[a^2=8 ; b^2=4$
$c^2=a^2m^2+b^2$
$c=\pm\sqrt{32+4}$
$\boxed{c=\pm 6}$

18. $FP+F'P=2a$
$=2\times 3$
$\boxed{FP+F'P=6}$
$4x^2+9y^2=36$
$\frac{x^2}{9}+\frac{y^2}{4}=1$
$[a^2=9 \; ; \; b^2=4$



∵ [FP + F'P = 2a

19. Divided by 144 ⇒ $\frac{x^2}{16}+\frac{y^2}{9}=1$ ∵ [The director circle is $x^2+y^2=\underbrace{a^2+b^2}_{r^2}$
$a^2=16$; $b^2=9$
Radius of the director circle $r^2=a^2+b^2=16+9=25$
$\boxed{r=5}$

20. $x^2+y^2=a^2$ Divided by 400 ⇒ ∵ [The locus is $x^2+y^2=a^2$
$\boxed{x^2+y^2=25}$
$\left[\frac{x^2}{25}+\frac{y^2}{16}=1 \quad a^2=25 ; b^2=16\right.$

21. $12(y^2+4y)-4(x^2+6x)-127=0$
$12[y^2+4y+2^2-2^2]-4[x^2+6x+3^2-3^2]-127=0$
$12(y+2)^2-48-4(x+3)^2+36-127=0$

$$12(y+2)^2-4(x+3)^2=139$$

$$\frac{12(y+2)^2}{139}-\frac{4(x+3)^2}{139}=1$$

$$\frac{(y+2)^2}{\frac{139}{12}}-\frac{(x+3)^2}{\frac{139}{4}}=1 \qquad \because \left[\frac{Y^2}{a^2}-\frac{X^2}{b^2}=1\right]$$

$$a^2=\frac{139}{12}\ ;\ b^2=\frac{139}{4}$$

$$e=\sqrt{1+\frac{b^2}{a^2}}$$

$$=\sqrt{1+\frac{139/4}{139/12}}$$

$$=\sqrt{1+3}$$

$$\boxed{e=2}$$

22. $\frac{2b^2}{a}=\frac{(2b)}{2}$

$\boxed{a=2b}$

$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{b^2}{4b^2}}=\sqrt{\frac{5}{4}}$$

$$\boxed{e=\frac{\sqrt5}{2}}$$

23. $FP\sim F'P=2a$

$24=2a$

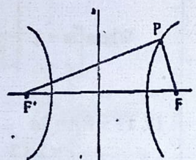
$\boxed{a=12}$

$e=2$

$$e=\sqrt{1+\frac{b^2}{a^2}}$$

$$e^2=1+\frac{b^2}{144}$$

$$4=1+\frac{b^2}{144}\Rightarrow \boxed{b^2=432}$$



$\because [FP\sim F'P=2a$

$\therefore$ equation $\boxed{\frac{x^2}{144}-\frac{y^2}{432}=1}$

24. $x^2-4(y-3)^2=16$

$$\frac{x^2}{16}-\frac{(y-3)^2}{4}=1 \qquad \because \left[\frac{X^2}{a^2}-\frac{Y^2}{b^2}=1\right.$$

Where $a^2=16$ $\quad X=x$

$b^2=14$ $\quad Y=y-3$

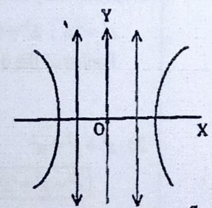
$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{4}{16}}=\frac{\sqrt5}{2}$$

Directrix

$$x=\pm\frac{a}{e}$$

$$x=\pm\frac{4}{\sqrt5/2}$$

$$\boxed{x=\pm\frac{8}{\sqrt5}}$$



25. $5x-2y+4k=0$

$2y=-5x-4k$

$$y=\frac{-5}{2}x-(2k)$$

$y=mx+c$

Where $m=\frac{-5}{2}$; $c=-2k$

$4x^2-y^2=36$

$$\frac{x^2}{9}-\frac{y^2}{36}=1$$

$a^2=9$; $b^2=36$

Condition for the tangency

$$c^2=a^2m^2-b^2$$

$$c=\pm\sqrt{a^2m^2-b^2}$$

$$-2k=\pm\sqrt{9\times\frac{25}{4}-36}$$

$$-2k=\pm\frac{9}{2}$$

$$\boxed{k=\frac{9}{4}\ (\text{or})\ -\frac{9}{4}}$$

26. $\frac{x^2}{16}-\frac{y^2}{9}=1$

$$\frac{xx_1}{16}-\frac{yy_1}{9}=1 \quad \because [P(x_1,y_1)=(2,1)$$

$$\frac{2x}{16}-\frac{y}{9}=1\Rightarrow \boxed{9x-8y-72=0}$$

27. $\theta=2\tan^{-1}\left(\frac{b}{a}\right)$ $\quad \because [a^2=16;b^2=9$

$$\theta=2\tan^{-1}\left(\frac{3}{4}\right)$$

Note: The equation of Hyperbola

(i) Not in the standard form

i.e., $ax^2+2hxy+by^2+2gx+2fy+c=0$

$$\theta=\tan^{-1}\left(\frac{2\sqrt{h^2-ab}}{a+b}\right)$$

(ii) In the standard form

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \theta=2\tan^{-1}\left(\frac{b}{a}\right)$$

28. $36y^2-25x^2+900=0$

$25x^2-36y^2=900$

Divided by $900\Rightarrow \frac{x^2}{36}-\frac{y^2}{25}=1$

$$\left(\frac{x}{6}\right)^2-\left(\frac{y}{5}\right)^2=1$$

The asymptotes are

$$\frac{x}{6}\pm\frac{y}{5}=0$$

$$\frac{y}{5}=\pm\frac{x}{6}$$

$$\boxed{y=\pm\frac{5}{6}x}$$

$\because$ [The asymptotes of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are

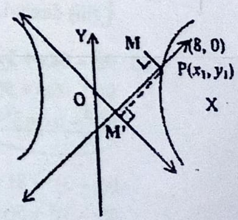
$$\frac{x}{a}\pm\frac{y}{b}=0$$

29. $PM.PM'=\frac{a^2b^2}{a^2+b^2}$ $\qquad \because [PM.PM'\ \frac{a^2b^2}{a^2+b^2}$

$$=\frac{64\times36}{100}$$

Where $a^2=64$; $b^2=36$

$$\boxed{=\frac{576}{25}}$$



30. $x^2+y^2=a^2-b^2$

$x^2+y^2=7$ [Where $a^2=16$, $b^2=9$ $\because$ [The locus of the point of intersection is $x^2+y^2=a^2-b^2$ a circle

31. ① x + ② $y-5=0$
② x − ① $y+5=0$ } These are perpendiculars $\because$ [If the asymptotes are perpendiculars then its hyperbola is called a R.H.

$\boxed{e=\sqrt{2}}$ $\because$ [In a R.H

32. $xy=8$

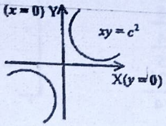
$xy=c^2$ $\because \left[c^2=\frac{a^2}{2}\right.$ $a \to$ semi – transverse axis

$8=\frac{a^2}{2}$

$a^2=16 \Rightarrow a=4$

33. $x=0, y=0$

y – axis x – axis



34. $xy=16$ $\because [c^2=16$

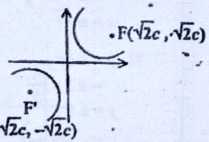
Vertices $(c, c)\ (-c, -c)$

$\boxed{=(4,4)\ (-4,-4)}$

35. $\because \left[\begin{array}{l} c^2=18 \\ c=3\sqrt{2}\end{array}\right.$

Foci $(\sqrt{2}c, \sqrt{2}c)\ (-\sqrt{2}c, -\sqrt{2}c)$

$\boxed{=(6,6)\ (-6,-6)}$



(OR)

$c^2=\frac{a^2}{2}=18 \Rightarrow a=6$

Foci $(a, a), (-a, -a)$ i.e.,(6, 6), (−6, −6)

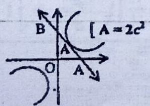
36. $xy=32$ $\because \left[\begin{array}{l} c^2=32 \\ c=4\sqrt{2}\end{array}\right.$ latus rectum $\because [LL'=2\sqrt{2}c$

$\boxed{LL'=2\sqrt{2}\times 4\sqrt{2}=16}$

37. Area $A=2c^2$ $\because [c^2=72$

$=2\times 72$

$\boxed{A=144}$



38. If the normal to the R.H at 't_1' meets the curve again at 't_2' then $t_1^3 t_2=-1$

$t_2=-\frac{1}{t_1^3}$. Where $c^2=9 \Rightarrow c=3$

$\left(ct_1, \frac{c}{t_1}\right)=\left(6, \frac{3}{2}\right)$ i.e., $\left(3t_1, \frac{3}{t_1}\right)=\left(6, \frac{3}{2}\right)$

$t_1=2$

$\therefore\ t_2=-\frac{1}{8}$

$\left(ct_2, \frac{c}{t_2}\right)=\left(\frac{-3}{8}, -24\right)$

39. to 44. } $y^2=4x$ $\left[\begin{array}{l}4a=4 \\ a=1\end{array}\right.$

$y^2=4ax$

(i) Axis :- x - axis $y=0$

(ii) Vertex:- $v(0,0)$

(iii) Focus:- F $(a, 0)=(1, 0)$

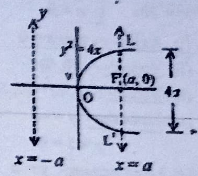
(iv) Directrix:- $x=-a$

$x=-1$

(v) Latus rectum:- $x=a$

$x=1$

(vi) $LL'=4a=4$



45. to 50. } $x^2=-4y$

$x^2=-4ay$ $\left[\begin{array}{l}4a=4 \\ a=1\end{array}\right.$

(i) Axis:- y - axis $x=0$

(ii) Vertex:- $v(0,0)$

(iii) Focus:- F $(0, -a)=(0, -1)$

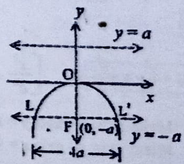
(iv) Directrix:- $y=a$

$y=1$

(v) Latus rectum:- $y=-a$

$y=-1$

(vi) $LL'=4a=4$



51. to 56. } $y^2=-8x$

$y^2=-4ax$ $\left[\begin{array}{l}4a=8 \\ a=2\end{array}\right.$

(i) Axis:- x - axis $y=0$

(ii) Vertex:- $v(0, 0)$

(iii) Focus:- $F(-a, 0)$

$= (-2, 0)$

(iv) Directrix:- $x = a$

$x = 2$

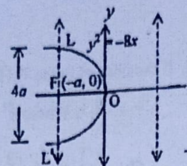
$x - 2 = 0$

(v) Latus rectum:- $x = -a$

$x = -2$

$x + 2 = 0$

(vi) $LL' = 4a = 8$



57. to 62. $x^2 = 4ay$, $x^2 = 20y$ $\left[\begin{array}{l} 4a = 20 \\ a = 5 \end{array} \right.$

(i) Axis :- y - axis $x = 0$

(ii) Vertex:- $v(0, 0)$

(iii) Focus:- $F(0, a) = (0, 5)$

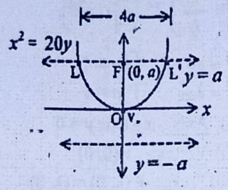
(iv) Directrix:- $y = -a$

$y + 5 = 0$

(v) Latus rectum:- $y = a$

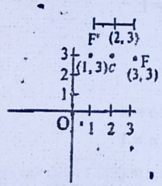
$y - 5 = 0$

(vi) $LL' = 4a = 20$



63. $F'(1,3)$

$F'C = FC = 1$ unit

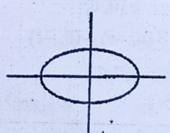


64. $\frac{x^2}{9} + \frac{y^2}{4} = 1$ $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right)$

$a > b$

Major axis: x - axis $y = 0$

Minor axis: y - axis $x = 0$

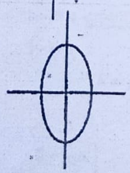


65. $\frac{x^2}{3} + \frac{y^2}{4} = 1$ $\left(\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1\right)$

$a > b$

Major axis: y - axis $\Rightarrow x = 0$

Minor axis: x - axis $\Rightarrow y = 0$



66. $a = 3$; $b = 2$

Length of major axis $= 2a = 6$

Length of minor axis $= 2b = 4$ **Ans: (4, 6)**

67. $a = 2$; $b = \sqrt{3}$

Length of major axis $= 2a = 4$

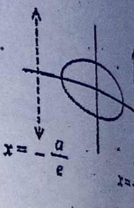
Length of minor axis $= 2b = 2\sqrt{3}$ **Ans: $(4, 2\sqrt{3})$**

68. $x = \pm \frac{a}{e}$

$= \pm \frac{4}{\frac{\sqrt{7}}{4}}$

$x = \pm \frac{16}{\sqrt{7}}$

$\left[a = 4 ; b = 3 \quad e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4} \right.$



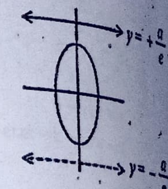
69. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

$y = \pm \frac{a}{e}$

$= \pm \frac{5}{\frac{4}{5}}$

$y = \pm \frac{25}{4}$

$\left[a = 5 ; b = 3 \quad e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5} \right.$



70. $x = \pm ae$

$= \pm 4 \times \frac{\sqrt{7}}{4}$

$x = \pm \sqrt{7}$

71. $y = \pm ae$

$= \pm 5 \times \frac{4}{5}$

$y = \pm 4$

72. Length $= \frac{2b^2}{a} = 2 \times \frac{9}{4} = \frac{9}{2}$

73. Length $= \frac{2b^2}{a} = 2 \times \frac{9}{5} = \frac{18}{5}$

74. 77. 79. 82. $\frac{x^2}{25} + \frac{y^2}{9} = 1$

$a = 5$; $b = 3$

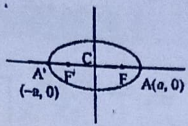
(i) $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

(ii) c (0, 0)

(iii) $F(\pm ae, 0) = (\pm 5 \times \frac{4}{5}, 0)$

$= (\pm 4, 0)$

(iv) $A(\pm a, 0) = (\pm 5, 0)$



75., 78., 80., 83. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$a = 3 ; b = 2$

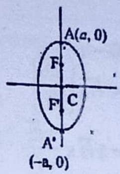
(i) $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$

(ii) c (0, 0)

(iii) $F(0, \pm ae) = \left(0, \pm 3 \times \frac{\sqrt{5}}{3}\right)$

$= (0, \pm\sqrt{5})$

(iv) $A(0, \pm a) = (0, \pm 3)$



76., 81., 84. $16x^2 + 25y^2 = 400$

$\frac{x^2}{25} + \frac{y^2}{16} = 1$ $\left[\begin{array}{l} a = 5 \\ b = 4 \end{array}\right.$

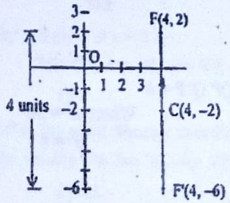
(i) $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

(ii) $F(\pm ae, 0) = (\pm 3, 0)$

(iii) $A(\pm a, 0) = (\pm 5, 0)$

85. F' (4, −6)

$\therefore$ FC = F'C = 4 units



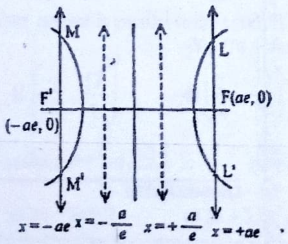
86., 90., 92., 94. $\frac{x^2}{9} - \frac{y^2}{4} = 1$ $\left[\begin{array}{l} a = 3 \\ b = 2 \end{array}\right.$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3}$

(i) transverse axis:- y axis $\Rightarrow x = 0$

conjugate axis:- x axis $\Rightarrow y = 0$

(ii) $x = \pm \frac{a}{e} = \pm \frac{3}{\frac{\sqrt{13}}{3}} = \pm \frac{9}{\sqrt{13}}$



(iii) $x = \pm ae = \pm\sqrt{13}$

(iv) $LL' = MM' = \frac{2b^2}{a} = \frac{8}{3}$

87., 91., 93. $\frac{y^2}{9} - \frac{x^2}{16} = 1$ $\left[\begin{array}{l} a = 3 \\ b = 4 \end{array}\right.$

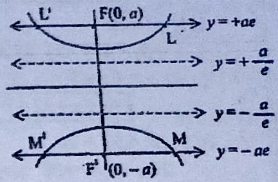
$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

(i) transverse axis:- x axis $\Rightarrow y = 0$

conjugate axis:- y axis $\Rightarrow x = 0$

(ii) $y = \pm \frac{a}{e} = \pm \frac{3}{5/3} = \pm \frac{9}{5}$

(iii) $y = \pm ae = \pm 3 \times \frac{5}{3} = \pm 5$



88. $144x^2 - 25y^2 = 3600$

Divided by 3600 $\Rightarrow \frac{x^2}{25} - \frac{y^2}{144} = 1$

Transverse axis:- x axis $\Rightarrow y = 0$

Conjugate axis:- y axis $\Rightarrow x = 0$

89. $8y^2 - 2x^2 = 16$

$\frac{y^2}{2} - \frac{x^2}{8} = 1$

Transverse axis:- y-axis $\Rightarrow x = 0$

Conjugate axis:- x-axis $\Rightarrow y = 0$

95. $\frac{y^2}{9} - \frac{x^2}{25} = 1$ $\left[\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1\right.$

$a = 3$ $b = 5$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{25}{9}} = \frac{\sqrt{34}}{3}$

96. $\frac{x^2}{16} - \frac{y^2}{25} = 1$ $\left[\begin{array}{l} a = 4 \\ b = 5 \end{array}\right.$

centre (0, 0)

97. $F(0, \pm ae) = \left(0, \pm 3 \times \frac{\sqrt{34}}{3}\right)$

$= (0, \pm\sqrt{34})$

98. $A(\pm a, 0) = (\pm 4, 0)$

99. $y^2 = 12x$

$yy_1 = 6x + 6x_1$ $\left[(x_1, y_1) = (3, -6)\right.$

$-6y = 6[x + 3]$

$\boxed{x + y + 3 = 0}$

100. $x^2 = 9y$

$xx_1 = 9\left(\frac{1}{2}y + \frac{1}{2}y_1\right)$ ∵ $[(x_1, y_1) = (-3, 1)]$

$-3x = \frac{9}{2}[y + 1]$

$\boxed{2x + 3y + 3 = 0}$

101. $y^2 = 8x$

$yy_1 = 4x + 4x_1$ $[(x_1, y_1) = (-3, 1)]$

$y = 4[x - 3]$

$\boxed{4x - y - 12 = 0}$

102. $2x^2 + 5y^2 = 20$

$2xx_1 + 5yy_1 = 20$ $[(x_1, y_1) = (2, 4)]$

$4x + 20y = 20$

$\boxed{x + 5y - 5 = 0}$

103. $4x^2 - 6y^2 = 24$

$4xx_1 - 6yy_1 = 24$ $[(x_1, y_1) = (5, 3)]$

$20x - 18y = 24$

$\boxed{10x - 9y - 12 = 0}$

104. $36x^2 - 25y^2 = 900$

$\frac{x^2}{25} - \frac{y^2}{36} = 1$

∴ The combined equation of the asymptotes $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

$\frac{x^2}{25} - \frac{y^2}{36} = 0$

$\boxed{36x^2 - 25y^2 = 0}$

105. $24x^2 - 8y^2 = 27$

$\frac{24x^2}{27} - \frac{8y^2}{27} = 1$

$\frac{x^2}{\left(\frac{9}{8}\right)} - \frac{y^2}{\left(\frac{27}{8}\right)} = 1$ $\left[a^2 = \frac{9}{8}\right.$; $b^2 = \frac{27}{8}$

∵ $\theta = 2\tan^{-1}\left(\frac{b}{a}\right)$

$= 2\tan^{-1}\left(\frac{3\sqrt{3}/2\sqrt{2}}{3/2\sqrt{2}}\right)$

$= \frac{2\pi}{2}$

110. $t_1 . t_2 = -1$

111. Equation of the normal at t_1

$xt_1 + y = 2at_1 + at_1^3$

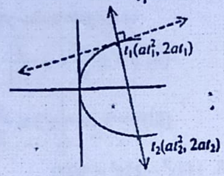
$at_2^2t_1 + 2at_2 = 2at_1 + at_1^3$ $[(x, y) = (at_2^2, 2at_2)]$

$at_1(t_2^2 - t_1^2) = 2a(t_1 - t_2)$

$at_1(t_2 - t_1)(t_2 + t_1) = -2a(t_2 - t_1)$

$t_2 + t_1 = \frac{-2}{t_1}$

$t_1 + \frac{2}{t_1} = -t_2$



119. Equation of the normal at t_1 is

$xt_1^2 - y = ct_1^3 - \frac{c}{t_1}$

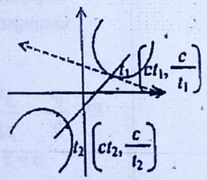
$ct_2t_1^2 - \frac{c}{t_2} = ct_1^3 - \frac{c}{t_1}$ $(x, y) = (ct_2, c/t_2)$

$\frac{1}{t_1} - \frac{1}{t_2} = t_1^3 - t_1^2t_2$

$\frac{t_2 - t_1}{t_1t_2} = t_1^2(t_1 - t_2)$

$\frac{1}{t_1t_2} = -t_1^2$

∵ $\boxed{-1 = t_1^3 . t_2}$



131. $t_2 + t_1 = \frac{-2}{t_1}$ [Ref:- Question no: 111

$-t_2 = t_1 + \frac{2}{t_1}$

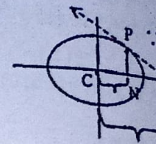
132. CN . CT $= a^2$

$= 6$

$\left[\frac{x^2}{6} + \frac{y^2}{3} = 1\right.$

$[a^2 = 6$; $b^2 = 3$

∵ [CN . CT $= a^2$



133. CG $= ae^3$

$= \sqrt{12}\left(\frac{\sqrt{2}}{\sqrt{3}}\right)$

$= 2\sqrt{3} \times \frac{2\sqrt{2}}{3\sqrt{3}}$

CG $= \frac{4\sqrt{2}}{3}$

$\left[x^2 + 3y^2 = 12\right.$

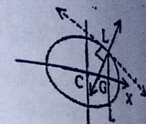
$\frac{x^2}{12} + \frac{y^2}{4} = 1$

$[a^2 = 12$; $b^2 = 4$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$= \sqrt{1 - \frac{4}{12}}$

$= \sqrt{\frac{2}{3}}$



134. Area of $F_1BF_2B' = 2abe$

$= 2 \times 2\sqrt{2} \times 2 \times \frac{1}{\sqrt{2}}$

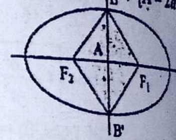
$\boxed{A = 8}$

$a^2 = 8$; $b^2 = 4$

$e = \sqrt{1 - \frac{4}{8}}$

$e = \frac{1}{\sqrt{2}}$

∵ [A = 2abe

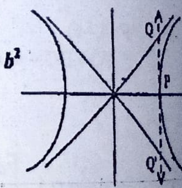


135. $QP.Q'P = b^2$

$= 4$ Where $a^2 = 36$

$b^2 = 4$

∵ $[QP.Q'P = b^2$



5. Differential Calculus - Applications I

Blue Print	Part - I 1 Mark		Part - II 6Marks		Part - III 10 Marks		Total Marks
	No. of Qns.	Marks	No. of Qns.	Marks	No. of Qns.	Marks	
	4	4	2	12	2	20	36

Choose the correct answer (Multiple Choice Questions)

I. Book Questions

1. The gradient of the curve $y=-2x^3+3x+5$ at $x=2$ is
   a) -20 b) 27 c) -16 d) -21

2. The rate of change of area A of a circle of radius r is *(Sep '08)*
   a) $2\pi r$ b) $2\pi r\frac{dr}{dt}$ c) $\pi r^2\frac{dr}{dt}$ d) $\pi\frac{dr}{dt}$

3. The velocity v of a particle moving along a straight line when at a distance x from the origin is given by $a+bv^2=x^2$ where a and b are constants. Then the acceleration is
   a) $\frac{b}{x}$ b) $\frac{a}{x}$ c) $\frac{x}{b}$ d) $\frac{x}{a}$ *(Mar '09)*

4. A spherical snowball is melting in such a way that its volume is decreasing at a rate of 1 cm³/min. The rate at which the diameter is decreasing when the diameter is 10 cm is *(Mar '10, Mar '11 & June '14)*
   a) $\frac{-1}{50\pi}$ cm/min b) $\frac{1}{50\pi}$ cm/min c) $\frac{-11}{75\pi}$ cm/min d) $\frac{-2}{75\pi}$ cm/min

5. The slope of the tangent to the curve $y=3x^2+3\sin x$ at $x=0$ is *(Mar '07, Jun '08, Jun '09, Mar '12 & June '14)*
   a) 3 b) 2 c) 1 d) -1

6. The slope of the normal to the curve $y=3x^2$ at the point whose x coordinate is 2 is *(Mar '06, Sep '06; Jun '07, Sep '09, Jun '13 & Mar '15)*
   a) $\frac{1}{13}$ b) $\frac{1}{14}$ c) $\frac{-1}{12}$ d) $\frac{1}{12}$

7. The point on the curve $y=2x^2-6x-4$ at which the tangent is parallel to the x-axis is *(Sep '10 & June '15)*
   a) $\left(\frac{5}{2},\frac{-17}{2}\right)$ b) $\left(\frac{-5}{2},\frac{-17}{2}\right)$ c) $\left(\frac{-5}{2},\frac{17}{2}\right)$ d) $\left(\frac{3}{2},\frac{-17}{2}\right)$

8. The equation of the tangent to the curve $y=\frac{x^3}{5}$ at the point $\left(-1,\frac{-1}{5}\right)$ is *(March '08)*
   a) $5y+3x=2$ b) $5y-3x=2$ c) $3x-5y=2$ d) $3x+3y=2$

9. The equation of the normal to the curve $\theta=\frac{1}{t}$ at the point $\left(-3,-\frac{1}{3}\right)$ is *(Sep '12 & Mar '14)*
   a) $3\theta=27t-80$ b) $5\theta=27t-80$ c) $3\theta=27t+80$ d) $\theta=\frac{1}{t}$

10. The angle between the curves $\frac{x^2}{25}+\frac{y^2}{9}=1$ and $\frac{x^2}{8}-\frac{y^2}{8}=1$ is *(Jun '07, Mar '09 & Sep '11)*
   a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

11. The angle between the curves $y=e^{mx}$ and $y=e^{-mx}$ for $m>1$ is *(Sep '13 & March '16)*
   a) $\tan^{-1}\left(\frac{2m}{m^2-1}\right)$ b) $\tan^{-1}\left(\frac{2m}{1-m^2}\right)$ c) $\tan^{-1}\left(\frac{-2m}{1+m^2}\right)$ d) $\tan^{-1}\left(\frac{2m}{m^2+1}\right)$

12. The parametric equations of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ are
   a) $x=a\sin^3\theta;\ y=a\cos^3\theta$ b) $x=a\cos^3\theta;\ y=a\sin^3\theta$
   c) $x=a^3\sin\theta;\ y=a^3\cos\theta$ d) $x=a^3\cos\theta;\ y=a^3\sin\theta$

13. If the normal to the curve $x^{2/3}+y^{2/3}=a^{2/3}$ makes an angle θ with the x-axis then the slope of the normal is *(Mar '11 & Mar '13)*
   a) $-\cot\theta$ b) $\tan\theta$ c) $-\tan\theta$ d) $\cot\theta$

14. If the length of the diagonal of a square is increasing at the rate of 0.1 cm/sec. What is the rate of increase of its area when the side is $\frac{15}{\sqrt{2}}$ cm? *(Sep '12)*
   a) 1.5 cm²/sec b) 3 cm²/sec c) $3\sqrt{2}$ cm²/sec d) 0.15 cm²/sec

15. What is the surface area of a sphere when the volume is increasing at the same rate as its radius? *(Jun '06 & Mar '10)*
   a) 1 b) $\frac{1}{2\pi}$ c) 4π d) $\frac{4\pi}{3}$

16. For what values of x is the rate of increase of x^3-2x^2+3x+8 is twice the rate of increase of x *(June '08 & Jun '12)*
   a) $\left(-\frac{1}{3},-3\right)$ b) $\left(\frac{1}{3},3\right)$ c) $\left(-\frac{1}{3},3\right)$ d) $\left(\frac{1}{3},1\right)$

17. The radius of a cylinder is increasing at the rate of 2 cm/sec and its altitude is decreasing at the rate of 3 cm/sec. The rate of change of volume when the radius is 3 cm and the altitude is 5 cm is *(June '07)*
   a) 23π b) 33π c) 43π d) 53π

18. If $y=6x-x^3$ and x increases at the rate of 5 units per second, the rate of change of slope when $x=3$ is *(March '13)*
   a) -90 units/sec b) 90 units/sec c) 180 units/sec d) -180 units/sec

19. If the volume of an expanding cube is increasing at the rate of 4 cm³/sec then the rate of change of surface area when the volume of the cube is 8 cubic cm is
   a) 8 cm²/sec b) 16 cm²/sec c) 2 cm²/sec d) 4 cm²/sec

20. The gradient of the tangent to the curve $y = 8 + 4x - 2x^2$ at the point where the curve cuts the y-axis is

a) 8 b) 4 c) 0 d) -4

21. The Angle between the parabolas $y^2 = x$ and $x^2 = y$ at the origin is *(Jun '06, Jun '10, Mar '14 & Sep '14)*

a) $2\tan^{-1}\left(\frac{3}{4}\right)$ b) $\tan^{-1}\left(\frac{4}{3}\right)$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

22. For the curve $x = e^t \cos t$; $y = e^t \sin t$ the tangent line is parallel to the x-axis when t is equal to *(Jun '12)*

a) $-\frac{\pi}{4}$ b) $\frac{\pi}{4}$ c) 0 d) $\frac{\pi}{2}$

23. If a normal makes an angle θ with positive x-axis then the slope of the curve at the point where the normal is drawn is *(September '07)*

a) $-\cot\theta$ b) $\tan\theta$ c) $-\tan\theta$ d) $\cot\theta$

24. The value of 'a' so that the curves $y = 3e^x$ and $y = \frac{a}{3}e^{-x}$ intersect orthogonally is *(Sep '10 & Jun '11)*

a) -1 b) 1 c) $\frac{1}{3}$ d) 3

25. If $S = t^3 - 4t^2 + 7$, the velocity when the acceleration is zero is *(Sep '06, Mar '07, Sep '09 & Jun '15)*

a) $\frac{32}{3}$ m/sec b) $\frac{-16}{3}$ m/sec c) $\frac{16}{3}$ m/sec d) $\frac{-32}{3}$ m/sec

26. If the velocity of a particle moving along a straight line is directly proportional to the square of its distance from a fixed point on the line. Then its acceleration is proportional to

a) s b) s^2 c) s^3 d) s^4 *(Sep '11)*

27. The Rolle's constant for the function $y = x^2$ on $[-2, 2]$ is

a) $\frac{2\sqrt{3}}{3}$ b) 0 c) 2 d) -2

28. The 'c' of Lagranges Mean Value Theorem for the function $f(x) = x^2 + 2x - 1$; $a = 0$, $b = 1$ is *(March '09, Jun '13, Sep '13 & Sep '14)*

a) -1 b) 1 c) 0 d) $\frac{1}{2}$

29. The value of c in Rolle's Theorem for the function $f(x) = \cos\frac{x}{2}$ on $[\pi, 3\pi]$ is *(Mar '06, Mar '08 & Mar '12)*

a) 0 b) 2π c) $\frac{\pi}{2}$ d) $\frac{3\pi}{2}$

30. The value of 'c' of Lagranges Mean Value Theorem for $f(x) = \sqrt{x}$ when $a = 1$ and $b = 4$ is *(Jun '10, Jun '11 & Sep '15)*

a) $\frac{9}{4}$ b) $\frac{3}{2}$ c) $\frac{1}{2}$ d) $\frac{1}{4}$

31. $\lim_{x \to \infty} \frac{x^2}{e^x}$ is = *(Sep '07 & Sep '08)*

a) 2 b) 0 c) ∞ d) 1

32. $\lim_{x \to 0} \frac{a^x - b^x}{c^x - d^x}$ *(Mar '07 & Sep '09)*

a) ∞ b) 0 c) $\log\frac{ab}{cd}$ d) $\frac{\log(a/b)}{\log(c/d)}$

33. If $f(a) = 2$; $f'(a) = 1$; $g(a) = -1$; $g'(a) = 2$ then the value of $\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$ is *(June '08 & Mar '16)*

a) 5 b) -5 c) 3 d) -3

34. Which of the following function is increasing in $(0, \infty)$? *(Mar '10 & Mar '13)*

a) e^x b) $\frac{1}{x}$ c) $-x^2$ d) x^{-2}

35. The function $f(x) = x^2 - 5x + 4$ is increasing in *(Mar '11 & Jun '11)*

a) $(-\infty, 1)$ b) $(1, 4)$ c) $(4, \infty)$ d) everywhere

36. The function $f(x) = x^2$ is decreasing in *(June '09 & Mar '15)*

a) $(-\infty, \infty)$ b) $(-\infty, 0)$ c) $(0, \infty)$ d) $(-2, \infty)$

37. The function $y = \tan x - x$ is *(June '14)*

a) an increasing function in $\left(0, \frac{\pi}{2}\right)$

b) a decreasing function in $\left(0, \frac{\pi}{2}\right)$

c) increasing in $\left(0, \frac{\pi}{4}\right)$ and decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

d) decreasing in $\left(0, \frac{\pi}{4}\right)$ and increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

38. In a given semi circle of diameter 4 cm a rectangle is to be inscribed. The maximum area of the rectangle is *(March '06 & Sep '14)*

a) 2 b) 4 c) 8 d) 16

39. The least possible perimeter of a rectangle of area 100 m^2 is *(Sep '07, Sep '08, Sep '10, Sep '11 & Jun '15)*

a) 10 b) 20 c) 40 d) 60

40. If $f(x) = x^2 - 4x + 5$ on $[0, 3]$ then the absolute maximum value is *(March '12)*

a) 2 b) 3 c) 4 d) 5

41. The curve $y = -e^{-x}$ is *(Mar '10 & Mar '13)*

a) concave upward for $x > 0$
b) concave downward for $x > 0$
c) everywhere concave upward
d) everywhere concave downward

42. Which of the following curves is concave down? (Mar '08, Jun '09 & Sep '13)

a) $y = -x^2$ b) $y = x^2$ c) $y = e^x$ d) $y = x^2 + 2x - 3$

43. The point of inflexion of the curve $y = x^4$ is at (June '13 & Mar '15)

a) $x = 0$ b) $x = 3$ c) $x = 12$ d) nowhere

44. The curve $y = ax^3 + bx^2 + cx + d$ has a point of inflexion at $x = 1$ then (Jun '06, Jun '10, Jun '12 & Mar '14)

a) $a + b = 0$ b) $a + 3b = 0$ c) $3a + b = 0$ d) $3a + b = 1$

II. COME Book Questions (PTA Questions)

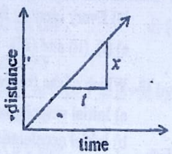
45. Let "h" be the height of the tank. Then the rate of change of pressure "p" of the tank with respect to height is

a) $\frac{dh}{dt}$ b) $\frac{dp}{dt}$ c) $\frac{dh}{dp}$ d) $\frac{dp}{dh}$

46. If the temperature θ° C of the certain metal rod of "l" metres is given by $l = 1 + 0.00005\,\theta + 0.0000004\,\theta^2$ then the rate of change of l in m/C° when the temperature is 100° C is

a) 0.00013 m/C° b) 0.00023 m/C° c) 0.00026 m/C° d) 0.00033 m/C°

47. The following graph gives the functional relationship between distance and time of a moving car in m/sec. The speed of the car is



a) $\frac{x}{t}$ m/s b) $\frac{t}{x}$ m/s c) $\frac{dx}{dt}$ m/s d) $\frac{dt}{dx}$ m/s

48. The distance - time relationship of a moving body is given by $y = F(t)$ then the acceleration of the body is the

a) gradient of the velocity/time graph b) gradient of the distance/time graph c) gradient of the acceleration/time graph d) gradient of the velocity/distance graph

49. The distance travelled by a car in "t" seconds is given by $x = 3t^3 - 2t^2 + 4t - 1$. Then the initial velocity and initial acceleration respectively are

a) (–4m/s, 4 m/s²) b) (4 m/s, –4 m/s²) c) (0, 0) d) (18.25 m/s, 23 m/s²)

50. The angular displacement of a fly wheel in radians is given by $\theta = 9t^2 - 2t^3$. The time when the angular acceleration zero is

a) 2.5 s b) 3.5 s c) 1.5 s d) 4.5 s

51. Food pockets were dropped from an helicopter during the flood and distance fallen in "t" seconds is given by $y = \frac{1}{2}gt^2$ ($g = 9.8$ m/s²). Then the speed of the food pocket after it has fallen for "2" seconds is

a) 19.6 m/s b) 9.8 m/s c) –19.6 m/s d) –9.8 m/s

52. An object dropped from the sky follows the law of motion $x = \frac{1}{2}gt^2$ ($g = 9.8$ m/s²). Then the acceleration of the object when $t = 2$ is

a) – 9.8 m/sec² b) 9.8 m/sec² c) 19.6 m/sec² d) – 19.6 m/sec²

53. A missile fired from ground level rises x metres vertically upwards in "t" seconds and $x = t(100 - 12.5\,t)$. Then the maximum height reached by the missile is (March '16)

a) 100 m b) 150 m c) 250 m d) 200 m

54. A continuous graph $y = f(x)$ is such that $f'(x) \to \infty$ as $x \to x_1$, at (x_1, y_1). Then $y = f(x)$ has a

a) vertical tangent $y = x_1$ b) horizontal tangent $x = x_1$ c) vertical tangent $x = x_1$ d) horizontal tangent $y = y_1$

55. The curve $y = f(x)$ and $y = g(x)$ cut orthogonally if at the point of intersection

a) slope of $f(x)$ = slope of $g(x)$ b) slope of $f(x)$ + slope of $g(x) = 0$ c) slope of $f(x)$ / slope of $g(x) = -1$ d) [slope of $f(x)$] [slope of $g(x)$] = –1

56. The law of the mean can also be put in the form (Sep '11 & Mar '14)

a) $f(a+h) = f(a) - hf'(a+\theta h)\ \ 0 < \theta < 1$ b) $f(a+h) = f(a) + hf'(a+\theta h)\ \ 0 < \theta < 1$ c) $f(a+h) = f(a) + hf'(a-\theta h)\ \ 0 < \theta < 1$ d) $f(a+h) = f(a) - hf'(a-\theta h)\ \ 0 < \theta < 1$

57. l'Hôpital's rule cannot be applied to $\frac{x+1}{x+3}$ as $x \to 0$ because $f(x) = x + 1$ and $g(x) = x + 3$ are (June '11)

a) not continuous b) not differentiable c) not in the indeterminate form as $x \to 0$ d) in the indeterminate form as $x \to 0$

58. If $\lim_{x \to a} g(x) = b$ and f is continuous at $x = b$ then

a) $\lim_{x \to a} g(f(x)) = f\left(\lim_{x \to a} g(x)\right)$ b) $\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$ c) $\lim_{x \to a} f(g(x)) = g\left(\lim_{x \to a} f(x)\right)$ d) $\lim_{x \to a} f(g(x)) \neq f\left(\lim_{x \to a} g(x)\right)$

59. $\lim_{x \to 0} \frac{x}{\tan x}$ is (Jun '06, Mar '08 & Sep '08)

a) 1 b) –1 c) 0 d) ∞

60. f is a real valued function defined on an interval $I \subset R$ (R being the set of real numbers) increases on I. Then

a) $f(x_1) \leq f(x_2)$ whenever $x_1 < x_2\ \ x_1, x_2 \in I$ b) $f(x_1) \geq f(x_2)$ whenever $x_1 < x_2\ \ x_1, x_2 \in I$ c) $f(x_1) \leq f(x_2)$ whenever $x_1 > x_2\ \ x_1, x_2 \in I$ d) $f(x_1) > f(x_2)$ whenever $x_1 > x_2\ \ x_1, x_2 \in I$

61. If a real valued differentiable function $y = f(x)$ defined on an open interval I is increasing then

a) $\frac{dy}{dx} > 0$ b) $\frac{dy}{dx} \geq 0$ c) $\frac{dy}{dx} < 0$ d) $\frac{dy}{dx} \leq 0$

62. f is a differentiable function defined on an interval I with positive derivative. Then f is *(June '07)*
a) increasing on I b) decreasing on I
c) strictly increasing on I d) strictly decreasing on I

63. The function $f(x) = x^3$ is
a) increasing b) decreasing c) strictly decreasing d) strictly increasing

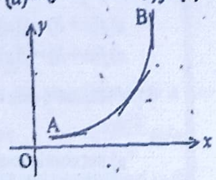
64. If the gradient of a curve changes from positive just before P to negative just after then "P" is a
a) minimum point b) maximum point c) inflexion point d) discontinuous point

65. The function $f(x) = x^2$ has *(March '16)*
a) a maximum value at $x = 0$ b) minimum value at $x = 0$
c) finite number of maximum values d) infinite number of maximum values

66. The function $f(x) = x^3$ has
a) absolute maximum b) absolute minimum c) local maximum d) no extrema

67. If f has a local extremum at a and if $f'(a)$ exists then *(Mar '10)*
a) $f'(a) < 0$ b) $f'(a) > 0$ c) $f'(a) = 0$ d) $f''(a) = 0$

68. In the following figure, the curve $y = f(x)$ is
a) concave upward
b) convex upward
c) changes from concavity to convexity
d) changes from convexity and concavity



69. The point that separates the convex part of a continuous curve from the concave part is *(Sep '12)*
a) the maximum point b) the minimum point c) the inflexion point d) critical point

70. f is a twice differentiable function on an interval I and if $f''(x) > 0$ for all x in the domain I of f then f is
a) concave upward b) convex upward c) increasing d) decreasing

71. $x = x_0$ is a root of even order for the equation $f'(x) = 0$ then $x = x_0$ is a *(Jun '08, Mar '11 & June '14)*
a) maximum point b) minimum point c) inflection point d) critical point

72. If x_0 is the x-coordinate of the point of inflection of a curve $y = f(x)$ then (Second derivative exists)
a) $f(x_0) = 0$ b) $f'(x_0) = 0$ c) $f''(x_0) \neq 0$ d) $f'''(x_0) \neq 0$

73. The statement "If f is continuous on a closed interval $[a, b]$ then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some number c and d in $[a, b]$" is *(Jun '12)*
a) The extreme value theorem b) Fermat's theorem c) Law of Mean d) Rolle's theorem

74. The statement : "If f has a local [illegible] (minimum or maximum) at c and if $f'(c)$ exists then $f'(c) = 0$" is *(March '09, Jun '13, Jun '15 & Sep '15)*
a) the extreme value theorem b) Fermat's theorem
c) Law of Mean d) Rolle's theorem

75. Identify the false statement *(March '12)*
a) all the stationary numbers are critical numbers
b) at the stationary point the first derivative is zero
c) at critical numbers the first derivative need not exist
d) all the critical numbers are stationary numbers

76. Identify the correct statement *(Sep '11)*
i) a continuous function has local maximum then it has absolute maximum
ii) a continuous function has local minimum then it has absolute minimum
iii) a continuous function has absolute maximum then it has local maximum
iv) a continuous function has absolute minimum then it has local minimum
a) (i) and (ii) b) (i) and (iii) c) (iii) and (iv) d) (i), (iii) and (iv)

77. Identify the correct statements *(Sep '10, Mar '13 & Sep '14)*
i) Every constant function is an increasing function
ii) Every constant function is a decreasing function
iii) Every identity function is an increasing function
iv) Every identity function is a decreasing function
a) (i), (ii) and (iii) b) (i) and (iii) c) (iii) and (iv) d) (i), (iii) and (iv)

78. Which of the following statement is incorrect? *(June '…)*
a) Initial velocity means velocity at $t = 0$
b) Initial acceleration means acceleration at $t = 0$
c) If the motion is upward, at the maximum height, the velocity is not zero
d) If the motion is horizontal, $v = 0$ when the particle comes to rest

79. Which of the following statements are correct (m_1 and m_2 are slopes of two lines) *(Sep '…)*
i) If the two lines are perpendicular then $m_1 m_2 = -1$
ii) If $m_1 m_2 = -1$ then the two lines are perpendicular
iii) If $m_1 = m_2$ then the two lines are parallel
iv) If $m_1 = -\frac{1}{m_2}$ then the two lines are perpendicular
a) (ii), (iii) and (iv) b) (i), (ii) and (iv) c) (iii) and (ii) d) (i) and (ii)

80. One of the conditions of Rolle's theorem is *(June '…)*
a) f is defined and continuous on (a, b) b) f is differentiable on $[a, b]$
c) $f(a) = f(b)$ d) f is differentiable on (a, b)

81. If a and b are two roots of a polynomial $f(x) = 0$ then Rolle's theorem says that there exists atleast

a) one root between a and b for $f'(x) = 0$
b) two roots between a and b for $f'(x) = 0$
c) one root between a and b for $f''(x) = 0$
d) two roots between a and b for $f''(x) = 0$

82. A real valued function which is continuous on $[a, b]$ and differentiable on (a, b) then there exists atleast one c in

a) $[a, b]$ such that $f'(c) = 0$
b) (a, b) such that $f'(c) = 0$
c) (a, b) such that $\frac{f(b) - f(a)}{b - a} = 0$
d) (a, b) such that $\frac{f(b) - f(a)}{b - a} = f'(c)$

83. In the law of mean, the value of 'θ' satisfies the condition *(March '15)*

a) $\theta > 0$ b) $\theta < 0$ c) $\theta < 1$ d) $0 < \theta < 1$

84. Which of the following statements are correct?

i) Rolle's theorem is a particular case of Lagranges law of mean
ii) Lagranges law of mean is a particular case of generalised law of mean (Cauchy)
iii) Lagranges law of mean is a particular case of Rolle's theorem
iv) Generalised law of mean is a particular case of Lagranges law of mean (Cauchy)

a) (ii), (iii) b) (iii), (iv) c) (i), (ii) d) (i), (iv)

III. Additional Questions

85. The luminous intensity I candela of a lamp at varying voltage V is given by $I = 4 \times 10^{-4} V^2$. Then the rate of change of light with respect to voltage is

a) $\frac{V}{1250}$ b) $\frac{1250}{V}$ c) 8×10^{-4} d) 8×10^4 V

86. The gradient of a curve at (1, 2) vanishes. Then a horizontal tangent at (1, 2) to the curve is

a) $x = 1$ b) $y = 2$ c) $x = 2$ d) $y = 1$

87. Equation of the tangent to the curve $y = x^3$ at (1, 1) is *(September '07)*

a) $y = 2x - 3$ b) $y = 2x + 3$ c) $y = 3x - 2$ d) $y = 3x + 2$

88. Equation of tangent to the curve $x = \cos\theta, y = \sin\theta$ at $\theta = \frac{\pi}{4}$ is

a) $x - y - \sqrt{2} = 0$ b) $x + y - \sqrt{2} = 0$ c) $x - y - \sqrt{2} = 0$ d) $x + y + \sqrt{2} = 0$

89. $y = f(x)$ and $y = g(x)$ are two curves with respective slopes $\tan\psi_1$ and $\tan\psi_2$. Then the angle between them is zero if

a) $\tan\psi_1 - \tan\psi_2 = 0$
b) $\tan\psi_1 + \tan\psi_2 = 0$
c) $\tan\psi_1 - \tan\psi_2 = -1$
d) $\tan\psi_1 . \tan\psi_2 = 1$

90. The condition for the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ to cut orthogonally is that *Deletion (March '06)*

a) $\frac{1}{a} + \frac{1}{c} = \frac{1}{b} + \frac{1}{d}$ b) $\frac{1}{a} - \frac{1}{c} = \frac{1}{b} - \frac{1}{d}$ c) $\frac{1}{a} + \frac{1}{c} = \frac{1}{b} - \frac{1}{d}$ d) $\frac{1}{a} - \frac{1}{c} = \frac{1}{b} + \frac{1}{d}$

91. If the Maclaurin's series expansion for $\log_e(1 + x)$ is $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$ then

a) $-1 \le x < 1$ b) $-1 \le x \le 1$ c) $-1 < x < 1$ d) $-1 < x \le 1$

92. The expansion of $\tan^{-1} x$ is

a) $x - \frac{x^3}{3} + \frac{x^5}{5} - ...$ b) $x + \frac{x^3}{3} - \frac{x^5}{5} + ...$ c) $x - \frac{x^3}{3} - \frac{x^5}{5} ...$ d) $x + \frac{x^3}{3} + \frac{x^5}{5} + ...$

93. The function $f(x) = \sin x$ is

a) increasing on $\left[\frac{\pi}{2}, \pi\right]$
b) increasing on $\left[0, \frac{\pi}{2}\right]$
c) increasing on $[0, \pi]$
d) decreasing on $[0, \pi]$

94. The inequality $\sin x < x < \tan x$ is true for all x in

a) $\left(-\frac{\pi}{2}, 0\right)$ b) $\left(0, \frac{\pi}{2}\right)$ c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ d) $(-\pi, \pi)$

95. The region of validity of the inequality $\log(1 + x) < x$ is

a) $(0, \infty)$ b) $(-\infty, 0)$ c) $(-\infty, \infty)$ d) $(-1, 1)$

96. The function $f(x) = \cos x$ has

a) only finitely many maximum points
b) infinitely many maximum points and finitely many minimum points
c) only finitely many minimum points
d) infinitely many minimum and maximum points

97. The function $f(x) = x^2, 0 < x < 2$ has neither maximum value nor minimum value because

a) f is not continuous
b) range of f is open
c) the interval (0, 2) is not closed
d) f is continuous

98. One of the critical numbers of $x^{3/5}(4 - x)$ is *Deletion (Sep '06)*

a) 0 b) 4 c) 4/3 d) 3/5

99. The stationary point of $f(x) = x^{3/5}(4 - x)$ occurs at *Deletion (Mar '07)*

a) 3/2 b) 2/3 c) 0 d) 4

100. The domain of concavity of the curve $y = 2 - x^2$ is

a) $(-\infty, 0)$ b) $(0, \infty)$ c) $(0, \infty)$ d) $(-\infty, \infty)$

101. The Taylor's theorem is obtained by putting $b - a = h$ in

a) Rolle's theorem
b) Lagranges law of mean
c) Extended law of mean
d) Generalised law of mean

102. The Maclaurin's theorem is obtained from the extended law of mean by putting

a) $a = 0, b = x$ b) $a = 0, b = h$ c) $a = x, b = 0$ d) $a = h, b = 0$

IV. Answers

Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.
1	d	16	d	31	b	46	a	61	b	76	a	91	d
2	b	17	b	32	d	47	a	62	c	77	a	92	a
3	c	18	a	33	a	48	a	63	d	78	c	93	b
4	b	19	a	34	a	49	b	64	b	79	a	94	b
5	a	20	b	35	c	50	c	65	b	80	c	95	a
6	c	21	c	36	b	51	a	66	d	81	a	96	d
7	d	22	a	37	a	52	b	67	c	82	d	97	c
8	b	23	a	38	b	53	d	68	a	83	d	98	a
9	c	24	b	39	c	54	c	69	c	84	c	99	a
10	d	25	b	40	d	55	d	70	a	85	a	100	d
11	a	26	c	41	d	56	b	71	c	86	b	101	c
12	b	27	b	42	a	57	c	72	c	87	c	102	a
13	b	28	d	43	d	58	b	73	a	88	b		
14	a	29	b	44	c	59	a	74	b	89	a		
15	a	30	a	45	d	60	a	75	d	90	b		

1. $y = -2x^3 + 3x + 5$

$$\frac{dy}{dx} = -6x^2 + 3$$

$$\left(\frac{dy}{dx}\right)_{x=2} = -6(2^2) + 3 = \boxed{-21}$$

2. Area $A = \pi r^2$

Differentiating $\frac{dA}{dt} = \pi . 2r . \frac{dr}{dt}$

$$\boxed{\frac{dA}{dt} = 2\pi r . \frac{dr}{dt}}$$

3. $a + by^2 = x^2$

Differentiating,

$$0 + 2bv . \frac{dv}{dt} = 2x . \frac{dx}{dt}$$

$$bv \frac{dv}{dt} = xv \quad \left\{\because v = \frac{dx}{dt}\right.$$

$$\boxed{\frac{dv}{dt} = \frac{x}{b}}$$

4. $\frac{dv}{dt} = -1$ $\quad v = \frac{4}{3}\pi r^3 \quad \because [D = 2r$

$D = 2r = 10$

$$= \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

$$v = \frac{4}{3}\pi \frac{D^3}{8}$$

$$\frac{dv}{dt} = \frac{4}{3}\pi \frac{3D^2}{8} \times \frac{dD}{dt}$$

$$-1 = \frac{\pi}{2} \times 10 \times 10 \times \frac{dD}{dt}$$

$$\Rightarrow \boxed{\frac{dD}{dt} = \frac{-1}{50\pi} \text{ cm/min}}$$. The diameter is decreasing by $\frac{1}{50\pi}$ cm/min

5. $y = 3x^2 + 3 \sin x$

$$\frac{dy}{dx} = 6x + 3 \cos x$$

$$\left(\frac{dy}{dx}\right)_{x=0} = 0 + 3 \cos 0 = \boxed{3}$$

6. $y = 3x^2$

$\frac{dy}{dx} = 6x$

$\left(\frac{dy}{dx}\right)_{x=2} = 6 \times 2 = 12$

Slope of the normal $= \frac{-1}{(dy/dx)}$

$\boxed{\text{Slope of the normal} = \frac{-1}{12}}$

7. $y = 2x^2 - 6x - 4$

$\frac{dy}{dx} = 4x - 6$

$0 = 4x - 6$

$\boxed{x = \frac{3}{2}}$

$y = 2\left[\frac{3}{2}\right]^2 - 6\left[\frac{3}{2}\right] - 4$

$\boxed{y = \frac{-17}{2}}$

Note:

∵ i) If the tangent is parallel to the x axis $\frac{dy}{dx} = 0$

ii) If the tangent is perpendicular to the x axis $\frac{dx}{dy} = 0$ (or) $\frac{dy}{dx} = \infty$

∴ Touching point $\boxed{(x, y) = \left(\frac{3}{2}, \frac{-17}{2}\right)}$

8. $y = \frac{x^3}{5}$

$\frac{dy}{dx} = \frac{3x^2}{5}$

$\left(\frac{dy}{dx}\right)_{\left(-1, \frac{-1}{5}\right)}^{x_1\ y_1} = \frac{3}{5}$

$\boxed{m = \frac{3}{5}}$

Equation of the tangent $y - y_1 = m(x - x_1)$

$y + \frac{1}{5} = \frac{3}{5}(x + 1)$

$5y + 1 = 3x + 3$

$\boxed{5y - 3x = 2}$

9. $\theta = \frac{1}{t}$

$\frac{d\theta}{dt} = \frac{-1}{t^2}$

$\left(\frac{d\theta}{dt}\right)_{\left(-3, \frac{-1}{3}\right)}^{t\ \ \theta} = \frac{-1}{9}$

$m = \frac{-1}{9}$ (say)

Equation of the normal $y - y_1 = \frac{-1}{m}(x - x_1)$

$\theta + \frac{1}{3} = 9(t + 3)$

$\boxed{3\theta = 27t + 80}$

$\theta = \frac{1}{t}$

(y) (x)

10. $\frac{x^2}{25} + \frac{y^2}{9} = 1$ | $\frac{x^2}{8} - \frac{y^2}{8} = 1$

$a = \frac{1}{25}; b = \frac{1}{9}$ | $a_1 = \frac{1}{8}$; $b_1 = \frac{-1}{8}$

∵ If the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ cut orthogonally then $\left(\theta = \frac{\pi}{2}\right)$

$\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$

$\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$

$25 - 9 = 8 + 8$

$16 = 16$

$\boxed{\therefore \theta = \frac{\pi}{2}}$

(OR)

If the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{c^2} - \frac{y^2}{d^2} = 1$ cut orthogonally, then $\boxed{a^2 = b^2 + c^2 + d^2}$

Here $25 = 9 + 8 + 8$ is true.

∴ They are perpendiculars

11. $y = e^{mx}$ —— (1)

$y = e^{-mx}$ —— (2)

Solve (1) and (2),

$e^{mx} = e^{-mx}$

$mx = -mx$

$\boxed{x = 0}$

$y = e^{mx}$; $y = e^{-mx}$

$\frac{dy}{dx} = me^{mx}$; $\frac{dy}{dx} = -me^{-mx}$

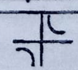
$\left(\frac{dy}{dx}\right)_{x=0} = m$; $\left(\frac{dy}{dx}\right)_{x=0} = -m$

$\boxed{m_1 = m}$; $\boxed{m_2 = -m}$

$\theta = \tan^{-1}\left|\frac{m + m}{1 - m^2}\right|$ ∵ [between the angle $\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

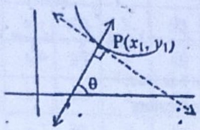
$\theta = \tan^{-1}\left|\frac{2m}{1 - m^2}\right|$ ⇒ $\boxed{\theta = \tan^{-1}\left(\frac{2m}{m^2 - 1}\right)}$ since $m > 1$

12. **Note:**

No.	Curve		Parametric equations
1.	$y^2 = 4ax$ (Parabola)		$x = at^2$; $y = 2at$
2.	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Ellipse)		$x = a\cos\theta$; $y = b\sin\theta$
3.	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (Hyperbola)		$x = a\sec\theta$; $y = b\tan\theta$
4.	$xy = c^2$ (Rectangular Hyperbola)		$x = ct$; $y = c/t$

5.	$x^2+y^2=a^2$ (Circle) 	$x=a\cos\theta$; $y=a\sin\theta$
6.	$x^{2/3}+y^{2/3}=a^{2/3}$ (Astroid (or) Acusked Hypocycloid) 	$x=a\cos^3\theta$; $y=a\sin^3$
7.	$\sqrt{x}+\sqrt{y}=\sqrt{a}$	$x=a\cos^4\theta$; $y=a\sin^4\theta$

13. If any line (whether it is tangent or normal) makes an angle θ with x axis then its slope is always tan θ.



14. $\frac{d}{dt}(AC)=0.1$; $x=\frac{15}{\sqrt{2}}$

$\frac{d}{dt}(\sqrt{2}\,x)=0.1$

$\sqrt{2}\frac{dx}{dt}=0.1$

$\boxed{\frac{dx}{dt}=\frac{0.1}{\sqrt{2}}}$

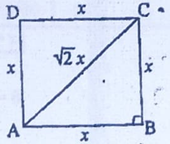
Area $A=x^2$

$\frac{dA}{dt}=2x.\frac{dx}{dt}$

$=2\times\frac{15}{\sqrt{2}}\times\frac{0.1}{\sqrt{2}}$

$\boxed{\frac{dA}{dt}=1.5\text{ cm}^2\text{ s}^{-1}}$

$\left[\begin{array}{l}AC^2=AB^2+BC^2\\ AC^2=2x^2\\ AC=\sqrt{2}\,x\end{array}\right.$



15. $\frac{dv}{dt}=\frac{dr}{dt}$

$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)=\frac{dr}{dt}$

$\frac{4}{3}\pi\,3r^2\frac{dr}{dt}=\frac{dr}{dt}$

$\boxed{4\pi r^2=1}$

$\because\left[\begin{array}{l}\text{Surface area}=4\pi r^2\\ \text{Volume } V=\frac{4}{3}\pi r^3\end{array}\right.$

16. Let $y=x^3-2x^2+3x+8$

$\frac{dy}{dt}=(3x^2-4x+3)\frac{dx}{dt}$

$\frac{dy}{dt}=2\left[\frac{dx}{dt}\right]$

$(3x^2-4x+3)\frac{dx}{dt}=2\frac{dx}{dt}$

$3x^2-4x+1=0$

$(x-1)(3x-1)=0$

$\boxed{x=1,\ \frac{1}{3}}$

17. $\frac{dr}{dt}=+2$; $r=3$

$\frac{dh}{dt}=-3$; $h=5$

$V=\pi r^2h$ $\because$ [$r, h\to$ variables]

$\frac{dv}{dt}=\pi\left[r^2\frac{dh}{dt}+h.2r\frac{dr}{dt}\right]$

$=\pi[9(-3)+2\times5\times3\times2]=33\pi\text{ cm}^3\text{ s}^{-1}$

$\boxed{\frac{dv}{dt}=33\pi\text{ cm}^3\text{ s}^{-1}}$

18. $y=6x-x^3$

$\frac{dy}{dx}=6-3x^2=m$ (say)

$m=6-3x^2$

$\frac{dm}{dt}=0-6x\frac{dx}{dt}$

$=-6\times3\times5=-90$

$\boxed{\frac{dm}{dt}=-90\text{ units s}^{-1}}$

$\because\left[\begin{array}{l}x=3\\ \frac{dx}{dt}=+5\end{array}\right.$

19. $\frac{dv}{dt}=+4$ $\left[\begin{array}{l}v=8\\ a^3=+8\\ a=2\end{array}\right.$

$v=a^3$

$\frac{dv}{dt}=3a^2\frac{da}{dt}$

$4=3\times4\times\frac{da}{dt}$

$\frac{da}{dt}=\frac{1}{3}$

(S) Surface Area $=6a^2$

$\frac{ds}{dt}=6\times2a\frac{da}{dt}$

$=12\times2\times\frac{1}{3}$

$\Rightarrow\boxed{\frac{ds}{dt}=8\text{ cm}^2\text{ s}^{-1}}$

20. If the curve cuts the y-axis $x=0$ $\Rightarrow$ $y=8+4x-2x^2$

$y=8+0-0=8$ $\therefore$ Point $(x_1,y_1)=(0,8)$

$y=8+4x-2x^2$

$\frac{dy}{dx}=4-4x$

$\left(\frac{dy}{dx}\right)_{(0,8)}=4(1-x)=4(1-0)$

$\boxed{m=4}$

21. $y^2=x$

$2y\frac{dy}{dx}=1$

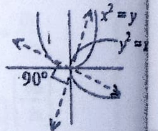
$x^2=y$

$\frac{dy}{dx}=2x$

$\theta=\tan^{-1}\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\theta=\tan^{-1}(\infty)$

$\theta=\frac{\pi}{2}$



$\left(\frac{dy}{dx}\right)_{(0,0)} = \frac{1}{2y}$ | $\left(\frac{dy}{dx}\right)_{(0,0)} = 0$

$\boxed{m_1 = \infty}$ $\boxed{m_2 = 0}$

22. $x = e^t \cos t$

$\frac{dx}{dt} = e^t.(-\sin t) + \cos t\, e^t$

$\boxed{\frac{dx}{dt} = e^t(\cos t - \sin t)}$

$y = e^t \sin t$

$\frac{dy}{dt} = e^t.\cos t + \sin t\, e^t$

$\boxed{\frac{dy}{dt} = e^t(\cos t + \sin t)}$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t + \sin t}{\cos t - \sin t}$

If the tangent parallel to the x axis $\frac{dy}{dx} = 0$

$\frac{\cos t + \sin t}{\cos t - \sin t} = 0$

$\cos t + \sin t = 0$

$\cos t = -\sin t \Rightarrow$

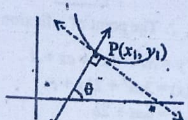
$\frac{\sin t}{\cos t} = -1$

$\tan t = -1 \Rightarrow t = \tan^{-1}(-1)$

$\boxed{t = -\frac{\pi}{4}}$

23. Slope of the tangent $-\frac{1}{m} = \frac{-1}{\tan\theta}$

$\therefore$ $\boxed{\text{Slope of the curve} = -\cot\theta}$



24. $y = 3e^x$

$\frac{dy}{dx} = 3e^x$

$\boxed{m_1 = 3e^x}$

$y = \frac{a}{3}e^{-x}$

$\frac{dy}{dx} = \frac{a}{3}(-e^{-x})$

$\boxed{m_2 = -\frac{a}{3}e^{-x}}$

If the curves cut orthogonally $m_1 \times m_2 = -1$

$3e^x \times \left(\frac{-a}{3}\right)e^{-x} = -1$

$\boxed{a = 1}$

25. $s = t^3 - 4t^2 + 7$

$v = \frac{ds}{dt} = 3t^2 - 8t$

$a = \frac{d^2s}{dt^2} = 6t - 8$

$a = 0 \Rightarrow 6t - 8 = 0$

$\boxed{t = \frac{4}{3}}$

$a = 0 \Rightarrow v = 3t^2 - 8t = 3 \times \frac{16}{9} - 8 \times \frac{4}{3}$

$\boxed{v = \frac{-16}{3}\, m.s^{-1}}$

26. $v \propto s^2 \Rightarrow v = ks^2$ (k - constant)

$\frac{dv}{dt} = k2s\frac{ds}{dt}$

$\frac{dv}{dt} = k.2s.v$ $\because \left[\frac{ds}{dt} = v\right.$

$= k.2s.ks^2$

$a = 2k^2s^3 \Rightarrow$ $\because \left[\frac{dv}{dt} = a \Rightarrow \boxed{a \propto s^3}\right.$

$a = 2k^2s^3$

27. If the given function is a quadratic function, then the value of c must be $\frac{a+b}{2}$

$\boxed{c = 0}$

28. $c = \frac{0+1}{1} = \frac{1}{2}$

29. $f(x) = \cos\frac{x}{2}$

$f'(x) = -\frac{1}{2}\sin\left(\frac{x}{2}\right)$

$f'(c) = \frac{-1}{2}\sin\left(\frac{c}{2}\right)$

$f'(c) = 0 \Rightarrow -\frac{1}{2}\sin\left(\frac{c}{2}\right) = 0$

$\sin\frac{c}{2} = 0$

$\frac{c}{2} = n\pi$ $\because$ [If $\sin\theta = 0 \Rightarrow \theta = n\pi$

$c = 2n\pi$ $n \in \mathbb{Z}$

$n = 0, 1, 2, 3, \ldots \Rightarrow c = 0, 2\pi, 4\pi, 6\pi, \ldots$

$\boxed{c = 2\pi} \in (\pi, 3\pi)$

30. $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(c) = \frac{1}{2\sqrt{c}}$

$f(a) = f(1) = 1$

$f(b) = f(4) = 2$

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$\frac{1}{2\sqrt{c}} = \frac{2-1}{3}$

$\sqrt{c} = \frac{3}{2} \Rightarrow \boxed{c = \frac{9}{4}} \in (1, 4)$

31. $\lim_{x\to\infty}\frac{x^2}{e^x} = \lim_{x\to\infty}\frac{2x}{e^x}$ $\because \left[\frac{\infty}{\infty}\text{ form}\right.$

$= \lim_{x\to\infty}\frac{2}{e^x} = \frac{2}{e^\infty} = \frac{2}{\infty} = 0$

32. $\lim_{x\to 0}\frac{a^x - b^x}{c^x - d^x} = \lim_{x\to 0}\frac{a^x.\log a - b^x.\log b}{c^x.\log c - d^x.\log d} = \frac{\log a - \log b}{\log c - \log d} = \boxed{\frac{\log(a/b)}{\log(c/d)}}$

33. $f(a) = 2;$ $f'(a) = 1;$ $g(a) = -1;$ $g'(a) = 2$

$\lim_{x\to a}\frac{g(x)f(a) - g(a)f(x)}{x - a} = \lim_{x\to a}\frac{f(a)g'(x) - g(a)f'(x)}{1}$

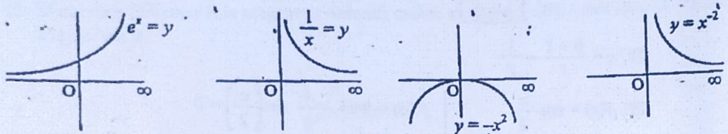
$= f(a).g'(a) - g(a).f'(a)$

$= 2 \times 2 - (-1) \times 1 = \boxed{5}$

34. $x \in (0, \infty)$

$f(x) = e^x$ $f'(x) = e^x$ $= +ve > 0$ For all values of x in $(0, \infty)$ increasing function	$f(x) = \frac{1}{x}$ $f'(x) = \frac{-1}{x^2}$ $= -ve < 0$ decreasing function	$f(x) = -x^2$ $f'(x) = -2x$ $= -ve < 0$ decreasing function	$f(x) = x^{-2}$ $f'(x) = -2x^{-3}$ $= \frac{-2}{x^3}$ $= -ve < 0$ decreasing function

Another method:



From these diagrams increasing function at $(0, \infty)$ is e^x only.

35. $f(x) = x^2 - 5x + 4$

$f'(x) = 2x - 5$

Substitute the answer in $f'(x)$,

i) In $(-\infty, 1)$ $x = 0 \Rightarrow f'(x) = 0 - 5 = -ve < 0$

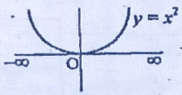
ii) In $(1, 4)$ $x = 2 \Rightarrow f'(x) = 4 - 5 = -ve < 0$

iii) In $(4, \infty)$ $x = 5 \Rightarrow f'(x) = 10 - 5 = +ve > 0$

increasing in $\boxed{(4, \infty)}$

36. $f(x) = x^2$

$y = x^2$



decreasing in $(-\infty, 0)$

increasing in $(0, \infty)$

37. $f(x) = \tan x - x$

$f'(x) = \sec^2 x - 1$

$= \tan^2 x$ $\quad \because [1 + \tan^2\theta = \sec^2\theta$

$= +ve > 0$

In $\left(0, \frac{\pi}{2}\right)$ increasing function.

38. $r = 2$ cm

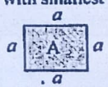
$A = r^2 \Rightarrow \boxed{A = 4}$ cm^2

Area $A = r^2$

39. **Note:** All the rectangles with a given area the one with smallest perimeter is a square.

$A = 100$

$a^2 = 100 \Rightarrow a = 10$ $\quad \boxed{4a = 40 \text{ m}}$

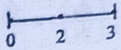


40. $f(x) = x^2 - 4x + 5$

$f'(x) = 2x - 4$

$f'(x) = 0 \Rightarrow 2x - 4 = 0$

$x = 2$



i) at $x = 0 \Rightarrow f(0) = 0 - 0 + 5 = 5$

ii) at $x = 2 \Rightarrow f(2) = 4 - 8 + 5 = 1$

iii) at $x = 3 \Rightarrow f(3) = 9 - 12 + 5 = 2$

$\boxed{\text{The absolute maximum value} = 5}$

41. $f(x) = -e^{-x}$

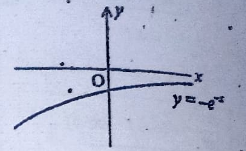
$f'(x) = +e^{-x}$

$f''(x) = -e^{-x}$

$= -\frac{1}{e^x}$

$= -ve$ [for all values of x

$f''(x) < 0$



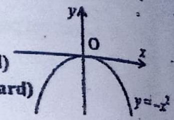
Hence, $f(x)$ is concave downward

42. $f(x) = -x^2$

$f'(x) = -2x$

$f''(x) = -2 = -ve < 0$

$\because$ [If $f''(x) > 0$ (concave upward)
If $f''(x) < 0$ (concave downward)



43. $f(x) = x^4$

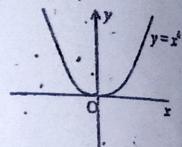
$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f''(x) = 0 \Rightarrow 12x^2 = 0$

$\boxed{x = 0}$

[i) at $(-\infty, 0)$ $f''(x) = +ve > 0$
ii) at $(0, \infty)$ $f''(x) = +ve > 0$



$\therefore$ The point of inflexion is no where.

44. $y = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$6ax + 2b = 0$

$6a + 2b = 0$ $\quad \because$ [at $x = 1$

$\boxed{3a + b = 0}$

If it has a point of inflexion $f''(x) = 0$

46. $l = 1 + 0.00005\,\theta + 0.0000004\,\theta^2$

$\frac{dl}{d\theta} = 0.00005 + 0.0000008\,\theta$

$\left(\frac{dl}{d\theta}\right)_{\theta = 100^\circ C} = 0.00005 + 0.00008$

$\boxed{= 0.00013 \text{ m } ^\circ C^{-1}}$

47. Velocity $= \frac{\text{displacement}}{\text{time}}$

$V = \frac{x}{t}$ ms^{-1}

49. $x = 3t^3 - 2t^2 + 4t - 1$

$\frac{dx}{dt} = V = 9t^2 - 4t + 4$

$\frac{d^2x}{dt^2} = a = 18t - 4$

i) $t = 0 \Rightarrow$ initial velocity

$\boxed{V = 4 \text{ m/s}}$

ii) $t = 0 \Rightarrow$ initial acceleration

$a = 0 - 4$ $\boxed{a = -4 \text{ m/s}^2}$

50. $\theta = 9t^2 - 2t^3$

$\omega = \frac{d\theta}{dt} = 18t - 6t^2$

$\propto = \frac{d^2\theta}{dt^2} = 18 - 12t$

[$\omega \to$ angular velocity
[$\propto \to$ angular acceleration

$\propto = 0 \Rightarrow 18 - 12t = 0$

$\boxed{t = \frac{3}{2} = 1.5 \text{ s}}$

51. $y = \frac{1}{2}gt^2$

$V = \frac{dy}{dt} = \frac{g}{2} \times 2t$

$= 9.8\, t$ $\because [g = 9.8$

$t = 2 \Rightarrow \boxed{V = 19.6 \text{ ms}^{-1}}$

52. $x = \frac{1}{2}gt^2$

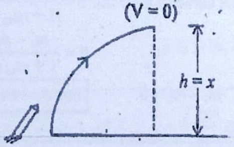
$V = \frac{dx}{dt} = gt$

$a = \frac{d^2x}{dt^2} = g$ (For any time)

$\boxed{a = 9.8 \text{ ms}^{-2}}$

53. $x = 100\,t - 12.5\,t^2$

$V = \frac{dx}{dt} = 100 - 25\,t$



At a maximum height $V = 0 \Rightarrow$

$100 - 25\,t = 0$

$t = 4$

$\therefore x = 4(100 - 50)$

$\boxed{x = 200 \text{ m}}$

54. $y = f(x_1)$

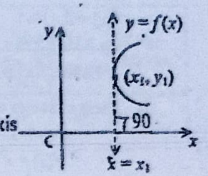
$f'(x) \to \infty$

$m = \frac{dy}{dx} = \infty$

$\tan\theta = \infty$ $\because [m = \tan\theta$

$\theta = 90°$ [The tangent makes an angle θ with x-axis

$\therefore$ Vertical tangent at $x = x_1$



56. $\boxed{f(a+h) = f(a) + h\,f'(a + \theta h)}$; $0 < \theta < 1$

57. $\underset{x \to 0}{Lt} \frac{x+1}{x+3} = \frac{0+1}{0+3} = \frac{1}{3}$

(not in the indeterminate form as $x \to 0$)

58. By using composite Function Theorem

$\boxed{\underset{x \to a}{Lt} f(g(x)) = f\left[\underset{x \to a}{Lt} g(x)\right]}$

59. $\underset{x \to 0}{Lt} \frac{x}{\tan x}$ $\left[\frac{0}{0}\right.$ form

$= \underset{x \to 0}{Lt} \frac{1}{\sec^2 x}$

$= \frac{1}{\sec^2 0} = 1$

62. $f'(x) > 0$ (strictly increasing function)

63. $f(x) = x^3$

Example:

$x_1 < x_2$ (say)

$-1 < 5$ $f(x_1) = f(-1) = -1$

$f(x_2) = f(5) = 125$

If $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

$-1 < 125$

Hence, $f(x)$ is strictly increasing in R.

Note:

$f(x) = x^3$ $\because$ [R – real numbers

$f'(x) = 3x^2$ For all values of x (+ve, –ve, 0)

i) If $x = 2$ (+ve) $\Rightarrow f'(x) = 12 > 0$

ii) If $x = -1$ (–ve) $\Rightarrow f'(x) = 3 > 0$

iii) If $x = 0 \Rightarrow f'(0) \geq 0$

It shows to be a increasing function not strictly increasing

[From this, If $f(x)$ is strictly increasing, $f'(x)$ is not strictly positive (> 0). If and only if is not true].

<u>Result:-</u> (concept)

If $f'(x) > 0 \Rightarrow f(x)$ is a strictly increasing function. It is true.

If $f(x)$ is a strictly increasing function $\Big\} \Rightarrow$ $f'(x) > 0$ is not true. $f'(x) \geq 0$ (may be)